



- **Disk Data Model**

a sequence of r same sized **stripe units** (“scsi” interface) with indices $0, 1, \dots, r-1$. Stripe units may be (groups of) sectors, tracks, cylinders, or \dots .

- **User Data Model**

a sequence of same sized data stripe units with indices $0, 1, 2, \dots$. Data stripe units typically created by file system, database management system, or other (sophisticated) user program.

- **Reliability Groups a.k.a. Stripes**

user data is partitioned into fixed sized groups with each group containing additional redundancy information.



■ Disk Array Data Layouts

Data Layouts: map stripe unit indices to disk sectors, tracks, or cylinders ...

n disks

r stripe units/disk

k stripe width

b number of stripes

g groups

$$n = k \cdot g$$

m user data stripe units/stripe

$c = k - m$ redundant stripe units/stripe

Total number of data stripe units $b(k - c)$.

Total number of parity stripe units $b \cdot c$.

The total number of stripe units within for a completely filled/utilized disk array is $bk = nr$.


 $d_0, d_1, d_2, d_3, \dots$

user data

 $\{d_0, d_1, d_2, c_0\}$

stripes

 $\{d_3, d_4, d_5, c_1\}$
 $\{d_6, d_7, d_8, c_2\}$
 \dots
 c_i redundant data

data stripe unit indices $DSUI = \{0, 1, 2, \dots, b(k-c)-1\}$

disk: $DSUI \mapsto \{0, 1, 2, \dots, n-1\}$

offset: $DSUI \mapsto \{0, 1, 2, \dots, r-1\}$

checkDisk: $DSUI \times \{0, 1, \dots, c-1\} \mapsto \{0, 1, 2, \dots, n-1\}$

checkOffset: $DSUI \times \{0, 1, \dots, c-1\} \mapsto \{0, 1, 2, \dots, r-1\}$



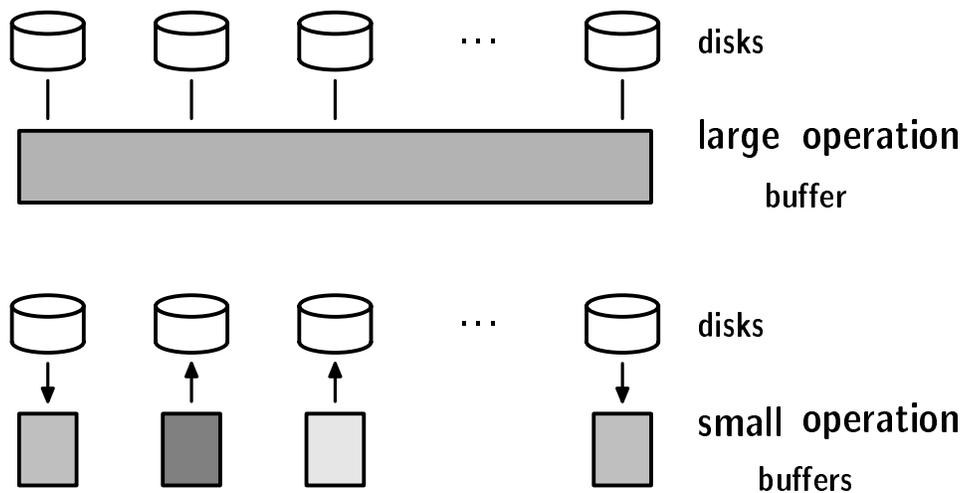
■ Layout Taxonomy

		typically
Level 0	just a bunch of disks JBOD, no redundancy	$m = k = 1$
Level 1	mirroring	$m = 1, k = 2$
Level 2	fine-grained interleaving with ECC error correction	$m = 10, k = 14$ $m = 20, k = 25$
Level 3	fine-grained interleaving with dedicated parity disk	$k = m+1$
Level 4	stripe unit interleaving; dedicated parity disk	$k = m+1$
Level 5	stripe-unit interleaving; distributed parity	$k = m+1$
Level 6	Level 5 with additional redundant stripe-units; typically one more	$k = m+2$

■ Workloads

Large Operations Parallel read or write accesses one stripe unit from each disk; high data transfer rates obtained.

Small Operations Independent read or write accesses one data stripe unit from each disk; high numbers of i/o operations obtained.

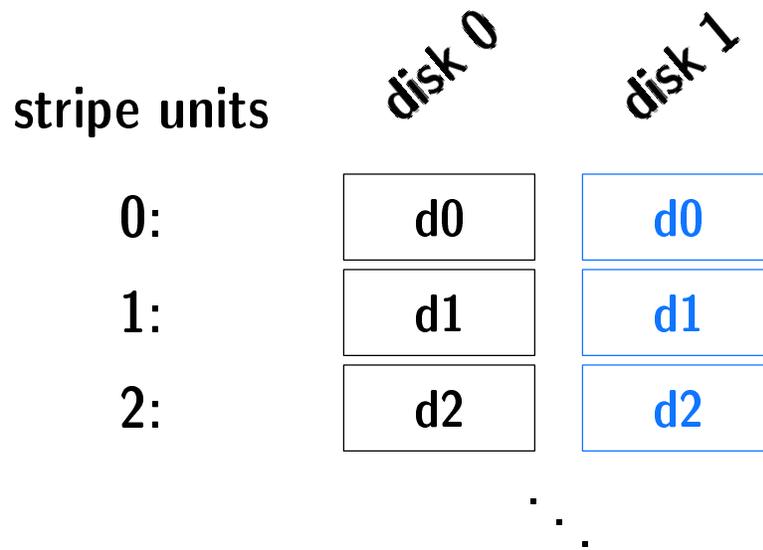


- **Operations** small reads, writes, read-modify-writes (r m w) & large reads, writes, read-modify-writes.

- **Relative Efficiency**
$$\frac{\text{RAID operations / sec.}}{\text{single disk operations / sec.}}$$



- RAID Level 1 mirroring



$$disk(a) = 0$$

$$offset(a) = a$$

$$checkDisk(a) = 1$$

$$checkOffset(a) = a$$



- RAID Level 1 mirroring

g stripes $n = 2 \cdot g$ disks

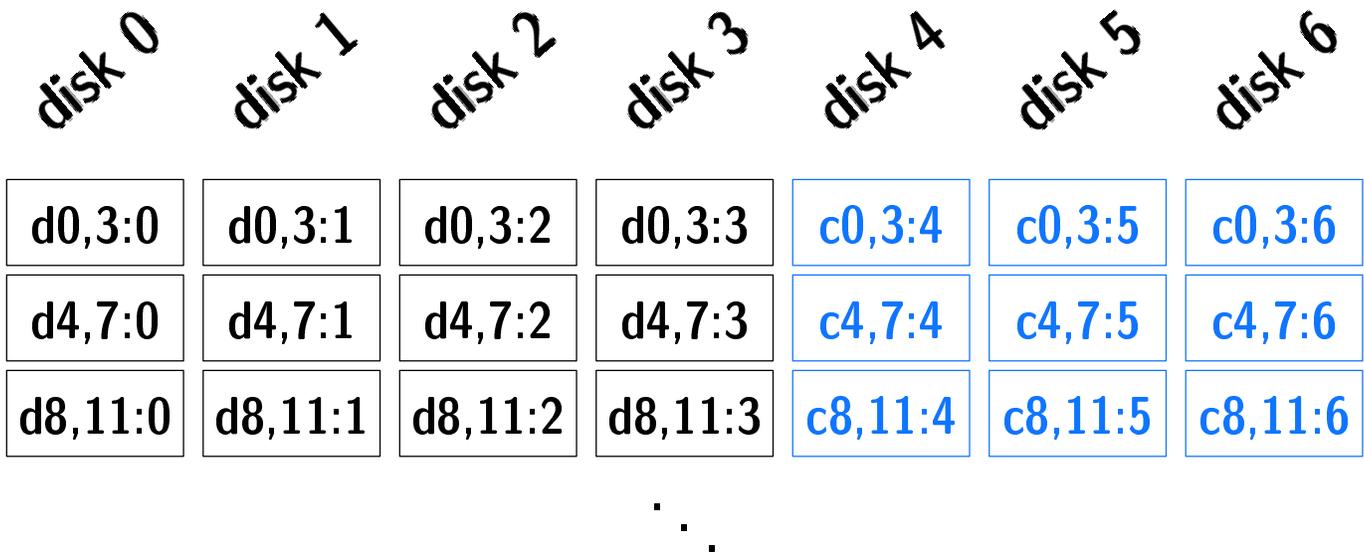
$k = 2, m = 1$

		Relative Efficiency
small	read:	$2g$
	write:	g
	r m w:	$9g/8$
large	read:	$2g/s$
	write:	g/s
	r m w:	$9g/8s$

s slowdown: Within large operations, all disks complete their individual tasks to finish the operation; s is less than 2.



- RAID Level 2 fine-grain interleaving with ECC



$$disk(a) = \{ 0, 1, \dots, m-1 \}$$

$$offset(a) = a/m$$

$$checkDisk(a) = \{ m, m+1, \dots, k-1 \}$$

$$checkOffset(a) = a/m$$



- RAID Level 2 fine-grain interleaving with ECC

g stripes $n = k \cdot g$ disks

$k = m + c$ stripe width

This level attempts to provide good performance with less redundant data.

		Relative Efficiency
small	read:	g/s
	write:	$g/2s$
	r m w:	g/s
large	read:	$g \cdot m/s$
	write:	$g \cdot m/s$
	r m w:	$g \cdot m/s$



Level 2 redundancy not needed; disk internal ecc guarantees one bit error in 10^{14} .

- RAID Level 3
fine-grained interleaving, dedicated parity disk

	disk 0	disk 1	disk 2	disk 3	disk 4
0:	d0,3:0	d0,3:1	d0,3:2	d0,3:3	c0,3
1:	d4,7:0	d4,7:1	d4,7:2	d4,7:3	c4,7
2:	d8,11:0	d8,11:1	d8,11:2	d8,11:3	c8,11
			⋮		

$$disk(a) = \{ 0, 1, \dots, m-1 \}$$

$$offset(a) = a/m$$

$$checkDisk(a) = k-1 = m$$

$$checkOffset(a) = a/m$$



- RAID Level 3 fine-grained interleaving,
dedicated parity disk

g stripes $n = k \cdot g$ disks

$k = m + 1$ stripe width

This level provides good performance with less redundant data. Level 2 and Level 3 performances are identical with one redundant disk.

		Relative Efficiency
		<hr/>
small	read:	g/s
	write:	$g/2s$
	r m w:	g/s
large	read:	$g \cdot m / s$
	write:	$g \cdot m / s$
	r m w:	$g \cdot m / s$



Un-interleave data to improve small operations.

- **RAID Level 4** stripe-unit interleaving, dedicated parity disk

	disk 0	disk 1	disk 2	disk 3	disk 4
0:	d0	d1	d2	d3	c0,3
1:	d4	d5	d6	d7	c4,7
2:	d8	d9	d10	d11	c8,11
					...

$$\text{disk}(a) = a \% m$$

$$\text{offset}(a) = a / m$$

$$\text{checkDisk}(a) = k-1 = m$$

$$\text{checkOffset}(a) = a / m$$



- RAID Level 4 stripe-unit interleaving,
dedicated parity disk

g stripes $n = k \cdot g$ disks

$k = m + 1$ stripe width

Level 4 provides better small operation performance than Level 3.

		Relative Efficiency

small	read:	$g \cdot m$
	write:	$g / 2$
	r m w:	g
large	read:	$g \cdot m / s$
	write:	$g \cdot m / s$
	r m w:	$g \cdot m / s$



- RAID Level 5
stripe-unit interleaving,
distributed parity stripe units

Level 4 with distributed parity stripe units.

	disk 0	disk 1	disk 2	disk 3	disk 4
0:	d0	d1	d2	d3	p0,3
1:	d4	d5	d6	p4,7	d7
2:	d8	d9	p8,11	d10	d11
3:	d12	p12,15	d13	d14	d15
4:	p16,19	d16	d17	d18	d19

⋮

$$P = m - Q \% k \quad R = a \% m \quad Q = a / m$$

$$disk(a) = \begin{cases} R & \text{if } R < P \\ R + 1 & \text{otherwise} \end{cases}$$

$$offset(a) = Q$$

$$checkDisk(a) = P$$

$$checkOffset(a) = Q$$



- RAID Level 5 stripe-unit interleaving,
distributed parity stripe units

g stripes $n = k \cdot g$ disks

$k = m + 1$ stripe width

Level 5 provides better than Level 4 small operation performance.

		Relative Efficiency
small	read:	$g \cdot k$
	write:	$g \cdot k / 4$
	r m w:	$g \cdot k / 2$
large	read:	$g \cdot m / s$
	write:	$g \cdot m / s$
	r m w:	$g \cdot m / s$



■ RAID Level 5 Left Symmetric layout

	disk 0	disk 1	disk 2	disk 3	disk 4
0:	d0	d1	d2	d3	p0,3
1:	d5	d6	d7	p4,7	d4
2:	d10	d11	p8,11	d8	d9
3:	d15	p12,15	d12	d13	d14
4:	p16,19	d16	d17	d18	d19
			⋮		

$$P = m - Q \% k \quad Q = a / m$$

$$\text{disk}(a) = a \% k$$

$$\text{offset}(a) = Q$$

$$\text{checkDisk}(a) = P$$

$$\text{checkOffset}(a) = Q$$



- RAID Level 6 stripe-unit interleaving, distributed check units

Level 5 with additional check stripe units.

	disk 0	disk 1	disk 2	disk 3	disk 4	disk 5
0:	d0	d1	d2	d3	c0,3:0	c0,3:1
1:	d4	d5	d6	c4,7:0	c4,7:1	d7
2:	d8	d9	c8,11:0	c8,11:1	d10	d11
3:	d12	c12,15:0	c12,15:1	d13	d14	d15
4:	c16,19:0	c16,19:1	d16	d17	d18	d19

...

$$P = m - Q \% k \quad R = a \% m \quad Q = a / m$$

$$disk(a) = \begin{cases} R & \text{if } R < P \\ R + c & \text{otherwise} \end{cases}$$

$$offset(a) = Q$$

$$checkDisk(a) = \{ P, P+1, \dots, P+(c-1) \}$$

$$checkOffset(a) = Q$$



- **RAID Level 6** stripe-unit interleaving,
distributed check stripe units

g stripes

$n = k \cdot g$ disks

$k = m+2$ typical stripe width

Level 6 provides better than Level 4 small operation performance as well as better reliability.

		Relative Efficiency
small	read:	$g \cdot k$
	write:	$g \cdot k / 4$
	r m w:	$g \cdot k / 2$
large	read:	$g \cdot m / s$
	write:	$g \cdot m / s$
	r m w:	$g \cdot m / s$



- RAID Level 6 “Left Symmetric” layout

	disk 0	disk 1	disk 2	disk 3	disk 4	disk 5
0:	d0	d1	d2	d3	c0,3:0	c0,3:1
1:	d6	d7	c4,7:0	c4,7:1	d4	d5
2:	c8,11:0	c8,11:1	d8	d9	d10	d11

...

assume $m = c\lambda$; $P = m - (Q\%k)\lambda$ $Q = a/m$

$disk(a) = a\%k$

$offset(a) = Q$

$checkDisk(a) = \{ P, P+1, \dots, P+(c-1) \}$

$checkOffset(a) = Q$