Hamiltonian Path

Συνδυαστική Βελτιστοποίηση

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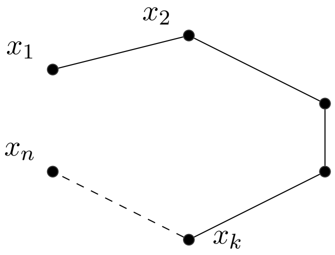
Hamiltonian path

Definition

A Hamiltonian path is a path in an undirected or directed graph that visits each vertex exactly once.

- $G = (V, E)$, weighted, non-directed
- $\deg_G(u)$
- $|V| = n$



Hamiltonian path

Spanning tree of G

- $T(V, E_T)$
- E_T: Closeness of T to HP

$$E_T = \sum_{\substack{u \in T, \\ \deg_T(u) > 2}} (\deg_T(u))$$

$E_T = 0$ for a HP

Shortest spanning tree

Problem

Find the shortest spanning tree $T^* = (V, E^*)$ of G so that the degree of no vertex exceeds 2

Shortest spanning tree

$$deg(u) = \begin{cases} 1 & (q) \\ 2 & (n-q) \end{cases}$$

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$$n - 1 = n - \frac{q}{2} \Rightarrow \begin{cases} q = 2 & (deg(u) = 1) \\ n - 2 & (deg(u) = 2) \end{cases}$$

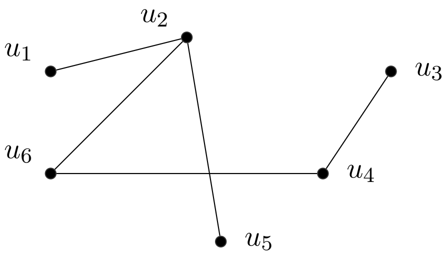
Branch and Bound (Hamiltonian path)

$G = (V, E, w)$

	1	2	3	4	5	
1	0	4	10	18	5	10
2	4	0	12	8	2	6
3	10	12	0	4	18	16
4	18	8	4	0	14	6
5	5	2	18	14	0	16
6	10	6	16	6	16	0

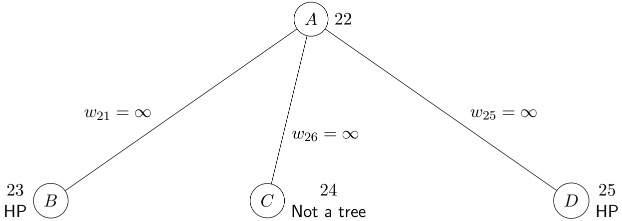
Table: Weights of G

Branch and Bound (Hamiltonian path)

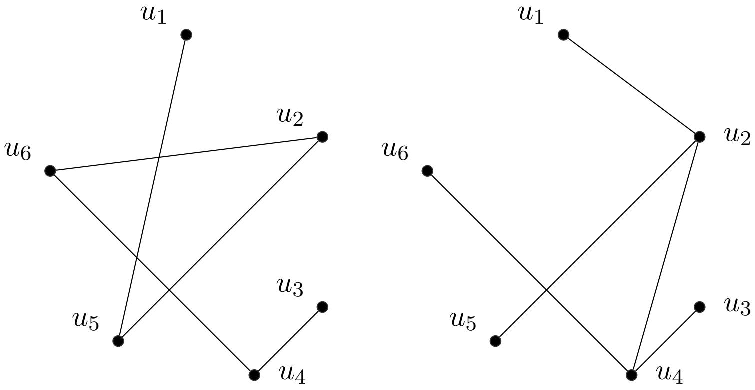


- T^*: cost $= 22$, $\deg(u_2) = 3$
- At least one of the edges (u_2, u_1), (u_2, u_6), (u_2, u_5) must be absent from the final answer (Shortest spanning tree with $\Delta(T) \leq 2$).

Branch and Bound (Hamiltonian path)



Branch and Bound (Hamiltonian path)



(a) $T^*(B) = 23$ (b) $T^*(C) = 24$

Branch and Bound (Hamiltonian path)

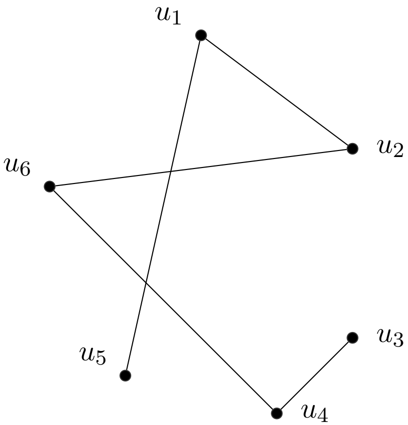


Figure: $T^*(D) = 25$ (Near optimal HP)