The class PLS

A local search problem P is in PLS if there are <u>3</u> polynomial-time algorithms Ap, Bp, Cp with :

1. Given a string x $\in \{0,1\}^*$, Ap determines if x $\in$ Dp and produces S_0
2. x , string s $\overset{Bp}{\Longrightarrow}$ if s is sol. and Computes the cost
3. x , sol. s $\overset{Cp}{\Longrightarrow}$ if s local opt and if it is not Cp outputs a neighbor s′ with (strictly) better cost

Theoretical results (for LS)

- Find a local optimum is – it EASY ?

<u>Example</u> Linear Programming

SIMPLEX . . .

<u>Class PLS</u> : polynomial time local search

(Johnson , Papadimitriou & Yannakakis ¨ 88¨)

-find initial solution in <u>polynomial time</u>

-cost in <u>polynomial time</u>

-find better solution in the neighborhood or not existence

in <u>polynomial Time</u>

Examples (even for simple neighborhoods !)

(Schäffer & Yannakakis " 91 ")

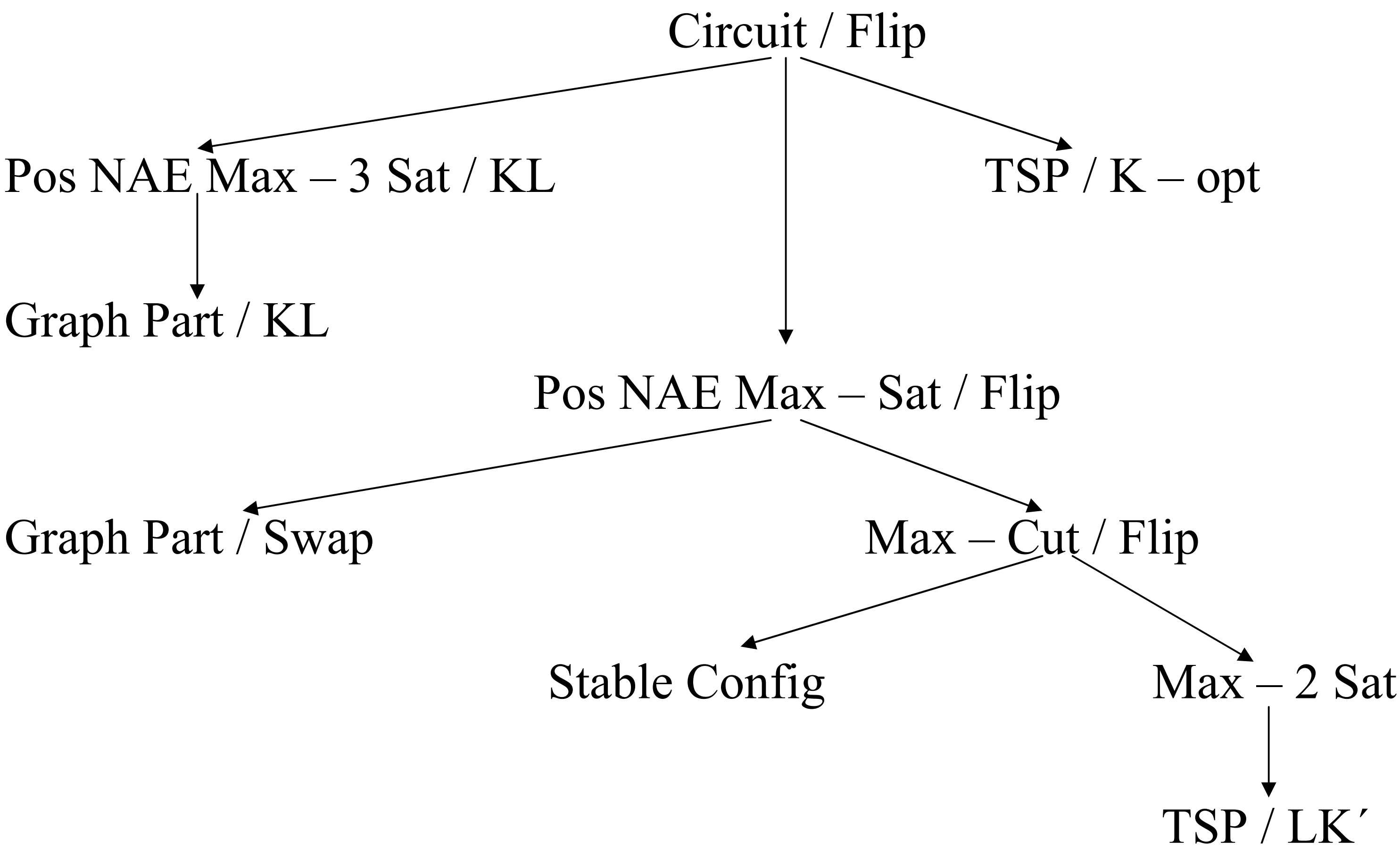
- 2 – SAT weighted
- TSP
- Bipartitioning of weighted graphs
- Max – Cut
- Stable configurations in Hopfield neural network model

PLS – complete problems

- If there exists a LS (in polynomial time) for a problem in the class then it will be the case for any other problem.

- ´´ For any problem in the class PLS – complete, the number of iterations with the standard local search is exponential unless P = NP ´´

Reductions


Circuit / Flip
Pos NAE Max – 3 Sat / KL
TSP / K – opt
Graph Part / KL
Pos NAE Max – Sat / Flip
Graph Part / Swap
Max – Cut / Flip
Stable Config
Max – 2 Sat
TSP / LK′

Which is the solutions quality of a LS ?

-for a given neighborhood

-even in exponential time !

Grover ´´ 92 ´´

- TSP with 2 – exchange
- Bipartitioning of a weighted graph with 2 – exchange

$$\overline{C_{loc}} \leq C_{AV}$$

local minimum

average cost of all solutions

Polynomial searchable Neighborhoods

- <u>Local Search ($P \neq NP$)</u>
  - **exact solution!**
  - **ε-approximate solution! (TSP Papad)**
  - **Local optimum**
    - Polynomial → **Unweighted cases**

      Approximation
    - Expo → **Weighted cases**

      Approximation? (PLS-complete)

Maximum Cut (Example with approximation 2 with N()=SWAP)

- Instance: $G=(V,E)$
- Solution: $V_1,V_2 \mid V_1 \cap V_2 = \emptyset, V_1 \cup V_2 = V$
- Measure: $Max|\{[u,v] \in E | u \in V_1, v \in V_2\}|$
- Initial solution: $V_1=\emptyset, V_2=V$ feasible

SWAP Neighborhood

$N(V_1,V_2)=\{(V_{1\kappa},V_{2\kappa}),\kappa=1,\ldots,|V|\}$

$V_{1\kappa}=V_1-\{v_\kappa\}$, $V_{2\kappa}=V_2\cup\{v_\kappa\}$ if $v_\kappa\in V_1$

$V_{1\kappa}=V_1\cup\{v_\kappa\}$, $V_{2\kappa}=V_2-\{v_\kappa\}$ if $v\in V_2$

2-approximation

- $(V_{1,}V_2)$ local optimum with N

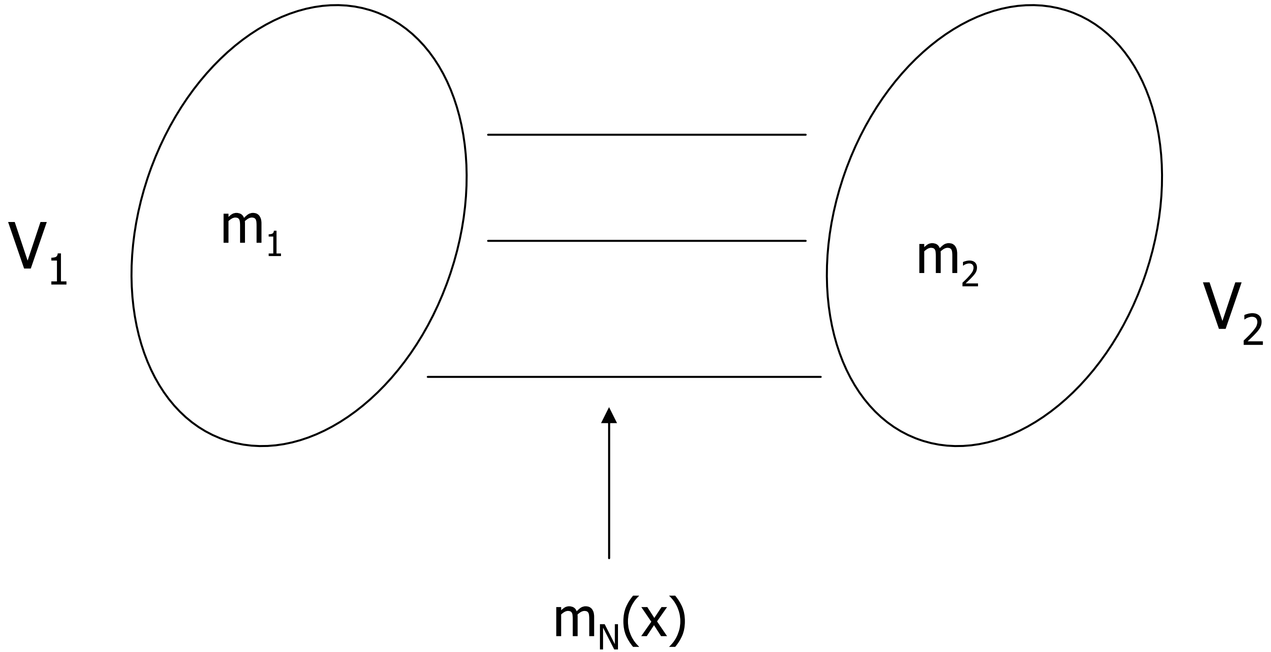
$$\Downarrow$$

$$\frac{m^*(x)}{m_N(x)} \leq 2$$

$\Rightarrow |E|=m \quad \Rightarrow m^*(x) \leq m$

if $m_N(x) \geq m/2 \Rightarrow$

Under a local optimum



$$m = m_1 + m_2 + m_N(x)$$

Under a local optimum

def ($\forall v_i$)

$m_{1i} = \{v | v \in V_1 \text{ and } (v, v_i) \in E\}$

$m_{2i} = \{v | v \in V_2 \text{ and } (v, v_i) \in E\}$

if (V_1,V_2) local Optimum

$\Downarrow \forall v_i \in V_1$

$|m_{1i}| - |m_{2i}| \leq 0$

$\Downarrow \forall v_i \in V_2$

$|m_{2j}| - |m_{1j}| \leq 0$

Under a local optimum

$$\sum_{\theta_i \in V_1} (|m_{1i}| - |m_{2i}|) = 2m_1 - m_N(x) \leq 0$$

$$\sum (|m_{2j}| - |m_{1j}|) = 2m_2 - m_N(x) \leq 0$$

$$\Downarrow$$

$$m_1 + m_2 - m_N(x) \leq 0 \Rightarrow m - m_N(x) - m_N(x) \leq 0 \Rightarrow$$

$$\boxed{m_N(x) \geq m/2}$$

FUTURE WORK

- ?!
- Proposed bound for QAP is it attained?
- TSP with 2-OPT is it in PLS - complete?
- Other problems with guaranteed LS
- Parallelization of PBs in PLS
- ′′ Good ′′ neighborhood structures
- Scheduling scheme in SA