PART B

ANALYSIS AND MODELLING OF INTERFERENCES ENCOUNTERED IN WPTS

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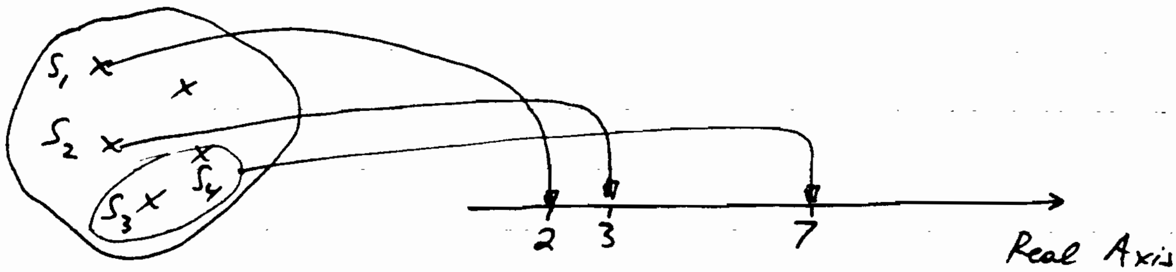
- ***Mathematical Preliminaries***
- ***Fading***
- ***Interference***

Order of Presentation

- MATHEMATICAL PRELIMINARIES
- FADING
- CO-CHANNEL INTERFERENCE (CCI)
- ADJACENT CHANNEL INTERFERENCE (ACI)

Random Variables (RV)

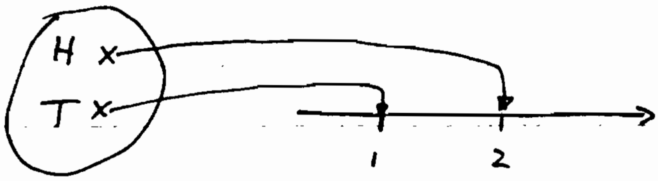
- Sample Space $\mathcal{S}$
- Assign a number to each point in $\mathcal{S}$



$X \triangleq$ Random Variable (RV)

$X = f(\mathcal{S}_i) \quad i = 1, 2, 3, \ldots$

Example Fair Coin Toss → $Prob[H] = Prob[T] = \frac{1}{2}$

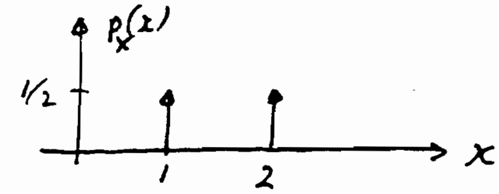


$p[x = 1] = 0.5$

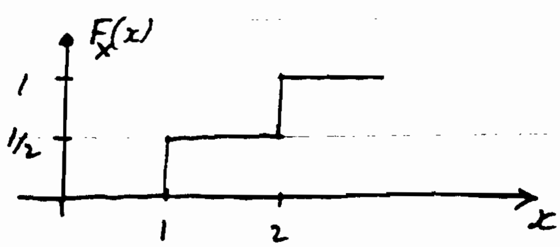
$p[x = 2] = 0.5$

$p[x = k;\ k \neq 1 \text{ or } 2] = 0$

Probability Density Function (pdf)



Probability Distribution Function or Cummulative Distribution Function (cdf)



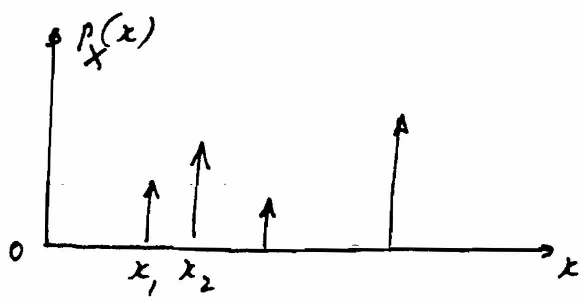
$$\boxed{p_X(x) = \frac{d F_X(x)}{dx}}$$

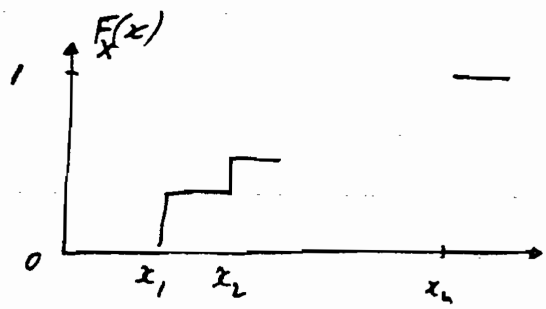
- Notations: R.V. $\rightarrow X$; Its value x
- R.V. $\rightarrow$ Discrete or Continuous

<u>Discrete R.V.</u>

pdf $\rightarrow$ $p_X(x_i)$
cdf $\rightarrow$ $F_X(x_i)$
$X = \{x_1, x_2, \ldots, x_n\}$
$\hookrightarrow$ range of the R.V.; each assigned a certain prob.

$\therefore$ $\text{Prob}[X = x_i] = p_X(x_i)$ and $\sum_{i=1}^{n} p_X(x_i) = 1$

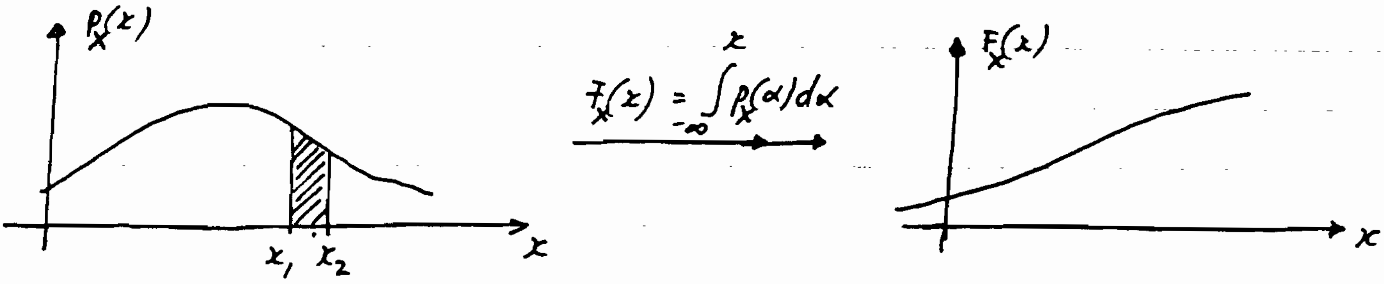




Continuous R.V.

X ; Prob$[x_1 \le X \le x_2]$.

$\therefore$ pdf $\rightarrow$ $p_X(x) = \lim\limits_{\Delta \to 0} \dfrac{p(x \le X \le x+\Delta)}{\Delta}$



Properties of $p(x)$; $F(x)$

1. $p_X(x) \ge 0$ $\quad p_X(x_i) \ge 0$.

2. $\int_{-\infty}^{\infty} p_X(x)\,dx = 1$; $\quad \sum\limits_{\text{all } i} p(x_i) = 1$

3. $F_X(x) = \int_{-\infty}^{x} p_X(\alpha)\,d\alpha = \text{Prob}[X \le x]$

   $F_X(x_j) = \sum\limits_{-\infty}^{j} p_X(x_i)$

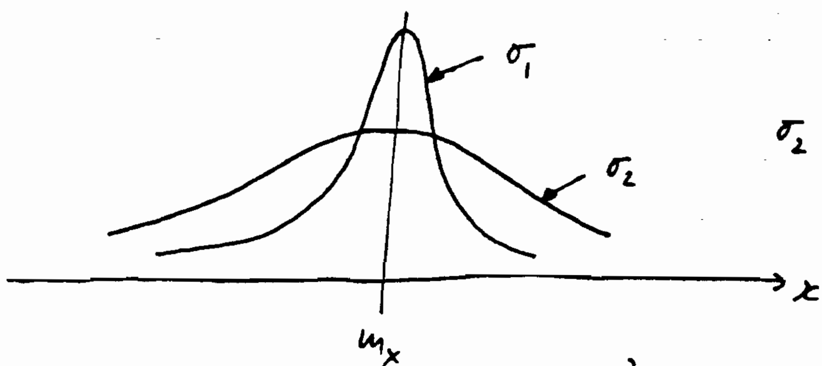
4. $F_X(-\infty) = 0$; $\quad F_X(\infty) = 1$

Gaussian R.V. (GRV)

- Also Normal.
- $$p_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-m_x)^2}{2\sigma^2}}$$

$m_x \doteq$ average, $\sigma_x^2 =$ variance } const.

$\sigma_x =$ standard deviation

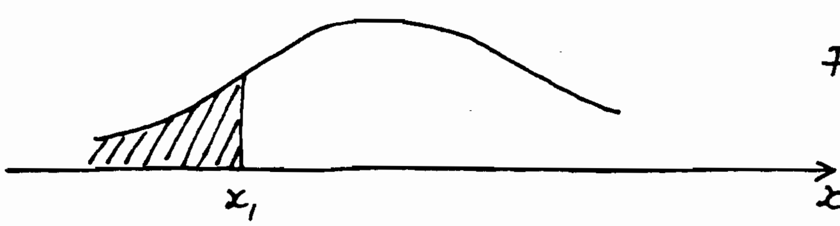


$\sigma_2 > \sigma_1$

$$F_X(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(\alpha-m_x)^2}{2\sigma_x^2}} d\alpha$$

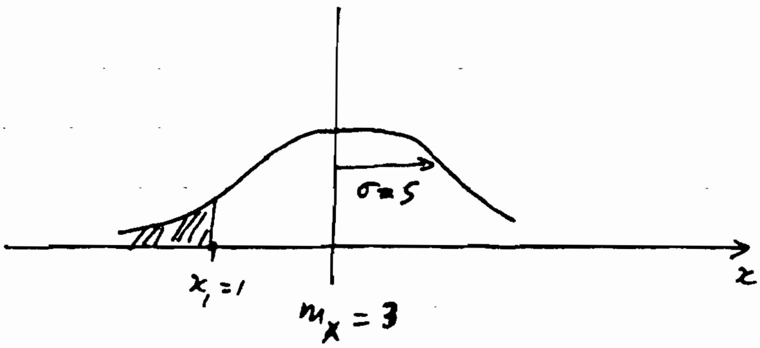
$$= \cdots = \frac{1}{2}\Big[1 + \underbrace{\frac{2}{\sqrt{\pi}} \int_0^{\gamma} e^{-t^2} dt}_{\triangleq \operatorname{erf}(\gamma)}\Big]$$

$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$; $Q(\gamma) = \frac{1}{2}\operatorname{erfc}\left(\frac{\gamma}{\sqrt{2}}\right)$



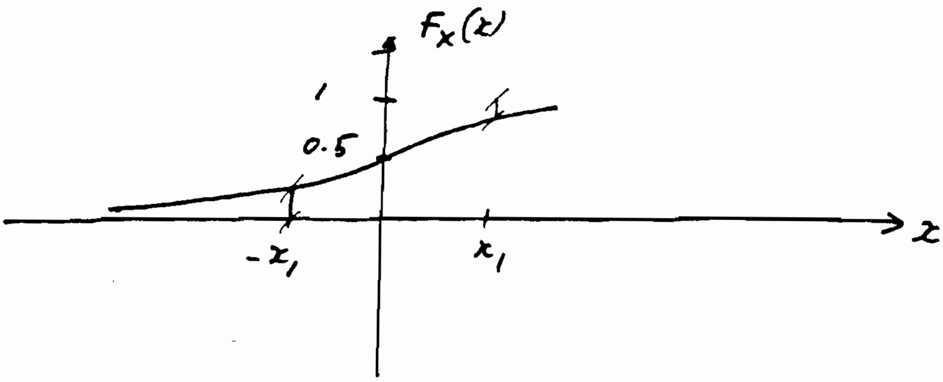
$F_X(x_1) = \text{Prob}[X \leq x_1]$

$$F_X(x) = 1 - \frac{1}{2}\,\mathrm{erfc}\left(\frac{x - m_X}{\sqrt{2}\,\sigma_X}\right)$$



$$\mathrm{Prob}\left[X \le 1\right] = 1 - \frac{1}{2}\,\mathrm{erfc}\left[\frac{1-3}{\sqrt{2}\cdot 5}\right]$$

For $m = 0$, $\sigma = 1$ → symmetry



Moments of R.V.

- Provide partial characterization of the R.V.

① Average $m_X = E\{X\} = \int_{-\infty}^{\infty} x\, p_X(x)\, dx$

② Variance $\sigma_X^2 = E\{(X - m_X)^2\} = \int_{-\infty}^{\infty} (x - m_X)^2 p_X(x)\, dx$

Note: $\sigma_X^2 = E\{X^2\} - [E\{X\}]^2$

– Higher order moments:

$$E\{X^n\} = \int_{-\infty}^{\infty} x^n p_X(x)\, dx \quad \text{— nth moment}$$

$$E\{(X - m_X)^n\} = \int_{-\infty}^{\infty} (x - m_X)^n p_X(x)\, dx \quad \text{- nth central moment}$$

Characteristic Function (C.F.)

$p_X(x) \longleftrightarrow \psi_X(v)$

$\therefore \psi_X(v) = E\{e^{jvX}\} = \int_{-\infty}^{\infty} e^{jvx} p_X(x)\, dx$

$\updownarrow$

$p_X(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-jvx} \psi_X(v)\, dv$

($g(t) \overset{F.T.}{\longleftrightarrow} G(f)$

$G(f) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$ (F.T.)

$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(f) e^{j\omega t} d\omega$ (I.F.T))

Note: The C.F. helps in the calculation of moments.

For the Gaussian R.V.

- k-th central moment : $E\{(X-m_X)^k\} = \mu_k = \begin{cases} 1\cdot 3\cdot 5 \cdots (k-1)\sigma_X^2 & k \text{ even} \\ 0 & k \text{ odd.} \end{cases}$

Also important point:

In general, $P_X(x)$ gives a complete characterization of a R.V.

m_X, σ_X^2 give only a partial characterization

- Notice : exception G.R.V.

Multiple R.V.

- $X, Y \longrightarrow$ joint pdf $p_{XY}(x, y)$

$$\text{Prob}[x_1 \le X \le x_2 ;\ y_1 \le Y \le y_2] = \int_{x_1}^{x_2}\int_{y_1}^{y_2} p_{XY}(x,y)\,dx\,dy$$

$$p_X(x) = \int_{-\infty}^{\infty} p_{XY}(x,y)\,dy \quad ; \quad p_Y(y) = \int_{-\infty}^{\infty} p_{XY}(x,y)\,dx$$

- Important Notice

$$p_{XY}(x,y) \underset{\not\longleftarrow}{\longrightarrow} p_X(x) \,;\, p_Y(y)$$

However, if X, Y independent then $p_{XY}(x,y) = p_X(x)\,p_Y(y)$

- Statistics

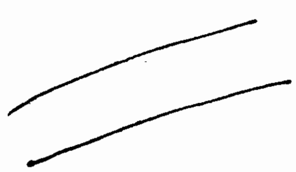
$$\text{cor}(X,Y) \triangleq E\{XY\} \qquad \text{(correlation)}$$
$$\text{cov}(X,Y) \triangleq E\{(X-m_X)(Y-m_Y)\} \qquad \text{(covariance)}$$

If $E\{XY\} = E\{X\}\,E\{Y\} \Rightarrow X, Y$ uncorrelated R.V.

- Important Notice.

$$\text{INDEPENDANCE} \underset{\not\longleftarrow}{\longrightarrow} \text{NO CORRELATION}$$

{ Correlation :



Uncorrelated : any value }

However for G.R.V $\rightleftarrows$

Central Limit Theorem

- Various versions

- Assume X_i $(i = 1, 2, \ldots, N)$ statistically independent and identically distributed R.V. with finite m_x and σ_x^2

Form $Y = \sum_{i=1}^{N} X_i$

As $N \to \infty$ Y becomes Gaussian.

- Example

Even for such small values of n, the two sides in (8-101) are remarkably close.

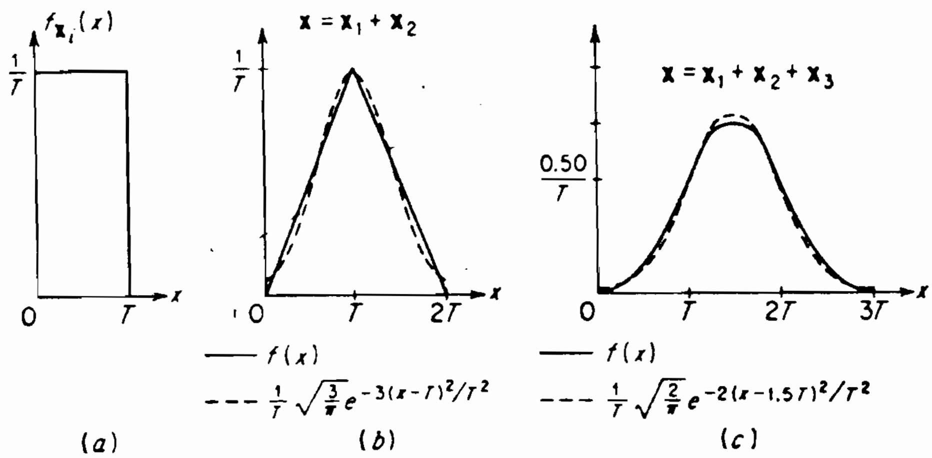
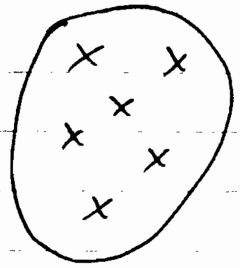


Fig. 8-3

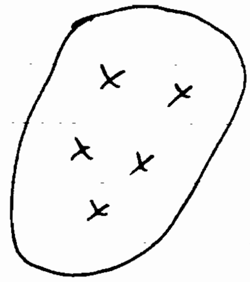
RANDOM PROCESSES (R.P.)

R.V.

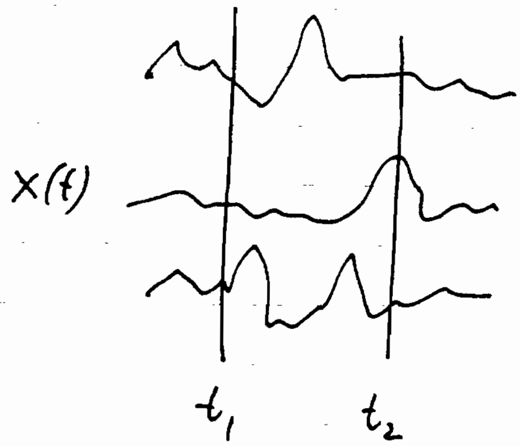


Sample point

R.P.



Sample functions



$X(t)$ is a R.P. $\Rightarrow$ $X(t_1)$ is a R.V. (x_1)

$X(t_2)$ is a R.V. (x_2)

$\vdots$

$x_1, x_2, \ldots, x_n$ are characterized by their <u>joint</u> pdf

i.e., $f_{x_1 x_2 \ldots x_n}$ (obviously complicated).

Consider 2 sample functions

$\underbrace{X_{t_1} ; X_{t_1+T}}_{\text{R.V.}}$ — $f_{X_{t_1} X_{t_1+T}}(x_1, x_2)$

If $f_{X_{t_1} X_{t_2+T}} = f_{X_{t_1+\tau} X_{t_2+T+\tau}}$ $\forall\ T, \tau$

then $X(t)$ is stationary (in the strict sense) – order 2

If $f_{X_{t_1}} = f_{X_{t_1+\tau}}$ $\longrightarrow$ order 1 (also generalization)

Wide Sense Stationary (W.S.S.) R.P.

- Only 2nd order statistics are considered.

$$\therefore \quad E\{X_{t_1}\} = E\{X_{t_1+\tau}\}$$

$$E\{X_{t_1} X_{t_1+T}\} = E\{X_{t_1+\tau} X_{t_1+T+\tau}\} \qquad \forall\ t_1, \tau, T$$

Ⓐ 1st moment

In general $E\{X(t)\} \triangleq$ mean value function $= m(t)$

but for w.s.s. $E\{X(t)\} = m = \text{const.}$

Ⓑ 2nd moment

$$E\{X_{t_1} X_{t_2}\} = R_{xx}(t_1, t_2) \triangleq \text{correlation function}$$

If w.s.s. $\Rightarrow R_{xx}(t_1, t_2) = R_{xx}(t_2 - t_1) = R_{xx}(\tau)$

Note: Only w.s.s. R.P. in this course.

Power Spectral Density

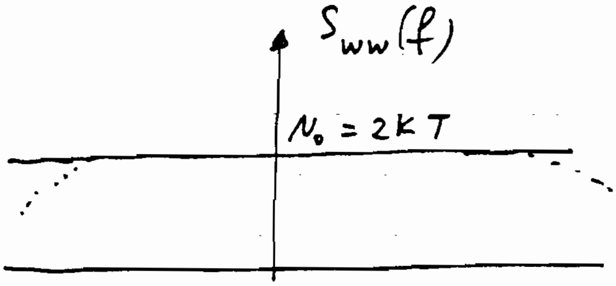
$$S_{xx}(f) \longleftrightarrow R_{xx}(\tau) \qquad \text{(SOS)}$$

Also Power $= \sigma_x^2 = E\{X_t X_t\} = R_{xx}(0)$

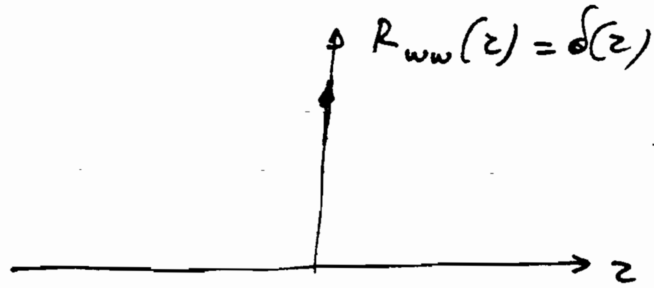
Complex R. P.

- Autocorrelation $R_{XX}(\tau) = E\{X(t+\tau)X^*(t)\}$
- Crosscorrelation $R_{XY}(\tau) = E\{X(t+\tau)Y^*(t)\}$

White Noise Process



$\longleftrightarrow$

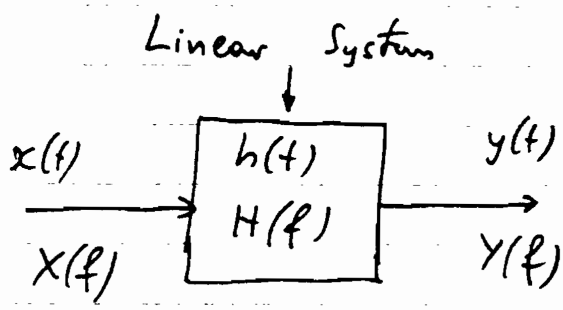


$$R_{ww}(\tau) = E\{w(t+\tau)w(t)\} = \begin{cases} 0 & \tau \neq 0 \\ N_0 & \tau = 0. \end{cases}$$

$\therefore$ Any 2 samples are uncorrelated

Not white $\rightarrow$ colored

Linear Systems



$$y(t) = x(t) \otimes h(t) \qquad \otimes : \text{convolution}$$
$$= \int x(\tau)\, h(t-\tau)\, d\tau$$

$$Y(f) = X(f)\, H(f)$$

$$S_Y(f) = |H(f)|^2 \; S_X(f)$$

BAND-PASS SIGNALS

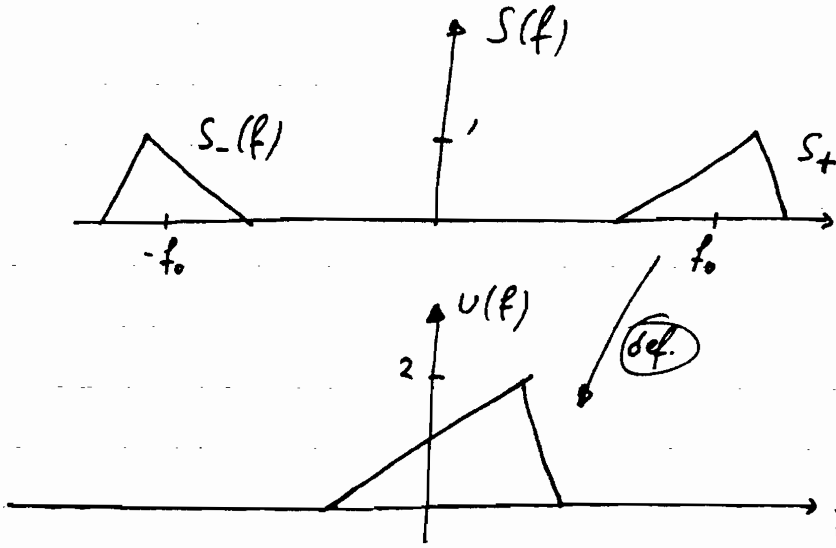
$$s(t) = a(t)\cos[2\pi f_0 t + \theta(t)]$$

$s(t)$ bandpass signal (i.e., its bandwidth $\ll f_0$), real.

Equivalently $s(t)$ can be rewritten as

$$s(t) = Re[u(t)e^{j2\pi f_c t}] \quad ; \quad u(t) = a(t)e^{j\theta(t)}$$

Let's see the spectrum.



$U(f) \longleftrightarrow u(t)$

(low pass equivalent, complex !!)

$s(t)$ real $\rightarrow$ $S(f) = S^*(-f) \Rightarrow S_+(f) = S_-^*(-f)$

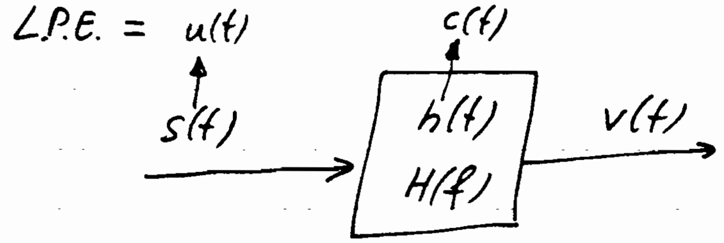
$$S(f) = S_+(f) + S_-(f) = \frac{1}{2}U(f-f_0) + \frac{1}{2}U^*(-f-f_0)$$

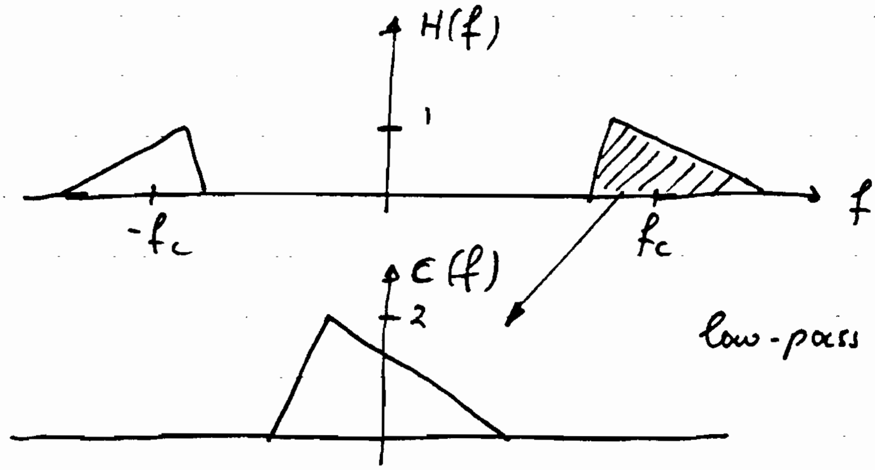
$$\therefore \quad s(t) = \frac{1}{2}u(t)e^{j2\pi f_0 t} + \frac{1}{2}u^*(t)e^{-j2\pi f_0 t} = Re[u(t)e^{j2\pi f_c t}]$$

Energy $\quad \mathcal{E} = \int_{-\infty}^{\infty} s^2(t)dt = \frac{1}{2}\int |u(t)|^2 dt = \frac{1}{2}\int |U(f)|^2 df.$ (HW)

$\therefore$ $s(t)$ can be completely represented by $u(t)$

BAND-PASS FILTERING





low-pass equivalent of $h(t)$

$c(t) \leftrightarrow C(f)$

$$H(f) = C(f-f_c) + C^*(-f-f_c)$$

$$s(t) = Re\left[u(t)\, e^{j2\pi f_c t}\right] \longleftrightarrow S(f) = \tfrac{1}{2} U(f-f_c) + \tfrac{1}{2} U^*(-f-f_c)$$

$$V(f) = \tfrac{1}{2} U(f-f_c)\, C(f-f_c) + \tfrac{1}{2} U^*(-f-f_c)\, C^*(-f-f_c)$$

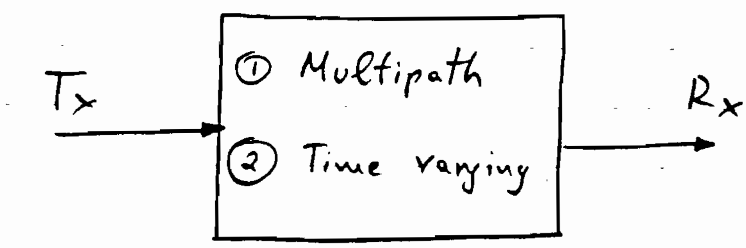
$$\updownarrow$$

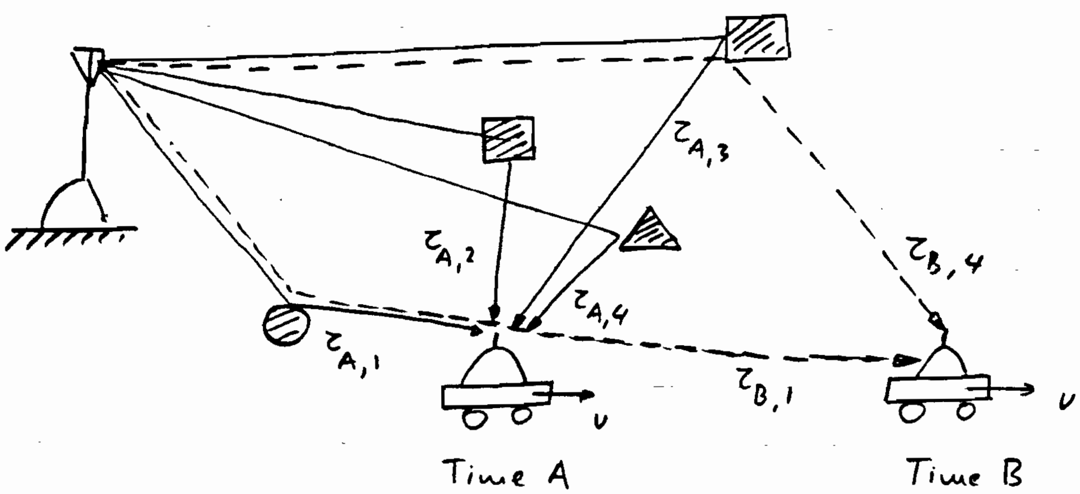
$$v(t) = Re\left[\underbrace{u(t) \otimes c(t)}\; e^{j2\pi f_c t}\right]$$

Convolution of LPEs!!

∴ We can use only Re LPE signals!!

FADING CHANNELS



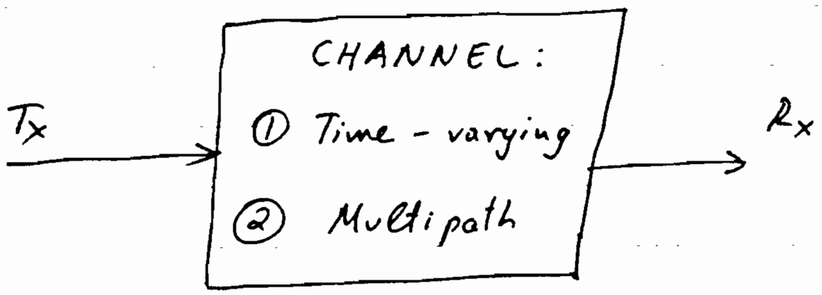


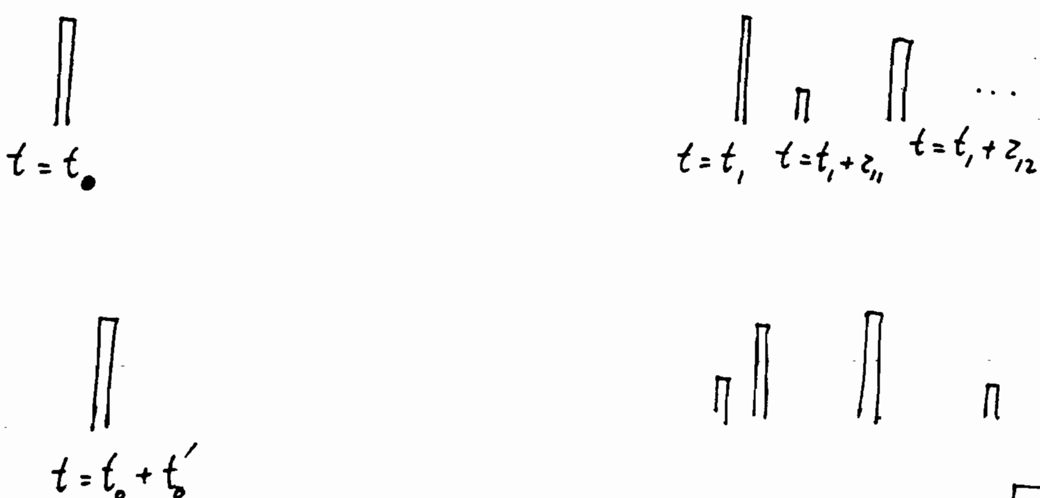
Max Doppler shift $f_d = v/\lambda$; v = velocity of vehicle

λ = wavelength of the carrier

(1 GHz → $\lambda = 30$ cm → f_d for 60 mi/hr $\simeq 90$ Hz).

CHARACTERIZATION OF FADING MULTIPATH CHANNELS





(Due to the multipath we have time-spread)

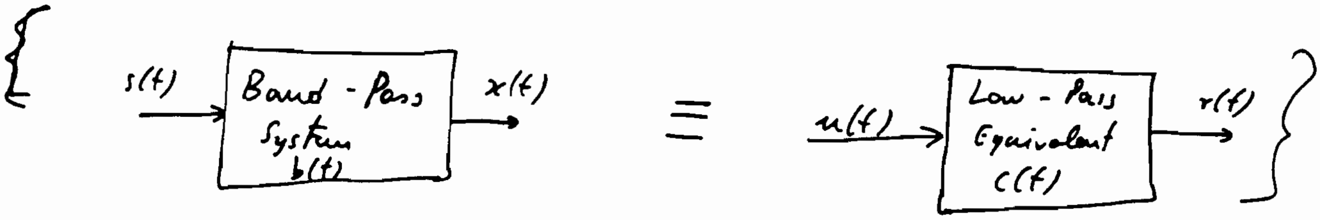
(Due to ~~the~~ time-varying of the structure of the medium we have <u>variable</u> time-spread)

Amplitude & Time

∴ Not deterministic ⟹ Model the channel <u>statistically</u>

Tx : $s(t) = \mathrm{Re}\left[u(t)\, e^{j2\pi f_c t}\right]$

Rx : $x(t) = \sum_n \alpha_n(t)\, s\left[t - \tau_n(t)\right]$ → n - rays. (n delay paths)



<u>LPE</u> : $r(t) = \sum_n \alpha_n(t)\, e^{-j2\pi f_c \tau_n(t)}\, u\left[t - \tau_n(t)\right]$

∴ <u>The channel</u> :

$$c(t;\tau) = \sum_n \alpha_n(t)\, e^{-j2\pi f_c \tau_n(t)}\, \delta\left[\tau - \tau_n(t)\right]$$ → { Discrete Multipath

[For a continuum of multipath components (example the tropospheric scatter channel) ⇒ $x(t) = \int_{-\infty}^{\infty} \alpha(\tau;t)\, s(t-\tau)\, d\tau$

∴ $c(\tau;t) = \alpha(\tau;t)\, e^{-j2\pi f_c \tau}$

(It represents the response of the channel at t due to an impulse applied at t].

For $u(t) = 1 \Rightarrow r(t) = \sum_n \alpha_n(t)\, e^{-j2\pi f_c \tau_n(t)} = \sum_n \alpha_n(t)\, e^{-j\theta_n(t)}$

with $\theta_n(t) = 2\pi f_c \tau_n(t)$

$\alpha_n(t)$ → large dynamic range

$\theta_n(t) \rightarrow (0, 2\pi]$ for $1/f_c$; f_c in MHz ⇒ random change. (unpredictable)

∴ $r(t)$ is a random, complex process.

Now, due to the large number of paths, the central limit theorem states that $r(t)$ is a complex Gaussian random process (cGrp) $\Rightarrow$

$\Rightarrow$ $c(z;t)$ is a time variant, cGrp.

If $E\{c(z;t)\} = 0 \Rightarrow$ The envelope $|c(z;t)|$ is Rayleigh

(In practice many terrestrial mobile applications $\rightarrow$ Rayleigh

If direct path (e.g mobile satellite) $\longrightarrow$ Ricean)

More details later on.

Assume that $c(\tau;t)$ is wide-sense-stationary (i.e., its 2nd order statistics (autocorrelation function) does not depend on a shift of the time origin. Equivalently it depends only on $\Delta t = t_1 - t_2$. Also the $E\{\cdot\} = ct$ or zero here)

∴ Autocorrelation function

$$\phi_c(\tau_1, \tau_2; \Delta t) = \frac{1}{2} E\left\{ c^*(\tau_1; t)\, c(\tau_2; t+\Delta t)\right\}$$

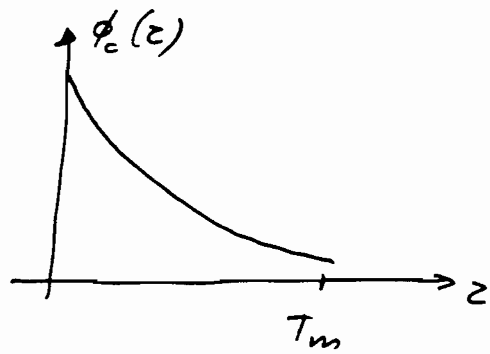
In most radio transmission applications, att. & phase, for two different path delays are <u>uncorrelated</u>. (UNCORRELATED SCATTERING

∴ $\phi_c(\tau_1, \tau_2; \Delta t) = \phi_c(\tau_1; \Delta t)\, \delta(\tau_1 - \tau_2)$

For $\Delta t = 0 \Rightarrow \phi_c(\tau) \equiv$ The average power of the channel as a function of the time delay

$\phi_c(\tau)$ is called <u>Delay Power Spectrum</u> or <u>Multipath Intensity Profile</u>

Typical :



$T_m \equiv$ multipath spread of the channel.

A completely equivalent (analogous) characterization of the time-varying multipath channel can be obtained in the frequency domain.

$$c(\tau;t) \longleftrightarrow C(f;t) = \int_{-\infty}^{\infty} c(\tau;t)\, e^{-j2\pi f\tau}\, d\tau$$

(Notice transformation of $\tau \longrightarrow f$; t stays!!)
(Since $c(\tau;t)$ c.G.r.p. $\Rightarrow$ $C(f;t)$ also c.G.r.p.)

∘ Autocorrelation function $\phi_C(f_1, f_2; \Delta t) = \frac{1}{2} E\{C^*(f_1;t)\, C(f_2; t+\Delta t)\}$

$$= \phi_C(\Delta f; \Delta t)$$

↳ due to <u>uncorrelated</u> scattering.

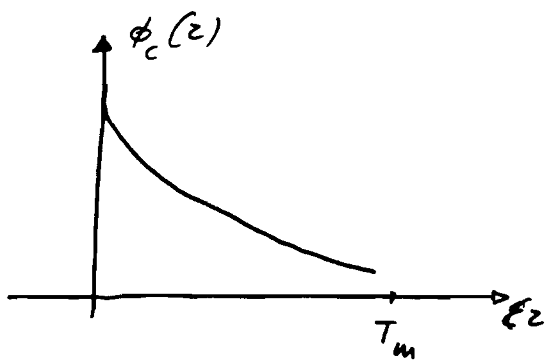
Its easy to show that (HW)

$$\phi_C(\Delta f; \Delta t) \longleftrightarrow \phi_c(\tau; \Delta t)$$

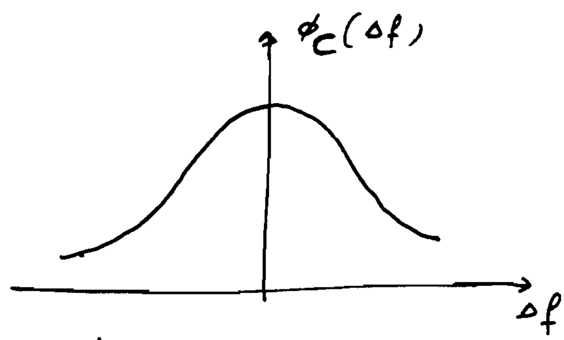
[To measure it, Tx two sinusoids separated by Δf & crosscorrelate the two separately Rx signals with a relative delay Δt]

For $\Delta t = 0 \Rightarrow \phi_C(\Delta f) \longleftrightarrow \phi_c(\tau)$

For example:



$\longleftrightarrow$



$$(\Delta f)_c \simeq \frac{1}{T_m}$$

↳ Coherence bandwidth of the channel.

Remarks

- Two sinusoids with freq. separation $> (\Delta f)_c$ are affected differently by the fading channel.
- Assuming that the info bearing signal has f_{max} & is transmitted through the fading channel.

  (A) If $(\Delta f)_c < f_{max} \Rightarrow$ Frequency selective channel

  (B) If $(\Delta f)_c > f_{max} \Rightarrow$ " nonselective " (flat fad…

Until now we considered τ & Δf.

Lets consider the other important parameter Δt (i.e., time variations of the channel)

Notice that the time variations are because the user is moving $\longrightarrow$ Doppler effect.

Def. the F.T. of $\phi_C(\Delta f;\Delta t)$ <u>w.r.t. Δt</u> as

$$S_C(\Delta f;\lambda) = \int_{-\infty}^{\infty} \phi_C(\Delta f;\Delta t)\, e^{-j2\pi\lambda\Delta t}\, d\Delta t$$

For $\Delta f = 0$ ↳

$$S_C(\lambda) = \int_{-\infty}^{\infty} \phi_C(\Delta t)\, e^{-j2\pi\lambda\Delta t}\, d\Delta t$$

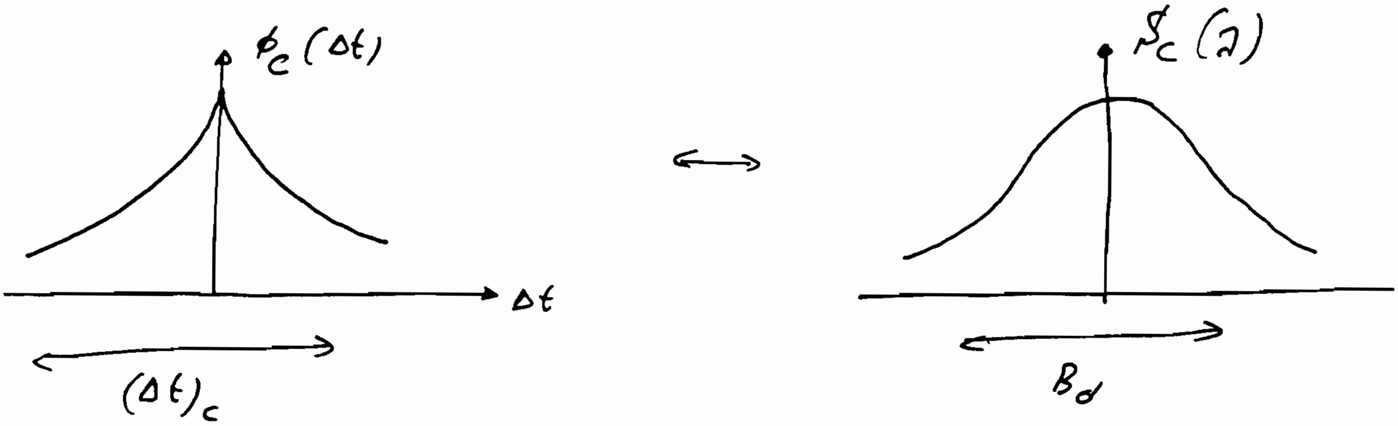
with $\lambda \equiv$ The Doppler frequency

and $S_C(\lambda) \equiv$ The Doppler power spectrum of the channel.

Note that for $\phi_C(\Delta t) = 1$ (i.e., No time variation) $\Rightarrow$
$\Rightarrow S_C(\lambda) = \delta(\lambda)$ (i.e., No spectral broadening observed in the transmission of a single tone !!)

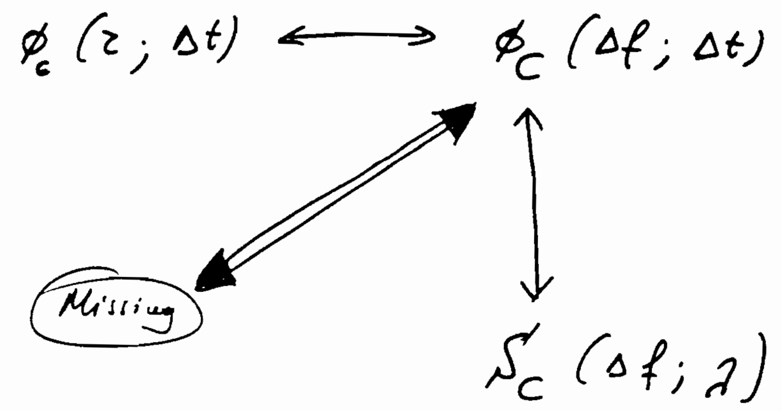
Max. frequency Doppler spread B_d (of the channel)

$$(\Delta t)_c \equiv \text{coherence time} \simeq \frac{1}{B_d}$$



Notice that for a slow changing channel $\Rightarrow B_d$ small or equivalently $(\Delta t)_c$ is high. (∴ It depends upon what kind of car you are driving and where you drive !!)

Until now:



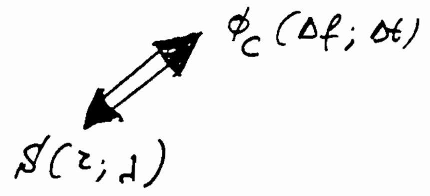
One more corner to complete the square.

$$S(\tau;\lambda) = \int_{-\infty}^{\infty} S_C(\Delta f;\lambda)\, e^{j2\pi\tau\Delta f}\, d\Delta f \qquad (\text{i.e. } \Delta f \to \tau)$$

The most general form:

$$S(\tau;\lambda) = \iint_{-\infty}^{\infty} \phi_C(\Delta f;\Delta t)\, e^{-j2\pi\lambda\Delta t}\, e^{-j2\pi\tau\Delta f}\, d\Delta t\, d\Delta f.$$

Connects :



(2-dim. F.T.)

9

① $\{T \gg T_m\} \Leftrightarrow$ {nonselective fading and no ISI}

② $\{B_d$ small$\} \Leftrightarrow$ {slowly changing channel}

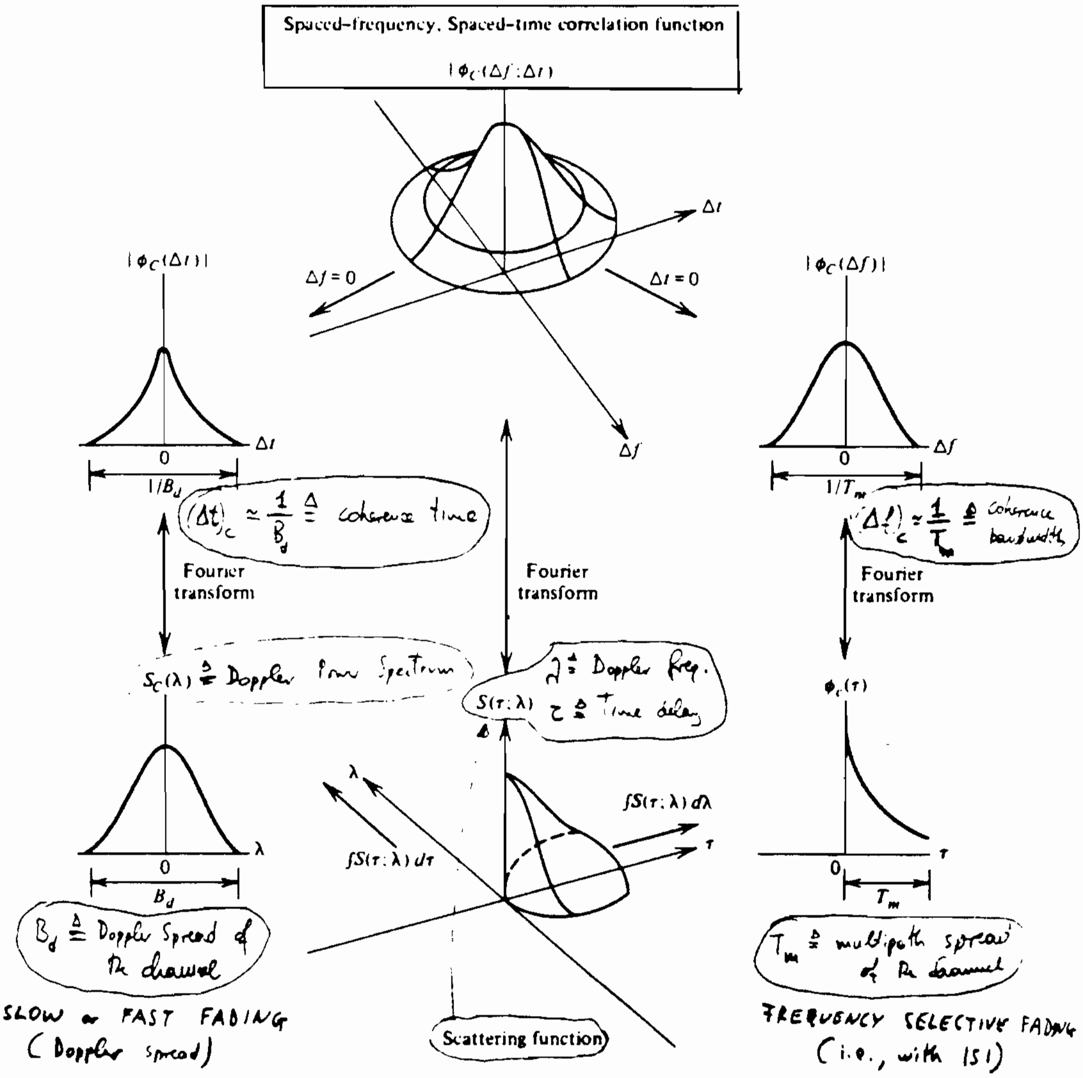


Figure 7.1.5 Relationships among the channel correlation functions and power spectra (from P. E. Green, Jr. [28], with permission).

Ref. Proakis, Digital Communications

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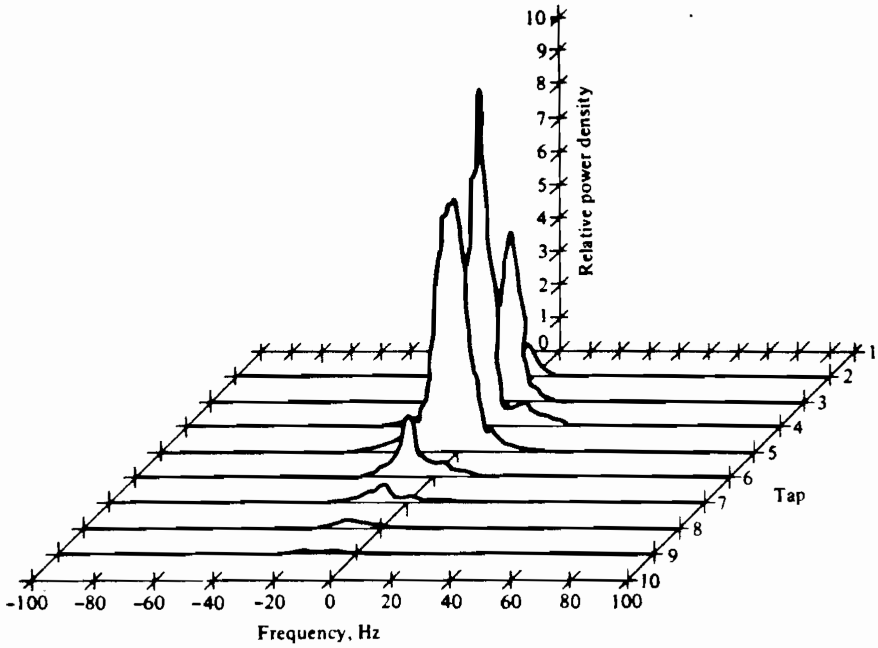


Figure 7.1.6 Scattering function of a medium-range tropospheric scatter channel.

Table 7.2.1 Multipath spread, Doppler spread, and spread factor for several time-variant multipath channels

Type of channel	Multipath duration	Doppler spread	Spread factor
Shortwave ionospheric propagation (HF)	10^{-3}–10^{-2}	10^{-1}–1	10^{-4}–10^{-2}
Ionospheric propagation under disturbed auroral conditions (HF)	10^{-3}–10^{-2}	10–100	10^{-2}–1
Ionospheric forward scatter (VHF)	10^{-4}	10	10^{-3}
Tropospheric scatter (SHF)	10^{-6}	10	10^{-5}
Orbital scatter (X band)	10^{-4}	10^{3}	10^{-1}
Moon at max. libration (f_0 = 0.4 kmc)	10^{-2}	10	10^{-1}

Ref. Proakis, Digital Communicati

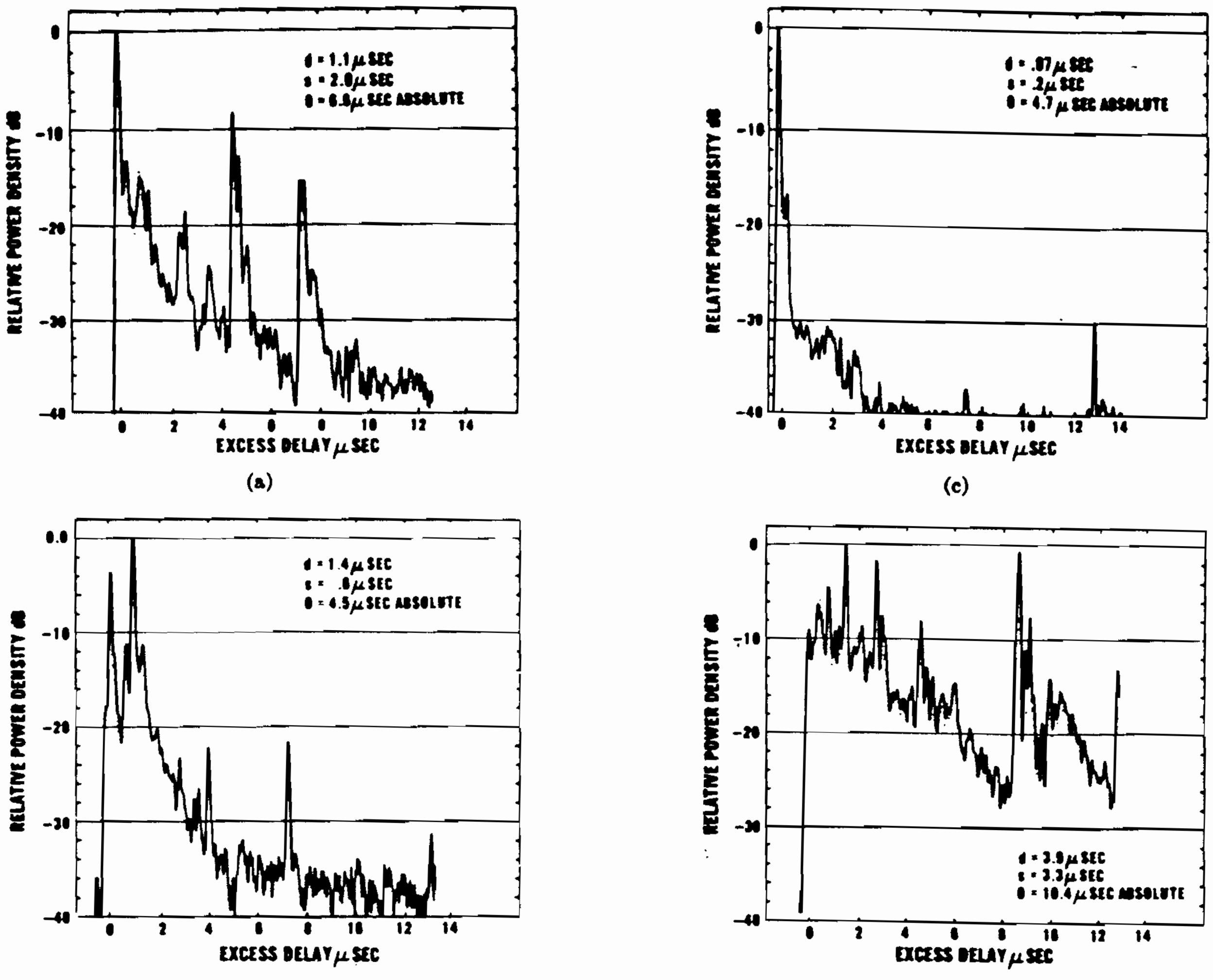


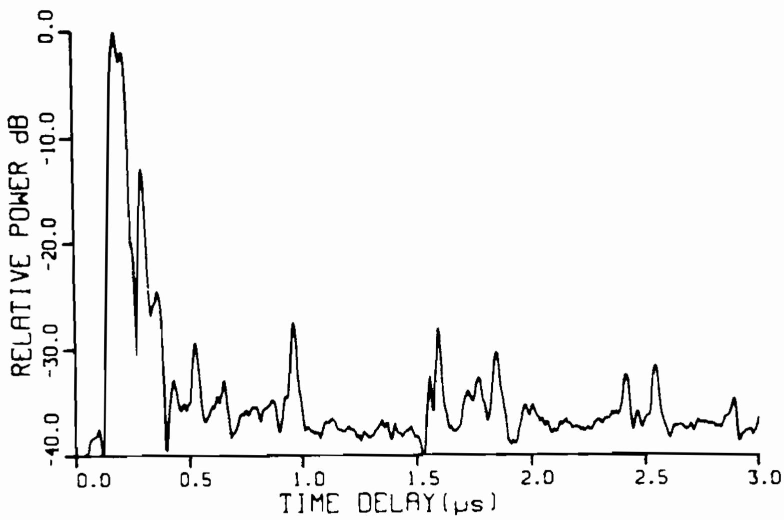
Fig. 6 Average power delay profiles. (a) Intersection of Bowery and Houston. (b) Broome Street between Thompson and Sullivan. (c) Intersection of Watt and Varick. (d) West 14th Street between 6th and 7th Avenues.

(a)

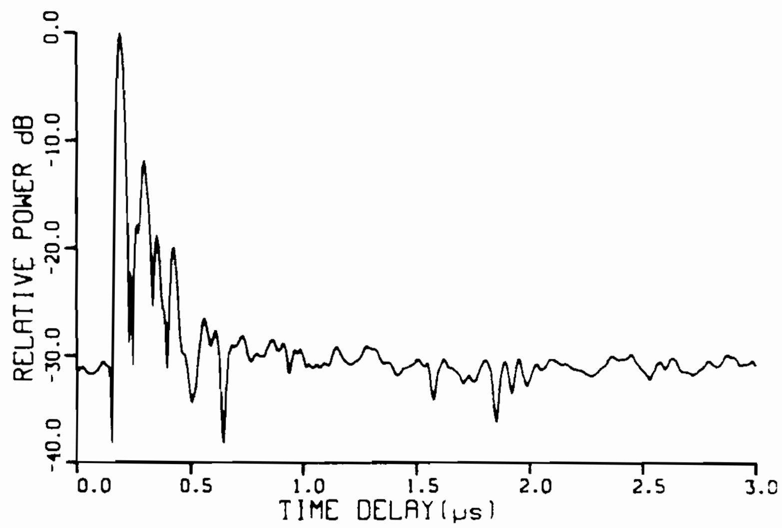
(b)

Fig. 7. (a) Looking south on Bowery near Houston. (b) Looking east on 14th Street toward intersection with 6th Avenue.

Ref. D. Cox & R. Leck, "Distribution of Multipath Delay Spread and Average Excess Delay for 910 MHz Urban Mobile Radio Paths" IEEE Trans. Ant. Prop., March 1975



(a)



(b)

Fig. 6. Envelope of the average for impulse response estimates recorded in a 2 min interval with CU transmit location 5 in: (a) the 900 MHz band; and (b) the 1.7 GHz band.

Ref. R. J. Bultitude et al., "A comparison of Indoor Radio Propagation Characteristics at 910 MHz and 1.75 GHz," J. Select. Areas Commun. vol. 7, pp. 20-30, Jan. 1989.
(JSAC)

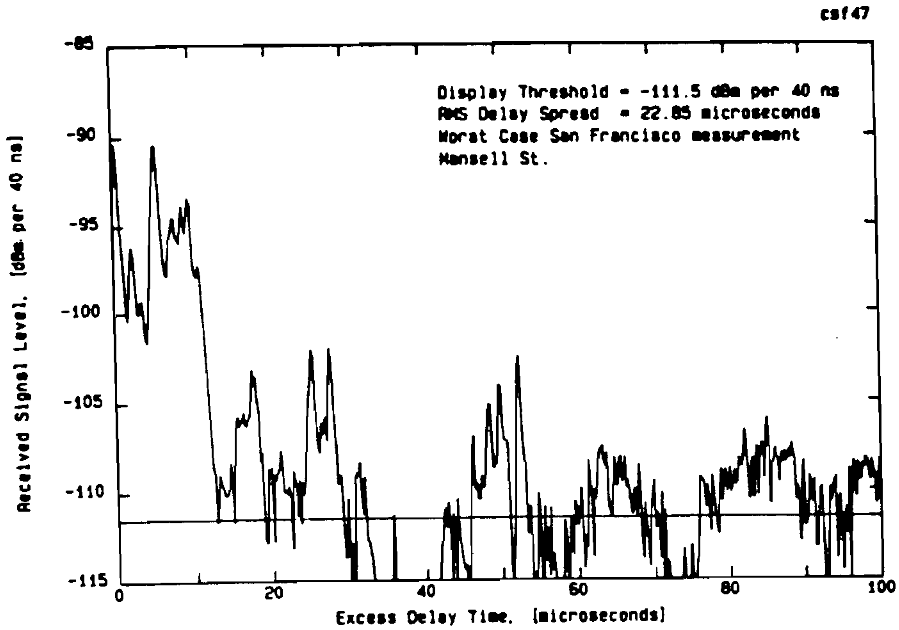


Fig. 8. One of the worst case multipath power delay profiles measured during campaign. Mansell Street in San Francisco.

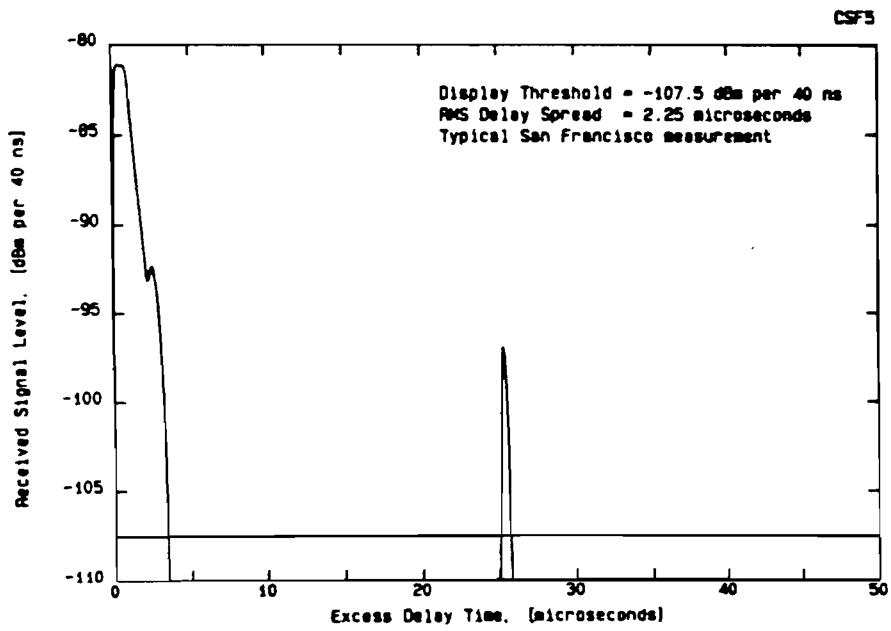


Fig. 7. Multipath power delay profile measured in South San Francisco, CA, with mobile at Bayshore Drive and US-101, near San Francisco International Airport.

Ref. T. S. Rappaport et. al., "900-MHz Multipath Propagation Measurements for U.S. Digital Cellular Radiotelephone," IEEE Trans. Veh. Tech., vol. VT-39, pp. 132-139, May 1990.

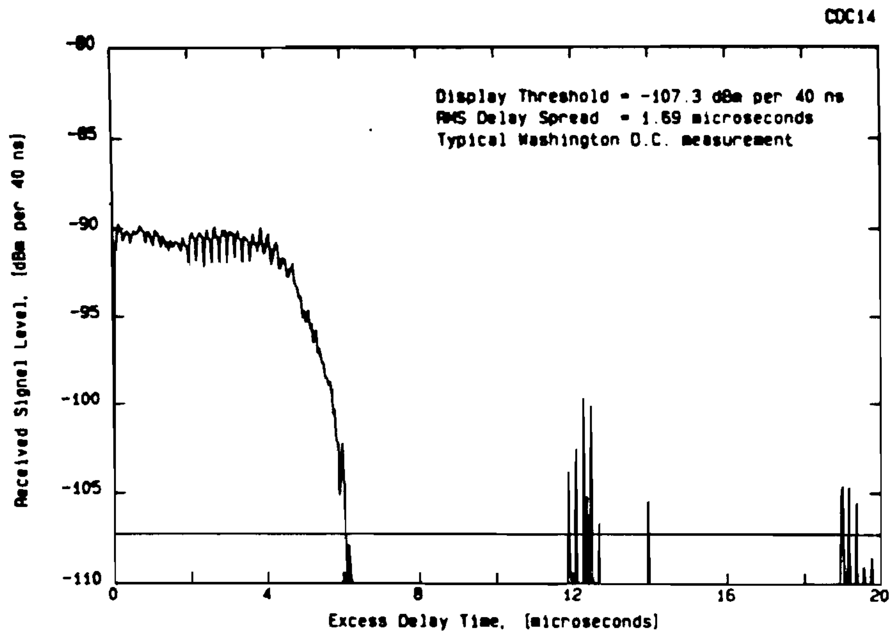


Fig. 4. Multipath power delay profile measured in Washington, DC, with mobile on DuPont Circle. Note the almost uniform distribution of multipath power out to 5 μs. This is common for channels confined to urban areas.

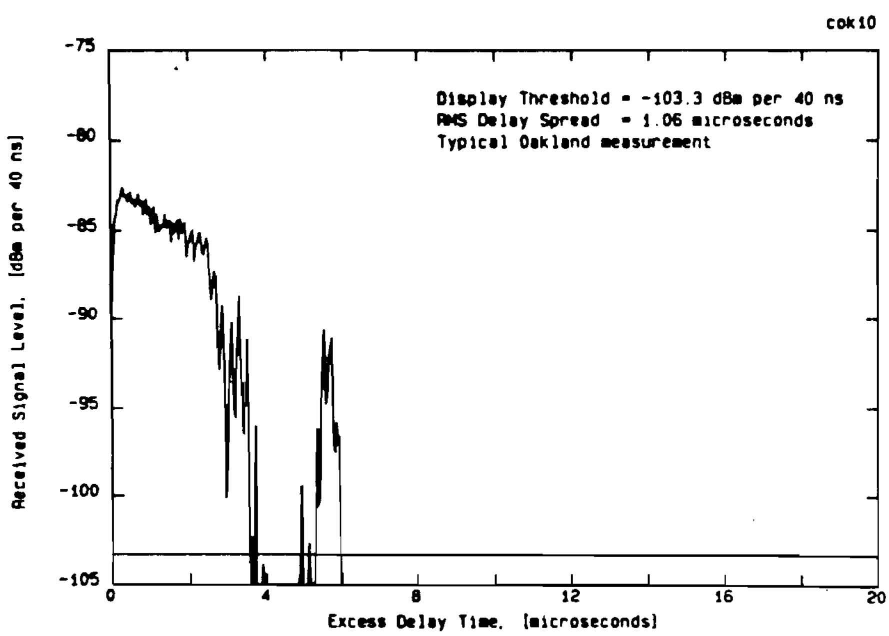


Fig. 5. Multipath power delay profile measured in Oakland, CA, with mobile on 8th and Western (urban commercial district). The profile displays nearly continuous power out to 4 μs. Distinct reflection at 6 μs.

FREQUENCY NONSELECTIVE FADING - FLAT FADING.

Flat fade $\Rightarrow$ multipath spread of the channel (T_m) is negligible

$$\therefore \quad \phi_c(\Delta t) \longleftrightarrow \underline{S_c(\lambda)} = \int_{-\infty}^{\infty} \phi_c(\Delta t)\, e^{-j2\pi\lambda \Delta t}\, d\Delta t$$

→ Doppler power spectrum of the channel

Max: B_d → coherence time of the channel $(\Delta t)_c \simeq \frac{1}{B_b}$

↓ more often used

Comments

① If the symbol rate of the info. signal is $\gg B_b \Rightarrow$ slow fading

- - - - - - - - is $\lesssim B_b \Rightarrow$ fast fade.

② Obviously, $\phi_c(\Delta t)$ is the autocorrelation function of the fading process. Depending upon the application, it can be modeled in various ways.

FADING MODEL	AUTOCORRELATION FUNCTION	SPECTRUM
Rectangular	$\sin(2\pi B_d \Delta t)/2\pi B_d \Delta t$	$\begin{cases} \frac{1}{2B_d} & \text{for } \lvert f \rvert < B_d \\ 0 & \text{otherwise} \end{cases}$
Gaussian	$\exp[-(\pi B_d \Delta t)^2]$	$\frac{1}{B_d\sqrt{\pi}} \exp[-(\frac{f}{B_d})^2]$
Land - Mobile	$J_0(2\pi B_d \Delta t)$	$\frac{1}{\pi\sqrt{f^2 - f_d^2}}$
1st order Butterworth	$\exp(-2\pi B_d \lvert\Delta t\rvert)$	$\frac{1}{\pi B_d (1 + \frac{f^2}{B_d^2})}$
2nd order Butterworth	$\exp(-\beta' \lvert\Delta t\rvert)[\cos(\beta' \Delta t) + \sin(\beta' \lvert\Delta t\rvert)]$	$\frac{1}{1 + 16(\frac{f}{B_d})^4}$

($J_0(\cdot) \equiv$ zero-order Bessel function of the first kind
$\beta' \equiv \pi B_d/\sqrt{2}$)

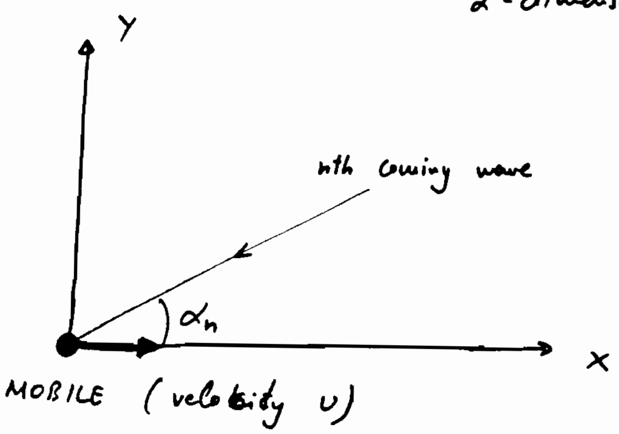
Ref.: L. J. Mason, "Error Probability Evaluation for Systems Employing Differential Detection in a Rician Fast Fading Environment and Gaussian Noise," IEEE Trans. Commun., vol. COM-35, pp. 39 – 46, Jan. 1987.

LAND - MOBILE FADING

- Fades of 40 dB or more below the mean level (minima occurring about every $\lambda/2$, i.e., ~ 15cm for 1GHz).

- This happens due:
  1. <u>Random Phase</u>, due to linear superposition of many scattered waves
  2. <u>Doppler Shift</u>, because of the vehicular speed.

2-dimensional model.



$$\omega_n = \beta v \cos\alpha_n$$

$$\omega_n \ \max = \pm \beta v$$

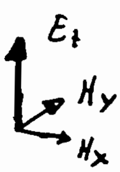
Vehicle motion introduces a Doppler shift (ω_n) in every way

$\omega_n = \beta v \cos\alpha_n$; $\beta = \frac{2\pi}{\lambda}$; λ wavelength of the Tx carrier

Example: 1 GHz → λ = 30cm → Doppler shift (Hz) for v = 60 mi/hr $\simeq$ 90 Hz.

(max in N.A.)
unless racing - lucky !!

Consider vertically polarized wave



Energy is detected at E_z.
(Antennas - practical: ① $\lambda/4$ (whip)
② $5\lambda/8$ + base loading coil)

$$E_z = E_0 \sum_{n=1}^{N} c_n \cos[\omega_c t + \omega_n t + \varphi_n] \qquad \varphi_n \sim [0, 2\pi] \text{ (uniform).}$$

Normalization of c_n's $\Rightarrow$ $\langle \sum_{n=1}^{N} c_n^2 \rangle = 1$ $\qquad \langle\rangle =$ ensemble average

∴ E_z can be described as a narrow-band random process and as a consequence of the central limit theorem for large N
→ E_z is a Gaussian random process.

∴ Following Rice [S.O. Rice, "Mathematical Analysis of Random Noise," B.S.T.J., July 1944 pp. 282-332 & Jan. 1945, pp. 46-

$$E_z = E_0 \sum_{n=1}^{N} c_n \cos\omega_c t \cos[\omega_n t + \varphi_n] - E_0 \sum_{n=1}^{N} c_n \sin\omega_c t \sin[\omega_n t + \varphi_n]$$

or $E_z = T_c(t)\cos\omega_c t + T_s(t)\sin\omega_c t$ with

$$T_c(t) = E_0 \sum_{n=1}^{N} c_n \cos[\omega_n t + \varphi_n]$$

$$T_s(t) = E_0 \sum_{n=1}^{N} c_n \sin[\omega_n t + \varphi_n]$$

} Gaussian Processes ∴ Mean & Variance

Variance of T_c (r.v. for fixed t)

$$\langle T_c^2 \rangle = \frac{E_0^2}{2} = \langle T_s^2 \rangle$$

$\langle\rangle$ = ensemble average, i.e., expected values not time average over ω_n, φ_n and c_n.

(HW) Proof

$$\langle T_c^2 \rangle = E_0^2 \langle c_1^2 \cos^2(K_1 + \varphi_1) + c_2^2 \cos^2(K_2 + \varphi_2) + \ldots + c_n^2 \cos^2(K_n + \varphi_n) + \ldots + \ldots + 2\Lambda_i \cos A \cos B + \ldots \rangle$$

$K_i = \omega_i t$

But $\langle \cos^2 \Gamma \rangle = 1/2$ and $\langle \sum c_n^2 \rangle = 1$ and $\langle 2\Lambda_i \cos A \cos B \rangle =$
$= \langle \cos(A-B) + \cos(A+B) \rangle = \emptyset$!!

Mean → zero.

T_c, T_s have prob. density function $= \frac{1}{\sqrt{2\pi b}} e^{-\frac{x^2}{2b}}$; $b = \frac{\mathcal{E}_0^2}{2}$ (signal mean power)

$r = \sqrt{T_c^2 + T_s^2}$ → envelope of $\mathcal{E}_t$

$\vartheta = \arctan \frac{T_s}{T_s}$ → phase of $\mathcal{E}_b$

Easy to show that $f_R(r) = \frac{r}{b} \exp[-\frac{r^2}{2b}]$ $r \geq 0$ – Rayleigh

(HW) $f_\Theta(\vartheta) = \frac{1}{2\pi}$ – Uniform.

Proof

Transformation $\left.\begin{array}{l} T_c = r\cos\vartheta \\ T_s = r\sin\vartheta \end{array}\right\} \Rightarrow J = \begin{vmatrix} \cos\vartheta & -r\sin\vartheta \\ \sin\vartheta & r\cos\vartheta \end{vmatrix} = r$

$$\left.\begin{array}{l} f_R(r) = \int_0^{2\pi} r\, f_{T_c T_s}(r\cos\vartheta, r\sin\vartheta)\, d\vartheta \\ T_s, T_c \text{ G.r.v independent} \end{array}\right\} \Rightarrow f_R(r) = \int_0^{2\pi} r \frac{1}{2\pi} \frac{1}{b} \exp[-\frac{1}{2b}(r^2\cos^2\vartheta + r^2 s \ldots] d\vartheta$$

$$\Rightarrow f_R(r) = \int_0^{2\pi} r \frac{1}{2\pi} \frac{1}{b} \exp(-\frac{r^2}{2b}) d\vartheta = \frac{r}{b} e^{-\frac{r^2}{2b}}$$

and

$$f_\Theta(\vartheta) = \int_0^\infty r\, f_{T_c T_s}(r\cos\vartheta, r\sin\vartheta)\, dr = \int_0^\infty \frac{r}{2\pi b} \exp(-\frac{r^2}{2b})\, dr = \frac{1}{2\pi}$$

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1.1.2 Probability Distributions

Since T_c and T_s are Gaussian, they have probability densities of the form

$$p(x)=\frac{1}{\sqrt{2\pi b}}e^{-x^2/2b} \tag{1.1-12}$$

where $b=E_0^2/2$ is the mean power, and $x=T_c$ or T_s.

The envelope of E_Z is given by

$$r=\left(T_c^2+T_s^2\right)^{1/2}, \tag{1.1-13}$$

and Rice[8] has shown that the probability density of r is

$$p(r)=\begin{cases}\dfrac{r}{b}e^{-r^2/2b}, & r\geqslant 0\\ 0, & r<0\end{cases} \tag{1.1-14}$$

which is the Rayleigh density formula. The Gaussian and Rayleigh densities are shown in Figure 1.1-3 for illustration.

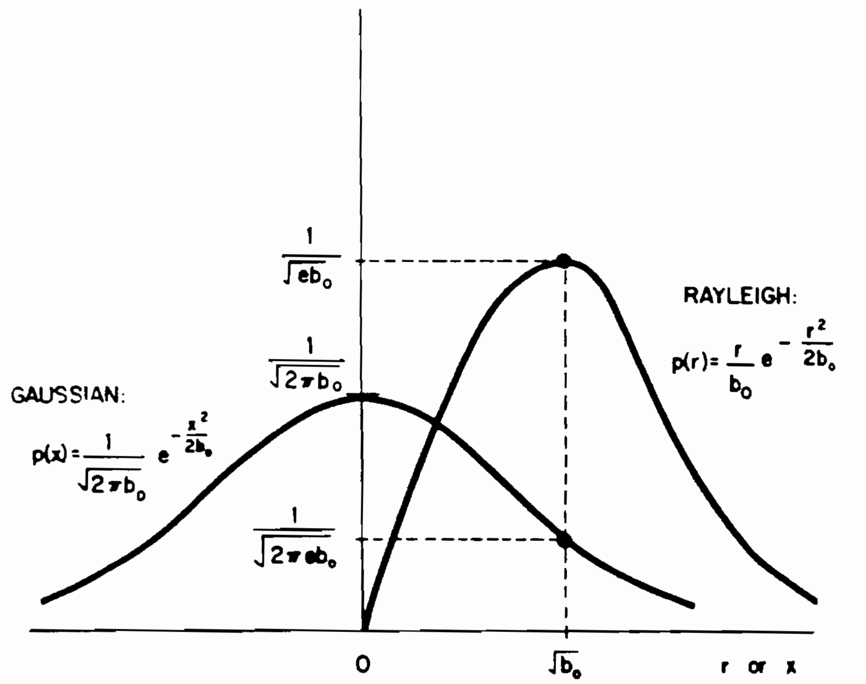


Figure 1.1-3 Gaussian and Rayleigh probability density functions.

Ref. Jakes, Microwave Mobile Communications

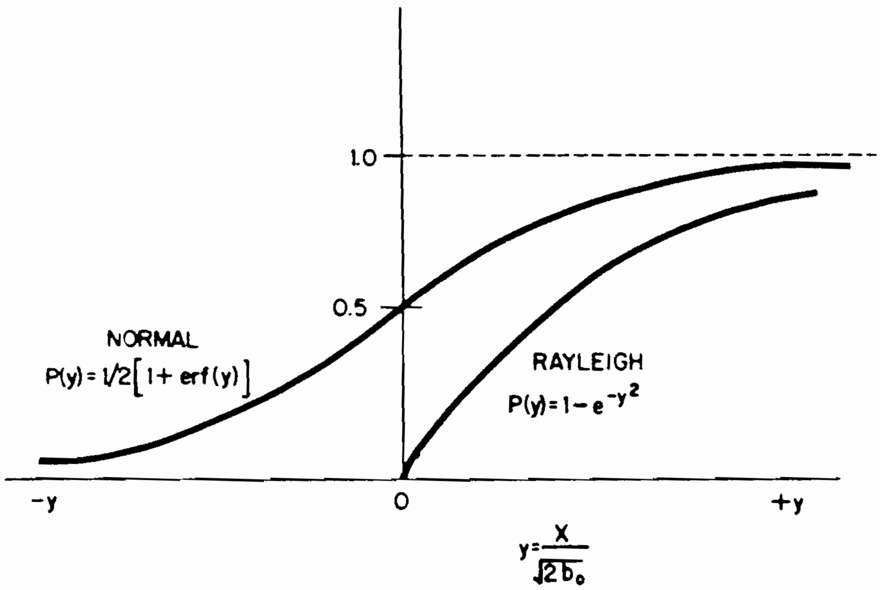


Figure 1.1-4 Normal and Rayleigh cumulative distributions.

Ref. Jakes, Microwave Mobile Communications

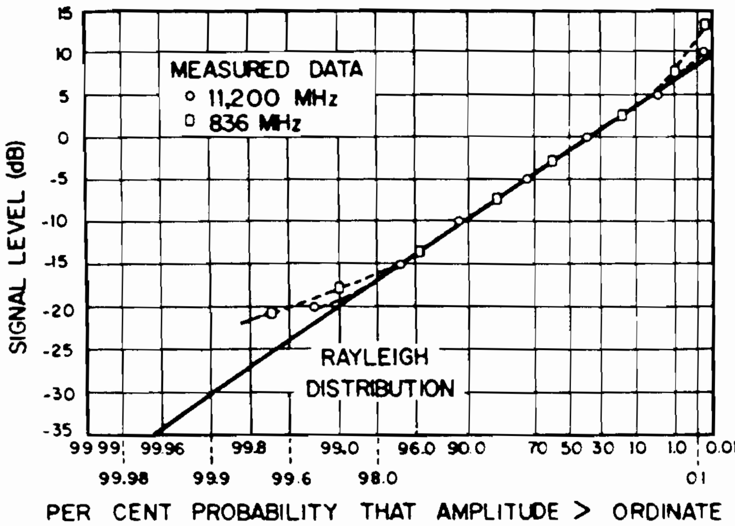
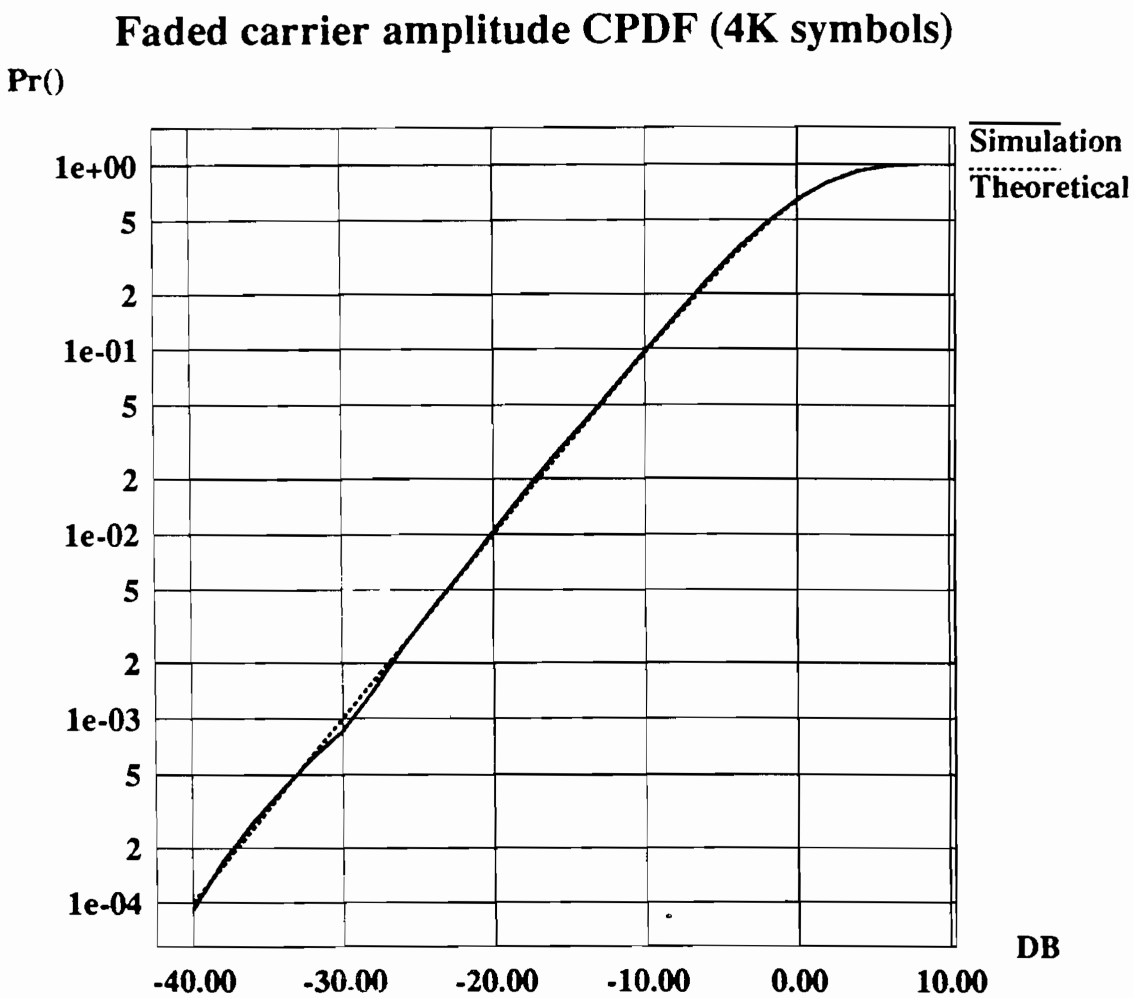


Figure 1.1-5 Cumulative probability distributions for 836 and 11,200 MHz.

Ref. Jakes, Microwave Mobile Communications


Faded carrier amplitude CPDF (4K symbols)
Pr()
Simulation
Theoretical
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5
2
1e-01
5
2
1e-02
5
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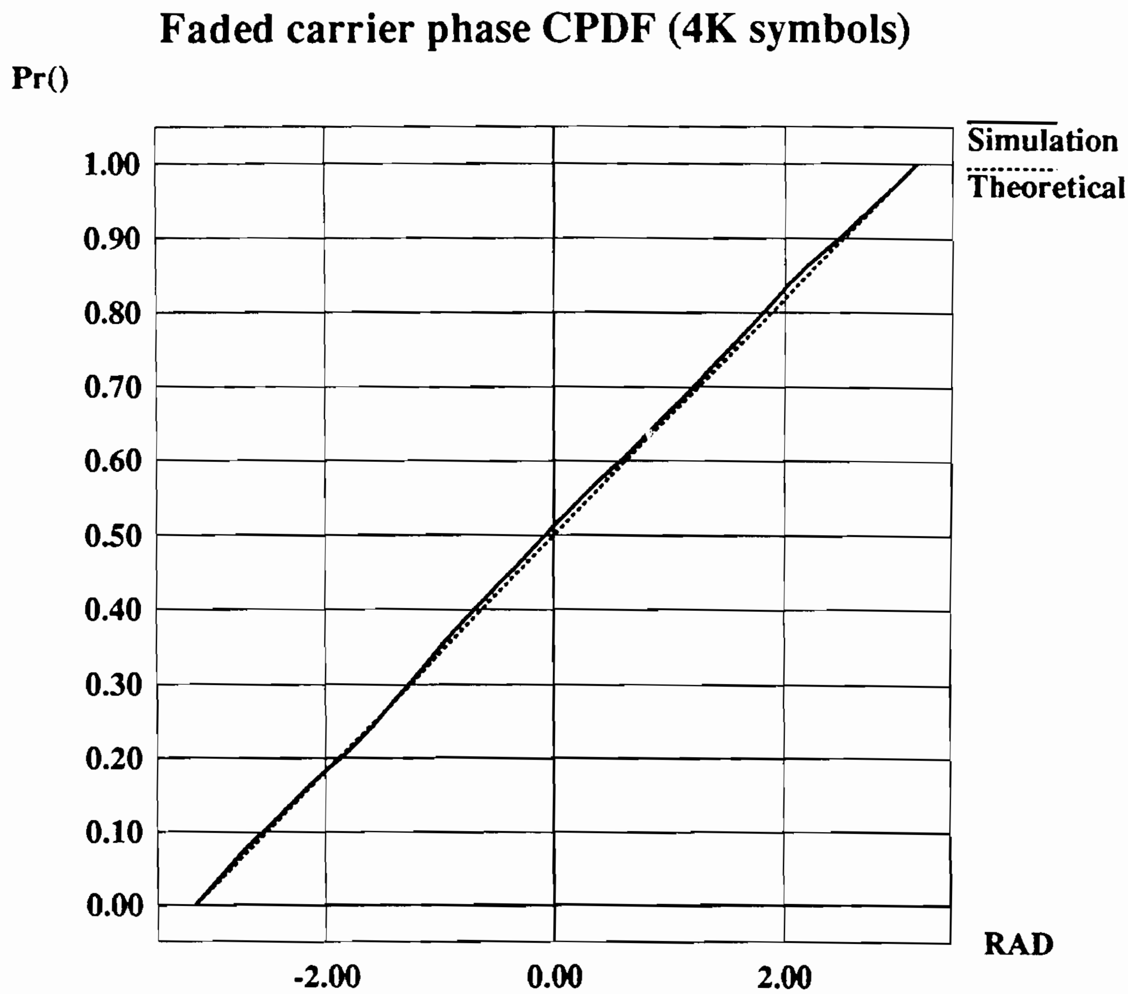
-10.00
0.00
10.00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0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
Simulation
Theoretical
RAD
-2.00
0.00
2.00

RF POWER SPECTRA OF THE FADING SIGNAL

- **RF Spectrum of the Field Components ($\rightarrow E_z$)**

(First spectrum $\longrightarrow$ then autocorrelation)

From the viewpoint of an observer on the mobile unit, the signal received from a CW transmission as the mobile moves with constant velocity may be represented as a carrier whose phase & amplitude are randomly varying with an effective bandwidth corresponding to twice the max Doppler shift of βv.

$\therefore$ Statistics of these processes through the power spectrum.

If $p(\alpha)\,d\alpha$ the fraction of the total incoming power within $d\alpha$ of α

$G(\alpha)$ the antenna power gain pattern

b the average power received by an isotropic antenna ($G(\alpha)=1$)

Then the differential variation of received power w.r.t. the angle α is

$$b\, G(\alpha)\, p(\alpha)\, d\alpha$$

But the relation between angle and phase is

$$f(\alpha) = f_m \cos\alpha + f_c = \frac{v}{\lambda}\cos\alpha + f_c \qquad (f(\alpha) = f(-\alpha))$$

$\therefore$ We can convert it to the variation of power w.r.t. frequency. as

$$S(f)\,|df| = b\left[p(\alpha)G(\alpha) + p(-\alpha)G(-\alpha)\right]|d\alpha|$$

But

$$|df| = f_m\,|-\sin\alpha\, d\alpha| = f_m \left| -\sqrt{1 - \frac{(f-f_c)^2}{f_m^2}}\, d\alpha \right|$$

$$= \sqrt{f_m^2 - (f-f_c)^2}\,|d\alpha| \qquad \left(\text{Notice } \cos\alpha = \frac{f-f_c}{f_m} \rightarrow \sin\alpha\right)$$

∴ In the general case

$$S(f) = \frac{b}{\sqrt{f_m^2 - (f-f_c)^2}} \left[p(\alpha) G(\alpha) + p(-\alpha) G(-\alpha) \right]$$

with $\alpha = \arctan\left[\frac{f-f_c}{f_m}\right]$ and $S(f) = \phi$ for $|f-f_c| > f_m$

For an omnidirectional $\lambda/4$ whip → $G(\alpha) \simeq 1.5$ (sensing E_z)

$$S_{E_z}(f) = \frac{1.5\, b}{\sqrt{f_m^2 - (f-f_c)^2}} \left[p(\alpha) + p(-\alpha) \right]$$

But α is uniformly distributed $(0, 2\pi]$

∴

$$S_{E_z}(f) = \frac{3b}{2\pi f_m} \frac{1}{\sqrt{1 - \left(\frac{f_c - f_c}{f_m}\right)^2}}$$

Similarly $S_{H_x}(f)$ & $S_{H_y}(f)$

It turns out that ① H_x is uncorrelated with both E_z & H_y ⇒ Corresponding cross spectra are zero.

② E_z, H_y are correlated

$$R_{E_z H_y}(\Delta t) = \frac{b}{\eta} \sin(\omega_c \Delta t)\, J_1(\beta v \Delta t)$$

At $\Delta t = 0$ ⇒ zero.

∴ For $\Delta t = 0$ all E_z, H_x, H_y are uncorrelated.

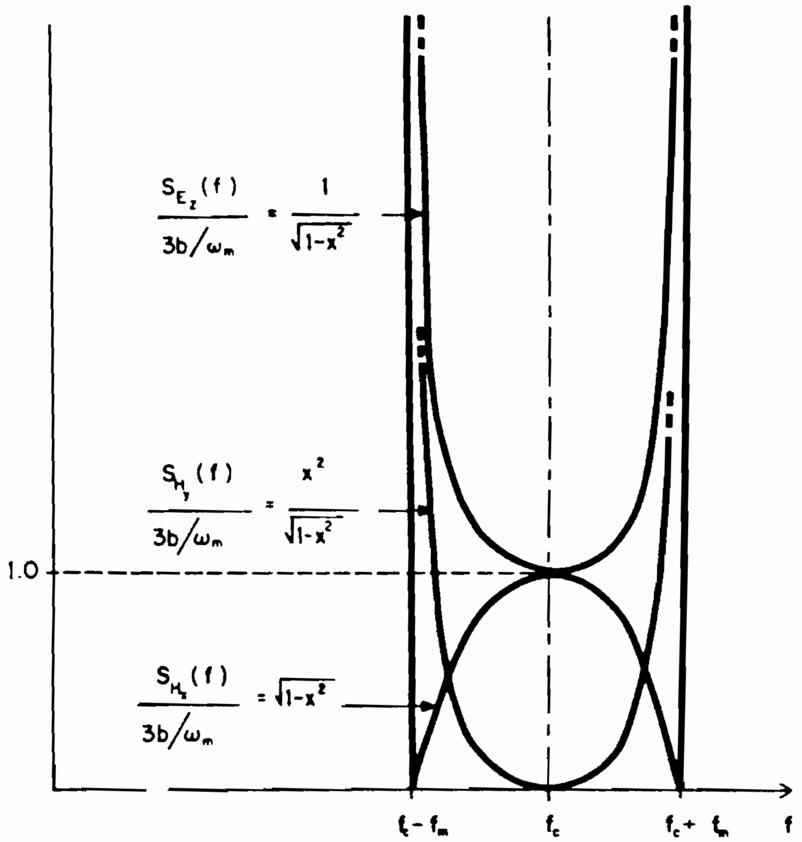

Figure 1.2-1 Power spectra of the three field components for uniformly distributed arrival angles. [$x = (f - f_c)/f_m$.]

Ref. Jakes, Microwave Mobile Communications

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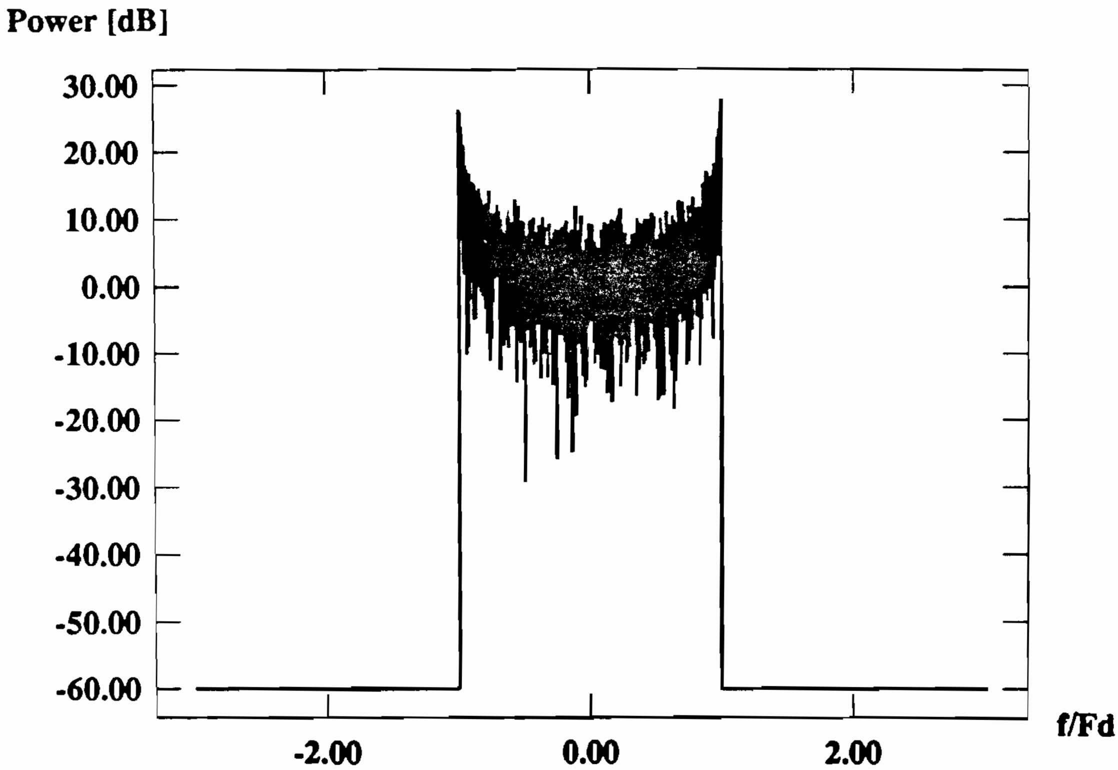
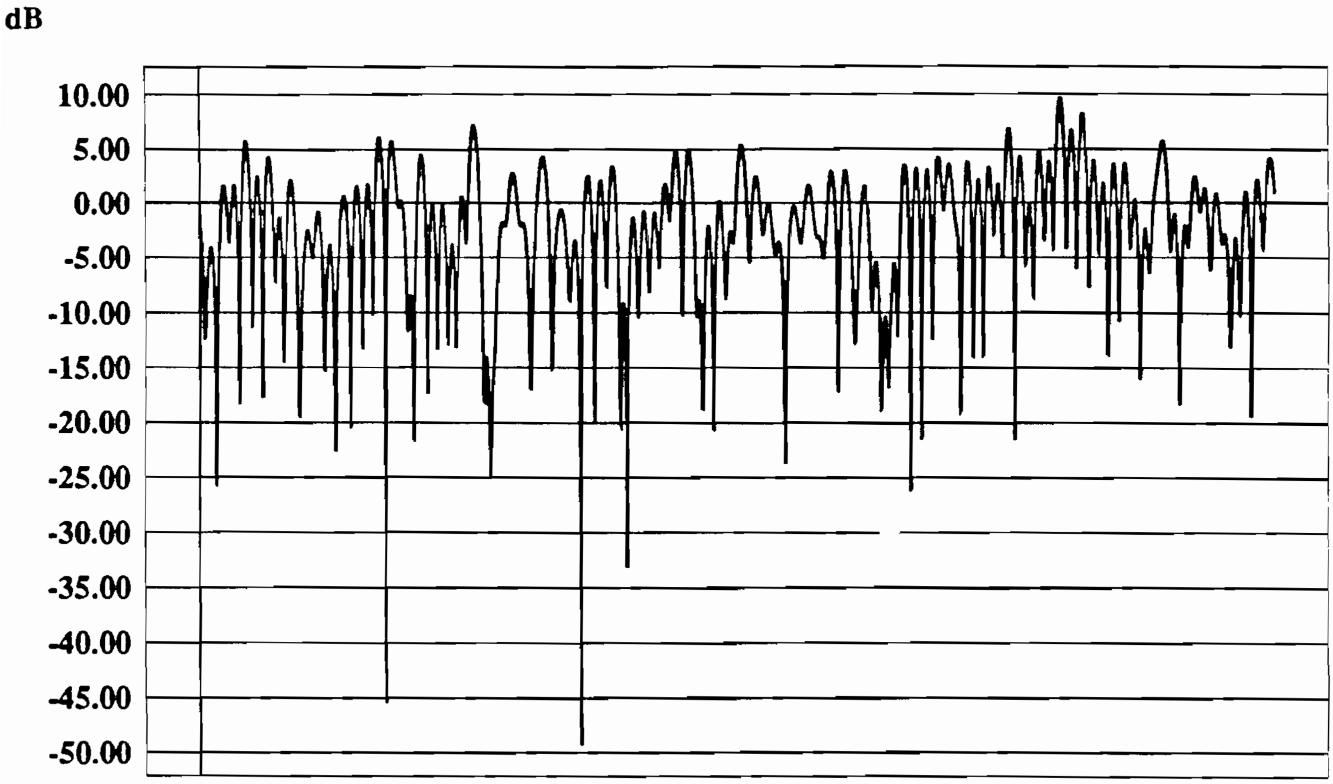
Faded carrier baseband equivalent spectrum



Faded carrier amplitude (20*log10[A/RMS])



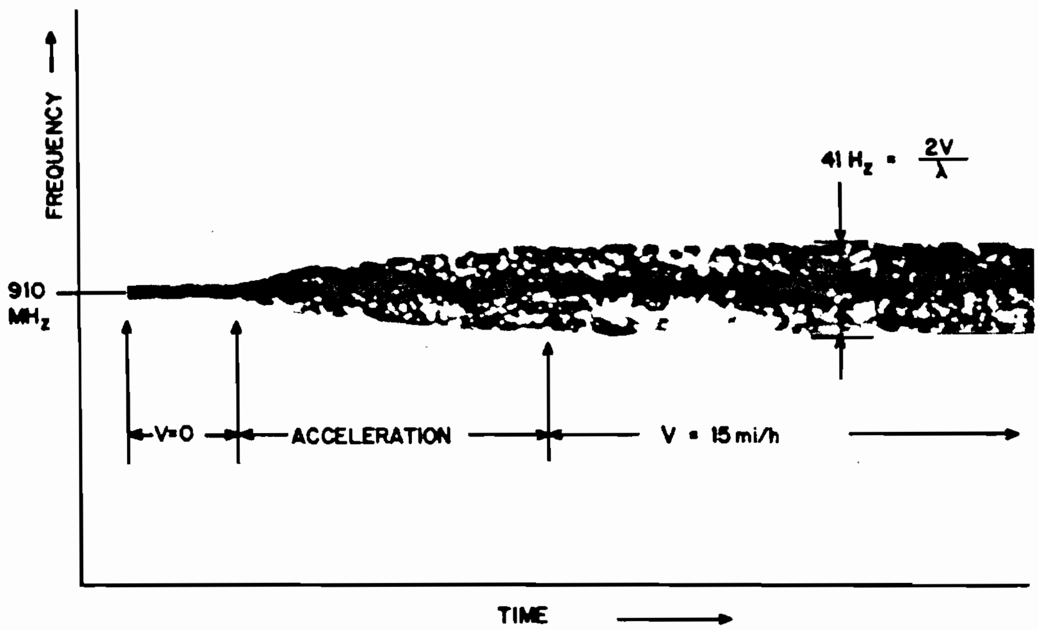


Figure 1.2-2 Frequency spectogram of RF signal at 910 MHz.

Ref. Jakes, Microwave Mobile Communications

A "Practical" Explanation of the Land-Mobile Channel

We have seen that

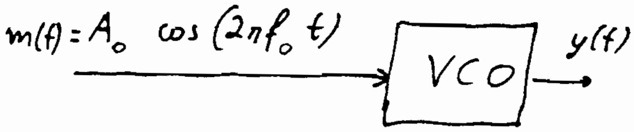
$$f(\alpha) = f_m \cos\alpha + f_c \quad ; \quad \alpha = \text{received angle of the multipath signal.}$$

f_m = max. Doppler freq. shift
f_c = carrier freq.

∴ FM modulation of f_c with $f_m \cos\alpha$.

Since in general α uniformly distributed $[0, 2\pi]$
∴ $-f_m \xrightarrow{\cos(\cdot)} f_m$

Equivalently:



$\beta = \frac{\Delta f}{f_0} \triangleq$ modulation index ; Δf = max. freq. (equivalent to f_m)

[Woodward's Theorem

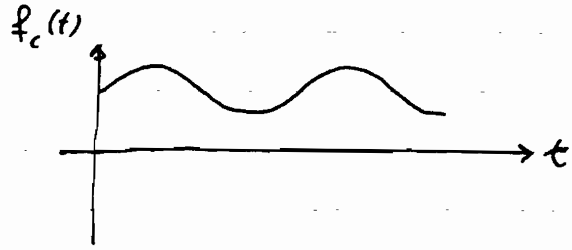
- For $\beta \gg 1$ (WBFM)
- Assume that $m(t)$ varies slowly enough so that the instanteneous frequency changes slowly enough so that the transients due to frequency changes are insignificant

Under these two conditions, we would expect that the strength of a spectral component will depend on the amount of time f_c spends at a given frequency.

∴ The spectrum will be limited to $\pm\Delta f$ (i.e., $\pm f_m$)]

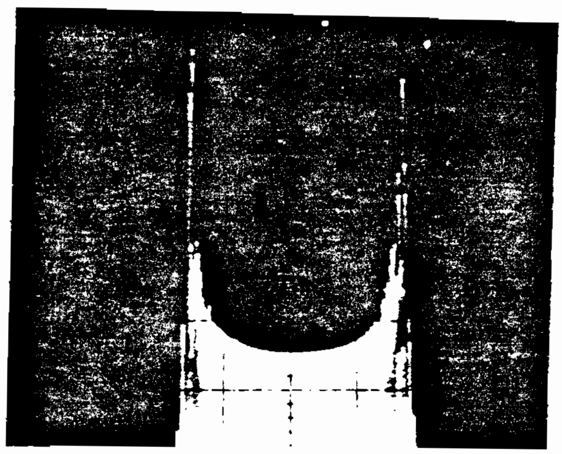
Applications

- cos $\rightarrow$ $f_c(t) = f_c + f_m \cos(\alpha)$



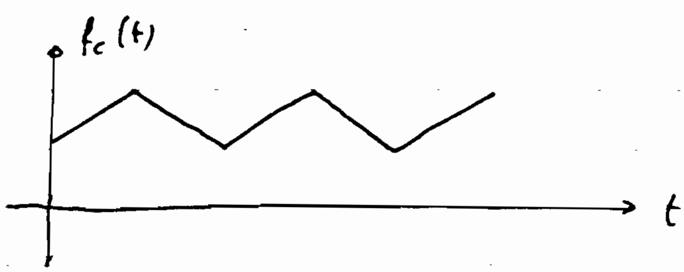
$f_c(t)$ spends more time on the "peaks" rather than the "lows" $\longrightarrow$ "twin peaks" of the spectrum.

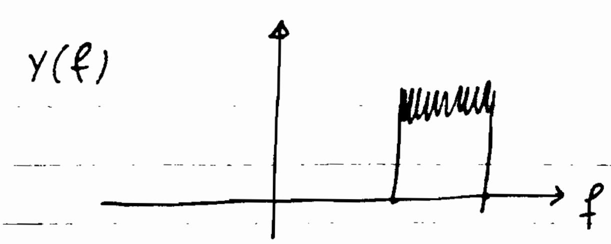
Y(f)



$\beta \gg 1$

- Linear





!!

POWER SPECTRUM AND PROPERTIES OF THE SIGNAL ENVELOPE

(A) Moments (In-phase & Quadrature)

$$\left[E_z = T_c(t)\cos\omega_c t - T_s(t)\sin\omega_c t \; ; \; T_c(t) = E_0 \sum_n c_n \cos(\omega_n t + \phi_n) \right]$$

$T_c \to s$; $\cos \to \sin$; $\phi_n \in (0, 2\pi]$

$T_{c_1} \longrightarrow T_c(t)$; $T_{c_2} \longrightarrow T_c(t+\tau)$

$T_{c_1} \to s_1$, $T_c \to s$, $T_{c_2} \to s_2$, $T_c \to s$

$$\langle T_{c_1} T_{c_2} \rangle = \langle T_{s_1} T_{s_2} \rangle = g(\tau) \; ; \; \langle T_{c_1} T_{s_2} \rangle = -\langle T_{s_1} T_{c_2} \rangle = h(\tau)$$

$$g(\tau) = \int_{f_c - f_m}^{f_c + f_m} S_i(f) \cos[2\pi(f - f_c)\tau]\, df \quad \text{(Autocorrelation)} \; ; \; h(\tau) = \int_{f_c - f_m}^{f_c + f_m} S_i(f) \sin[2\pi(f - f_c)\tau]\, df \quad \text{(Cross-correlation)}$$

At $\tau = 0 \Rightarrow$ The following moments can be obtained

$$b_n = (2\pi)^n \int_{f_c - f_m}^{f_c + f_m} S_i(f)(f - f_c)^n\, df \qquad (\text{Time} \leftrightarrow \text{Frequency})$$

REMARKS

① For all E_z, H_x, $H_y \to h(\tau) = 0$ (due to the symmetry around f_c)

② $E_z \to$

$$b_n = b_0\, \omega_m^n \frac{1\cdot 3\cdot 5 \cdots (n-1)}{2\cdot 4\cdot 6 \cdots n}$$

$b_0 = \langle T_c^2 \rangle = \langle T_s^2 \rangle$ (i.e., power of T_c or T_s)

$= 1.5\, b = \frac{3}{4} E_0^2$

$$\boxed{g(\tau) = b_0 J_0(\omega_m \tau)}$$

(Similar formulas for H_x, H_y)

(B) Envelope Autocorrelation and Spectrum

The autocorrelation function of the envelope r of a narrowband Gaussian process can be expressed in terms of a (hypergeometric) function as

$$R_r(\tau) = \frac{\pi}{2} b_0 F\left[-\frac{1}{2}; -\frac{1}{2}; 1, \rho^2(\tau)\right]$$

$$\rho^2(\tau) = \frac{1}{b_0^2}\left[g^2(\tau) + h^2(\tau)\right].$$

(At $\tau = 0 \rightarrow \rho^2(0) = 1$, and $R_r(0) = \langle r^2 \rangle = \frac{\pi}{2} b_0 F\left[-\frac{1}{2}; -\frac{1}{2}; 1; 1\right] = 2 b_0$)

Series expansion $\longrightarrow$ $R_r(\tau) = \frac{\pi}{2} b_0 \underbrace{\left[1 + \frac{1}{4}\rho^2(\tau)\right.}_{\simeq R_r(\tau)} \left. + \frac{1}{64}\rho^2(\tau) + \ldots\right]$

(At $\tau = 0 \rightarrow R_r(0) = \frac{5\pi}{8} b_0 \simeq 1.964\, b_0 \simeq 2 b_0$!!)

To find the spectrum it is (not !!) very straightforward.

Two steps:

1. It can be shown (not very easy) that in general and for positive frequencies the <u>baseband</u> envelope spectrum is

$$S_0(f) = \frac{\pi}{8 b_0} \int_{f_c - f_m}^{f_c + f_m - f} S_i(x)\, S_i(x+f)\, dx \qquad 0 \le f \le 2 f_m$$

(limits go: $f_c \rightarrow f_c \rightarrow f_c + f_m$)

2. $$S_{0E_z}(f) = \frac{b_0}{4\omega_m} K\left[\sqrt{1 - \left(\frac{f}{2f_m}\right)^2}\right]$$ K = the complete eliptic integral of first kind

(Similar expressions for H_x & H_y)

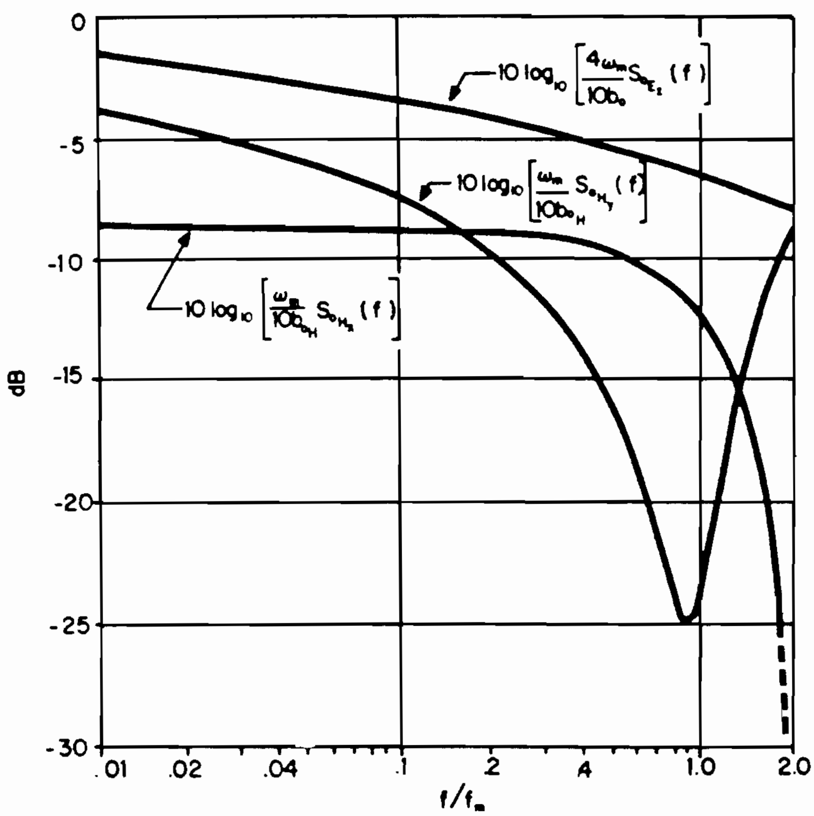


Figure 1.3-1 Baseband spectra of the field envelopes.

Ref. Jakes, Microwave Mobile Communications

If we have an <u>additional</u> <u>direct</u> wave from the Tx ($\equiv$ Rician fading) arriving at an angle α_0

$$\therefore \quad f_a = f_m \cos \alpha_0$$

$$\therefore \quad S^{\otimes}_{E_z}(f) = S_{E_z}(f) + B\, \delta(f - f_c - f_a)$$

(B = const.)

Using previous equations

$$S^{\otimes}_{E_z}(f) = \text{Previous Spectrum} + B_1 \left[S_{E_z}(f_c + f_a + f) + S_{E_z}(f_c + f_a -$$

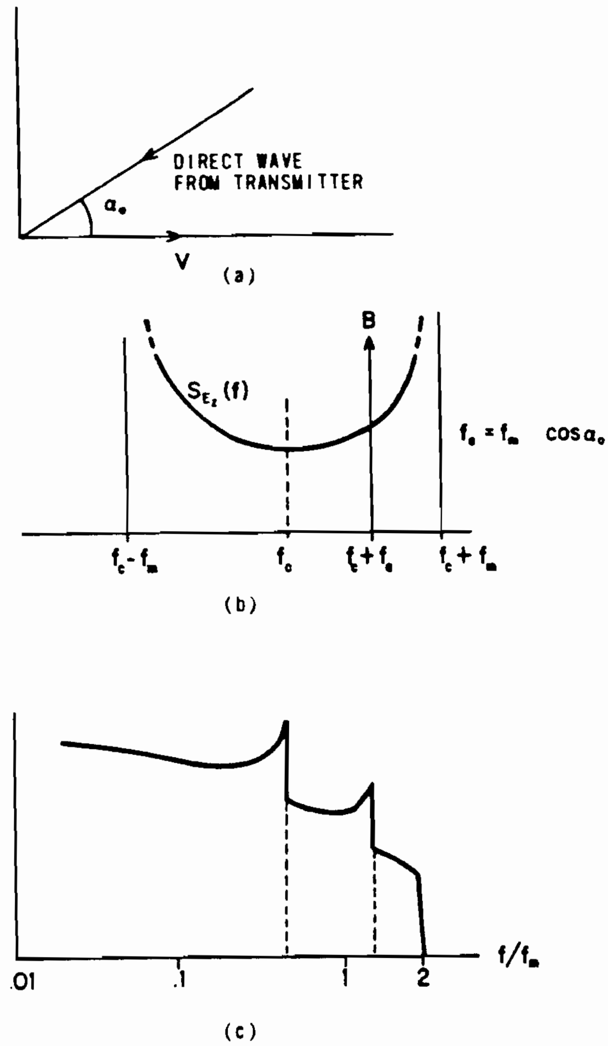


Figure 1.3-2 Effect of the addition of a direct wave from the transmitter on the envelope spectrum. (*a*) Geometry, (*b*), input spectrum, (*c*) spectrum of the envelope.

Ref. Jakes, Microwave Mobile Communications

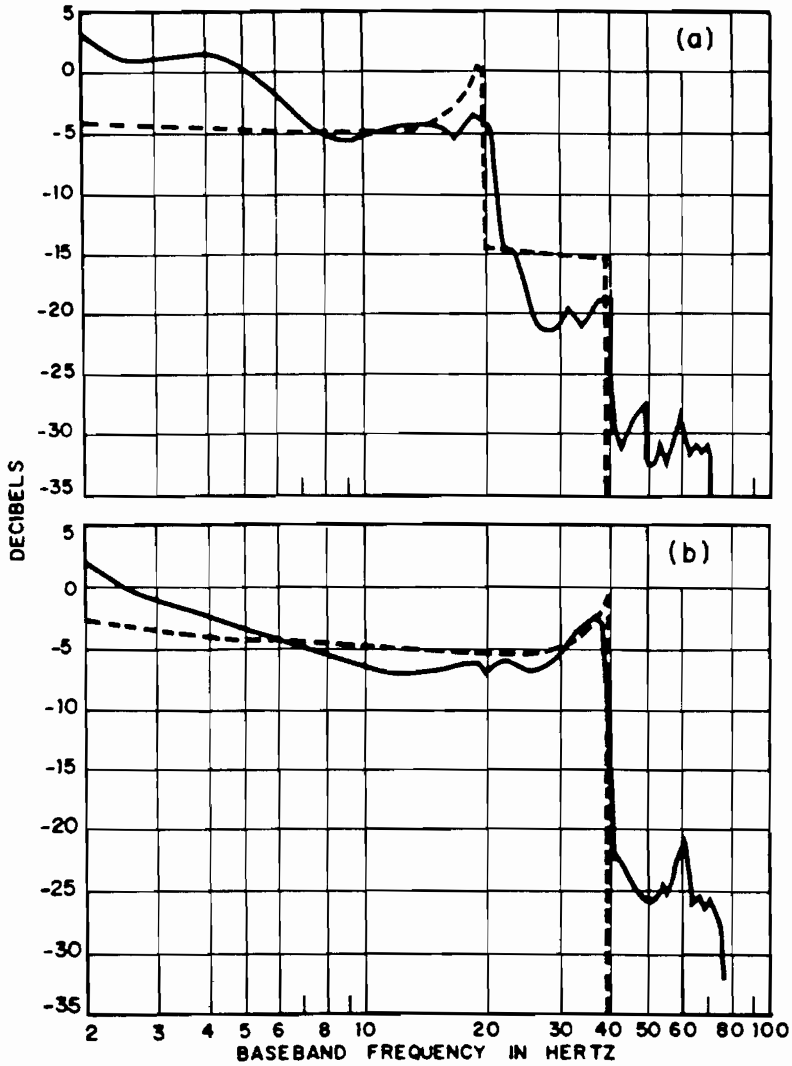
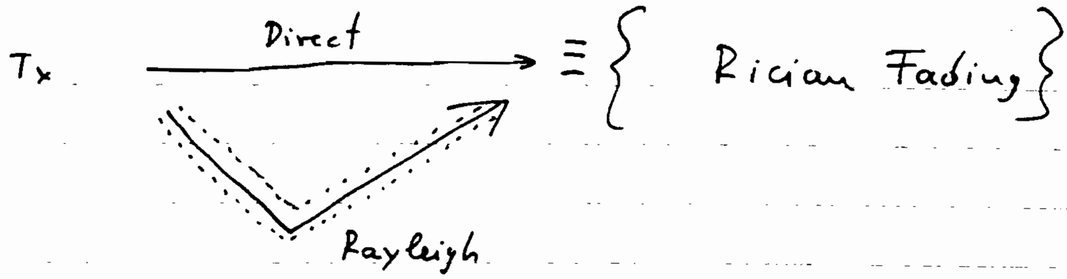


Figure 1.3-3 Comparison of theoretical (dashed) and experimental baseband spectra with direct wave from transmitter. (*a*) $\alpha_0 = 90°$, (*b*) $\alpha_0 = 0°$.

Ref. Jakes, Microwave Mobile Communications

RICIAN FADING



① Its envelope, v (>0), has the following pdf.

$$p(v) = 2v\sqrt{\frac{1+K}{S}}\,\exp\left[-K-(1+K)v^2\right] I_0\left[2v\sqrt{K(1+K)}\right]$$

K = power ratio between direct & reflected signals
[$K \to \infty \equiv$ No fading ; $K = 0 \equiv$ Rayleigh]

$I_0[\cdot]$ = modified Bessel function of zero order

S = power of the reflected signal.

② Phase statistics, θ, of the Rician fading

$$p(\theta) = \frac{e^{-K}}{2\pi} + \frac{\sqrt{K}\cos\theta\,\exp(-K\sin^2\theta)}{2\sqrt{\pi}}\left[2 - \mathrm{erfc}(\sqrt{K}\cos\theta)\right]$$

Applications

- Mobile - satellite
- Cellular in rural areas
- Must be a line-of-sight path.

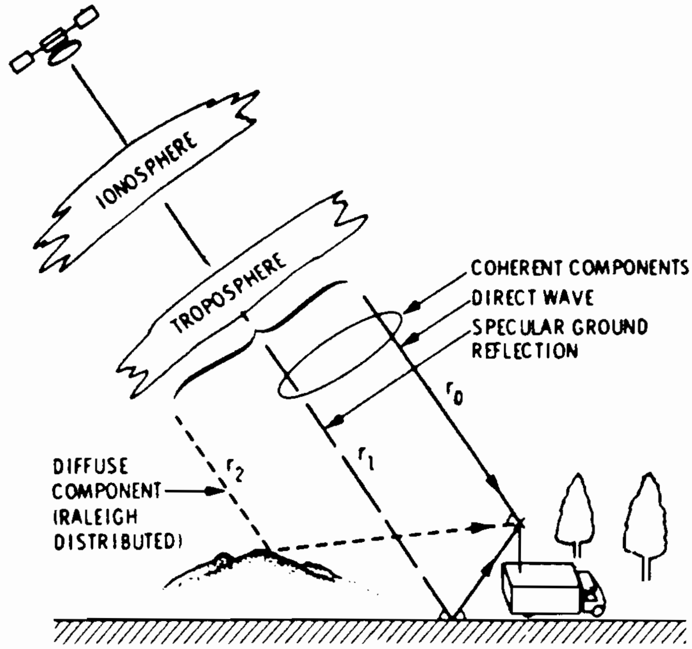


Fig. 1. Propagation model for mobile-satellite channels.

r_0 : direct path

r_1 : ground reflected path

r_2 : diffused path

Measured results indicate that for the MSAT channel

K is ~10 dB (elevation angle 20°) or ~20 dB (40°)

<u>Ref</u>. Davarian, "Channel simulation ... "

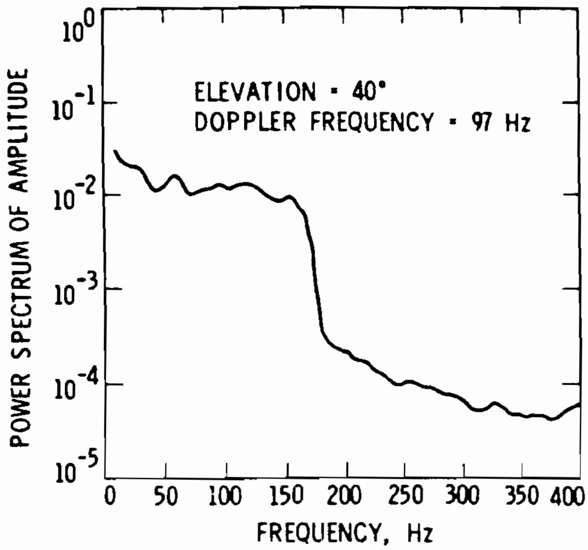


Fig. 6. The power spectrum of the received carrier (unmodulated) amplitude is determined by a field experiment. The elevation angle, incident Doppler, and balloon position were 40°, 97 Hz, and ahead of the vehicle, respectively.

Ref. Davarian.

REALIZATION OF MULTIPATH FADING INTERFERENCE

- (A) Tape recordings
- (B) Hardware and/or software
- (C) Computer simulation

- (A) Tape recordings of actual fading signals would be used!! (baseband ; frequencies < 200 Hz).

- (B) Several designs
  - I) Analog
    (e.g., i) Jakes, Microwave Mobile Communications
    ii) Arredondo et. al., "A multipath fading simulator for mobile radio," IEEE Trans. Veh. Techn., vol. VT-2 pp. 241-244, Nov. 1973
    iii) Caples et al., "A UHF channel simulator for digital mobile radio," IEEE Trans. Veh. Techn., vol. VT-29, pp. 281-289, May 1980.)
  - II) Digital (baseband)
    (e.g., i) Casas and Leung, "A simple digital fading simulator for mobile radio," IEEE Trans. Veh. Tech vol. VT-39, pp. 205-212, Aug. 1990.)

- (C) Usually, not explicitly given

ANALOG DESIGNS

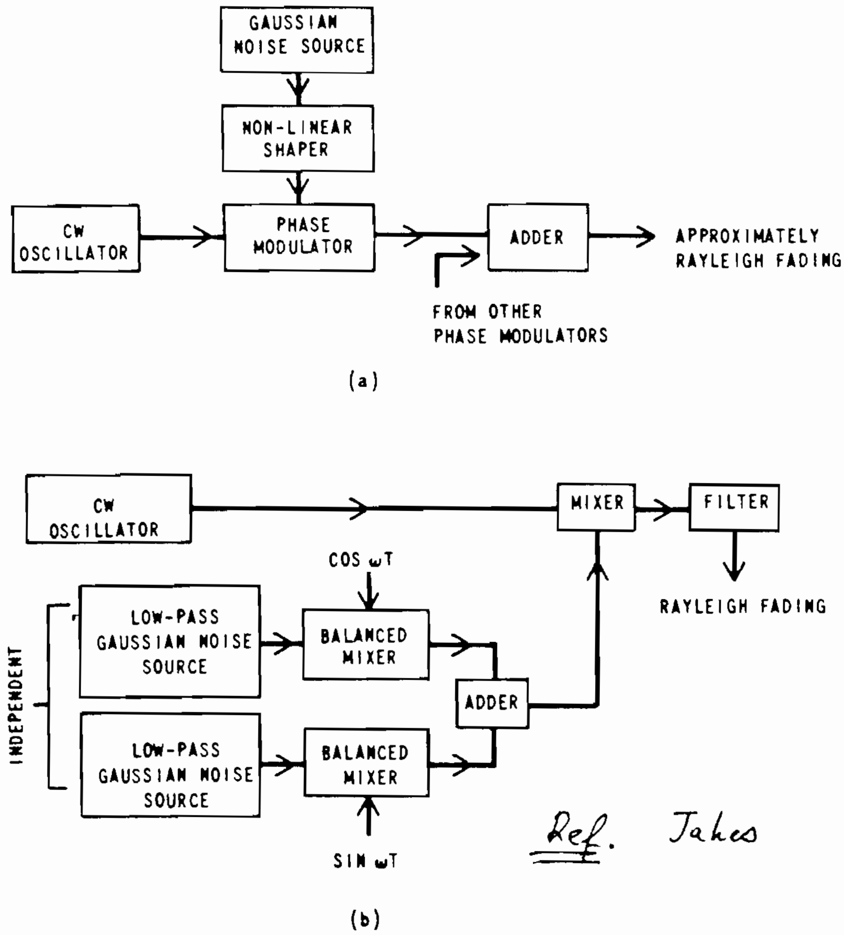


Figure 1.7-1 Two types of fading simulators. (*a*) Simulator using uniform phase modulation. (*b*) Simulator using quadrature amplitude modulation.

(a) Use of non-linear shaper to change the Gaussian process so that uniformly distributed phase modulation can be obtained

(b) • QAM modulation. → (change of the position of filter!!)
• Frequency selective fading → Add more <u>delayed</u> paths.

(c) Using frequency generators. (→ insight)

$$E(t) = \mathrm{Re}\left[T(t)\, e^{j\omega_c t} \right]$$

with $T(t) = E_0 \sum_{n=1}^{N} c_n\, e^{j(\omega_m t \cos\alpha + \phi_n)}$; $c_n^2 = p(\alpha_n)\, d\alpha_n = \frac{1}{2\pi}\, d\alpha$

ϕ_n = r.v. uniformly distributed over $(0, 2\pi]$

$\therefore\ d\alpha = \frac{2\pi}{N} \Rightarrow c_n^2 = \frac{1}{N} \Rightarrow \alpha_n = \frac{2\pi}{N} n \qquad n = 1, 2, 3, \ldots, N$

For symmetry $\frac{N}{2}$ odd integer, so that

$$T(t) = \frac{E_0}{\sqrt{N}} \Big\{ \underbrace{e^{j(\omega_m t + \phi_N)}}_{Ⓐ} + \underbrace{e^{-j(\omega_m t + \phi_{-N})}}_{Ⓑ} + \sum_{n=1}^{\frac{N}{2}-1} \Big[\underbrace{e^{j(\omega_m t \cos\alpha_n + \phi_n)}}_{Ⓒ} + \underbrace{e^{-j(\omega_m t \cos\alpha_n + \phi_{-n})}}_{Ⓓ} \Big] \Big\}$$

Ⓐ : Doppler shift : ω_m

Ⓑ : " " : $-\omega_m$

Ⓒ : " "s : $\omega_m \cos\left(\frac{2\pi}{N}\right) \xrightarrow[1 \cdots \frac{N}{2}-1]{n=} -\omega_m \cos\left(\frac{2\pi}{N}\right)$

Ⓓ : " "s : $-\omega_m \cos\left(\frac{2\pi}{N}\right) \longrightarrow \omega_m \cos\left(\frac{2\pi}{N}\right)$

Obviously Ⓒ, Ⓓ overlap ⇒ only nonoverlapping terms are really needed.

$$T(t) = \frac{E_0}{\sqrt{N}} \Big\{ e^{j(\omega_m t + \phi_N)} + e^{-j(\omega_m t + \phi_{-N})} +$$

$$+ \sqrt{2} \sum_{n=1}^{N_0} \Big[e^{j(\omega_m t \cos\alpha_n + \phi_n)} + e^{-j(\omega_m t \cos\alpha_n + \phi_{-n})} \Big]$$

$$N_0 = \frac{1}{2}\left(\frac{N}{2} - 1\right)$$

<u>Central Limit Theorem</u>

As $N \to \infty \Rightarrow T(t) \simeq$ c. G. r. p. $\Rightarrow |T(t)| \simeq$ Rayleigh (as desired)

It turns out that for $N \geq 6$ "almost" Rayleigh !!

How about the autocorrelation fuction? (i.e., spectrum of fadi…

$$R(\tau) = \langle E(t)\, E(t+\tau) \rangle_{\phi_n, \phi_m} = \frac{1}{2} \mathrm{Re}\Big[\langle T(t)\, T(t+\tau)\, e^{j\omega_c(2t+\tau)} \rangle + \langle T^*(t)\, T(t+\tau)\, e^{j\omega_c \tau} \rangle \Big]$$

$$= \frac{b_0}{N} \cos(\omega_c \tau) \Big[4 \sum_{n=1}^{N_0} \cos\Big(\omega_m \tau \cos\Big(\frac{2\pi n}{N}\Big)\Big) + 2\cos(\omega_m \tau) \Big]$$

(Only terms involving $\phi_n - \phi_m$; $m = n$ ‖ $b_0 = \langle T^2 \rangle$)

$$R(\tau) = g(\tau) \cos(\omega_c \tau)$$

$\cos(\omega_c \tau)$ → carrier factor

$g(\tau)$ → low frequency factor

We have seen that for a uniformly scattered field

$$g(\tau) = b_0 \, J_0(\omega_m \tau) \quad ; \quad J_0(x) = \frac{2}{\pi} \int_0^{\pi/2} \cos(x \cos\alpha) \, d\alpha$$

For $N \to \infty \Rightarrow$ the sum will become an integral
∴ We expect that (using the discrete approximation of the Bessel function – Riemann sum):

$$\underbrace{2 \sum_{n=1}^{N_0} \cos\left(\omega_m \tau \cos \frac{2\pi n}{N}\right) + \cos(\omega_m \tau)}_{\circledast} \stackrel{!}{=} \underbrace{\frac{N}{2} J_0(\omega_m \tau)}_{\circledast\circledast}$$

Parameters: $\omega_m \tau$ & N

Nummerical evaluation shows that for $N = 34$

$$\circledast = \circledast\circledast$$ (up to eight significant digits for $\omega_m \tau \leq 15$

$$\therefore \quad N_0 = \frac{1}{2}\left(\frac{34}{2} - 1\right) = 8$$

so that 8 frequency components are enough to produce an excellent approximation of the spectrum of the land-mobile channel, i.e.,

$$\left[1 - \left(\frac{f - f_c}{f_m}\right)\right]^{-1/2}$$

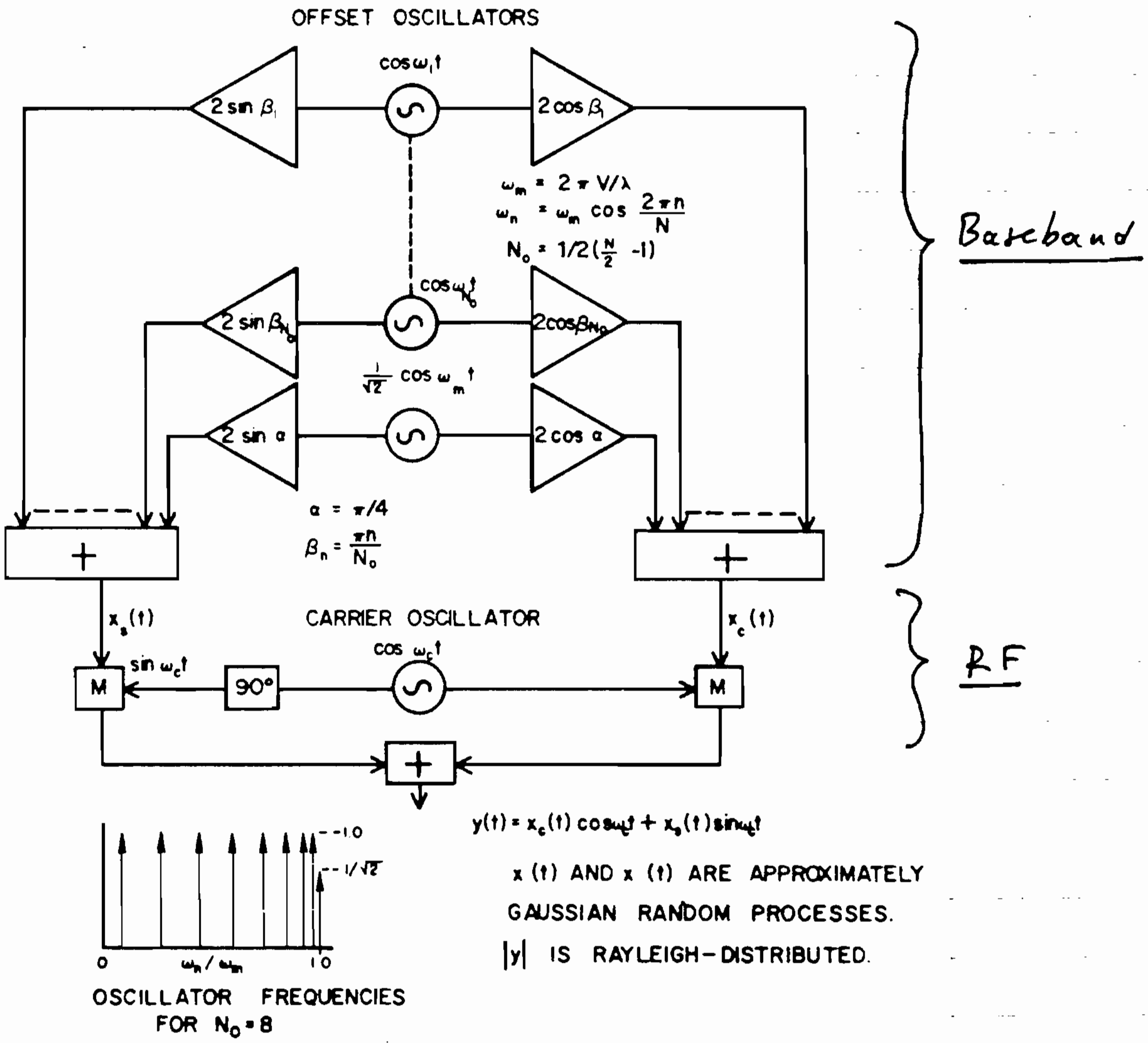


Figure 1.7-2 Simulator that duplicates mobile radio spectrum.

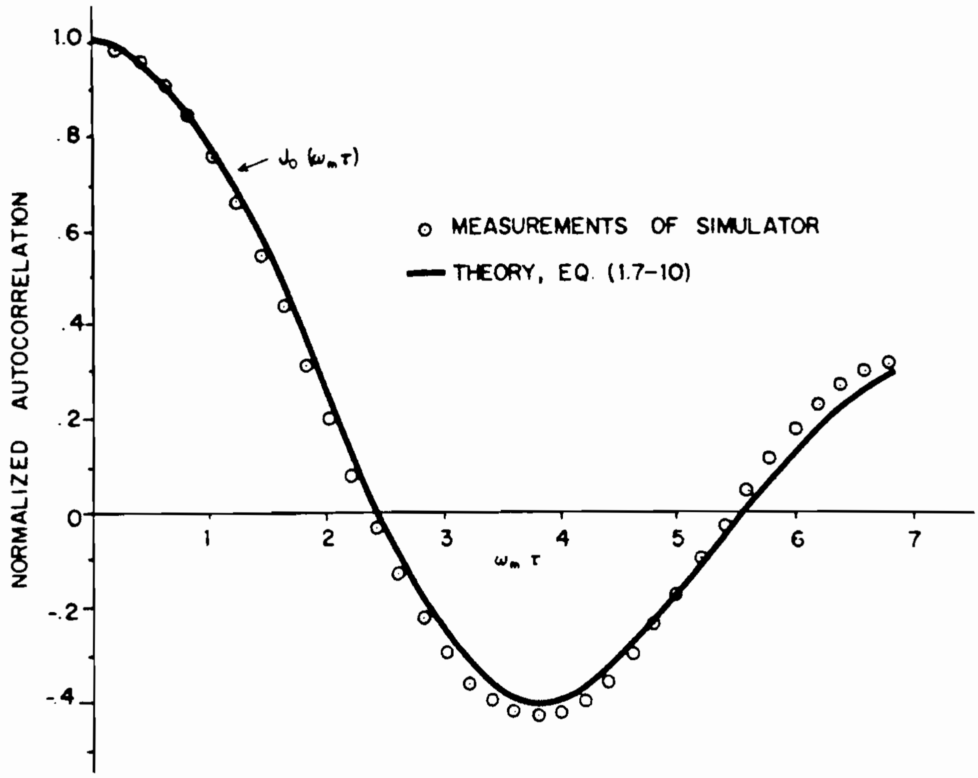


Figure 1.7-4 Comparison of theoretical autocorrelation function of the fading signal with data from a laboratory simulator.

Ref. Jakes

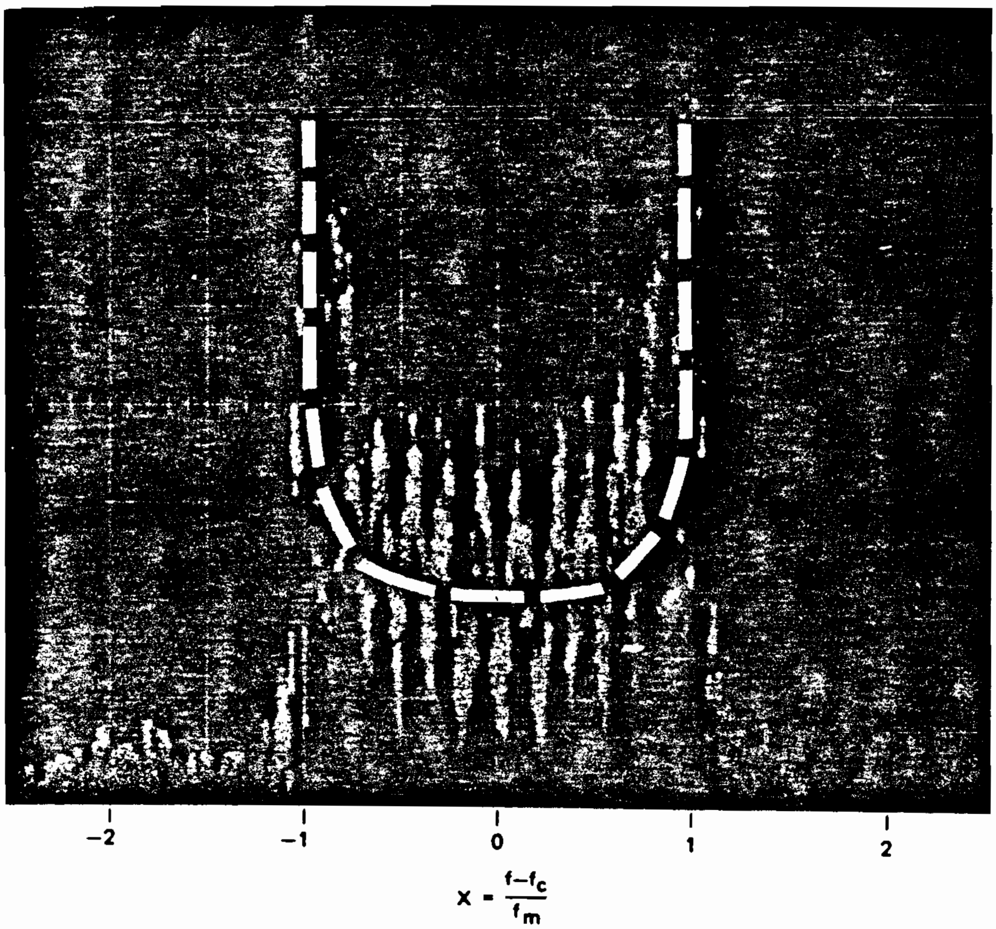


Figure 1.7-5 RF Spectrum of simulated fading carrier. Dashed line is the theoretical spectrum, $(1-X^2)^{-1/4}$.

Ref. Jakes

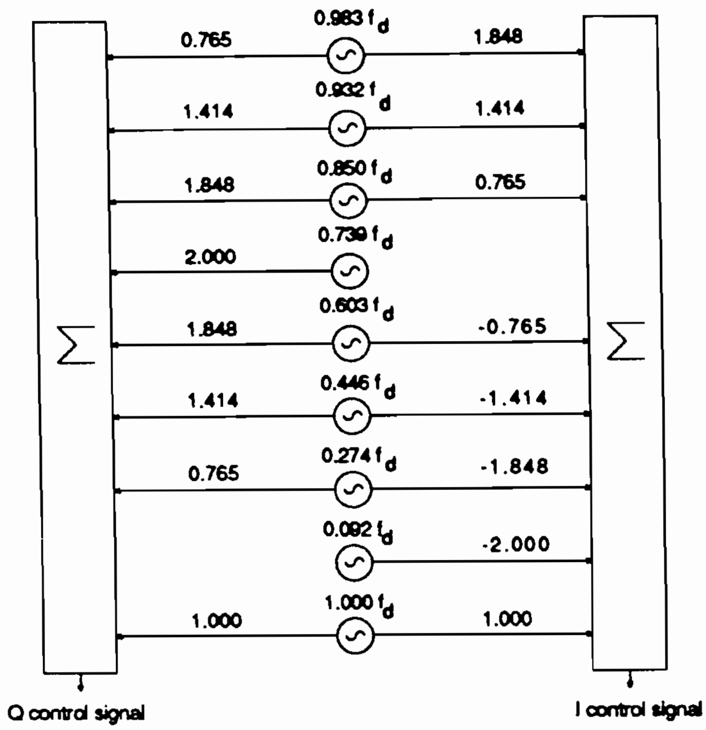


Fig. 4. Generation of the pseudo-random control signals showing the frequencies and magnitudes of the sinusoids. The frequencies of the sinusoids are shown as a fraction of the Doppler rate, f_d. The quantities along the connecting lines are the amplitudes.

Ref. Casas & Leung.

Generators (& math. operat.) are implemented digitally

$N_0 = 8 \Rightarrow 9$ frequencies all together

$f_9 = f_d \; (= f_m)$; $f_1 = f_m \cos\left(\frac{2\pi}{34}\right) = 0.983 \, f_m$

$f_2 = f_m \cos\left(2 \times \frac{2\pi}{34}\right) = 0.932 \, f_m$

$\vdots$

$f_8 = f_m \cos\left(8 \times \frac{2\pi}{34}\right) = 0.0922 \, f_m$

Also : $\beta_n = \frac{\pi}{N_0} n$; $\beta_1 = \frac{\pi}{8} \rightarrow 2\cos\left(\frac{\pi}{8}\right) = 1.848$

$\vdots$

$\beta_8 = \pi \rightarrow 2\cos(\pi) = -2$

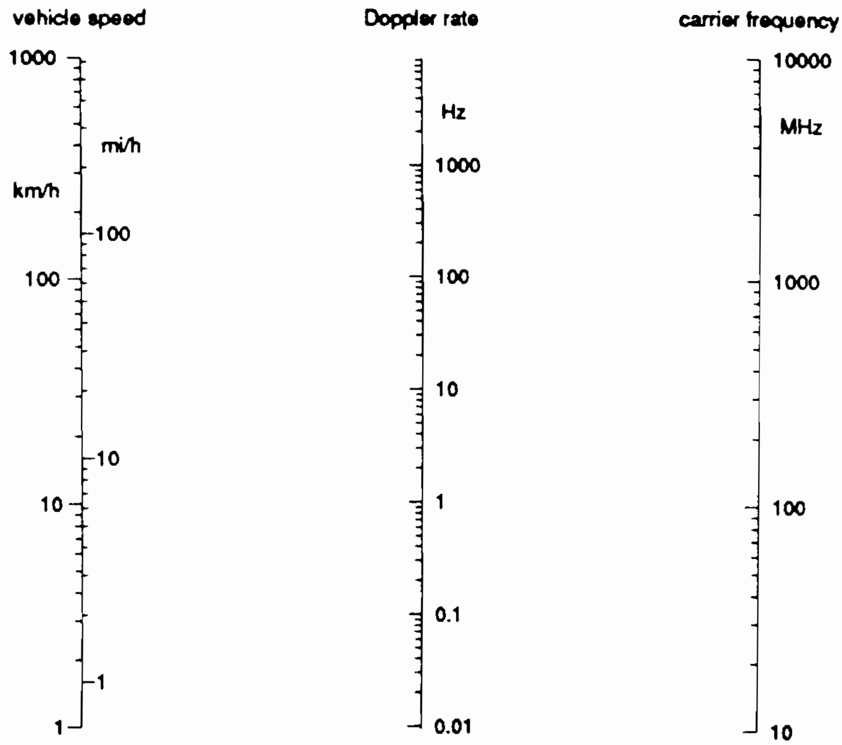
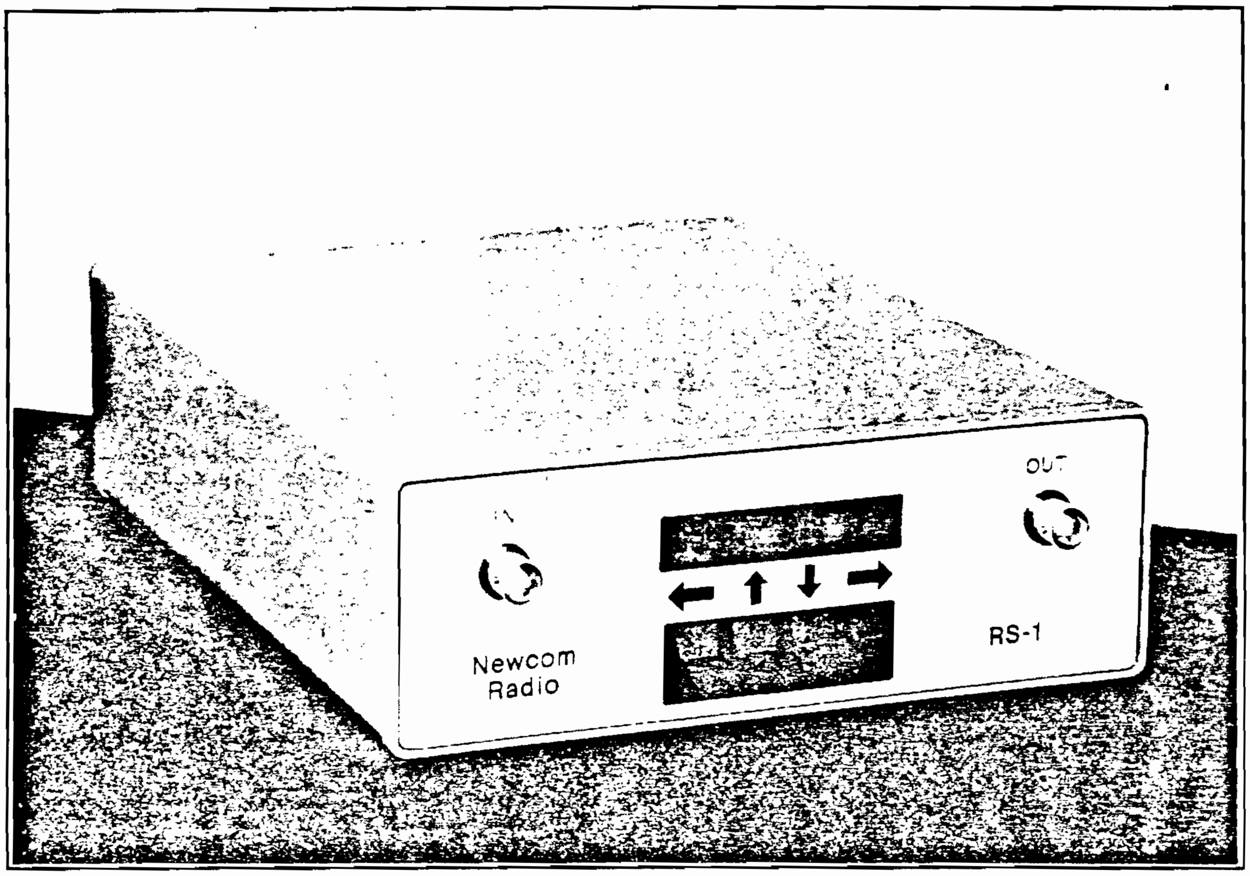


Fig. 3. Nomograph showing the relationship between maximum Doppler rate (f_d), vehicle speed (v), and carrier frequency (f_c). Draw a straight line intersecting the appropriate axes at the two known quantities. The unknown is read at the intersection of the line with the axis of the unknown variable.

Ref. Casas & Leung

Newcom Radio
RS-1 Fading Simulator



Simple noise tests are not adequate to determine the performance of mobile radio equipment due to multipath fading. To properly evaluate mobile equipment performance, the radio engineer has two choices: perform expensive field tests, or use a simulated radio channel. The latter has the advantage of being controllable, repeatable and less expensive.

RS-1 Features:

Low cost. Priced at $2500, the RS-1 delivers a simulated mobile radio channel at a fraction of the cost of other simulators.

Simple to use, with keypad entry and an LCD display.

Suitable for any narrow-band modulation type.

Accurate and repeatable simulations, giving you high confidence in your equipment evaluations.

Operates with standard 70 MHz IF noise test equipment to simplify your test setups.

Simulates Doppler rates up to 127 Hz (equivalent to a car travelling over 90 mph using a cellular radio).

Site 2, Comp. 22, R.R.1, Anmore, B.C. V3H 3L8 **Telephone (604) 469-1474**

RS-1 SPECIFICATIONS

ELECTRICAL CHARACTERISTICS:

Fading type	Rayleigh, flat (non-frequency selective)
Doppler fade rate	0 to 127 Hz, in 1 Hz steps
Input level	–20dBm or lower
Fading dynamic range	+10 dB to –40 dB
Nominal gain	–10 dB +–3 dB
Input and output port impedance	50 Ohms
Return loss	>15 dB
Standard frequency range	60-80 MHz
Data input	Front panel keypad. Simple menu interface.
Display	16 character LCD
Line power	115 Volts +–10% 60 Hz

PHYSICAL CHARACTERISTICS:

Input/Output connectors	BNC
Operating temperature range	10 C–30 C (50F–85F)
Size	24cm x 18.5cm x 6.5cm (9.5" x 7.25" x 2.5")
Weight	2 kg (4 lb. 6 oz.)

WARRANTY:

........One year parts and labour

PRICE:

........$2,500

Specifications subject to change without notice.

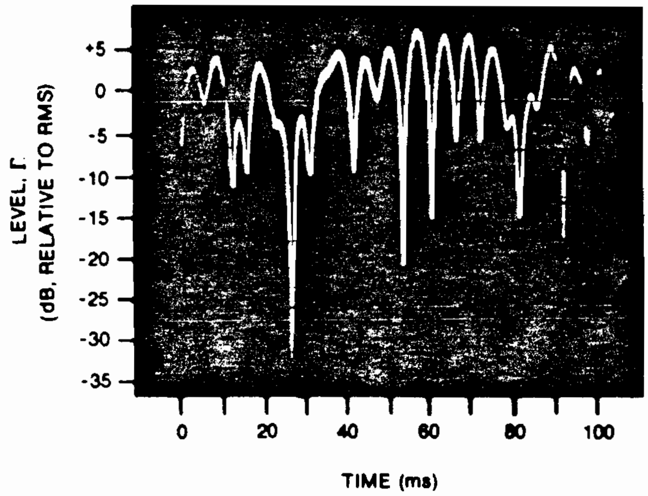


Fig. 11. Signal envelope from Rayleigh fading generator ($\hat{f}_D$ = 160 Hz).

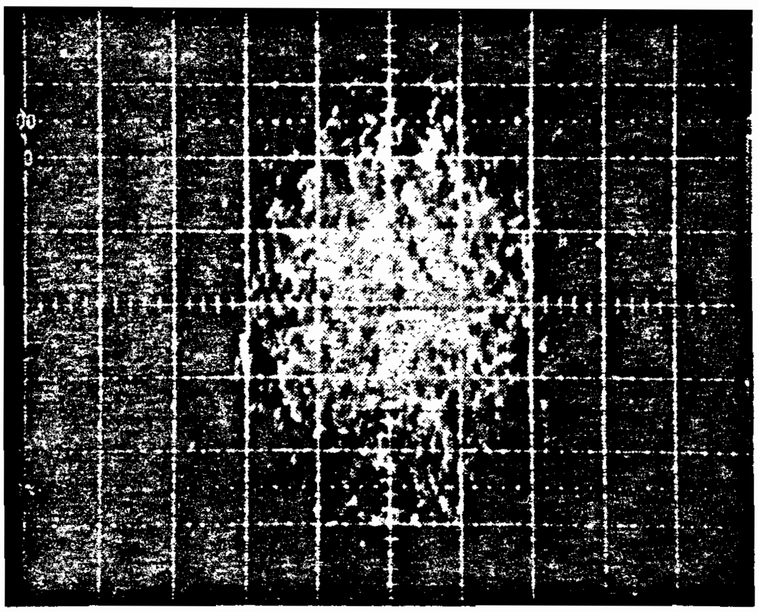


Fig. 12. Polar plot of in-phase and quadrature noise sources after shaping ($\hat{f}_D$ = 160 Hz).

Ref. E. L. Caples et al., "A UHF Channel Simulator for Digital Mobile Radio," IEEE Trans. Veh. Techn., vol. VT-29, pp. 281-289, May 1980.

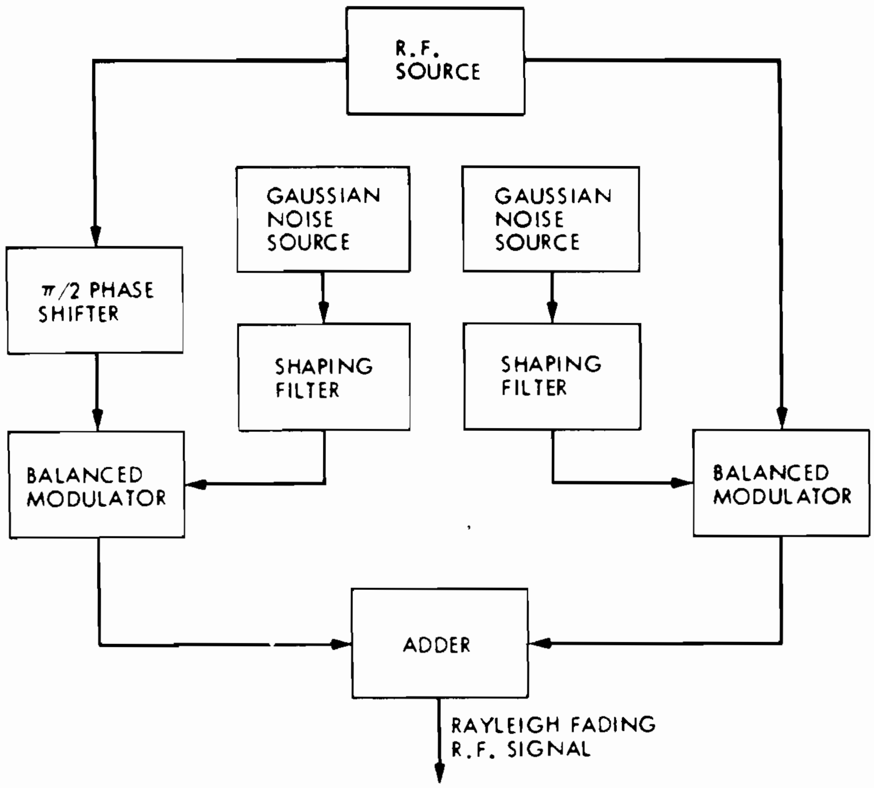


Fig. 7. Rayleigh fading simulator block diagram.

Ref. Davarian

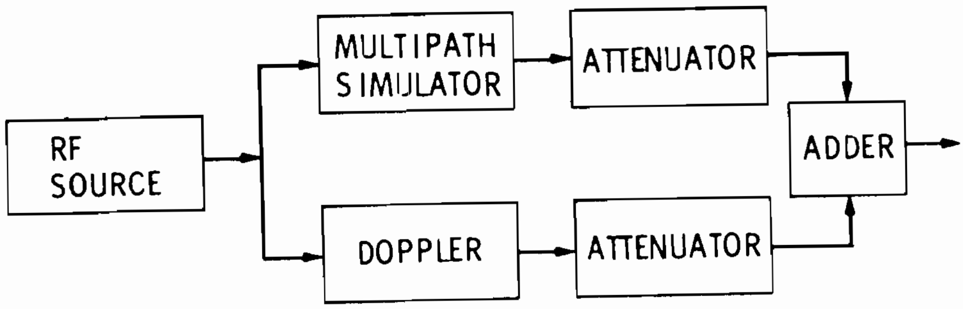


Fig. 10. Rician fading simulator block diagram.

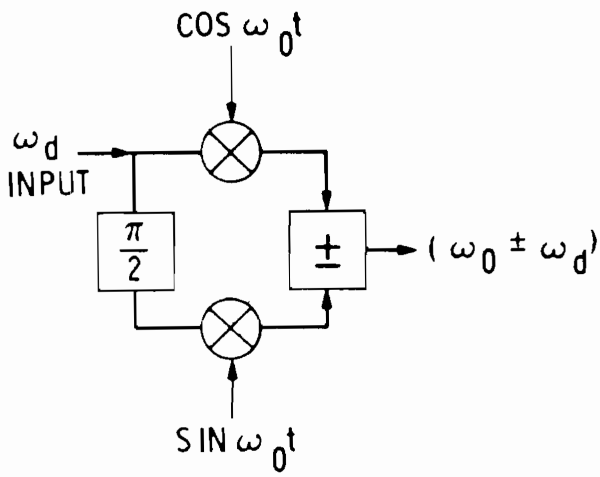


Fig. 11. The circuit block diagram for frequency shifting of the direct path. ω_0 is the angular frequency of the carrier and ω_d denotes the desired shift.

Ref. Davarian

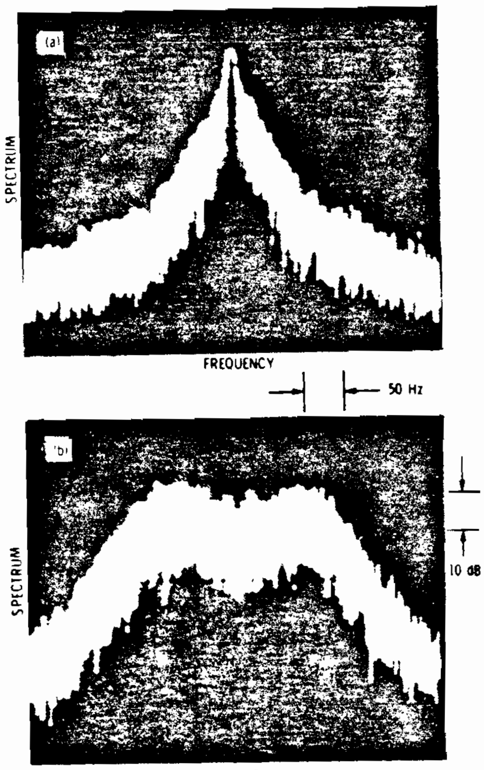


Fig. 9. Measured power spectrum of an unmodulated carrier. (a) No fading. (b) Rayleigh fading with f_D = 104 Hz.

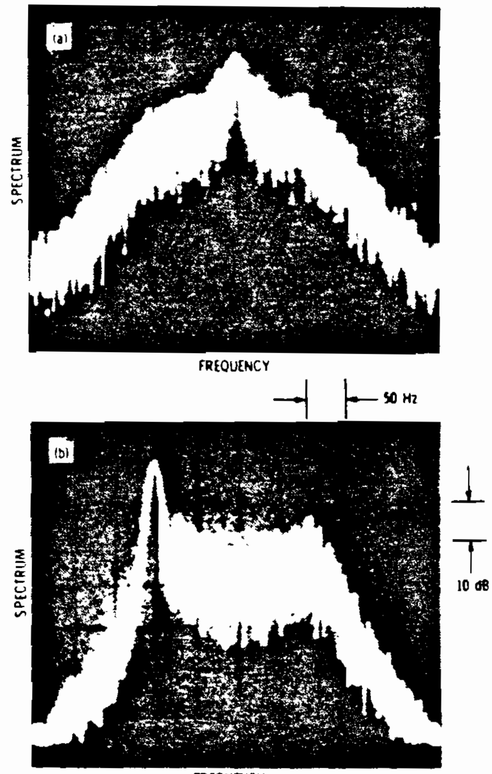


Fig. 12. Measured power spectrum of an unmodulated carrier in the presence of Rician fading when f_D = 104 Hz. (a) Direct path is not frequency shifted and K = 6 dB. (b) Direct path is shifted by f_D and K = 10 dB.

Ref. Davarian

Ref. Dowanim.

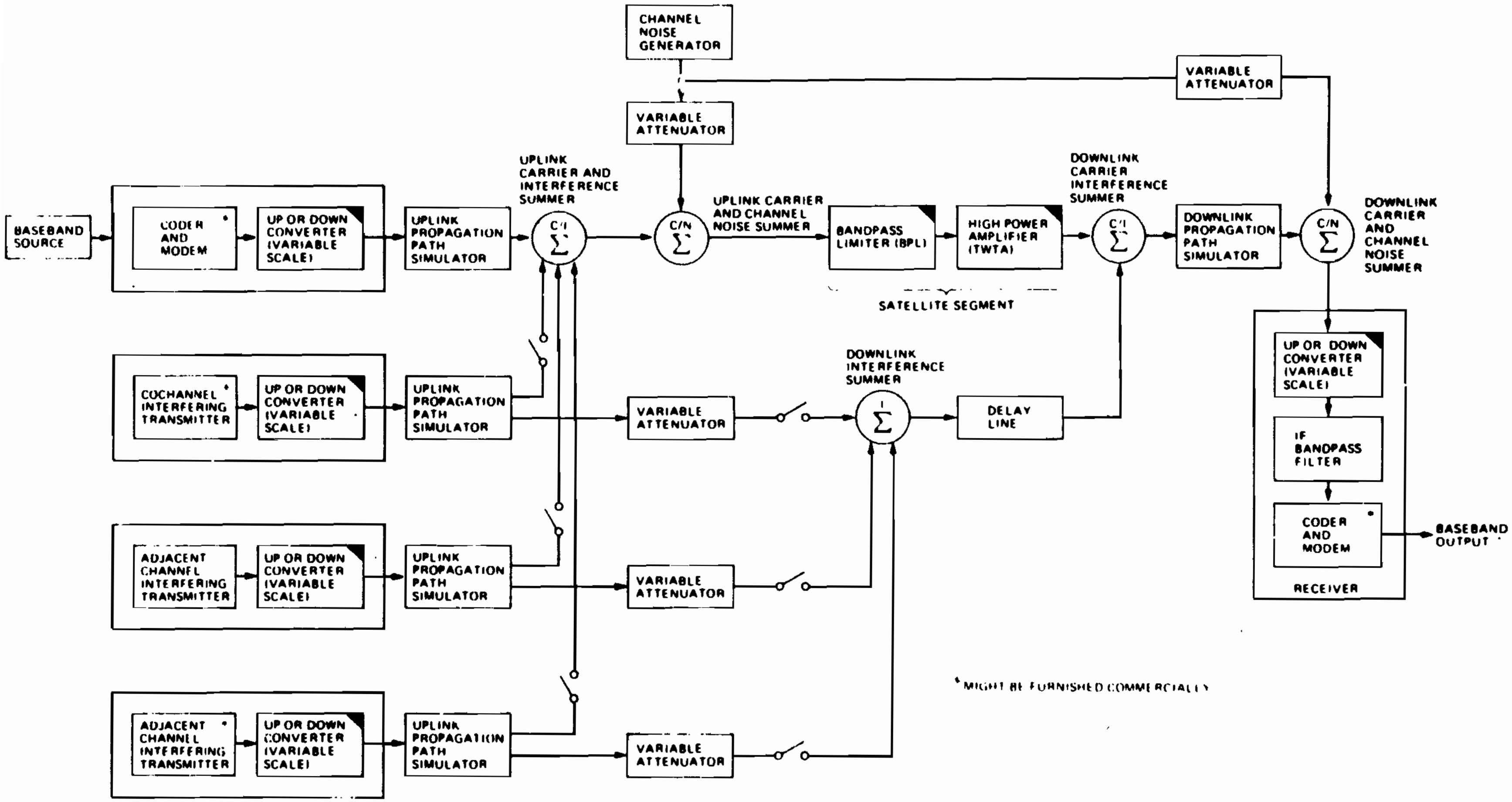
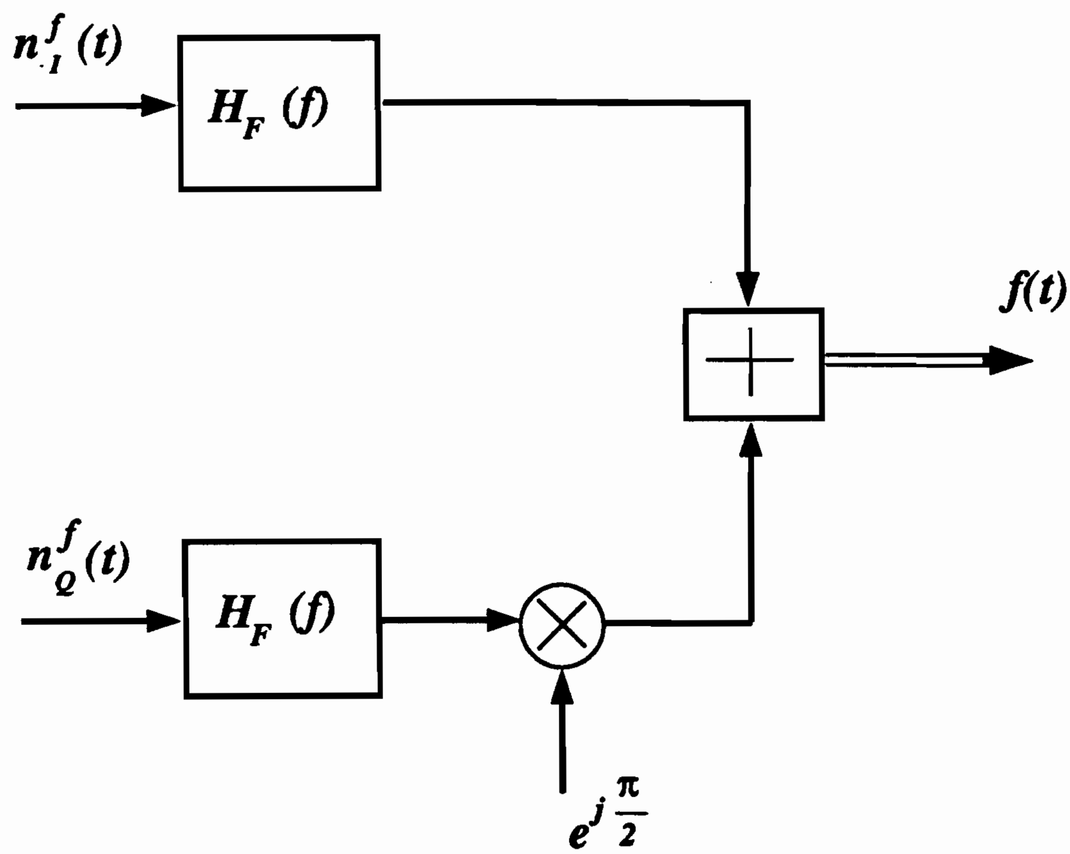


Fig. 13. Channel simulator block diagram.

(C) COMPUTER SIMULATION



Ref. D. Makrakis and P. T. Mathiopoulos, "Prediction/Cancellation Techniques for Fading Broadcasting Channels – Part I: PSK Signals," IEEE Trans. Broadcasting, vol. 36, pp. 146–155, June 1990.

CONSEQUENCES THAT FADING HAS ON THE BIT ERROR RATE (BER) PERFORMANCE OF DIGITAL WIRELESS COMMUNICATION SYSTEMS

① Degradation of the average BER
[caused by amplitude and phase fluctuations of the fading process]

amplitude fluctuation ≡ envelope fluctuation (e.g., Rayleigh)
phase " ≡ random FM, Doppler shift, phase jitter

② Irreducible error rates or error floors (i.e., degradated BER which cannot be improved by increasing the $\frac{C}{N}$)

[caused exclusively by the fading caused phase fluctuation]

Note the error floors exist even for small values of Doppler shift (i.e., slow fading) e.g. $f_d T_s = 0.01$

Comment : Fading is a "good" and "useful" phenomenon !!

Some Comments

- There is not a clear cut borderline between slow and fast fading.

Slow fading

(A) - Phase jitter due to fading might be assumed to be compensated (pilot-aided carrier recovery or differential detection)

- Under this assumption, the BER degradation is exclusively due to the effect of fading on the amplitude (envelope) of the received signal.

- ∴ In this case, we need to take into account only the first order statistics of the fading process (i.e., the fact that its envelope is Rayleigh).
  Second order statistics (i.e., autocorrelation ≡ spectrum) are not needed.

NOT A CORRECT APROACH !!

(B) slow fading ⟹ correlation ⟹ burst of error !!

- Analysis is more difficult

- Interleaving of bits (symbols) will uncorrelate the fading. However, the receiver required is more complex.
  (In fact interleaving is one way of improving the performance in a fading environment)

Fast fading

- In this case, both first and second order statistics have to be taken into account

- Depending upon how fast the fading is, correlation will influence the system BER performance differently.

SUMMARY

PRACTICAL EFFECTS OF FADING

1) *Gaussian Characteristics*

Central Limit Theorem: $x = x_1 + x_2 + \ldots + x_N$ (independent).

As $N \to \infty$ then x becomes Gaussian.

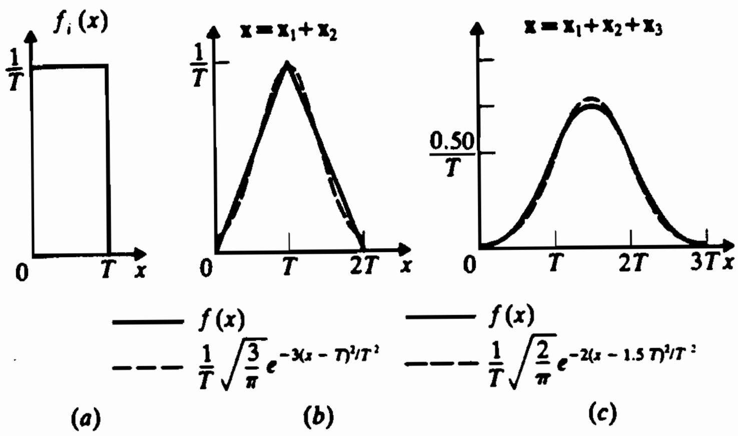


Figure 8-5

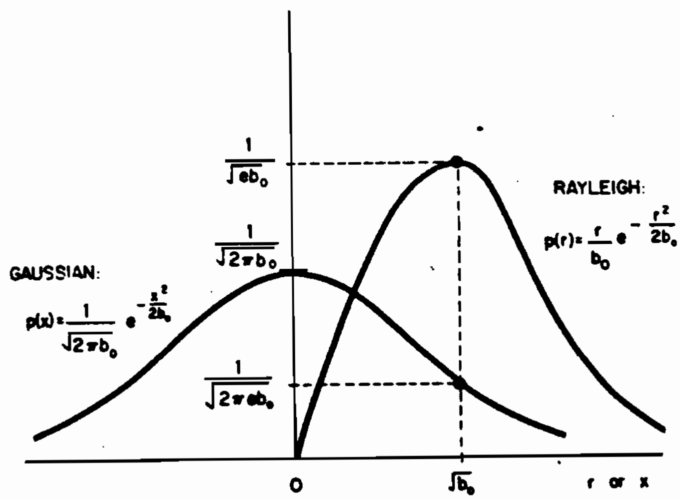


Figure 1.1-3 Gaussian and Rayleigh probability density functions.

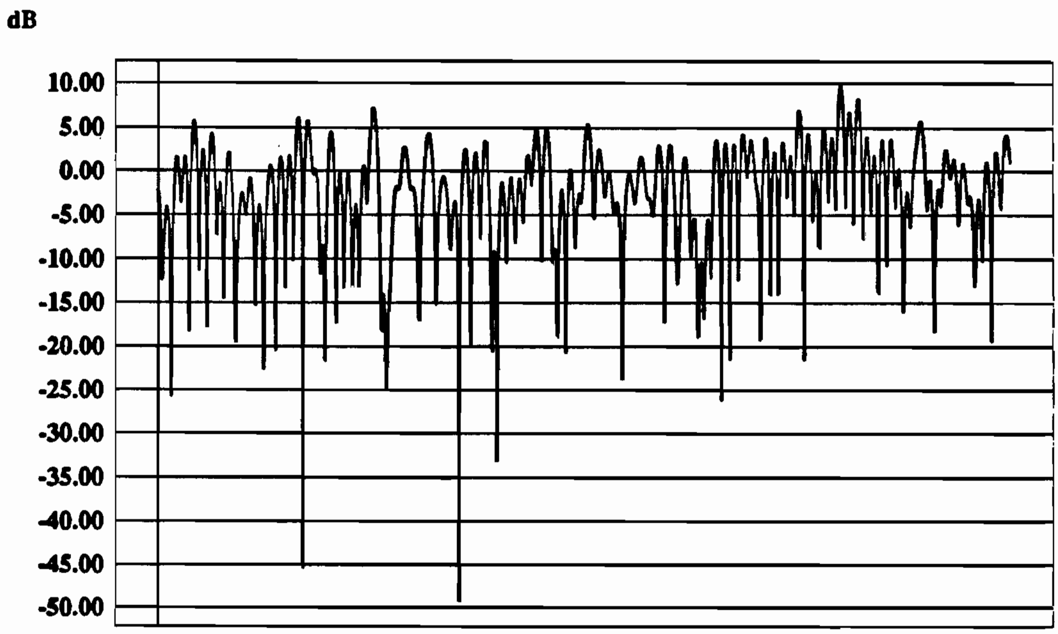
$$f(t) = \cos(2\pi f_c t + 2\pi f'_d t + \phi)$$
$$= \cos(2\pi f'_d t + \phi)\cos(2\pi f_c t) - \sin(2\pi f'_d t + \phi)\sin(2\pi f'_d + \phi)$$

$$T_c(t) = \cos(2\pi f'_d t + \phi); T_s(t) = \sin(2\pi f'_d t + \phi)$$

The envelope $r = \sqrt{T_c^2 + T_s^2} \rightarrow$ ***Rayleigh.***

The phase $\theta = \arctan(T_s/T_c) \rightarrow$ ***uniform.***

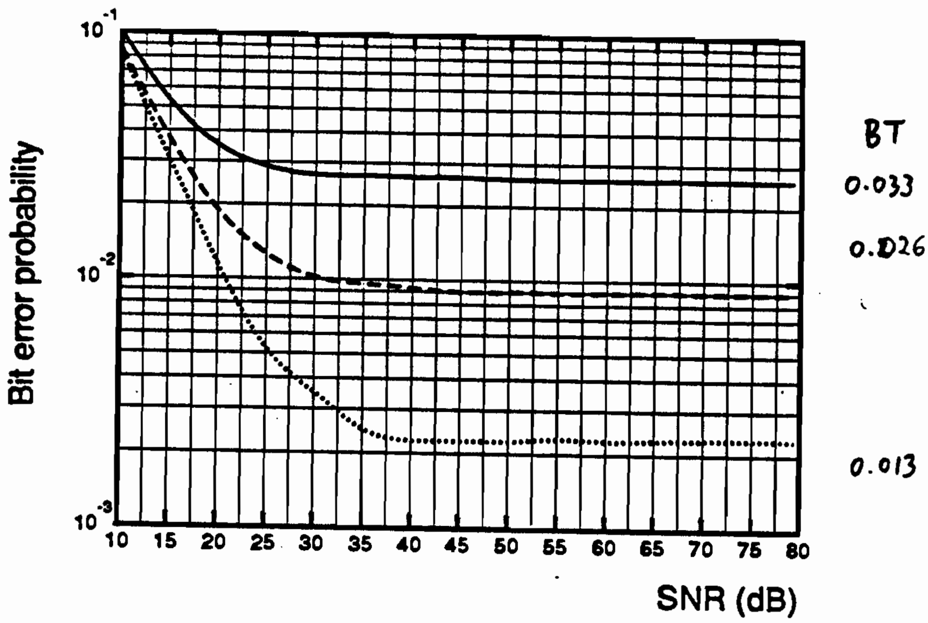
2) **Envelope Fluctuation**



3) **Random Phase Shift** (Random FM)

$g \in [0 \to 2\pi)$

<u>Error Floors</u>



4) **Correlation**

5) **Delay Spread**

If total delay spread $\tau \geq 0.1\,T \to$ Frequency selective fading.

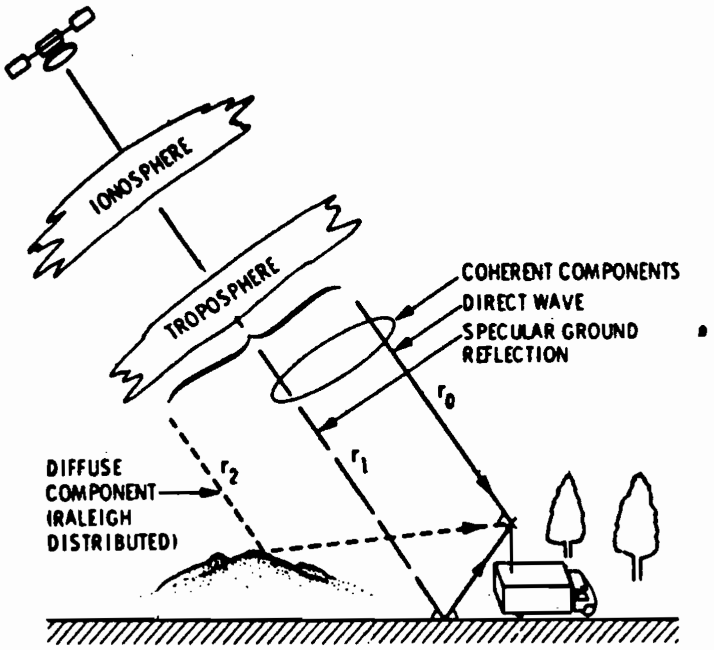


Fig. 1. Propagation model for mobile-satellite channels.

RICIAN FADING

Envelope

$$p(v)=2v\sqrt{\frac{(1+K)}{S}}\ \exp\,[-K-(1+K)v^2]\,I_0[2v\sqrt{K(1+K)}]$$

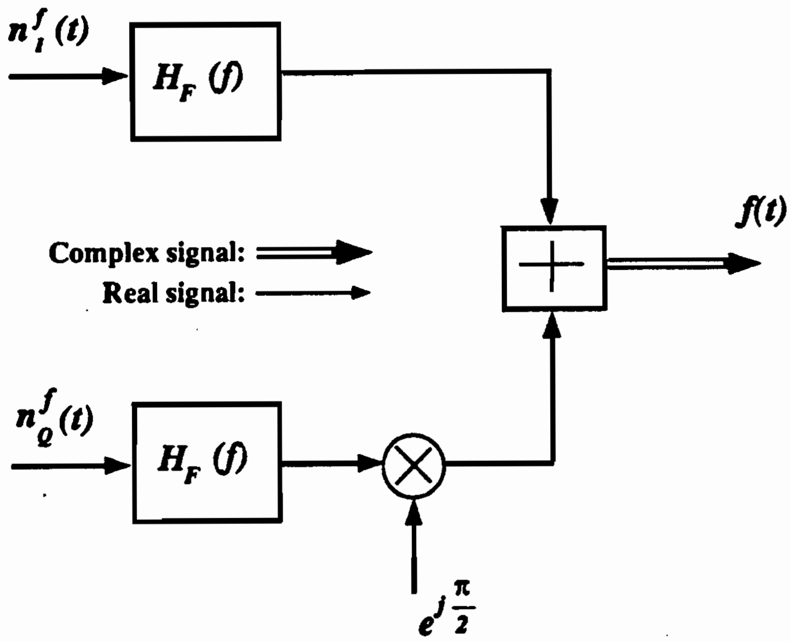
Phase

$$p(\theta)=\frac{e^{-K}}{2\pi}+\frac{\sqrt{K}\cos\theta\,\exp\,(-K\sin^2\theta)}{2\sqrt{\pi}}\cdot[2-\mathrm{erfc}\,(\sqrt{K}\cos\theta)]$$

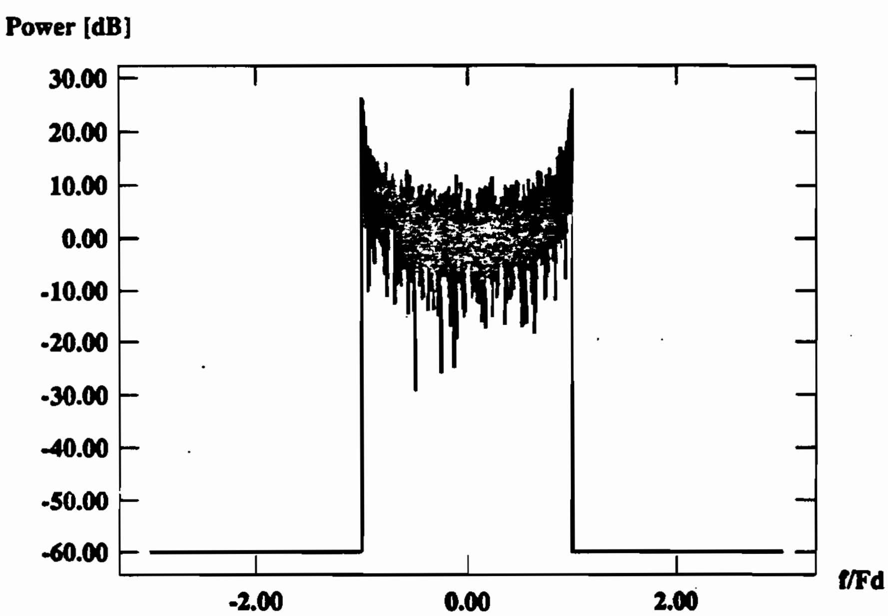
$$K \triangleq \frac{\text{Power of Direct Path}}{\text{Power of the diffused (faded) path}}$$

$K \to \infty$ (No fading) ; $K \to -\infty$ (Rayleigh fading)

Fading Simulator Implementation



Faded carrier baseband equivalent spectrum



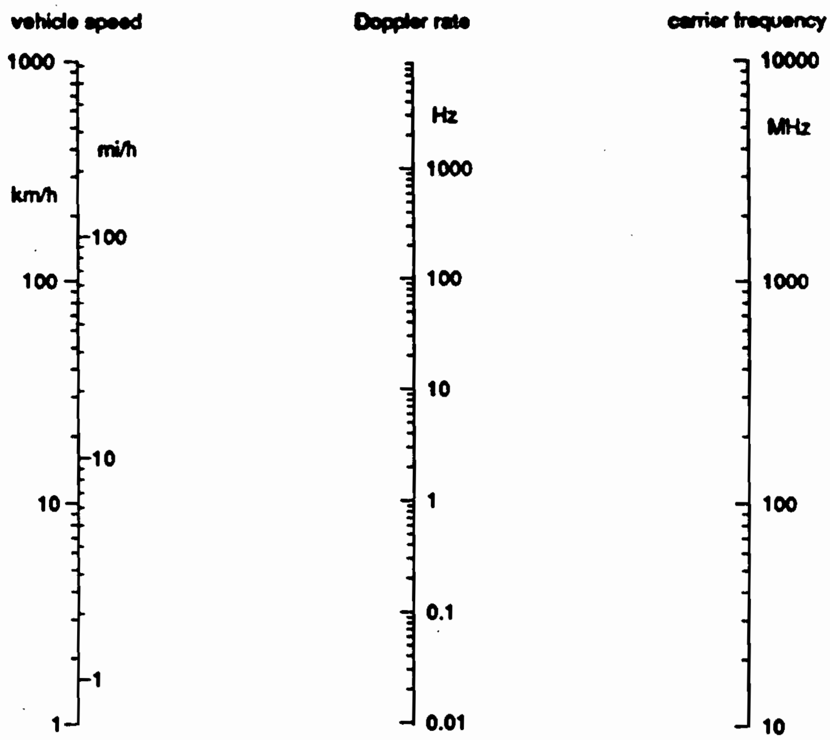
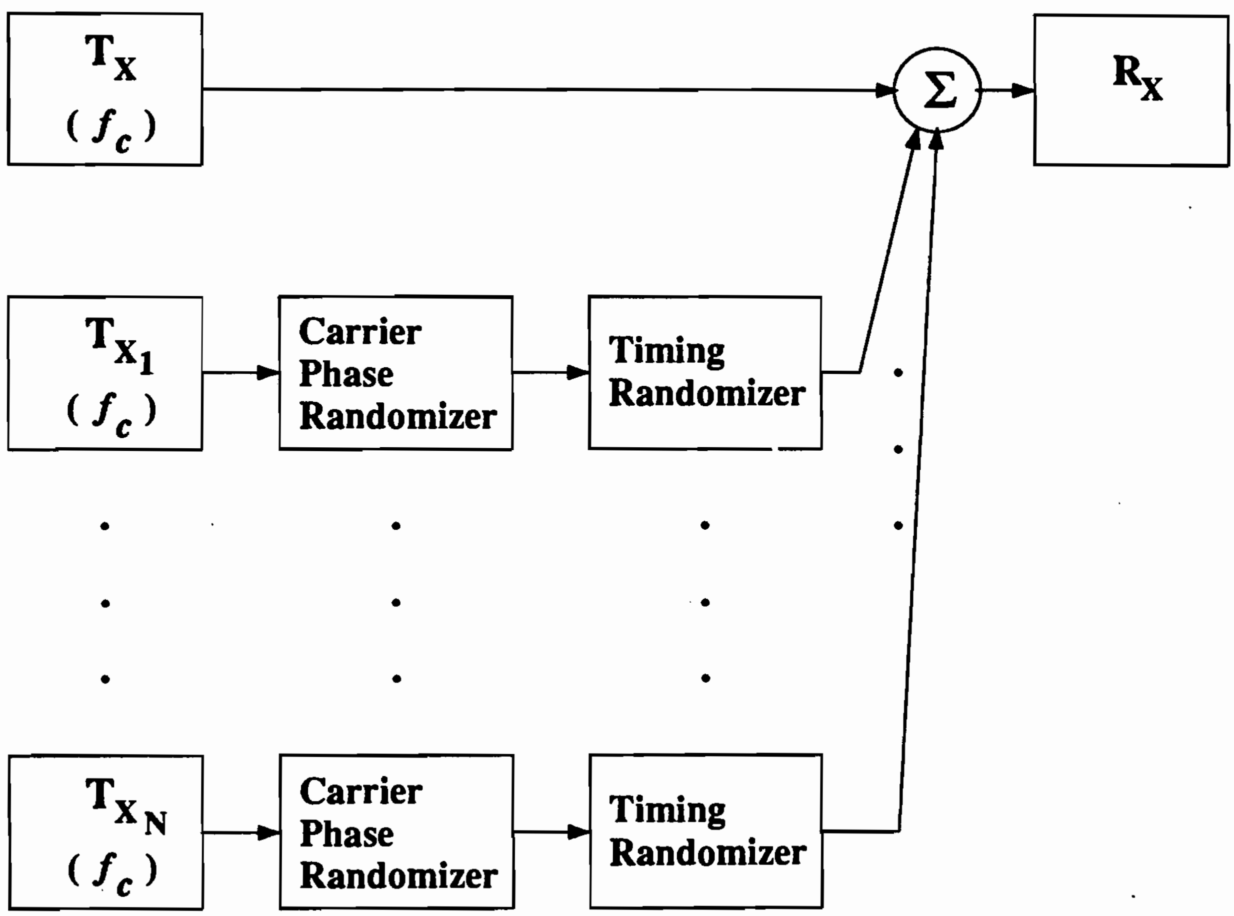
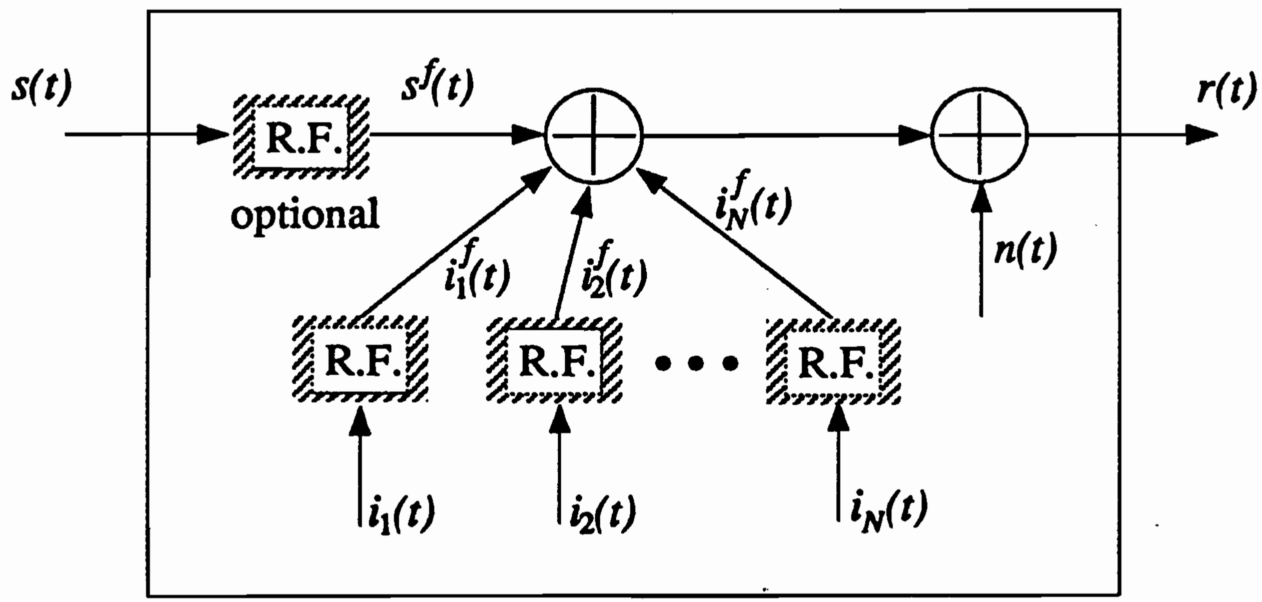


Fig. 3. Nomograph showing the relationship between maximum Doppler rate (f_d), vehicle speed (v), and carrier frequency (f_c). Draw a straight line intersecting the appropriate axes at the two known quantities. The unknown is read at the intersection of the line with the axis of the unknown variable.

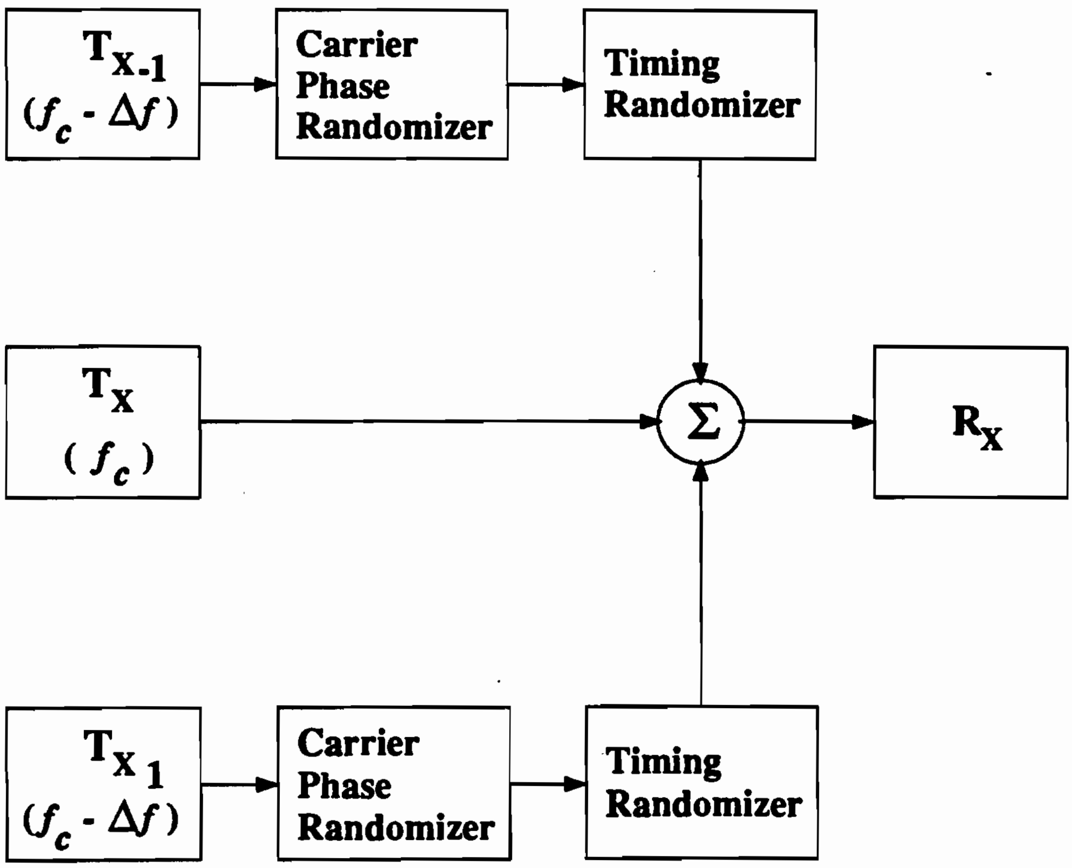
Co-Channel Interference (CCI)

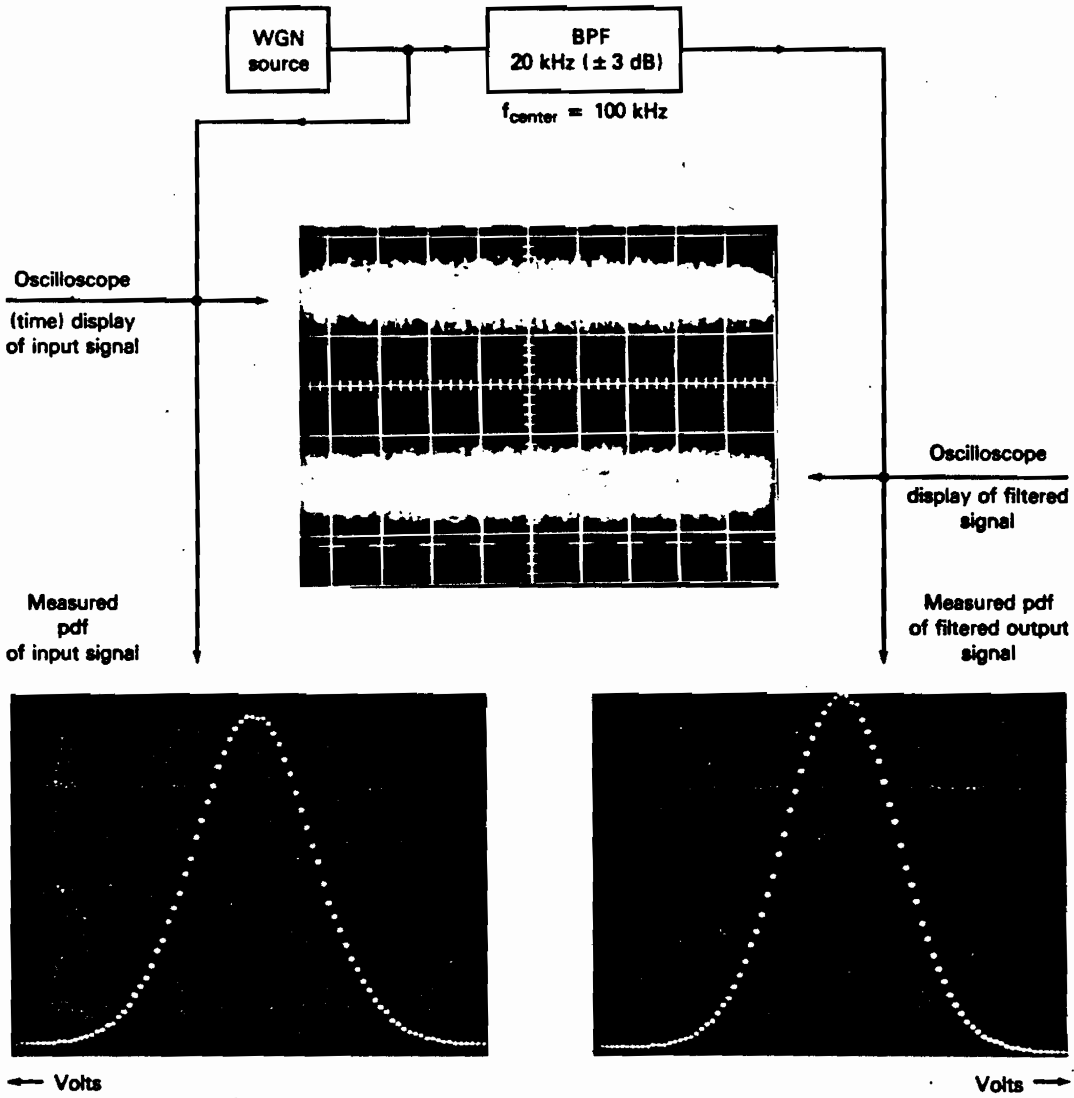




CHANNEL (R.F. = Rayleigh Fading)

Adjacent Channel Interference (ACI)




WGN
source
BPF
20 kHz (± 3 dB)
f_{center} = 100 kHz
Oscilloscope
(time) display
of input signal
Oscilloscope
display of filtered
signal
Measured
pdf
of input signal
Measured pdf
of filtered output
signal
← Volts
Volts →