Για το Test 8 Ιουνίου 2012

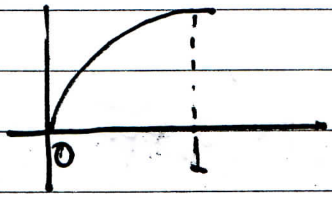
Θ1) Γ - καμπύλη ("καλή") (Καρτεσιανή εξίσωση ή Τομή επιφανειών ή πολ κυλ/Σφαιρ. εξίσωση)

1) Παραμέτρηση της Γ

π.χ $\Gamma: y^2 = x$, $x \in [0,1]$

$\vec{r}(y) = \vec{r}(y^2, y)$, $y \in [0,1]$

$\vec{r}(t) = \vec{r}(t^2, t)$, $t = y$, $x = t$



$y = συν^2(x + e^x)$

$\vec{r}(x) = (x, συν^2(x + e^x))$

$(x-2)^{2/3} + (y-12)^{2/3} = 1$

$y - 12 = ημ^3 t$, $t \in [0, 2$

$\vec{r}(t) = (2 + συν^3 t, 12 + ημ^3 t)$

2) $\vec{r}(t)$, $t \in [a, b]$, $\vec{r}(t) = (x(t), y(t), z(t))$

$\vec{r}'(t) = (x'(t), y'(t), z'(t))$

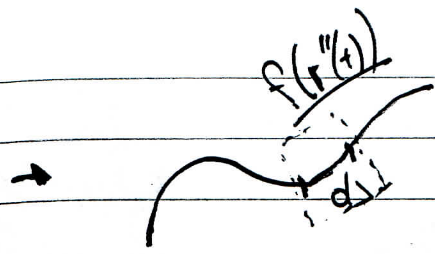
3) Μήκος $\ell(\Gamma) = \int_a^b \|\vec{r}'(t)\| dt$, $ds = \|\vec{r}'(t)\| dt$

$\|\vec{r}'(t)\| = \left[(x'(t))^2 + (y'(t))^2 + (z'(t))^2\right]^{1/2}$

$ds = \|r'(t)\| dt$

$f: \mathbb{R}^3 \, (ή \, \mathbb{R}^2) \to \mathbb{R}$ $\int f \, ds$

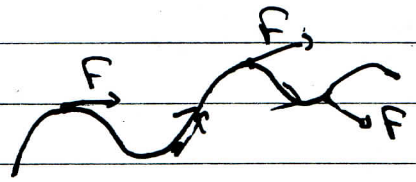
$\|\vec{r}'(t)\|$



$$\int_\Gamma f\,ds = \int_a^b f(\vec{r}(t))\,\|\vec{r}'(t)\|\,dt$$

$\vec{F} : \mathbb{R}^3 (\acute{\eta}\ \mathbb{R}^2) \to \mathbb{R}^3 (\acute{\eta}\ \mathbb{R}^2)$, $\vec{F} = (P, Q, R)$ δ.π

$$\int_\Gamma \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t)\,dt$$



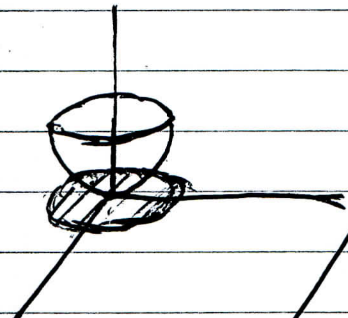
$d\vec{r} =$

0,7

Θ2 S επιφάνεια του $\mathbb{R}^3$

1) Παραμέτρηση της S

π.χ $S = \{(x,y,z) : x^2+y^2 = z,\ x^2+y^2 \le 1\}$



$\vec{r}(r,\theta) = (r\cdot\sigma\upsilon\nu\theta,\ r\cdot\eta\mu\theta,\ r^2)$

$(r,\theta) \in [0,1] \times [0,2\pi]$

π.χ: $S = \{(x,y,z) : x^2+(y+1)^2+(z+2)^2 = 1\}$: Άρα βλέπω ότι:

$x = \sigma\upsilon\nu\theta\cdot\eta\mu\varphi$

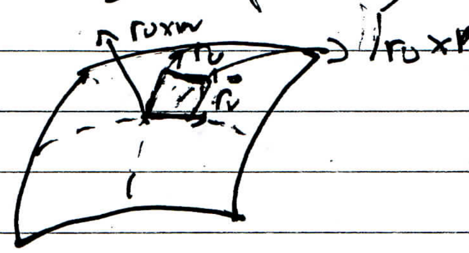
~~$\vec{r}(\theta,\varphi) = (\sigma\upsilon\nu\theta\cdot\eta\mu\varphi,\ \eta\mu\theta\cdot\eta\mu\varphi - 1,$~~

$y+1 = \eta\mu\theta\cdot\eta\mu\varphi$ / Άρα

$z+2 = \sigma\upsilon\nu\varphi$

$\to \vec{r}(\theta,\varphi) = (\sigma\upsilon\nu\theta\cdot\eta\mu\varphi,\ \eta\mu\theta\cdot\eta\mu\varphi - 1,\ \sigma\upsilon\nu\varphi - 2)$ / $(\theta,\varphi) \in [$

2) $\vec{r}(u,v) = (x(u,v), y(u,v), z(u,v))$, $(u,v) \in D$ (= απλό $\subseteq \mathbb{R}$)



$$\vec{r}_u = \left(\frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u}\right)_{(u,v)}$$

$$\vec{r}_v = \left(\frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v}\right)_{(u,v)}$$

$\vec{r}_u \times \vec{r}_v\,(u,v)$ = το κάθετ

3) Εμβαδόν επιφανείας S

$$A(S) = \iint_D \|\vec{r}_u \times \vec{r}_v\|(u,v)\, du\, dv$$

$$dS = \|\vec{r}_u \times \vec{r}_v\|\, du \cdot dv$$

$f: A \to \mathbb{R}$, $S \subseteq A$

$$I = \iint_S f\, dS = \iint_D f(\vec{r}(u,v))\,\|\vec{r}_u \times \vec{r}_v\|\, du\, dv$$

TETTRIS

~~30~~ $\vec{F} = (P, Q, R) : \mathbb{R}^3 \to \mathbb{R}^3$

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_D \vec{F}(\vec{r}(u,v)) \cdot (\vec{r}_u \times \vec{r}_v)\, du\, dv$$

$d\vec{S} = (\vec{r}_u \times \vec{r}_v)\, du\, dv$

$\boxed{0,7}$

Θ 3) $f : \mathbb{R}^3 \to \mathbb{R}$, $\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$, Κλίση/Ανάδ

π.χ $f(x,y,z) = x^2 \cdot y + x \cdot y^2 + e^2 + \text{τοξεφ}(xz) + 100^z + \ln[(z^{15} + 20$

Να βρούμε το ∇ στο (1, 1, 2)

$\ln\left(\text{τοξεφ}(\text{συν}^2(10^{xyz} + e^{\text{τοξημ}(\ln(x^9 + \text{εφ}(xy))})\right)$

→ $\vec{F}(x,y) = (P, Q)_{(x,y)}$

Στροβιλισμός

$\nabla \times \vec{F}(x,y) = \left(0, 0, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)_{(x,y)} = \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\vec{k}$

→ $\vec{F} = (P, Q, R)$, $\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$ Στροβιλισμός

$\text{div}\vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$ Απόκλιση

0,6

Καλή Επιτυχ