PART C

DIGITAL MODULATION - DEMODULATION (MODEM) TECHNIQUES

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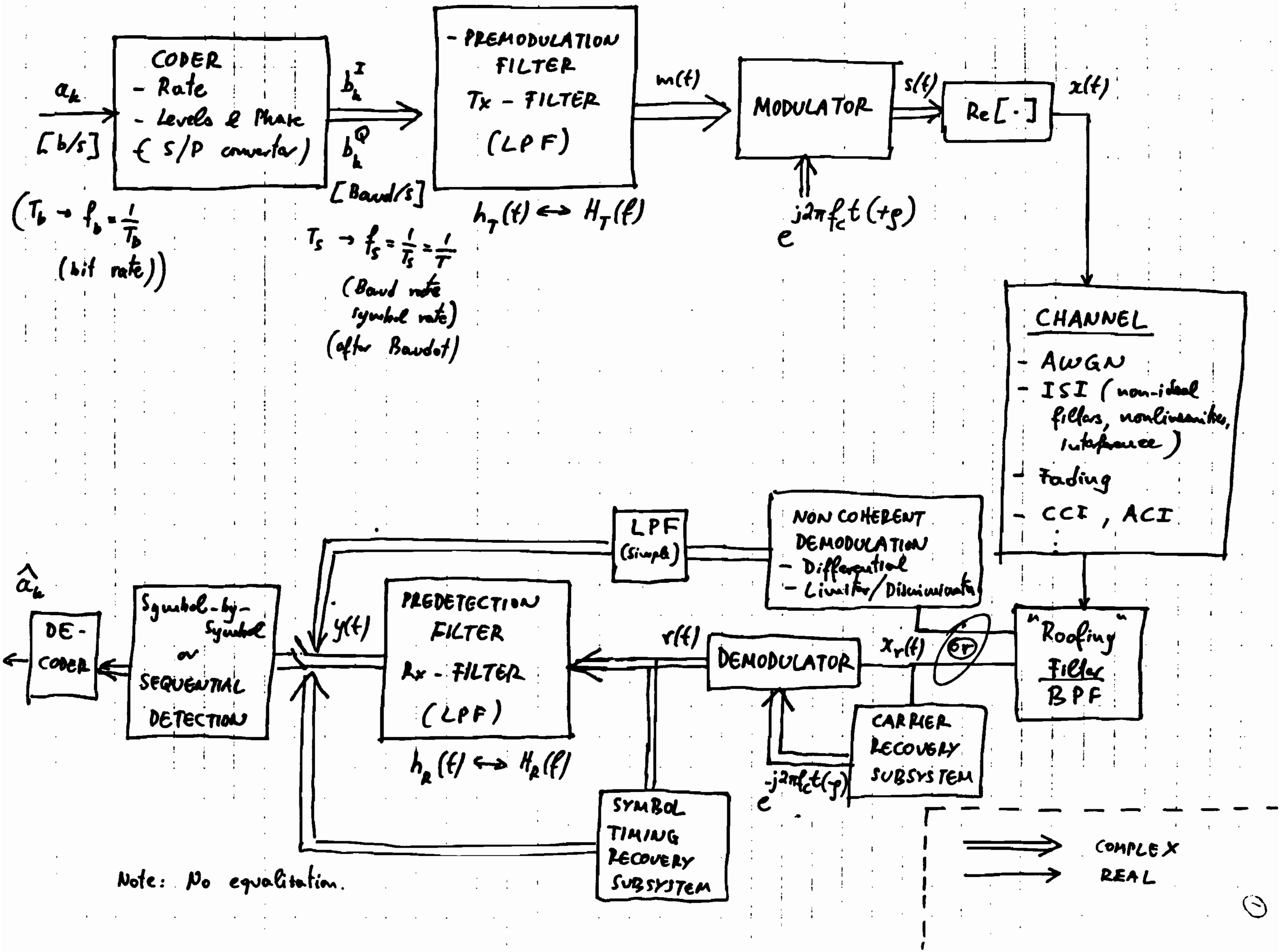
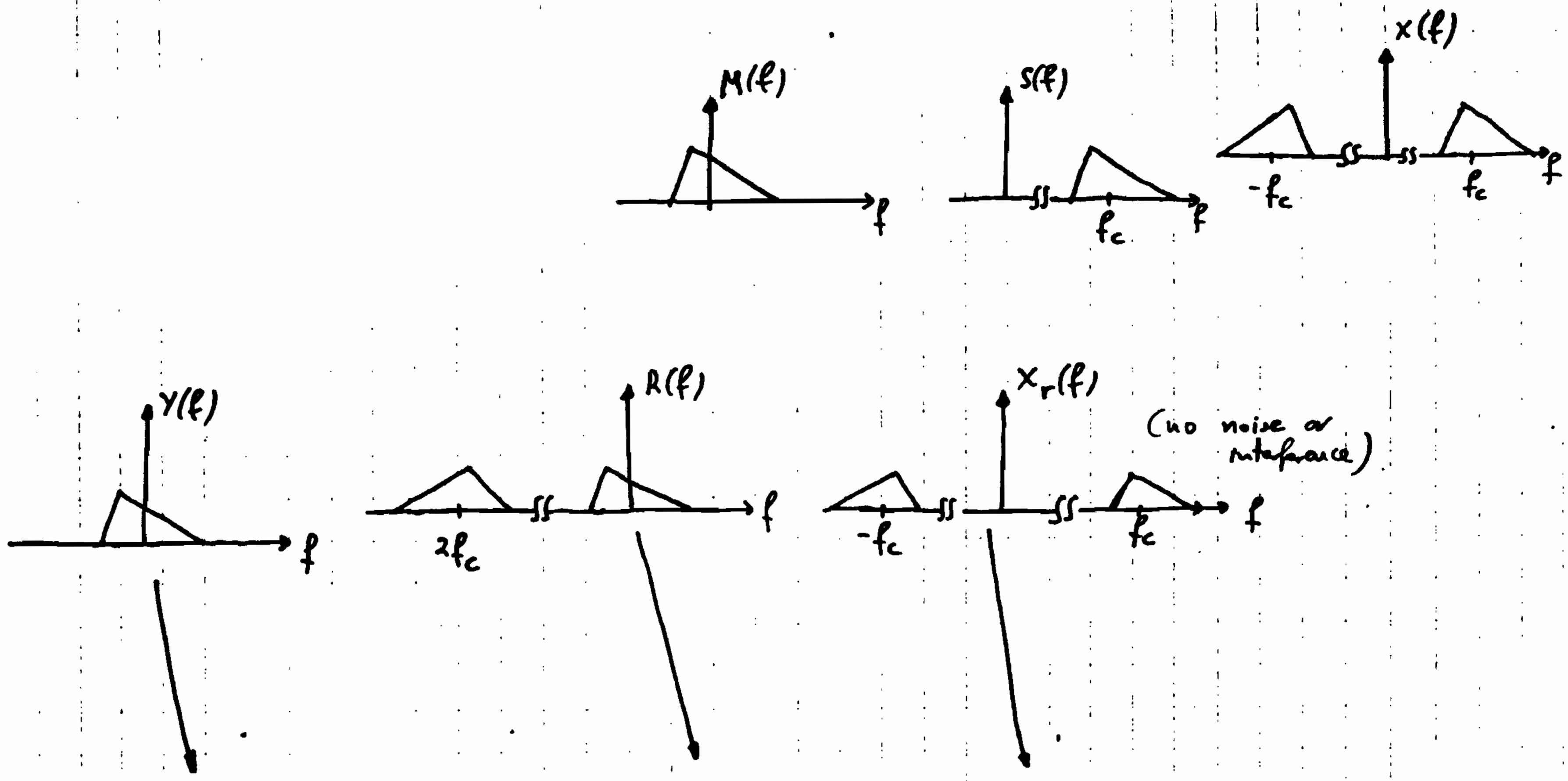
- ***Non-constant and Constant Envelope Signals***
- ***Performance (noise, fading, interference, non-linearities)***
- ***Techniques for Performance Improvement (Coherent and Non-coherent)***
- ***Modem Implemetation and Testing***

- PERFORMANCE
  - noise
  - fading
  - co-channel interference
  - nonlinearities
- TECHNIQUES TO IMPROVE THE PERFORMANCE
  - Combat interferences
- MODEM IMPLEMENTATION
  - hardware
  - software
  - DSP



Note: No equalization.


M(f)
S(f)
x(f)
$-f_c$
f_c
f
$X_r(f)$
(no noise or interference)
R(f)
$2f_c$
Y(f)

$$s(t) = e^{j2\pi f_c t} \sum_{k=-\infty}^{\infty} c_k \, h_T(t-kT)$$

$$c_k = b_k^I + j b_k^Q = a_k e^{j\theta_k} \rightarrow \begin{cases} b_k^I = a_k \cos\theta_k \\ b_k^Q = a_k \sin\theta_k \end{cases}$$

$$s(t) = [\cos(2\pi f_c t) + j\sin(2\pi f_c t)] \sum_k [b_k^I + j b_k^Q] \, h_T(t-kT)$$

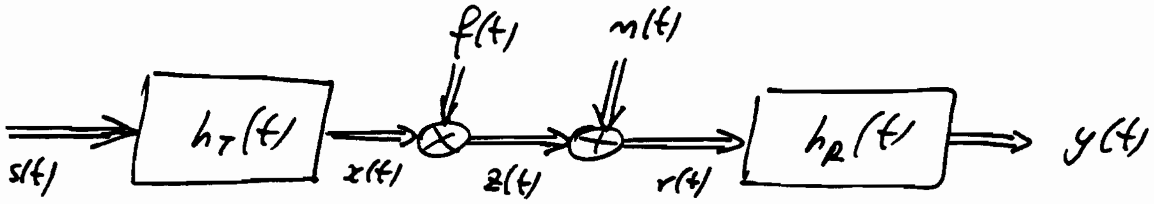
$$= \cos(2\pi f_c t) \sum_k b_k^I h_T(t-kT) - \sin(2\pi f_c t) \sum_k b_k^Q h_T(t-kT)$$

$$+ j[\cos(2\pi f_c t) \sum_k b_k^Q h_T(t-kT) + \sin(2\pi f_c t) \sum_k b_k^I h_T(t-kT)]$$

$$\therefore \boxed{x(t) = Re[x(t)] = \cos(2\pi f_c t) \sum_k b_k^I h_T(t-kT) - \sin(2\pi f_c t) \sum_k b_k^Q h_T(t-kT)}$$

Example for Computer Simulation [AWGN $n(t)$ & fading $f(t)$]

1) Mathematical Model (usually convolution)

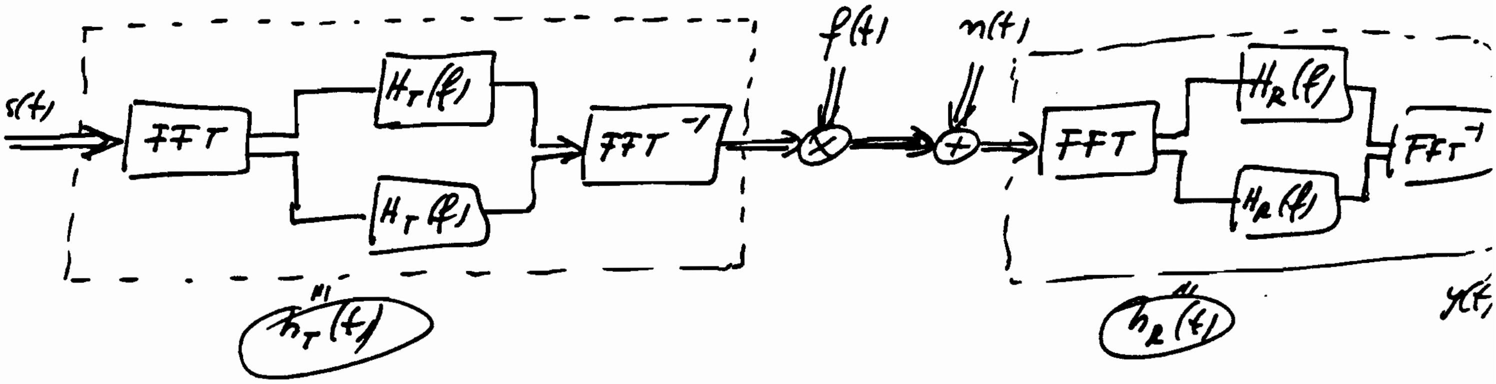


$s(t) = s_I(t) + j\, s_Q(t)$; $f(t) = f_I(t) + j\, f_Q(t)$; $n(t) = n_I(t) + j\, n_Q(t)$

(Although theoretically $h_T(t)$, $h_R(t)$ can be complex, in computer simulation they are real.)

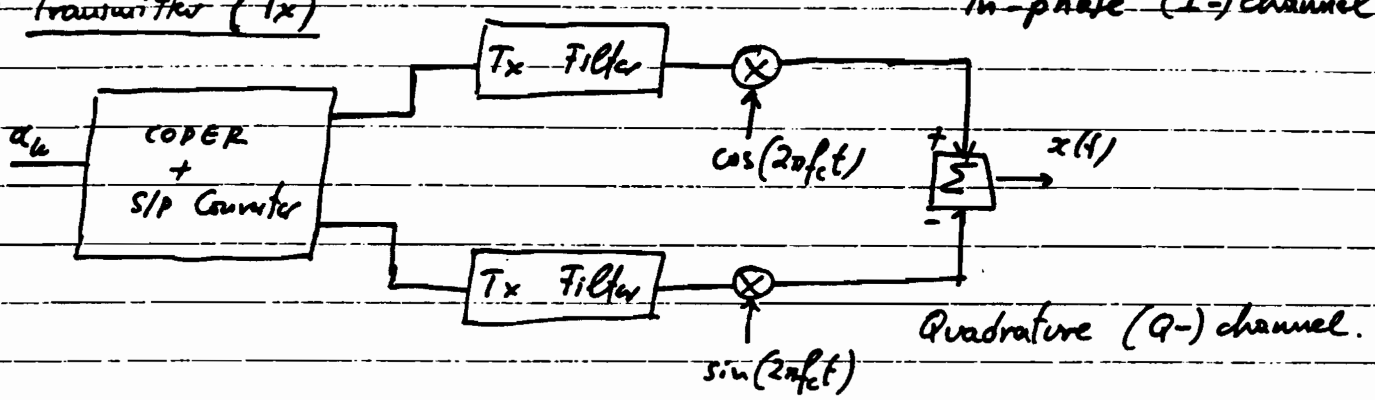
$h_T(t) \leftrightarrow H_T(f)$; $h_R(t) \leftrightarrow H_R(f)$

2) Computer Simulation Implementation (usually only multiplication)

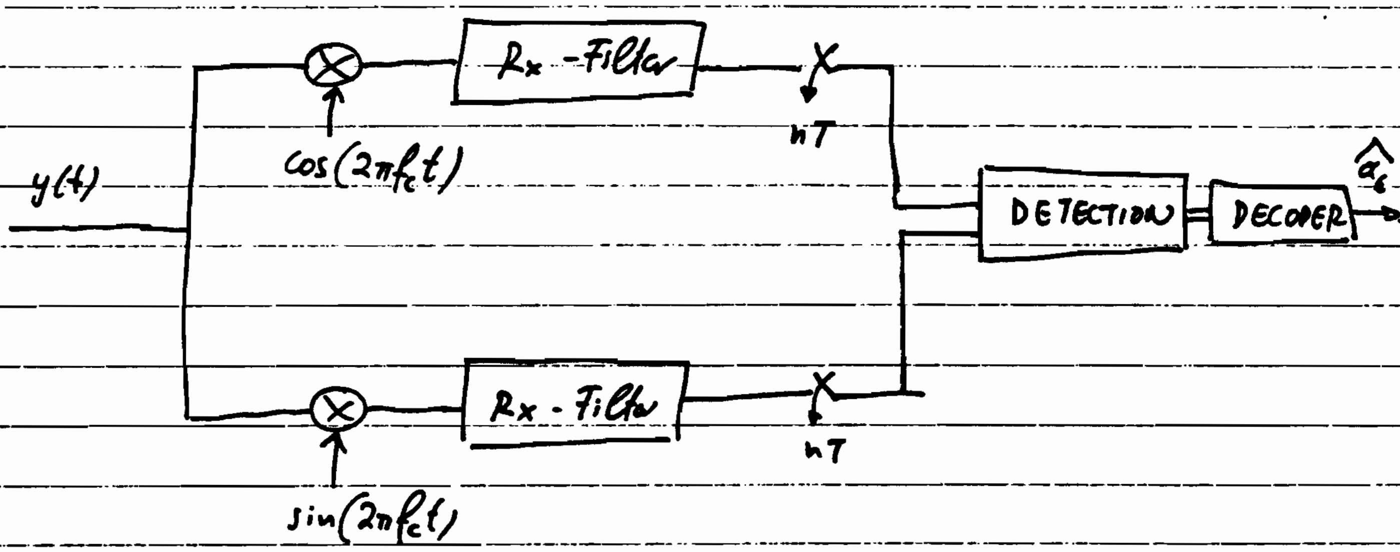


IMPLEMENTATION.

① Transmitter (Tx)



② Receiver (Rx)



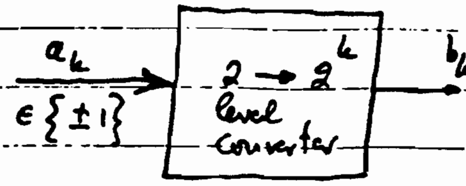
[QAM - Quadrature Amplitude Modulation]

MOST POPULAR "UNCODED" MODULATION SCHEMES

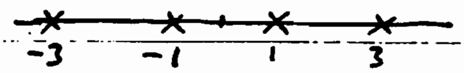
[Baseband] - unfiltered

① M-ary PAM (Pulse Amplitude Modulation)

$\boxed{M=4}$

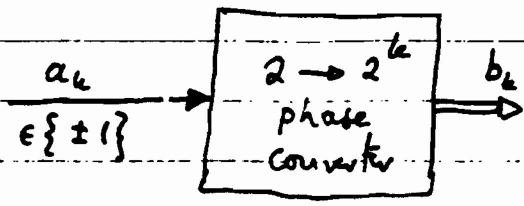


$b_k \in \{\pm 1, \pm 3, \ldots, \pm 2^{k-1}\}$

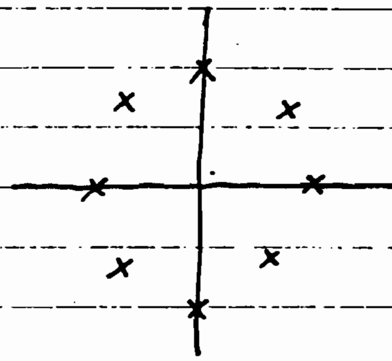


② M-ary PSK (Phase Shift Keying)

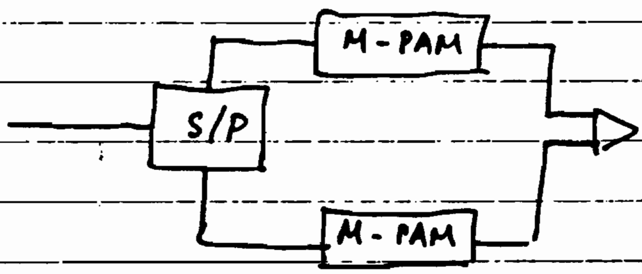
$\boxed{M=8}$

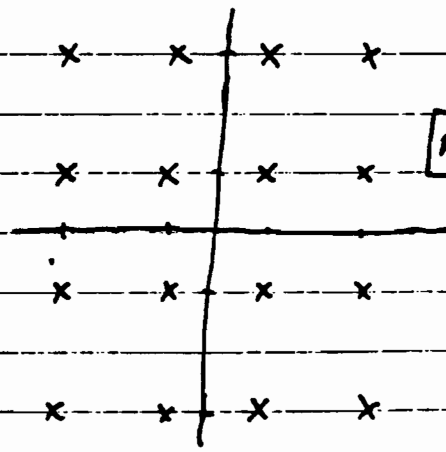


$b_k \in \left\{0, \frac{1}{M}2\pi, \ldots, \frac{M-1}{M}2\pi\right\}$



③ M^2-QAM [belongs to the APK (Amplitude & Phase Shift Keying) family]





$\boxed{M^2=16}$

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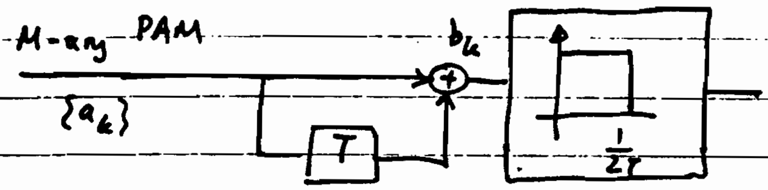
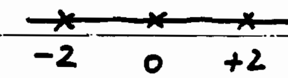
$(2M-1)$ - PR (Partial Response)



Example: $a_k \in \{\pm 1\} \longrightarrow b_k \in \{+2, 0, -2\}$ $\boxed{2M-1=3}$

Probs: $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$



- class-I PR
- Adding +1 or -1 $\Rightarrow$ controlled amount of interference
- Comparison with M-PAM
  - Worst Performance
  - Better Spectrum
- QPR
-

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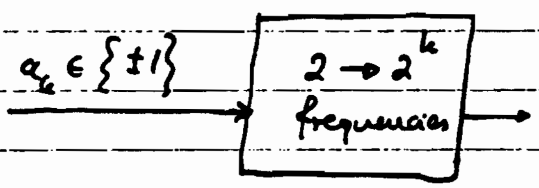
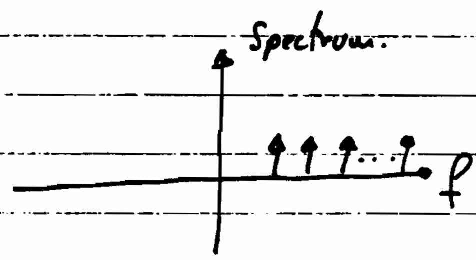
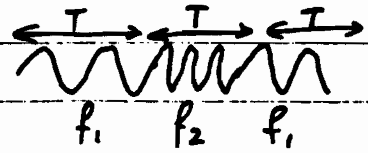
M-FSK [Frequency Shift Keying]





- Continuous Phase FSK (e.g., MSK $f_2 - f_1 = \frac{1}{2T}$)



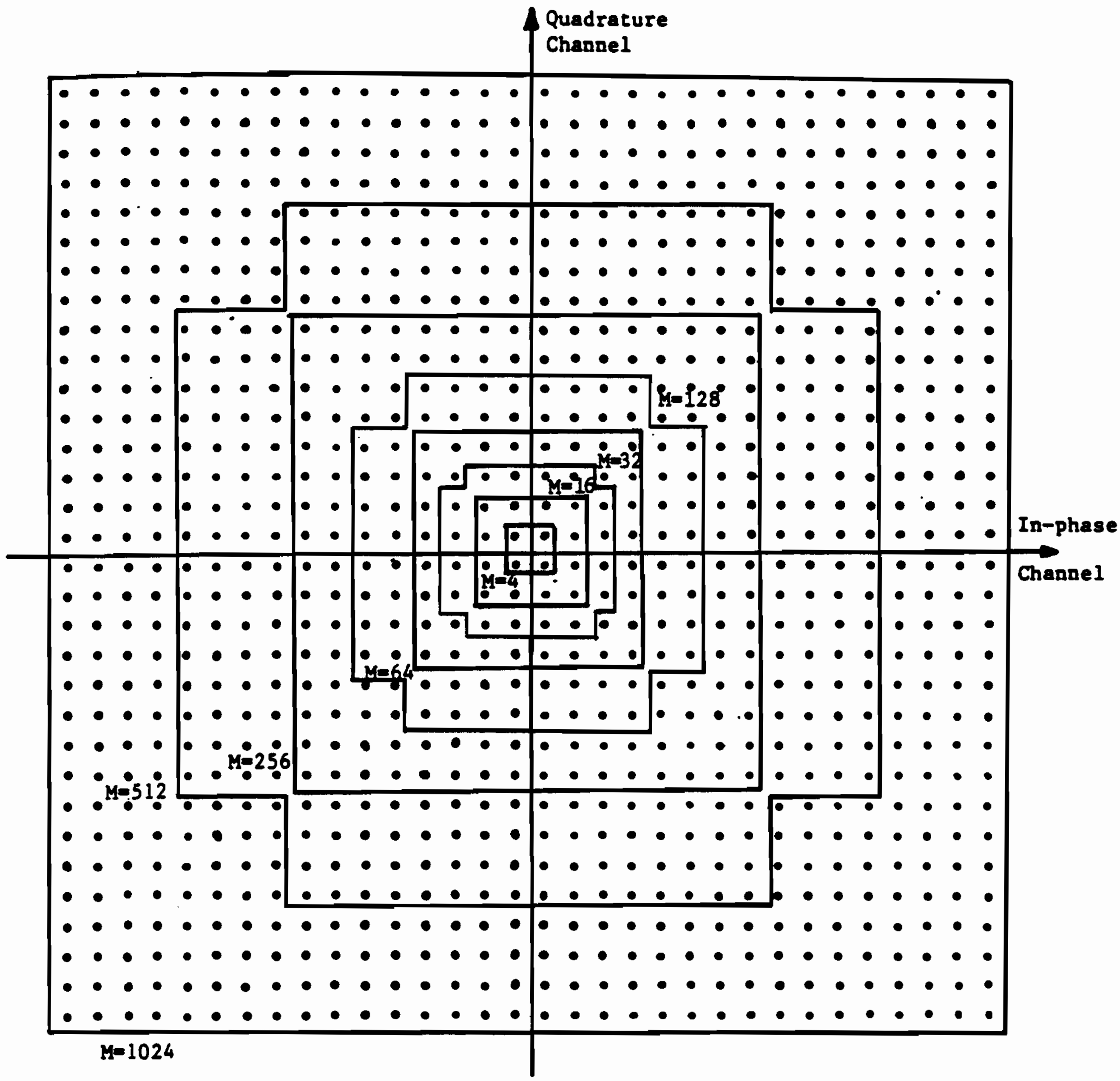
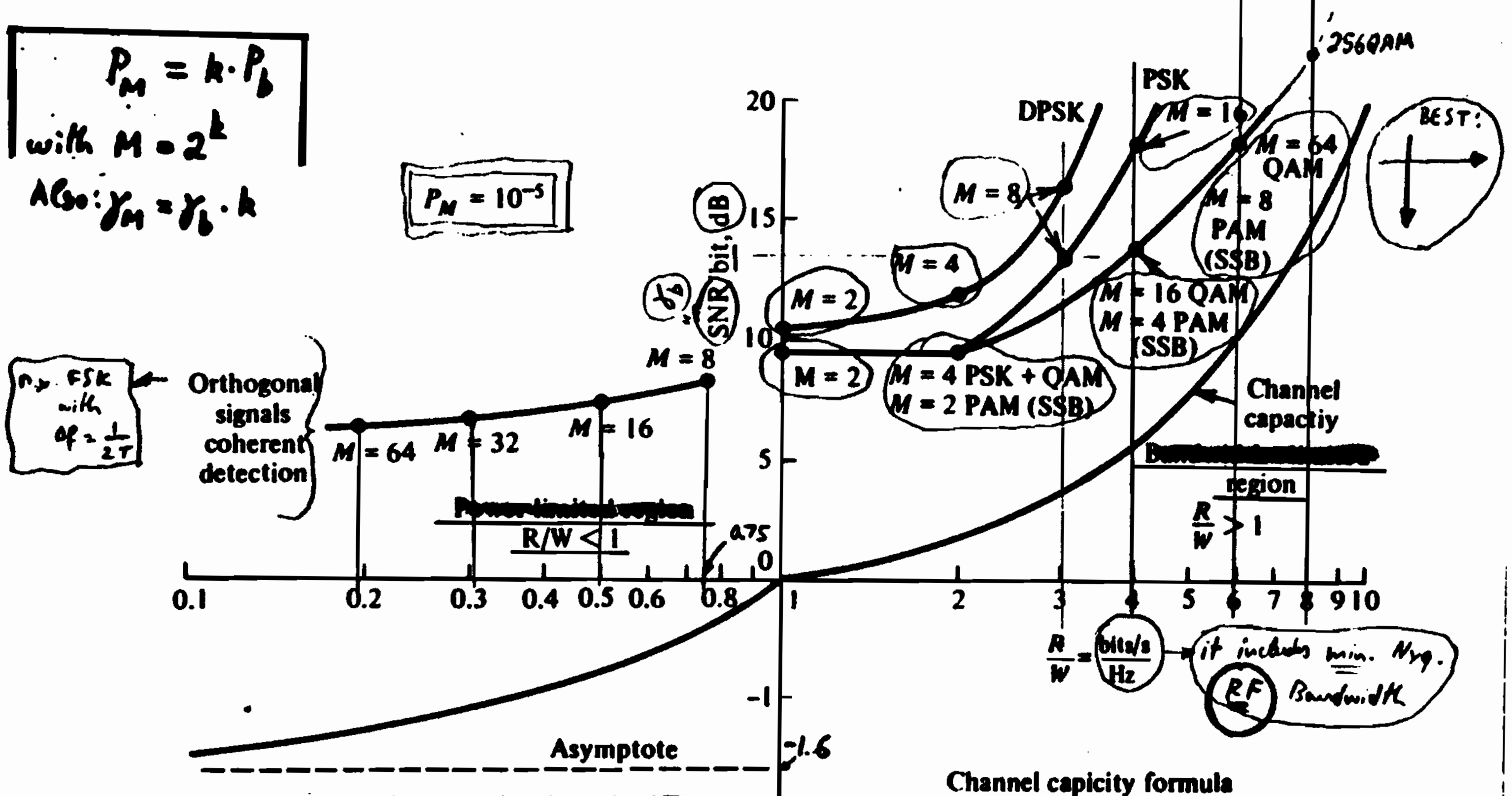


Figure 1.1: Constellations of M-ary QAM schemes with M = 4, 16, 32, 64,..., 1024.
I: in-phase carrier (channel) ; Q: quadrature-phase carrier (channel).

Ref. P. Mathiopoulos, "On bandwidth efficient QAM transmission systems," Ph.D. Thesis, University of Ottawa, Dept. of Electrical Engineering, 1989.

R : Information Rate in $\frac{bits}{sec}$

$$\boxed{\begin{aligned} P_M &= k \cdot P_b \\ \text{with } M &= 2^k \\ \text{Also: } \gamma_M &= \gamma_b \cdot k \end{aligned}}$$



$$\gamma_b = \frac{E_b}{N_0} = \frac{C \times T_b}{N_0} = \frac{C \times (T_M/k)}{N_0} = \underbrace{\frac{C}{N_0 f_M}}_{\text{SNR in min. Nyq. BW}} \times \frac{1}{k}$$

Channel capicity formula

$$\frac{C}{W} = \log_2\left(1 + \frac{C}{W}\frac{E_b}{N_0}\right)$$

$$\gamma_b = \frac{E_b}{N_0} = \text{SNR/bit}$$

Figure 4.2.31 Comparison of several modulation methods at 10^{-5} symbol error probability.

Ref. Proakis

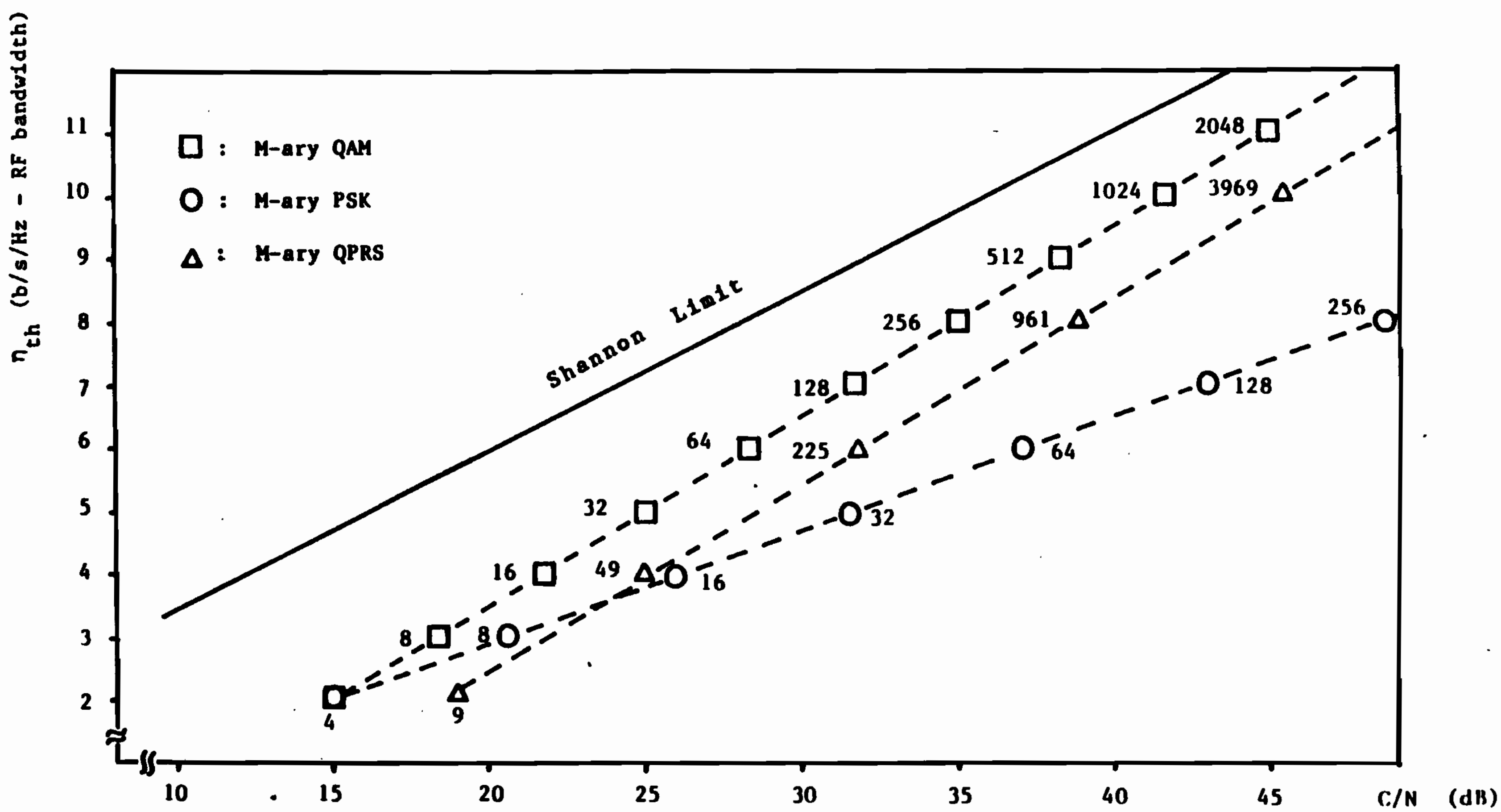


Figure 1.2: Theoretical spectral efficiency η_{th} in b/s/Hz of RF bandwidth and C/N requirement at $P_S = 10^{-8}$ of various spectrally efficient modulation systems. The average C/N is specified in the double-sided Nyquist bandwidth which equals the symbol rate. Since we refer to η_{th} ideal $\alpha = 0.0$ raised-cosine filters have been assumed.

Ref. P. Mathiopoulos, "On bandwidth efficient QAM transmission systems," Ph.D. Thesis, University of Ottawa, Dept. of Electrical Engineering, 1987.

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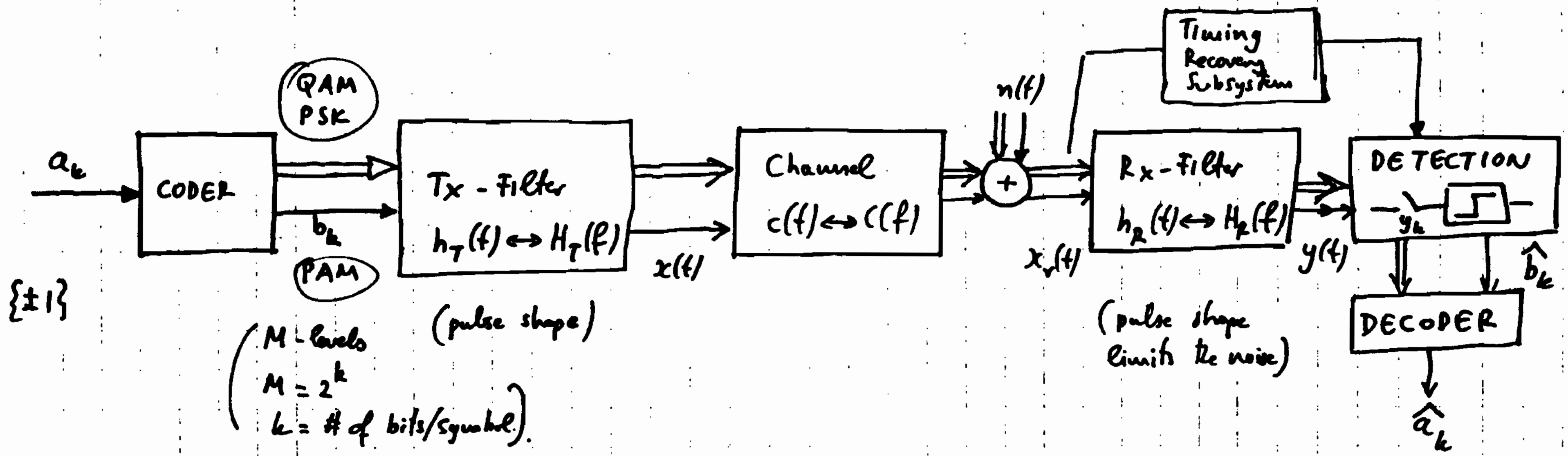
PERFORMANCE IN THE AWGN CHANNEL

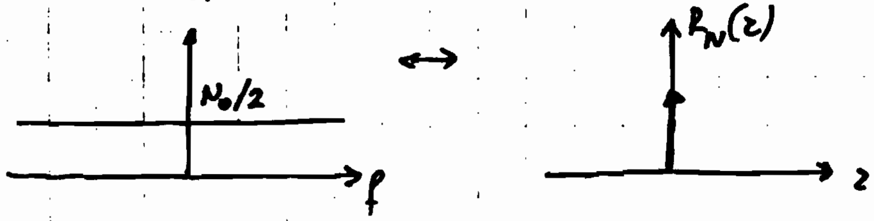
- Baseband Systems.



$n(t)$: white Gaussian noise (additive)

$\updownarrow$

$N(f)$



white $\Leftrightarrow$ uncorrelated
Gaussian
zero mean, σ^2.

$$x(t) = \sum_k b_k\, h_T(t-kT) \quad ; \quad (b_k \text{ takes equiprobable values from an } M\text{-ary alphabet, e.g., } \pm d, \pm 3d, \ldots, \pm (M-1)d)$$

$$x_r(t) = \sum_k b_k\, g(t-kT) + n(t) \quad ; \quad g(t) = h_T(t) \otimes c(t) = \int_{-\infty} h_T(z)\, c(t-z)\, dz$$

(Filtering: time dispersion
reduction of bandwidth)

$$G(f) = H_T(f)\, C(f)$$

$$y(t) = \sum_k b_k\, h(t-kT) + \tilde{n}(t) \quad ; \quad h(t) = h_T(t) \otimes c(t) \otimes h_R(t)$$

$$H(f) = H_T(f)\, C(f)\, H_R(f)$$

$$\tilde{n}(t) = n(t) \otimes h_R(t)$$

Sample at $t = nT + t_0$ results:

$$y(nT+t_0) = \sum_k b_k\, h(nT - kT + t_0) + \tilde{n}(nT+t_0)$$

Assume ideal timing ($\Rightarrow t_0 = 0$)

$$y_n = y(nT) = \sum_k b_k\, h_{n-k} + \tilde{n}_n$$

$$= h_0 \Big(\underbrace{b_n}_{\text{useful signal}} + \underbrace{\frac{1}{h_0} \sum_{\substack{k \\ k \neq n}} b_k\, h_{n-k}}_{\text{Inter-Symbol-Interference (ISI)}} + \underbrace{\frac{\tilde{n}_n}{h_0}}_{\text{noise}} \Big)$$

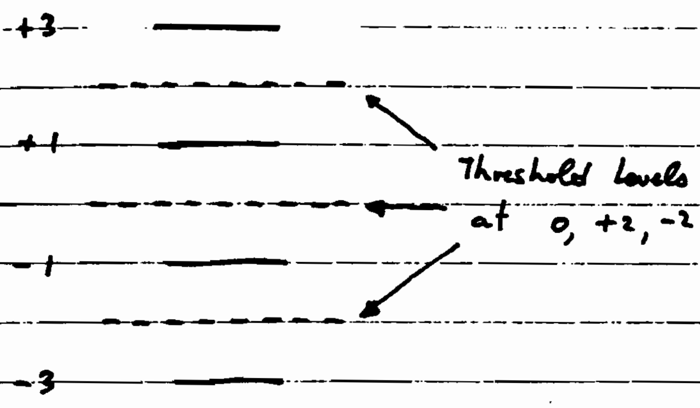
h_0 = gain or attenuation through the system (easily compensated $\therefore h_0 = 1$)

$$\boxed{\text{An error occurs when } \left| \frac{1}{h_0} \Big[\sum_{\substack{k \\ k \neq n}} b_k\, h_{n-k} + \tilde{n}_n \Big] \right| > d}$$

$2d$ is the geometrical distance between the points

Example

Assume $d=1$, $h_0=1$



4-PAM

Problems: (ISI & Noise)
(or "useful" phenomena!?)

NYQUIST CRITERION FOR ZERO ISI

There are 3 Nyquist criteria.
The first deals with pulse shapes (essentially filters) which result in zero ISI at the sampling instant.

$$\text{If } \{h(t)=0 \quad \text{for} \quad t=nT \ (n\neq 0)\} \Longleftrightarrow \{h_n=0 \quad \text{for } n\neq 0\}$$

$$\Longleftrightarrow \{h_n = h(nT) = \delta_n\} \Longrightarrow y_n = b_n + \breve{n}_n \ (\text{i.e., no ISI})$$

$\therefore$ For no ISI $\Longrightarrow \sum_k h(kT)\,\delta(t-kT) = \delta(t)$

Nyquist First Criterion $(SOS)^2$

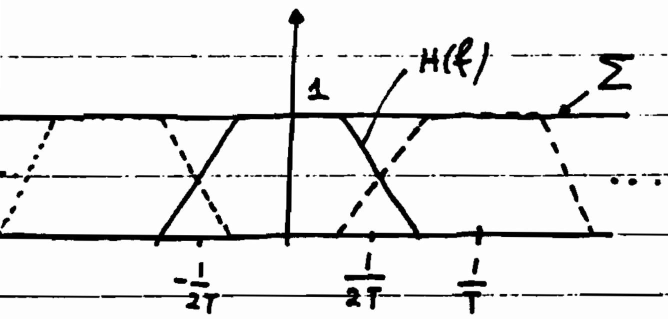
$$\boxed{\underbrace{h(t)\sum_k \delta(t-kT) = \delta(t)}_{\text{Time Domain}} \longleftrightarrow \underbrace{\frac{1}{T}\sum_m H\left(f-\frac{m}{T}\right) = 1.}_{\text{Frequency Domain}}}$$

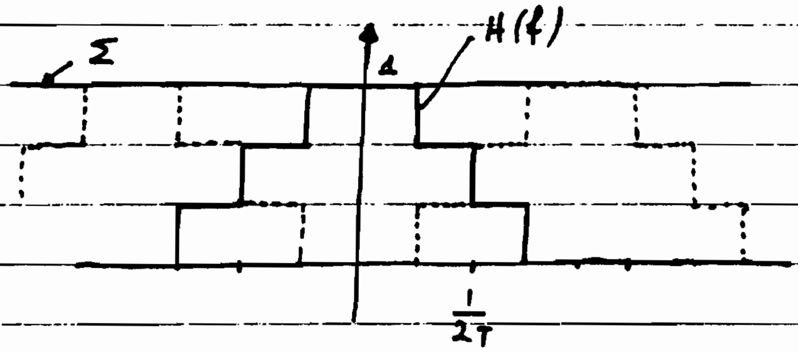
Comments

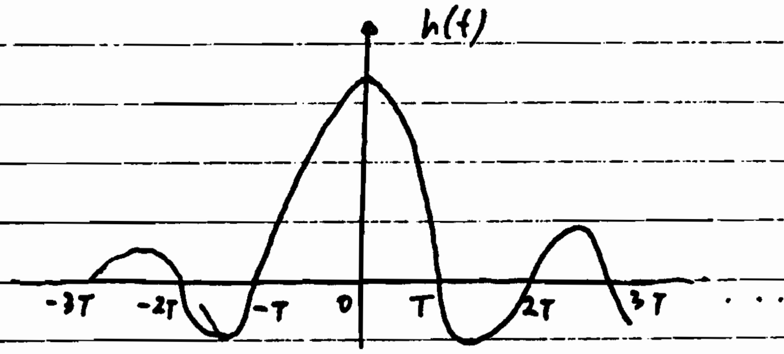
① Impulse response includes Tx-filter, Channel and Rx-filter.
(For the Nyquist it does not matter how they are "split")

② In the time domain, we force the overall impulse response $h(t)$ to be zero at all sampling instances (expect of course $t=0$).

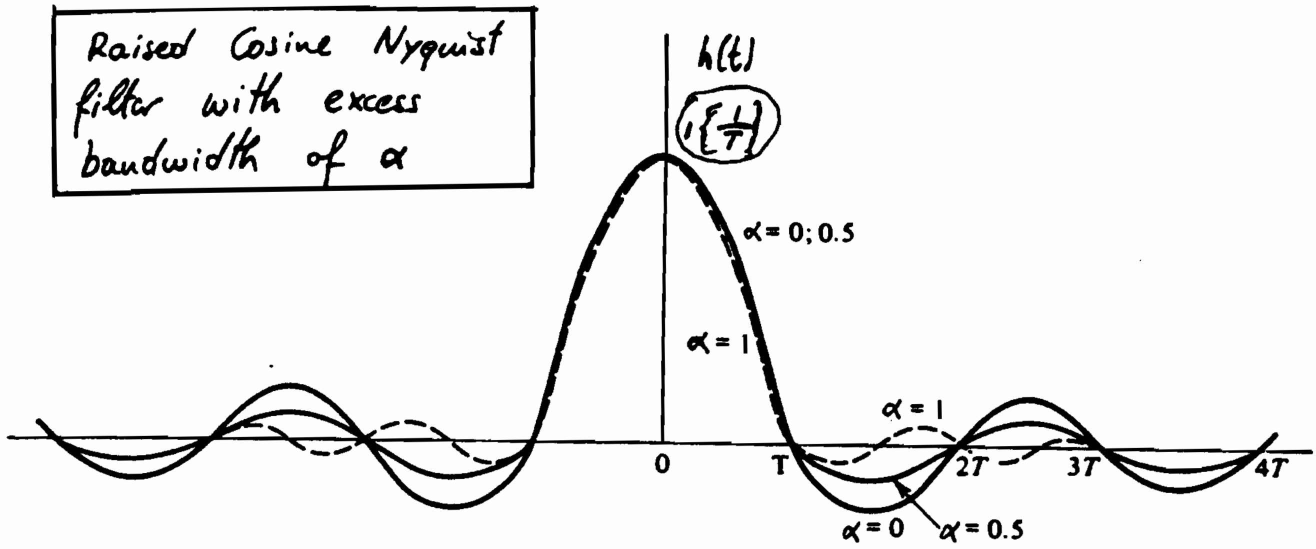
③ In the frequency domain, this is equivalent of having an even symmetry around the Nyquist frequency $f_N = \frac{1}{2T}$

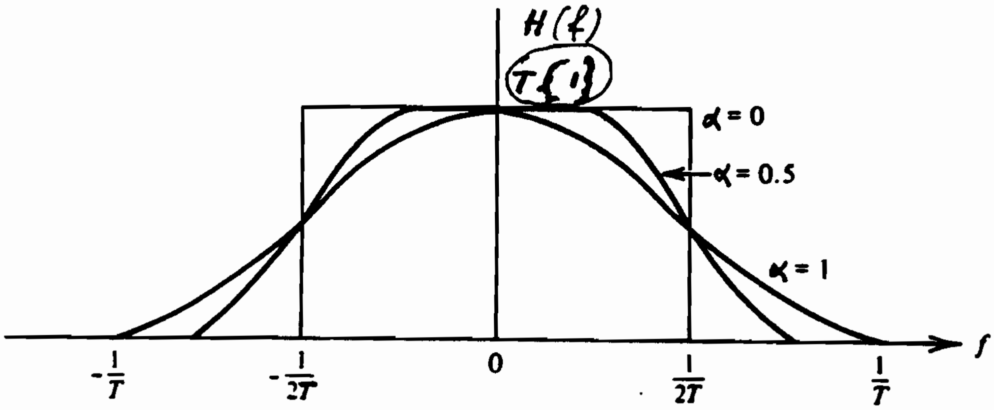






Raised Cosine Nyquist filter with excess bandwidth of α

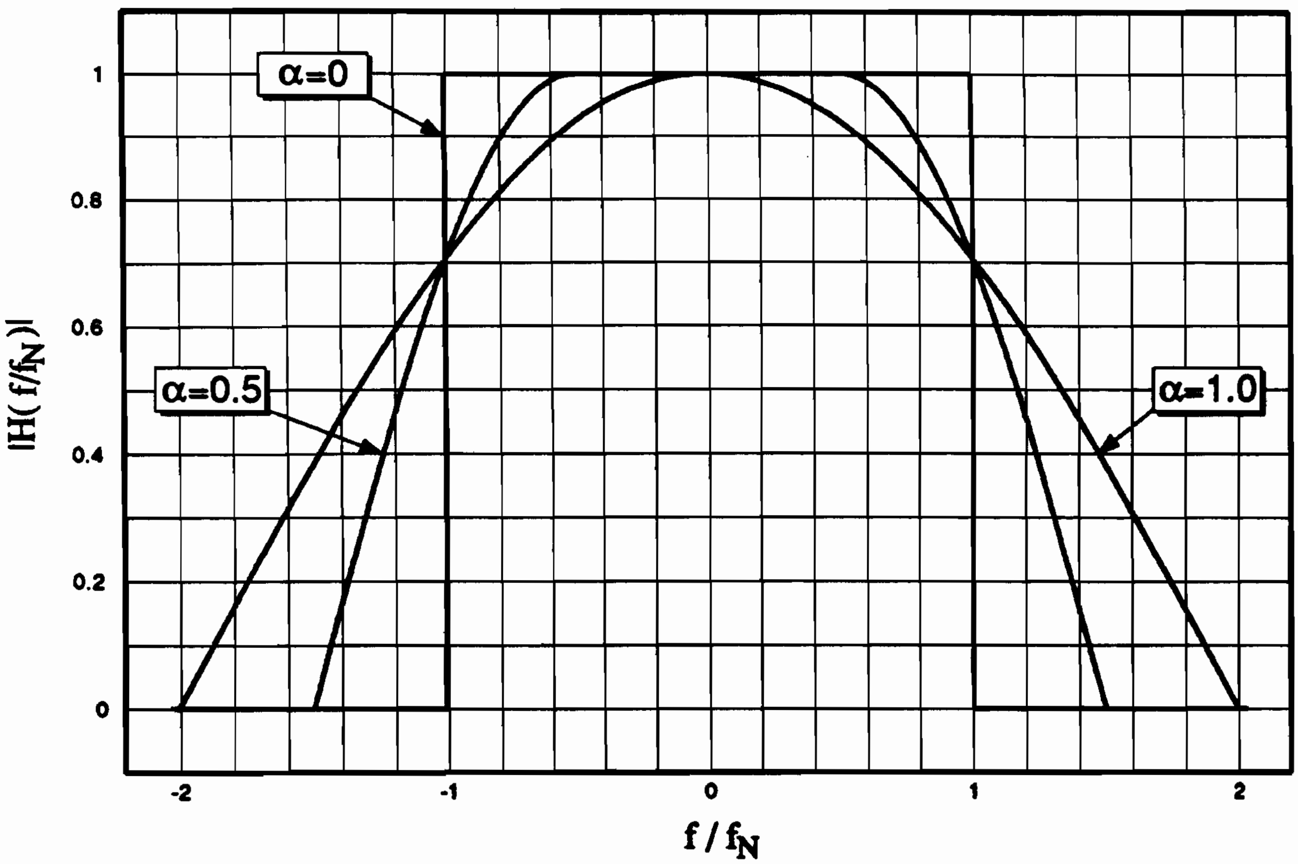




$$H(f) = \begin{cases} 1 & \text{for} \quad 0 \leq f < \frac{1-\alpha}{2T} \\ \cos^2\left[\frac{\pi T}{2\alpha}\left(f - \frac{1-\alpha}{2T}\right)\right] & \text{for} \quad \frac{1-\alpha}{2T} \leq f < \frac{1+\alpha}{2T} \\ 0 & \text{for} \quad f \geq \frac{1+\alpha}{T} \end{cases}$$

$$h(t) = \frac{\sin\left(\frac{\pi t}{T}\right)}{\frac{\pi t}{T}} \; \frac{\cos\left(\frac{\alpha \pi t}{T}\right)}{1 - \frac{4\alpha^2 t^2}{T^2}}$$

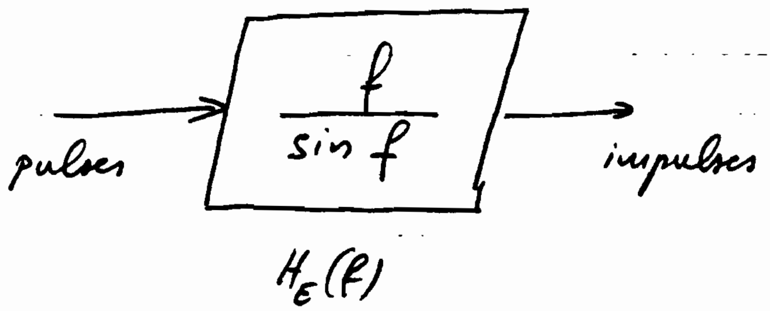
Root of raised-cosine transfer function



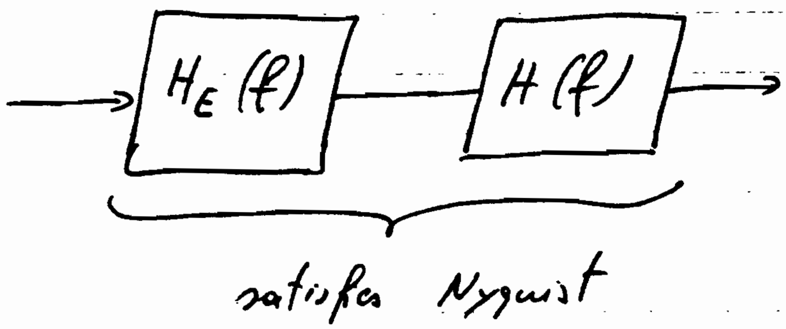
Important Note

The Nyquist filters (or equivalently impulse responses) have been derived assuming <u>impulse</u> transmission.

If we have pulse, which is the case for most practical applications, then we need a filter which transforms the pulses $\longrightarrow$ impulses

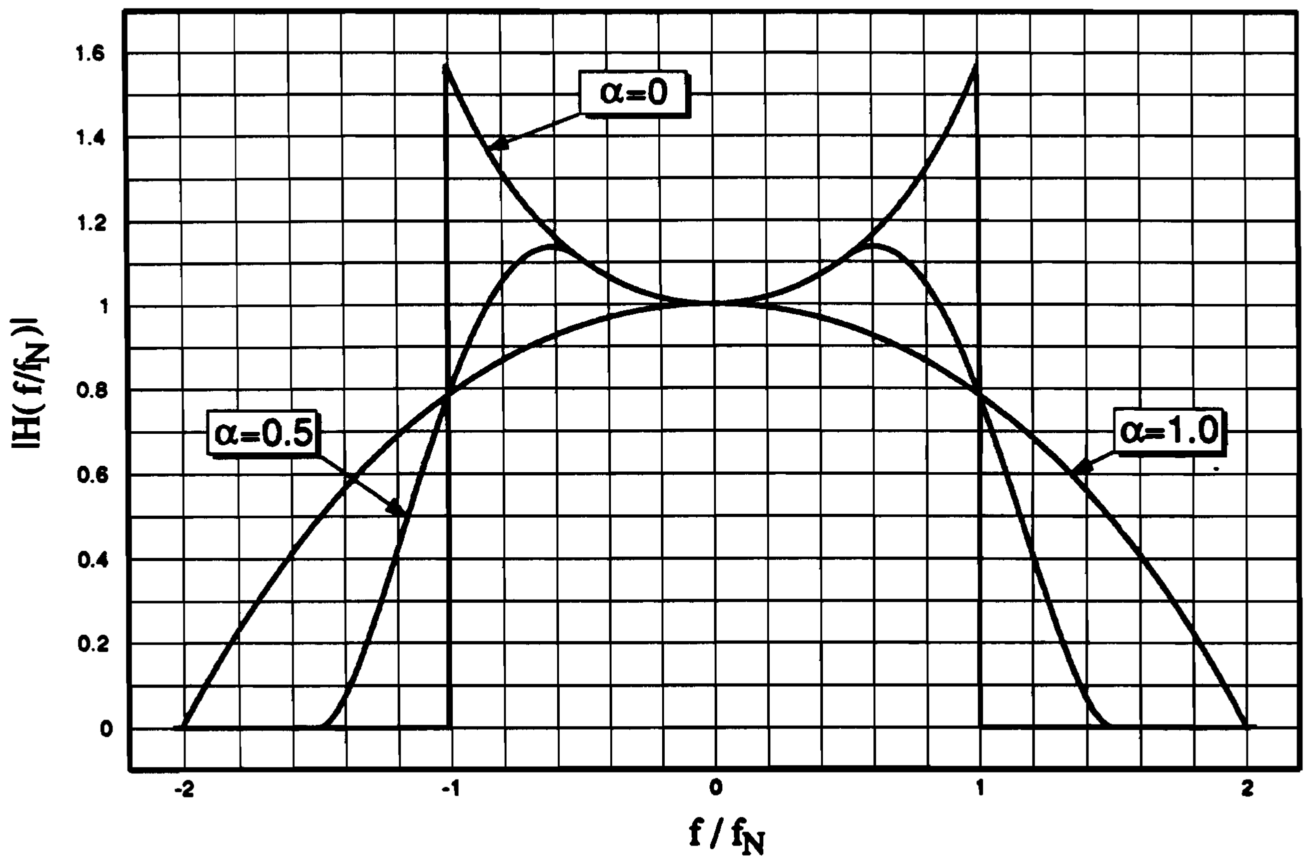


$\therefore$ For pulse transmission:

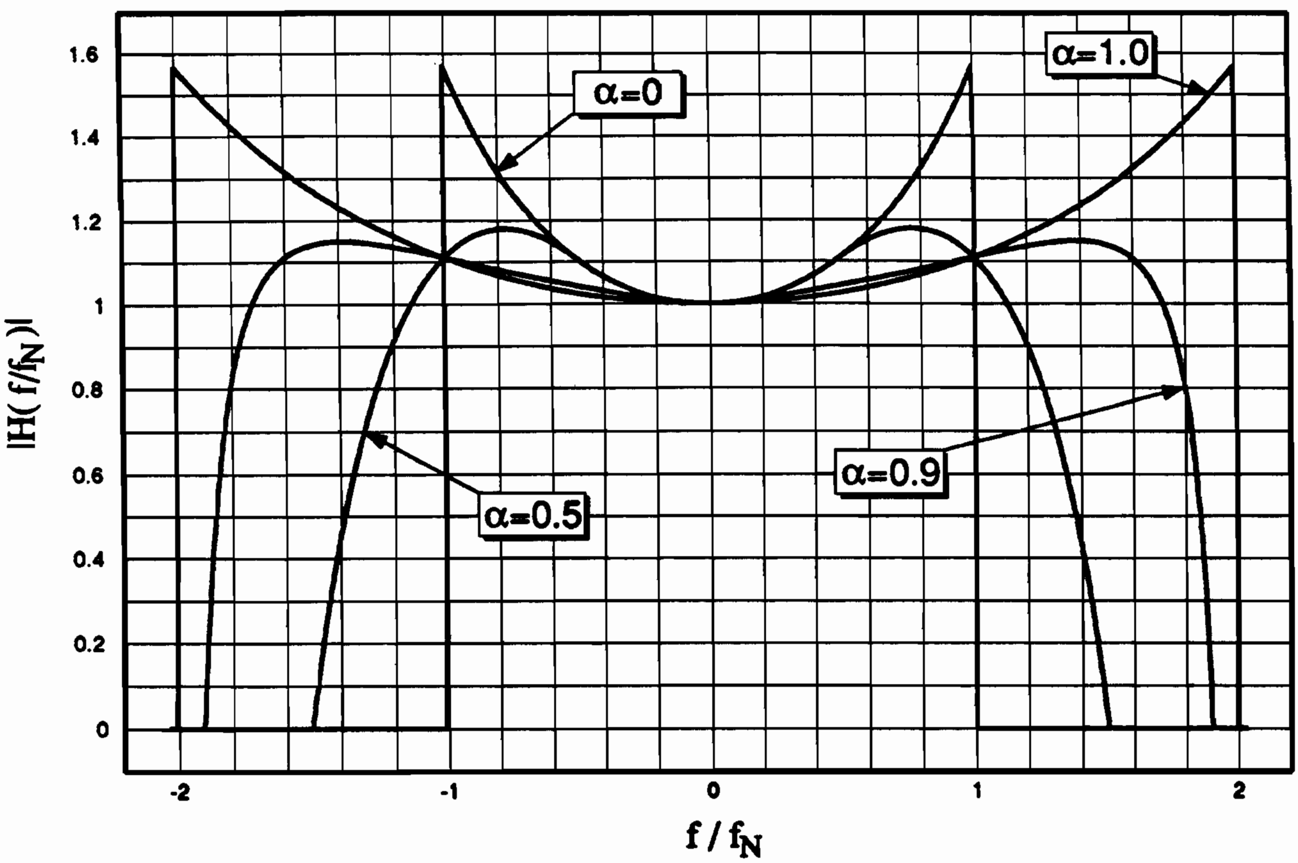


Also linear phase $\Rightarrow$ constant group delay.

x/sin(x) equalized, raised-cosine transfer function



x/sin(x) equalized, root of raised-cosine transfer function



PARTIAL RESPONSE SIGNALS

- Bandlimited signals with controlled amount of Intersymbol Interference (ISI).

- For a raised cosine (α) pulse we have:

$$h(nT) = \begin{cases} 1 & \text{for } n=0 \\ 0 & \text{for } n \in \{\pm 1, \pm 2, \pm 3, \dots\} \end{cases}$$

- Consider now that:

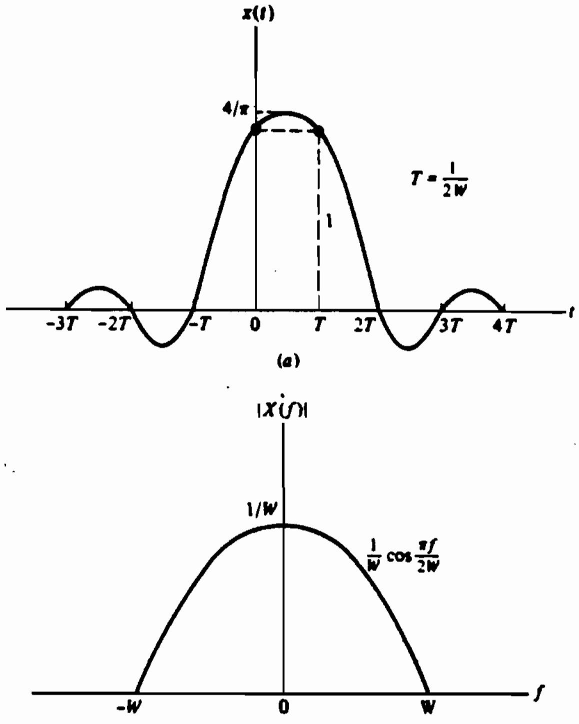
1)
$$h(nT) = \begin{cases} 1 & \text{for } n=0,1 \\ 0 & \text{otherwise.} \end{cases}$$

[duobinary signalling]

2)
$$h(nT) = \begin{cases} 1 & \text{for } n=-1 \\ 1 & \text{for } n=1 \\ 0 & \text{otherwise} \end{cases}$$

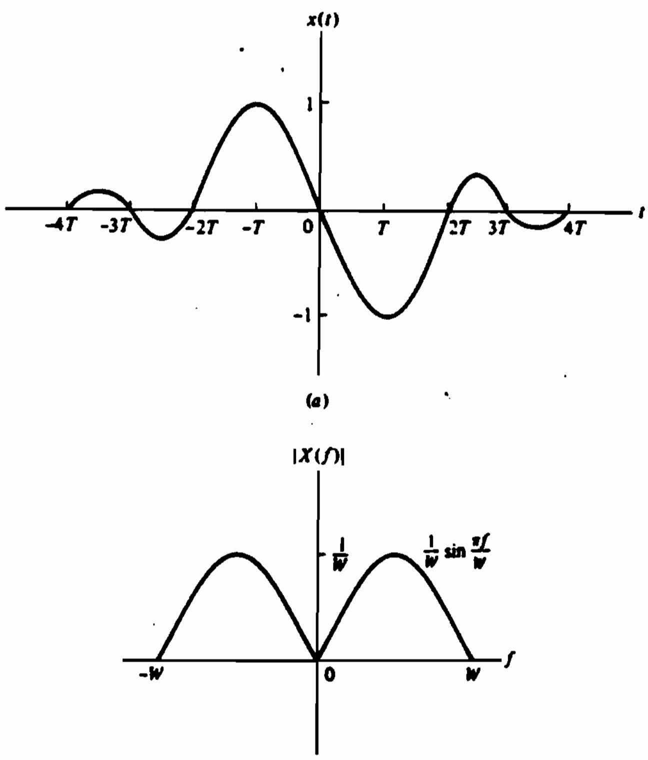
[modified duobinary signalling]

18B


x(t)
4/π
1
T = 1/2W
-3T -2T -T 0 T 2T 3T 4T
t
|X(f)|
1/W
1/W cos πf/2W
-W 0 W
f

(a)

(b)


x(t)
1
-1
-4T -3T -2T -T 0 T 2T 3T 4T
t
|X(f)|
1/W
1/W sin πf/W
-W 0 W
f

(a)

(b)

- Problem with Error Propagation

Duobinary : $B_n = A_n + A_{n-1}$

$$A_n \in \{\pm 1\} \Rightarrow B_n \in \{\pm 2, 0\}$$

To decode A_n use $B_n - A_{n-1}$ (Note: Subtraction of ISI)

If A_{n-1} in error $\Rightarrow$ A_n in error too ...

$\therefore$ Error propagation.

- Solution : Precoding

Essentially, it is a differential encoding in which we are transmitting the "difference in symbols" rather that the symbols themselves

Binary signaling with duobinary pulses

Data sequence D_n:		1	1	1	0	1	0	0	1	0	0	0	1	1	0	1
Precoded sequence P_n:	0	1	0	1	1	0	0	0	1	1	1	1	0	1	1	0
Transmitted sequence A_n:	−1	1	−1	1	1	−1	−1	−1	1	1	1	1	−1	1	1	−1
Received sequence B_n:		0	0	0	2	0	−2	−2	0	2	2	2	0	0	2	0
Decoded sequence D_n:		1	1	1	0	1	0	0	1	0	0	0	1	1	0	1

$$D_n \in \{0, 1\}$$

$$P_n = D_n \ominus P_{n-1}$$

$$A_n = 2P_n - 1$$

$$B_n = A_n + A_{n-1} = 2(P_n + P_{n-1} - 1)$$

$$D_n = \frac{B_n}{2} \oplus 1 \qquad [D_n = P_n \oplus P_{n-1}]$$

($\ominus$ denotes mod-2)

Four-level signal transmission with duobinary pulses

Data sequence D_n:		0	0	1	3	1	2	0	3	3	2	0	1	0
Precoded sequence P_n:	0	0	0	1	2	3	3	1	2	1	1	3	2	2
Transmitted sequence A_n:	−3	−3	−3	−1	1	3	3	−1	1	−1	−1	3	1	1
Received sequence B_n:		−6	−6	−4	0	4	6	2	0	0	−2	2	4	2
Decoded sequence D_n:		0	0	1	3	1	2	0	3	3	2	0	1	0

$$D_n \in \{0, 1, 2, 3\}$$

Four-level signal transmission with modified duobinary pulses

Data sequence D_n:			0	1	3	1	2	0	3	3	2	0	1	0
Precoded sequence P_n:	0	0	0	1	3	2	1	2	0	1	2	1	3	1
Transmitted sequence A_n:	−3	−3	−3	−1	3	1	−1	1	−3	−1	1	−1	3	−1
Received sequence B_n:			0	2	6	2	−4	0	−2	−2	4	0	2	0
Decoded sequence D_n:			0	1	3	1	2	0	3	3	2	0	1	0

In general: $D_n \in \{0, 1, 2, \dots M-1\}$; $\bigcirc \triangleq$ mod-M

$$P_n = D_n \ominus P_{n-1} \quad \boxed{\text{Duobinary}} \quad \Big\| \quad P_n = D_n \oplus P_{n-2} \quad \boxed{\text{Modified Duobinary}}$$

$$A_n = 2P_n - (M-1)$$

$$D_n = \frac{B_n}{2} + (M-1) \quad \boxed{\text{Duobinary}} \quad \Big\| \quad D_n = \frac{B_n}{2} \quad \boxed{\text{Modified Duobinary}}$$

Generalization of a Technique for Binary Data Communication

E. R. KRETZMER, SENIOR MEMBER, IEEE

Abstract—A technique for binary data transmission is described, in which each binary symbol is chosen to be a prescribed superposition of n impulses of form $(\sin 2\pi Ft)/2\pi Ft$, spaced at intervals $1/2F$. Such superposition leads to more than two received levels with binary input, but in return permits realization of the Nyquist rate ($2F$ symbols/s in bandwidth F). Appropriate choice of the superposition coefficients provides a variety of spectral distributions to suit individual applications. The most interesting classes of system functions are defined, the associated coding procedures are described, and the performance characteristics are summarized.

Introduction

A useful generalization of the so-called biternary, duobinary, or polybinary [1]–[6] technique for binary data transmission is reported herewith. It is based on recognizing that one may choose an end-to-end system function $H(f)$ whose transform $h(t)$, the overall response per binary symbol, is a prescribed weighted linear superposition of n impulses of form $(\sin 2\pi Ft)/2\pi Ft$, spaced at intervals $1/2F$. Use of such a superposition rather than a single impulse leads to more than two received levels with binary input, but in return it renders practical the attainment of the Nyquist rate [7] ($2F$ symbols/s in bandwidth F), since $H(f)$ is zero for $f \geq F$ and is also continuous at $f = F$. Resulting constraints on level transitions in the received signal make possible a limited amount of error detection. Furthermore, appropriate choice of the superposition coefficients makes available a variety of spectral distributions to suit individual applications. This communication will define the most interesting classes of system functions (channels), describe the associated coding procedures, and summarize the performance characteristics.

Definition

The transmission channel will be characterized by the superposition relationship mentioned: A received sample value c_n depends on n successive transmitted sample values $b_1 \ldots b_n$ as follows:

$$c_n = k_1 b_n + k_2 b_n - 1 \ldots + k_n b_1, \qquad (1)$$

where the k's are integer weighting coefficients, with the smallest $|k|$ equal to unity.

The corresponding channel frequency function is

$$H(f) = \int_{-\infty}^{\infty} \left[k_1 \delta(t) + k_2 \delta\left(t - \frac{1}{2F}\right) \ldots + k_n \delta\left(t - \frac{n-1}{2F}\right)\right] e^{-j2\pi ft}\, dt \qquad (2)$$

$[0 < |f| < F]$; the band limitation permits the use of the delta function instead of $(\sin 2\pi Ft)/2\pi Ft$.

With these two characterizations in mind we can now tabulate those channels which have been found of greatest interest (see Table I).

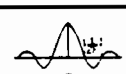
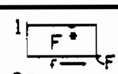
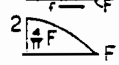
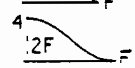
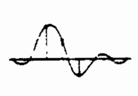
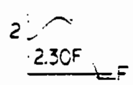
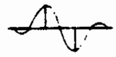
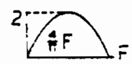
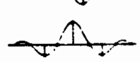
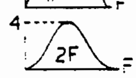
Each row of this table defines just one member of a class, namely the one having the smallest possible number n of k's and hence of received levels. Thus, Class 1 is the polybinary class of systems [4] having all unity-weight coefficients, while Class 2 has only linearly tapered distributions of coefficient values. As n is increased within each class, the number of received levels also rises and $H(f)$ becomes increasingly "concentrated," yielding a more drastic spectral tapering. However, the full band F is still required to convey $2F$ bps, and performance margins are substantially reduced. To ensure the essential condition of continuity in $H(f)$ at $f = F$, certain superpositions must be ruled out, e.g., odd values of n in Class 1.

Decoding or Precoding

The received multilevel signal can be interpreted with a digital version of McColl's apparatus [8], in which the contributions of $n - 1$ preceding (transmitted) sample values—all binary in our case—are subtracted from the present received sample value. A binary decision is then made on the difference, which is either 0 or k_1 in the absence of noise. A shift register stores the resulting binary digit as well as the $n - 1$ preceding ones (see Fig. 1). An error in any received sample tends to propagate until c_n reaches the top or bottom level.

Alternatively, a precoding operation can be undertaken at the transmitter, such that the nth binary digit a_n of the original sequence is the modulo-2 summation of the nth binary digit b_n actually transmitted and the $n - 1$ preceding digits transmitted, all weighted by their assigned coefficients, i.e.,

TABLE I

Class	k_1	k_2	k_3	k_4	k_5	$h(t)$		$H(f)$* $[0 < f < F]$	No. of Rec. Levels
Binary (ideal)	1							1	2
1 ($n = 2$)	1	1						$2 \cos \frac{\pi f}{2F}$	3
2 ($n = 3$)	1	2	1					$4 \cos^2 \frac{\pi f}{2F}$	5
3 ($n = 3$)	2	1	-1					$2 + \cos \frac{\pi}{F} f - \cos \frac{2\pi f}{F} + j\left[\sin \frac{\pi}{F} f - \sin \frac{2\pi f}{F}\right]$	5
4 ($n = 3$)	1	0	-1					$2 \sin \pi \frac{f}{F}$	3
5 ($n = 5$)	-1	0	2	0	-1			$4 \sin^2 \pi \frac{f}{F}$	5

* Entry in figure is area under curve.

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The author is with Bell Telephone Labs., Inc., Holmdel, N. J.

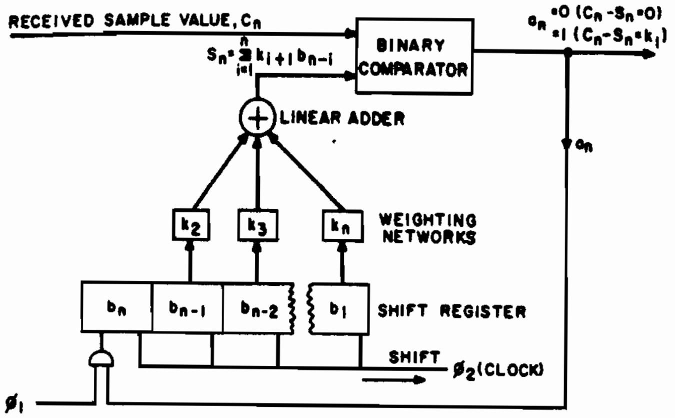
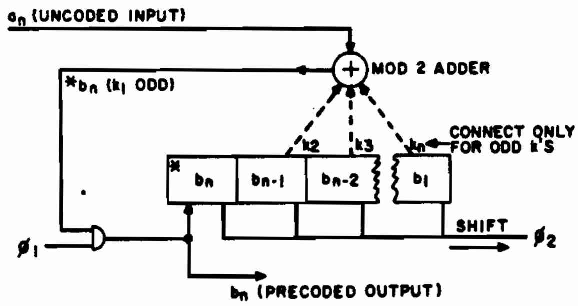


Fig. 1. Decoder.



*NOTE: FOR k_1 EVEN, b_{n-1} REPLACES b_n, ETC

Fig. 2. Precoder.

$$a_n \equiv [k_1 b_n + k_2 b_{n-1} \ldots + k_n b_1] \bmod 2. \tag{3}$$

Comparing this with (1), we find that

$$a_n \equiv c_n \bmod 2, \tag{4}$$

i.e., an even-numbered value of the received sample implies a zero in the original sequence, an odd-numbered value a one. The precoding operation thus simplifies the interpretation of the received signal and eliminates error propagation; on the other hand, it limits the "decision distance" (difference in signal values to be distinguished by detector) to unity instead of k_1—a matter of consequence only where k_1 exceeds unity, as in Class 3. The precoding implementation appears in Fig. 2.

Performance

Performance is appraised by two criteria:

1) the speed tolerance, i.e., the percentage increase over the Nyquist rate which will just cause overlap between adjacent levels ("eye closure"); and
2) the required increase in signal-to-noise ratio over a reference binary system (operating at like bit rate) for a fixed error rate in the presence of white Gaussian noise (ignoring both error propagation and error detection capability).

Criterion 1 has been evaluated [9] by computer simulation using "worst possible" data sequences (see Table II). Criterion 2 is readily formulated in terms of the "decision distance" and the "noise bandwidth" of $[H(f)]^{1/2}$. The exponent 1/2 arises from a geometrically equal apportionment of $H(f)$ between transmitter

TABLE II

Class	k_1	k_2	k_3	k_4	k_5	Speed Tolerance (in percent)	S/N Increase Req. at $2F$ b/
Ideal binary	1					0	0
1 ($n = 2$)	1	1				43	2.1 dB
2 ($n = 3$)	1	2	1			40	6.0 dB
3 ($n = 3$)	2	1	−1			38	1.2 dB 7.2 dB (precode
4 ($n = 3$)	1	0	−1			15	2.1 dB
5 ($n = 5$)	−1	0	2	0	−1	8	6.0 dB

and receiver. Consequently the ratio of the area under $H(f)$ relati to the area for the binary reference system gives a direct meas of the required increase in signal power. The area for each $H(f$ given in Table I. The "decision distance" is unity in all ca except two for Class 3 when precoding is not used, since k_1 = contributing a 6-dB improvement to the performance of this p ticular system [10]. The performance figures appear in Table

The speed tolerances listed in Table II may give some indicat of other allowable departures from the prescribed transfer functi Further analysis is under way, including simulation of typi channel impairments superimposed on $H(f)$. For Classes 4 and for example, this will test the practicability of a data baseba format achieving very efficient band utilization without dc tra mission.

The virtues of the channels described are mainly that they aff relatively simple means of attaining $2F$ bps in bandwidth F w relatively gentle filter cutoffs, and that one can choose from number of differently tapered spectral distributions which may advantageous in specific applications. Instead of the basic ($2\pi Ft)/2\pi Ft$ impulse, any other functions satisfying Nyqui first criterion [7] may be used as channel response compone An infinite number of such functions exist, their bandwidth rang from F to $2F$.

Acknowledgment

The author acknowledges the contributions of F. K. Becker a H. O. Burton in clarifying some of the preceding concepts.

References

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[2] A. P. Brogle, "A new transmission method for pulse-code modula communication systems," *IRE Trans. on Communications Systems*, CS-8, pp. 155–160, September 1960.
[3] O. E. Ringelhaan, "System for transmission of binary information at tv the normal rate," U. S. Patent 3 162 724, December 1964.
[4] A. Lender, "The duobinary technique for high-speed data transmissic *IEEE Trans. on Communication and Electronics*, vol. 82, pp. 214– May 1963. See also A. Lender, "Correlative digital communication te niques," *IEEE Trans. on Communication Technology*, vol. COM-12, 128–135, December 1964.
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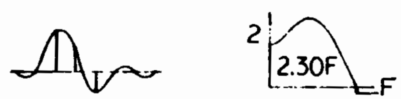
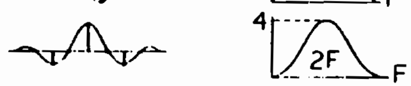
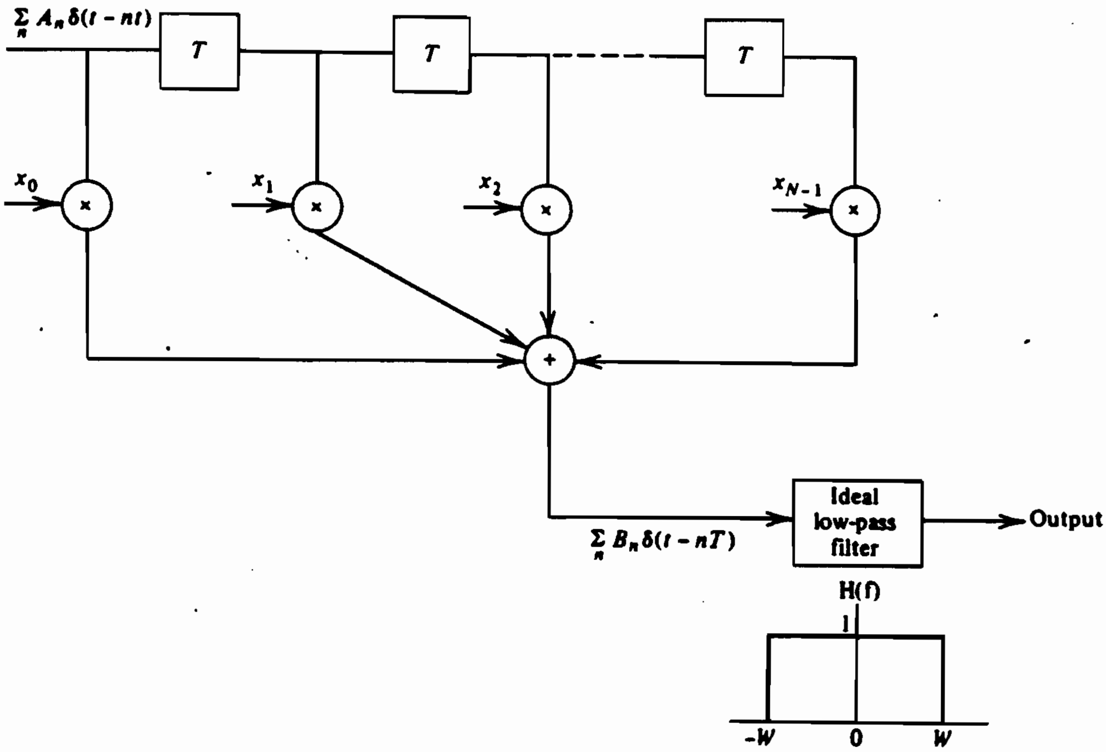
[7] H. Nyquist, "Certain topics in telegraph transmission theory," *Tr AIEE*, vol. 47, pp. 617–644, April 1928.
[8] L. A. McColl, "Signalling method and apparatus," U. S. Patent 2 056 October 1936.
[9] W. R. Bennett and J. R. Davey, *Data Transmission*. New York: McGr Hill, 1965, ch. 7. See also, S. Habib, internal communication.
[10] R. L. Townsend, "A class of partial response systems," internal c munication.

TABLE I

Class	k_1	k_2	k_3	k_4	k_5	$h(t)$	$H(f)$* $[0 < f < F]$	No. of Rec. Levels
Binary (ideal)	1						1	2
1 $(n = 2)$	1	1					$2 \cos \frac{\pi f}{2F}$	3
2 $(n = 3)$	1	2	1				$4 \cos^2 \frac{\pi f}{2F}$	5
3 $(n = 3)$	2	1	−1				$2 + \cos \frac{\pi}{F} f - \cos \frac{2\pi f}{F} + j\left[\sin \frac{\pi}{F} f - \sin \frac{2\pi f}{F}\right]$	5
4 $(n = 3)$	1	0	−1				$2 \sin \pi \frac{f}{F}$	3
5 $(n = 5)$	−1	0	2	0	−1		$4 \sin^2 \pi \frac{f}{F}$	5

* Entry in figure is area under curve.

18F

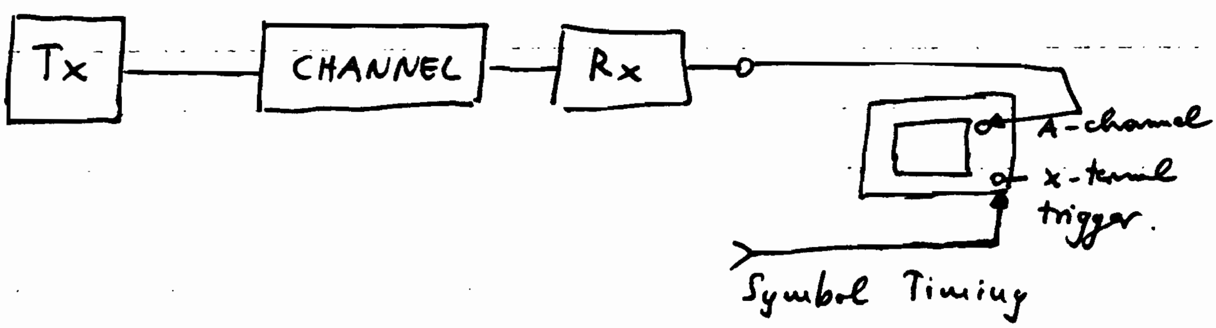


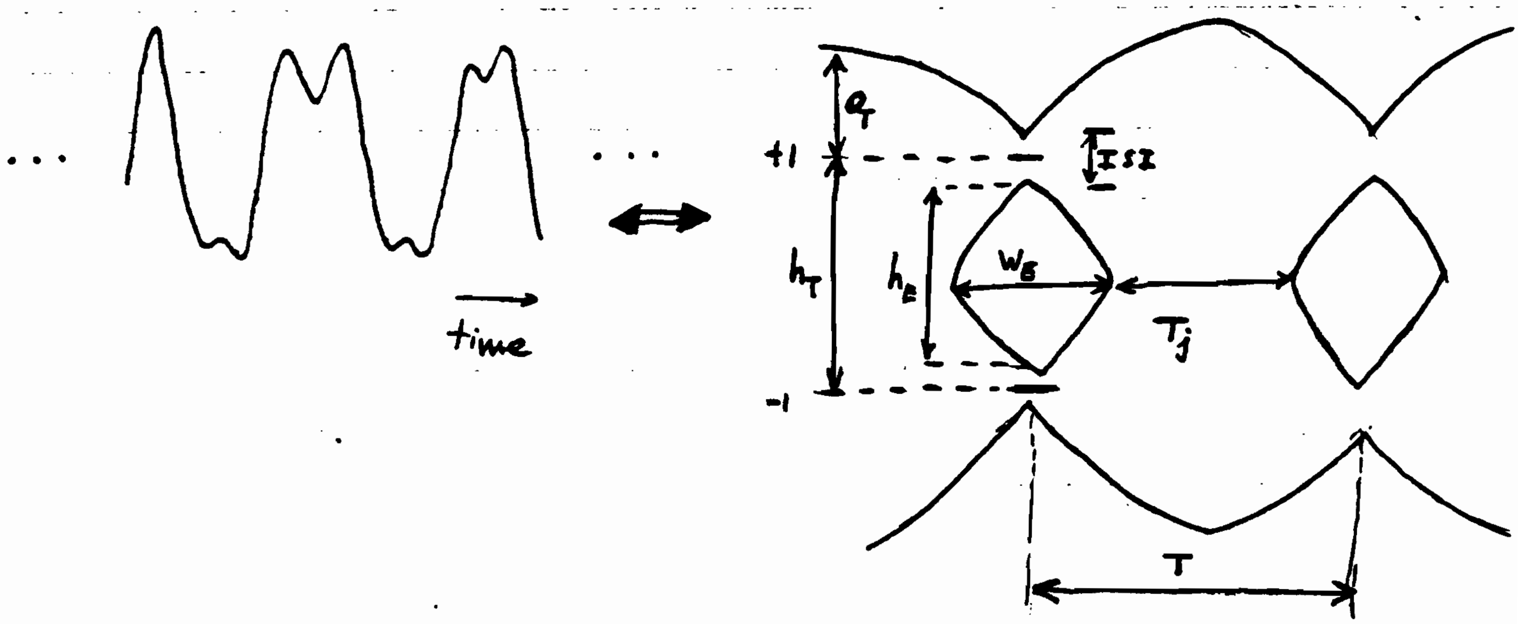
Implementation: Correlative Encoder.

EYE DIAGRAMS.

- Imperfections ⟶ degradation
  - (Filtering, nonlinearities, carrier and symbol recovery)

- Graphical illustration of this degradation : Eye-diagram
  - Quick check of the performance of a modem in the field
  - Useful in the analysis and simulation.

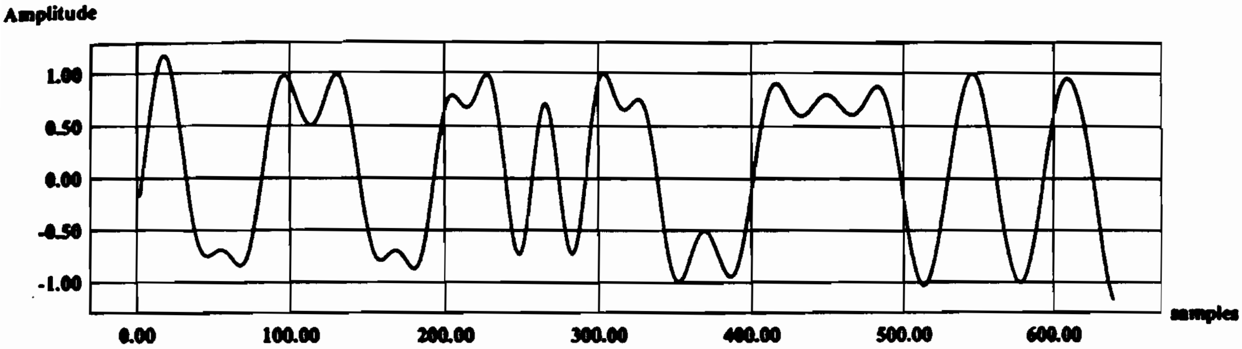




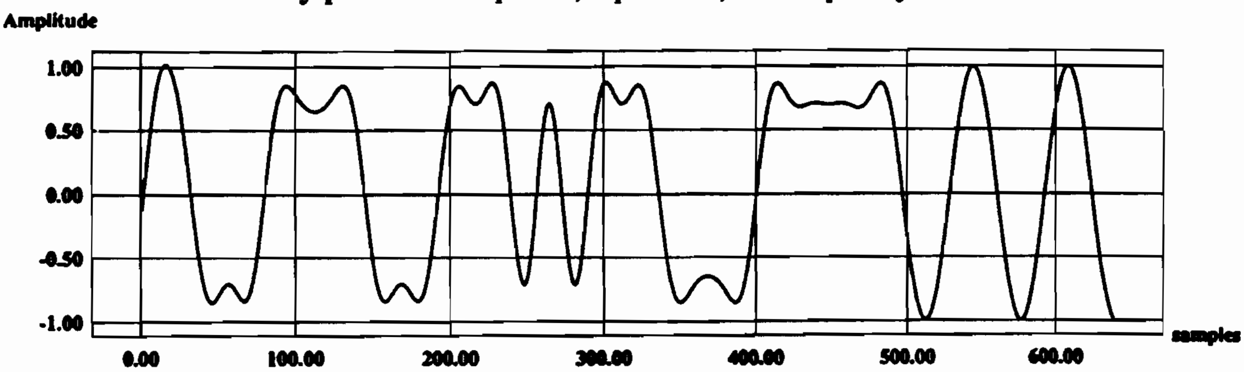
h_E = eye height ; w_E = eye width
O_T = overshoot
T_j = jitter

(20)

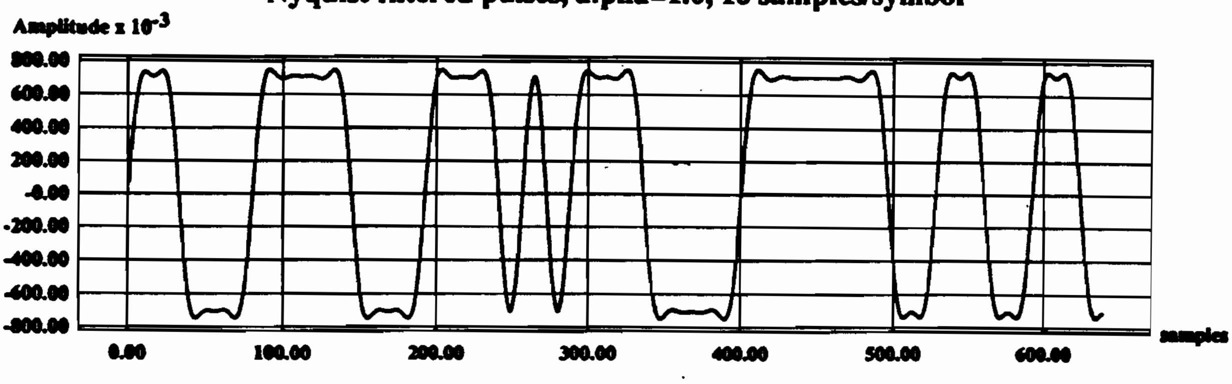
Nyquist filtered pulses, alpha=0.2, 16 samples/symbol

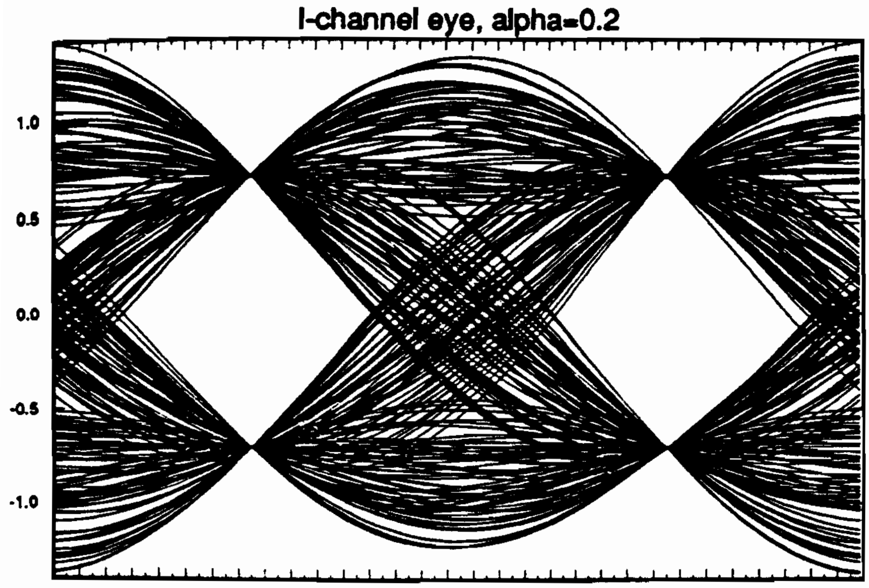


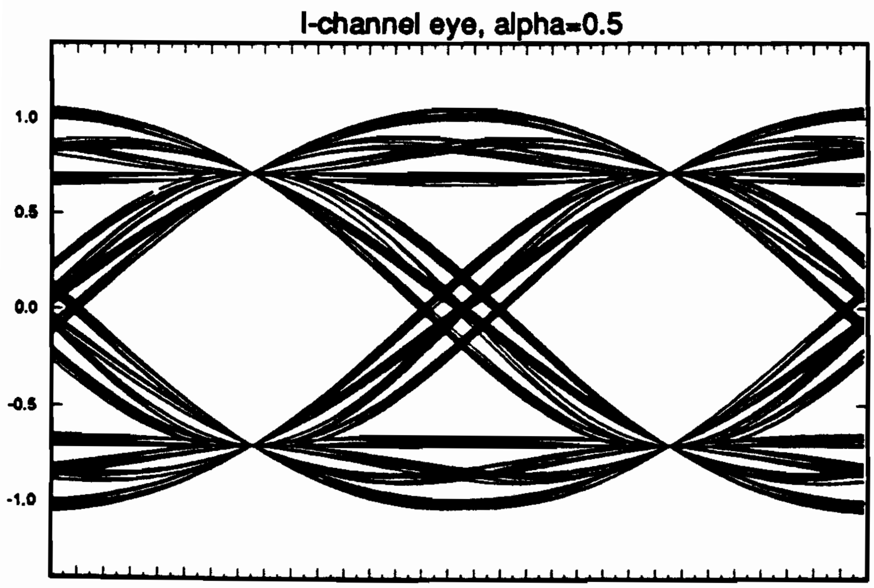
Nyquist filtered pulses, alpha=0.5, 16 samples/symbol

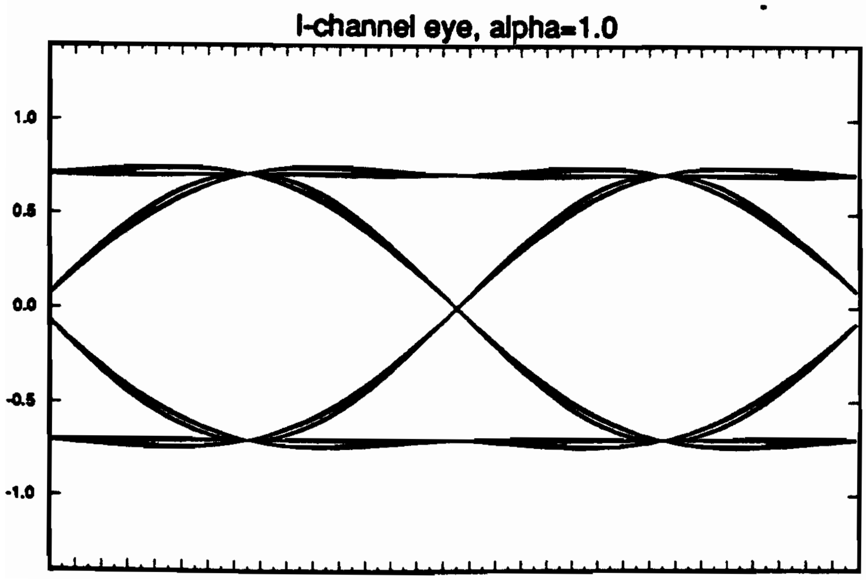


Nyquist filtered pulses, alpha=1.0, 16 samples/symbol



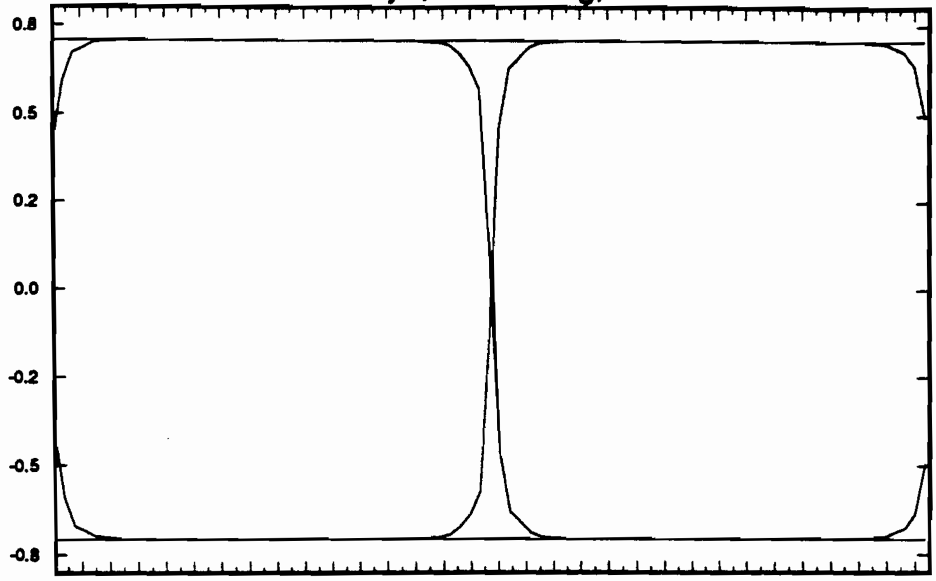

I-channel eye, alpha=0.2
1.0
0.5
0.0
-0.5
-1.0


I-channel eye, alpha=0.5
1.0
0.5
0.0
-0.5
-1.0

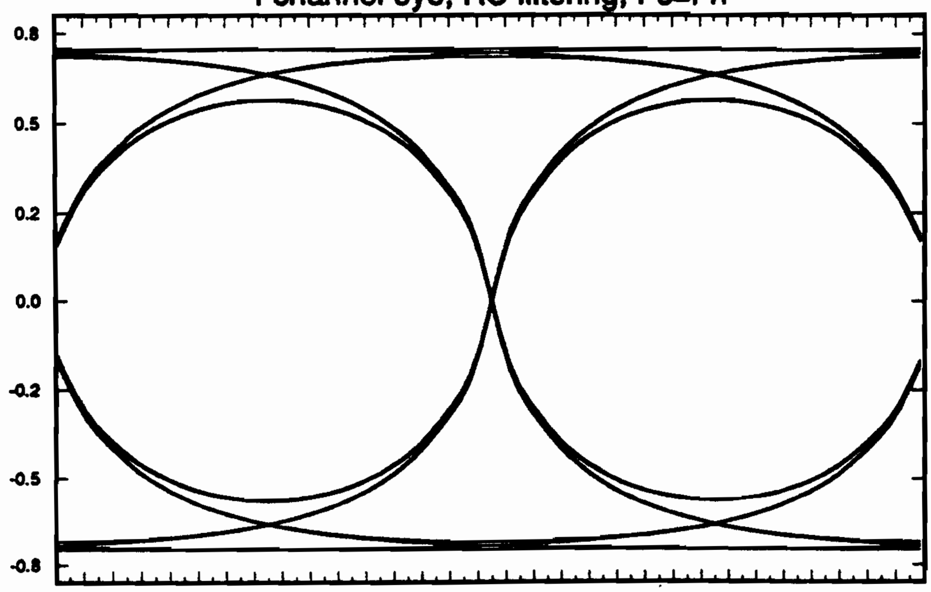

I-channel eye, alpha=1.0
1.0
0.5
0.0
-0.5
-1.0

21A

I-channel eye, RC filtering, Fc=10*Fn



I-channel eye, RC filtering, Fc=Fn



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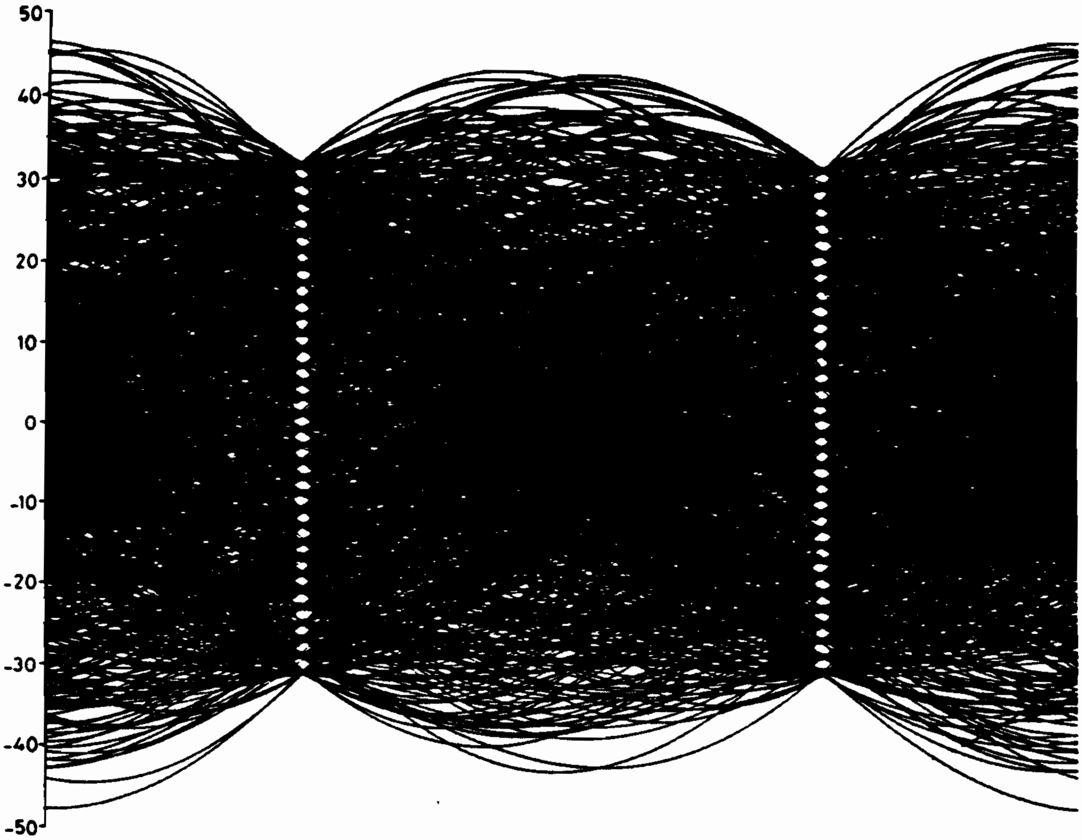


Fig. 3 *Eye diagram of the in-phase channel of the 1024-QAM using $\alpha = 0.2$ raised cosine filters and a channel with parabolic amplitude distortion*

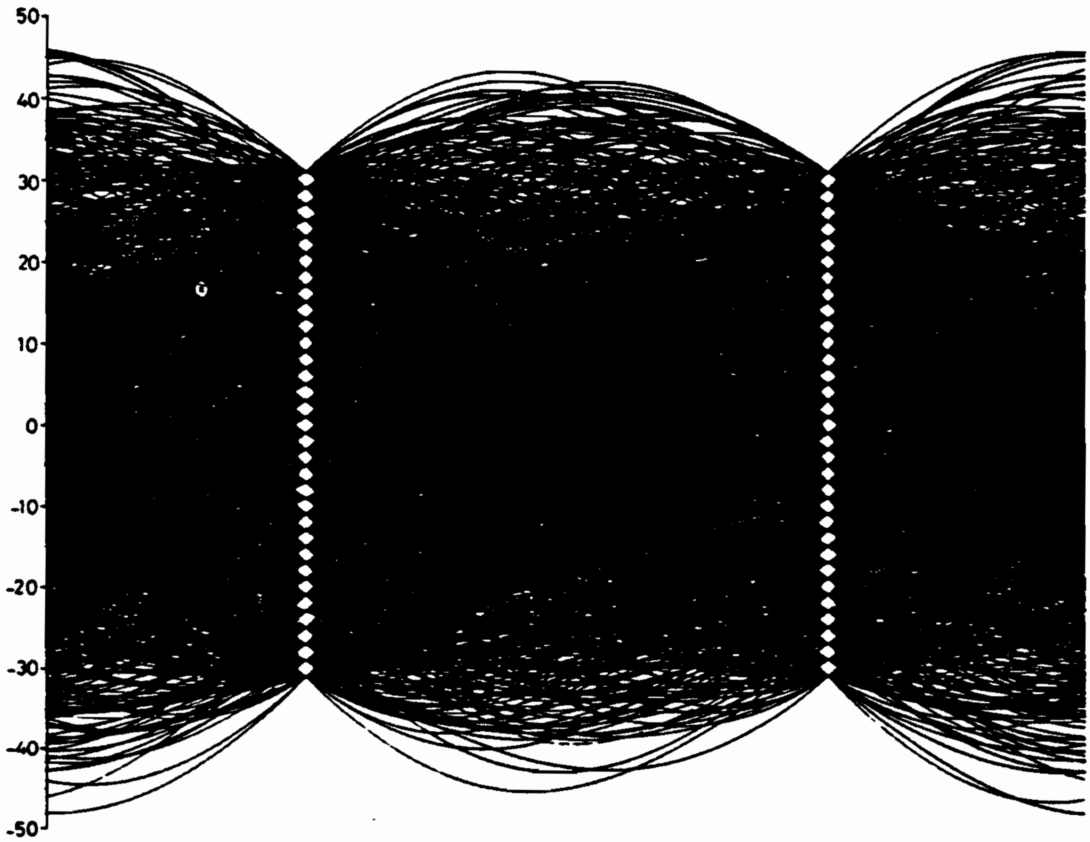
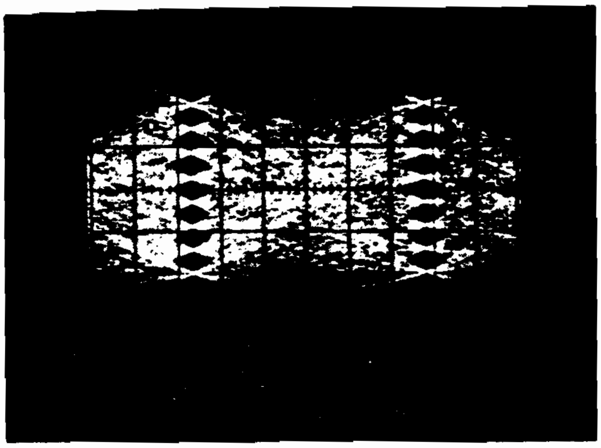


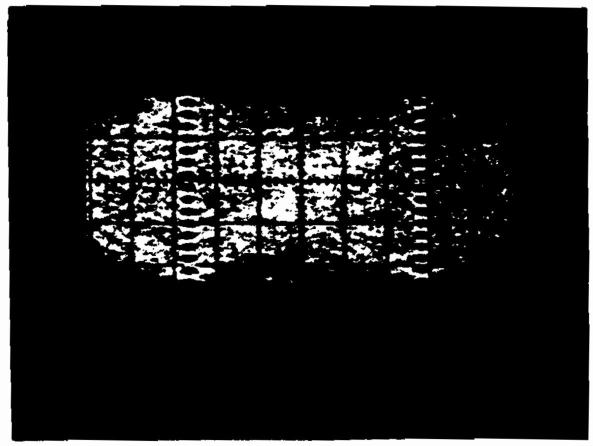
Fig. 2 *Eye diagram of the in-phase channel of the 1024-QAM system using raised cosine filters with a roll-off factor of $\alpha = 0.2$*

Ref. P. Mathiopoulos, H. Ohnishi and K. Feher, "Study of 1024-QAM system performance in the presence of filtering imperfections," IEE Proc., Pt I, Apr'e 1990

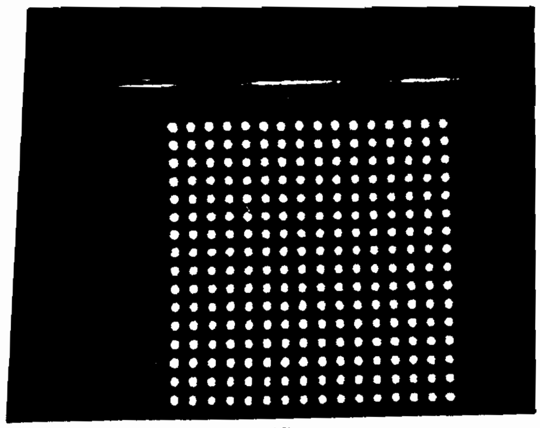
EXPERIMENTAL EYES & STATE-SPACE DIAGRAMS

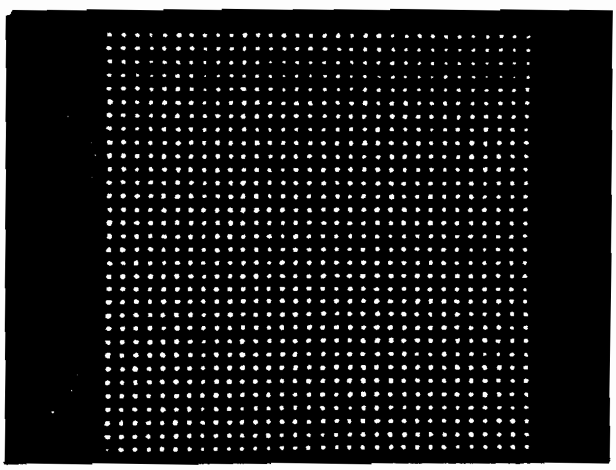


64-QAM

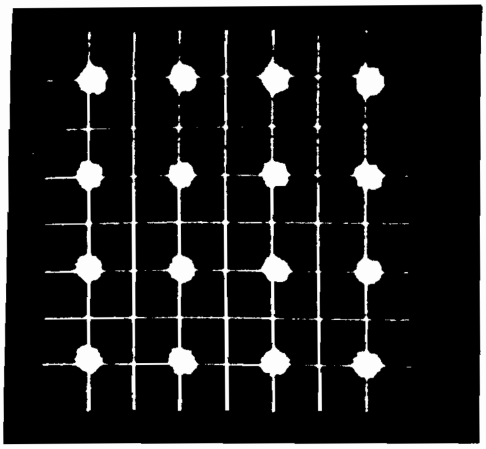


256-QAM

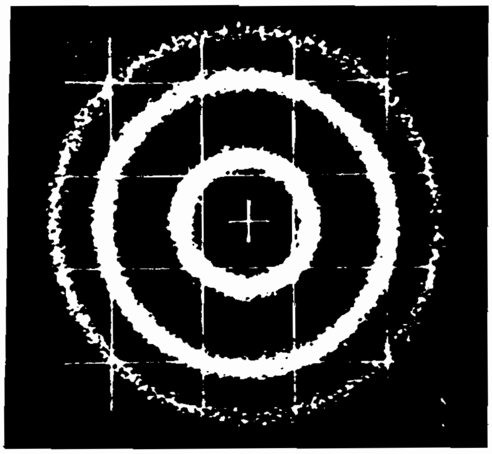




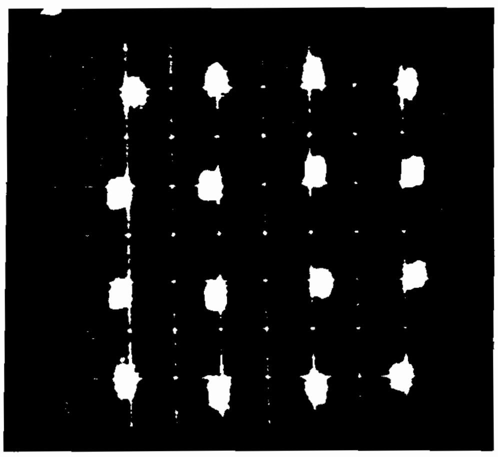
1024-QAM



(a)



(b)

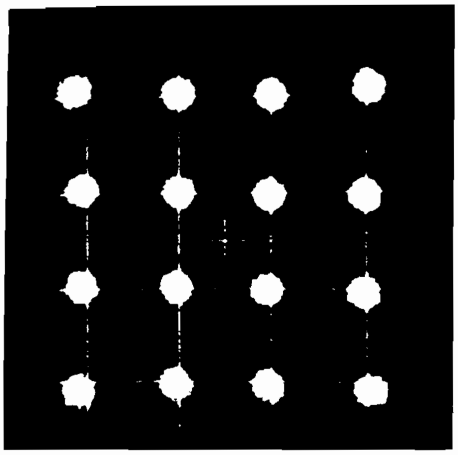


(c)

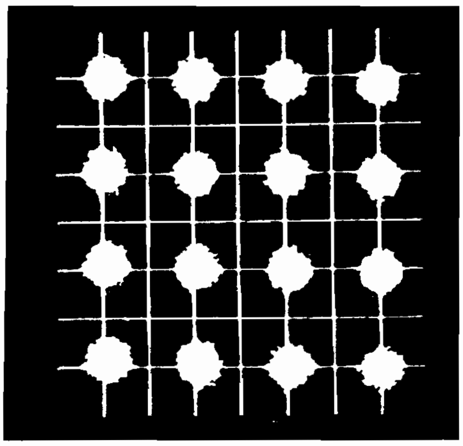
Figure 7.22 16-QAM constellation with geometric defects due to specific faults. (a) 16-QAM radio: Normal constellation plus eye-closure data. (b) Receiver out of lock. (c) 16-QAM radio: 3-dB transmitter power overdrive (AM-AM and AM-PM).

(continued)

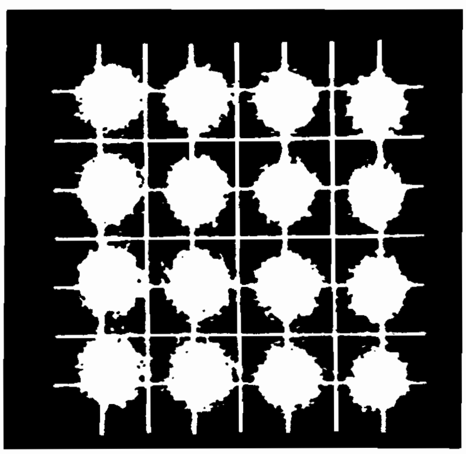
Ref. Feher/Eng. of HP, Telecommunications, Measurement Analysis and Instrumentation, , Prentice Hall, 1987.



(d)



(e)

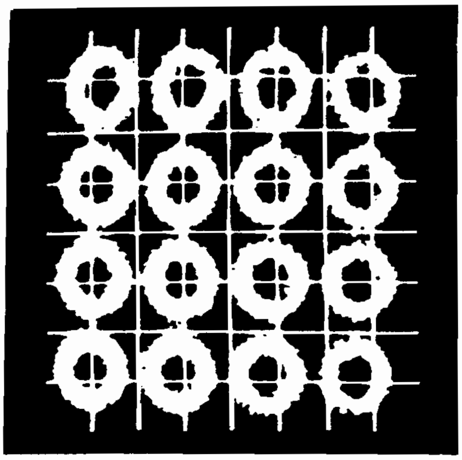


(f)

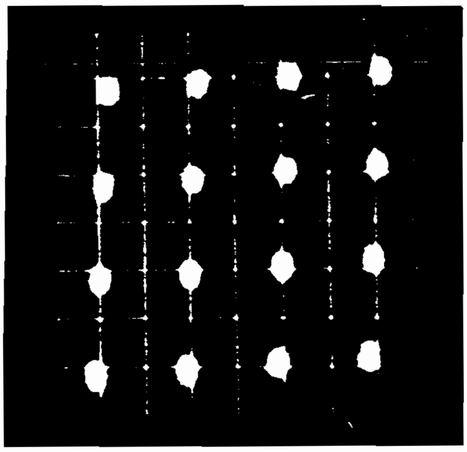
Figure 7.22 (continued) (d) Expansion due to underdrive of TWT. (e) 16-QAM radio: C/N ratio 20 dB (setup using 3708A). (f) 16-QAM radio: C/N ratio 15 dB (setup using 3708A).

(continued)

25



(g)



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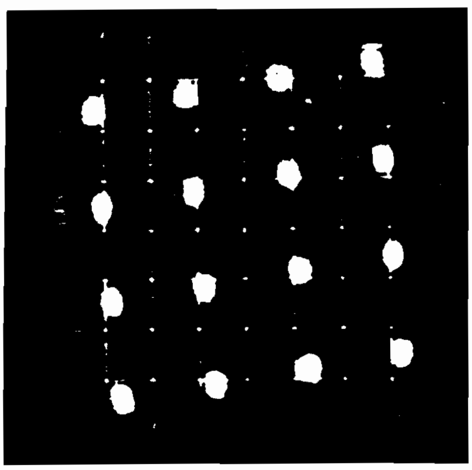
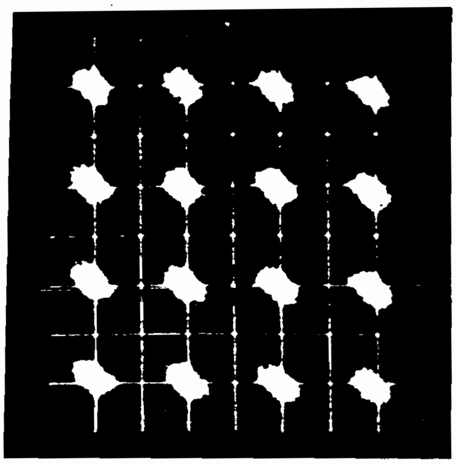


(i)

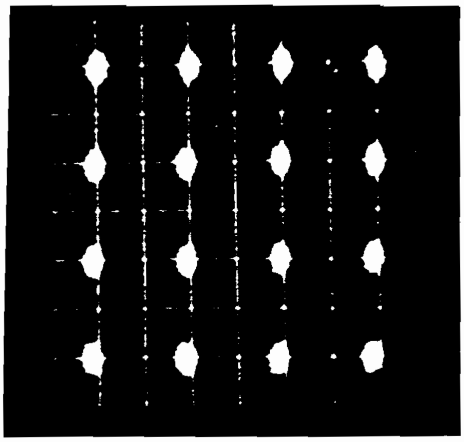
Figure 7.22 (continued) (g) 16-QAM radio: interferer tone present. (h) 16-QAM radio: recovered carrier *I* vs. *Q* phase nonorthogonal. (i) 16-QAM radio: carrier recovery loop lock misadjusted.

(continued)

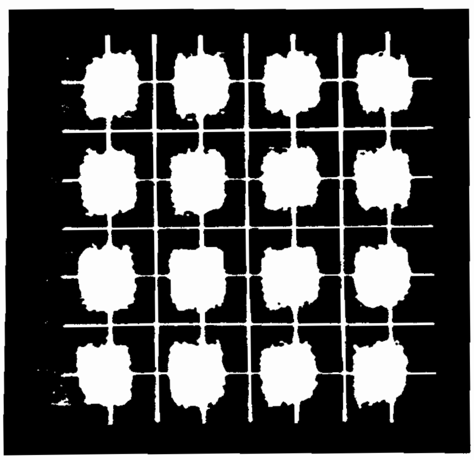
25A



(j)



(k)



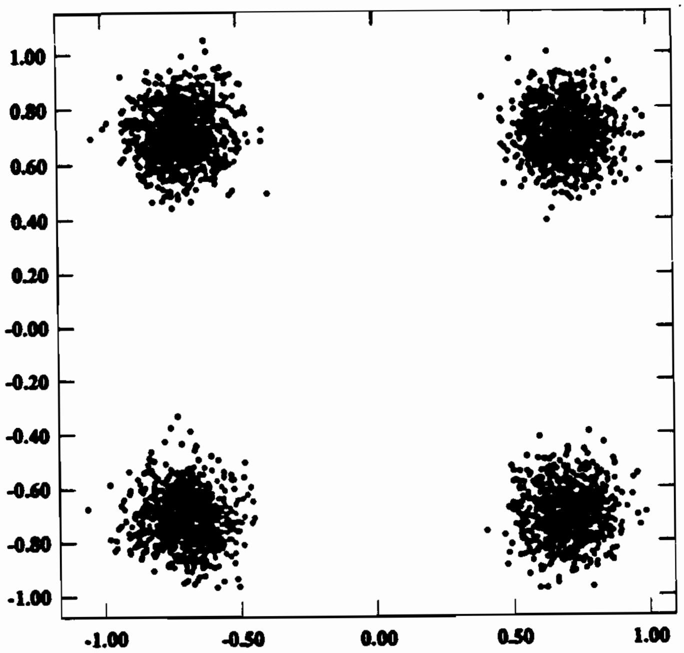
(l)

Figure 7.22 (continued) (j) 16-QAM radio: multipath fade (+6 dB slope, 50 MHz to 90 MHz). (k) 16-QAM radio: multipath fade (6 dB symmetrical notch, 70 MHz). (l) 16-QAM radio: multipath fade (10 dB symmetrical notch, 70 MHz).

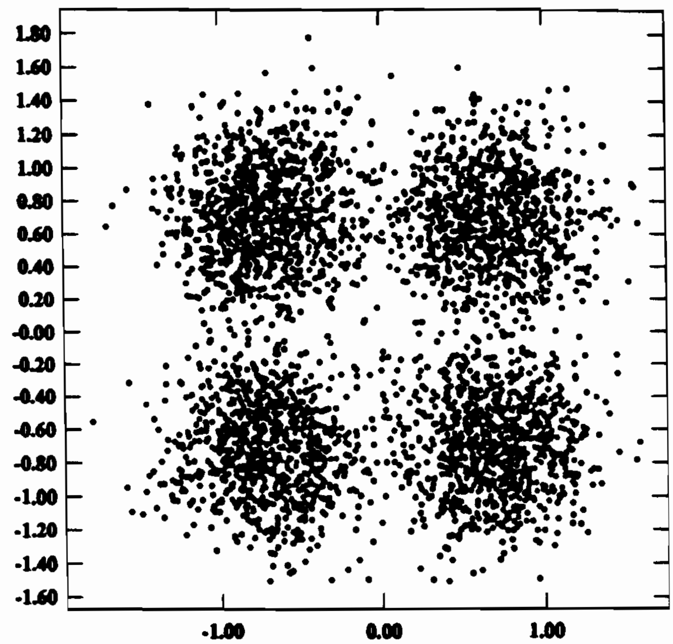
TABLE 7.7 16-QAM Degradation Allocation at BER = 10^{-8} (from [Bates, 7.7] with permission of the IEEE)

	16 QAM
Quadrative carrier offset amplitude imbalance	9%
Nyquist filters	15%
Timing recovery	6%
Carrier recovery	21%
Power amplifier	15%
Radio	2%
Total eye closure	68%
Resulting eye opening	32%

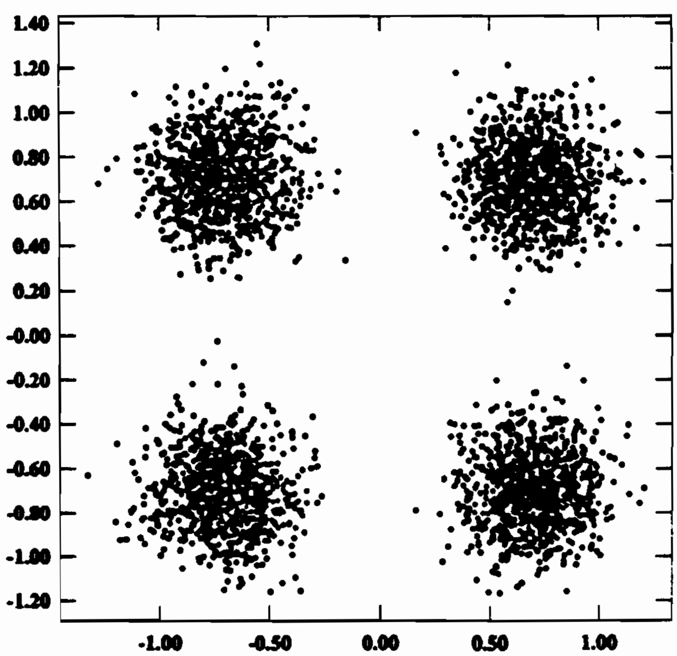
QPSK, 4K symbols, alpha=0.35, Eb/No=15dB



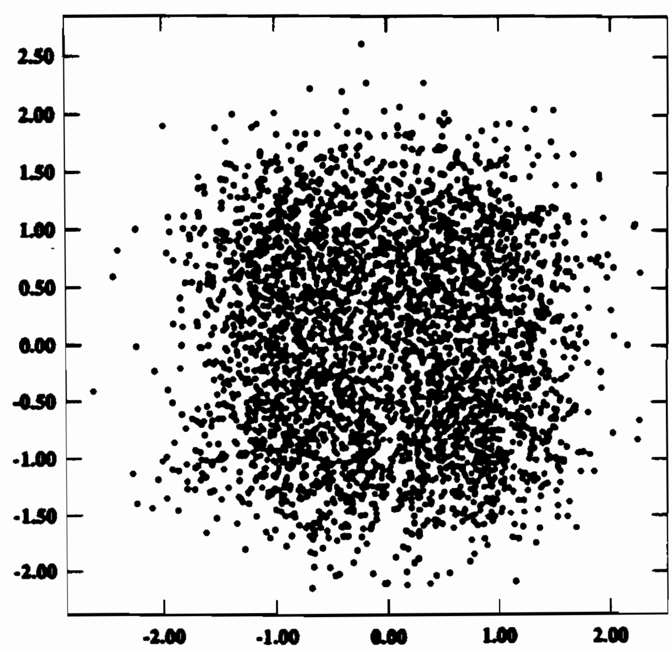
QPSK, 4K symbols, alpha=0.35, Eb/No=5dB



QPSK, 4K symbols, alpha=0.35, Eb/No=10dB

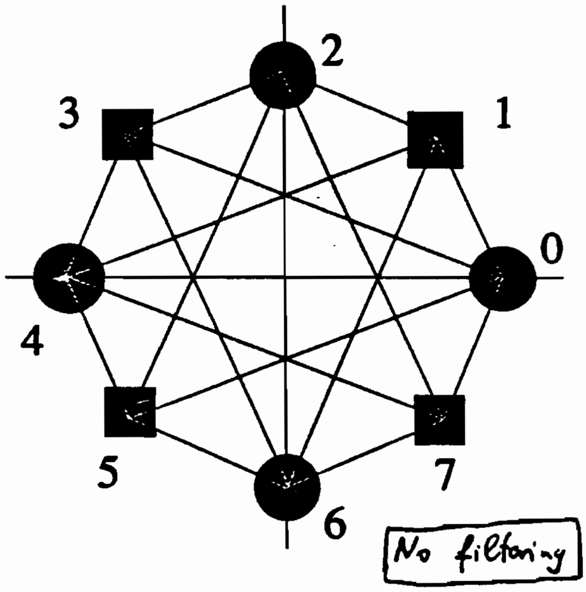


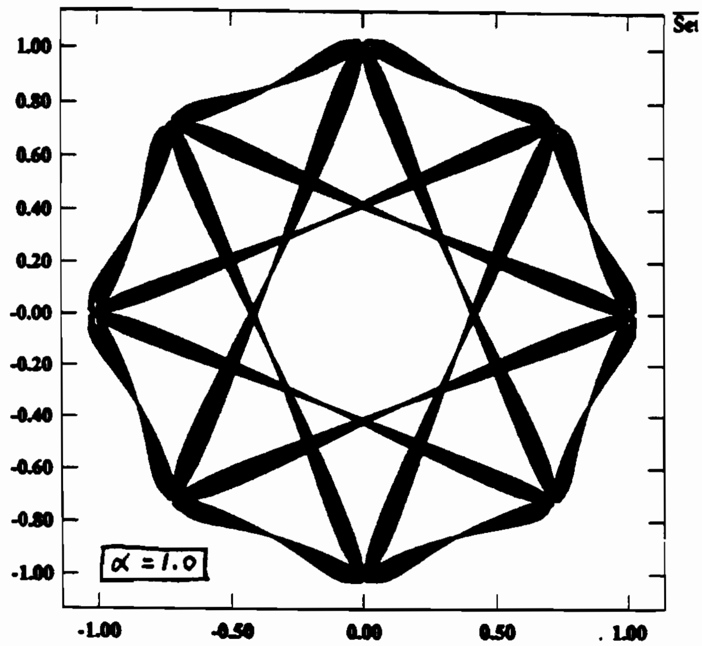
QPSK,4K symbols, alpha=0.35, Eb/No=0dB



$\frac{\pi}{4}$ - QPSK : State-space diagrams

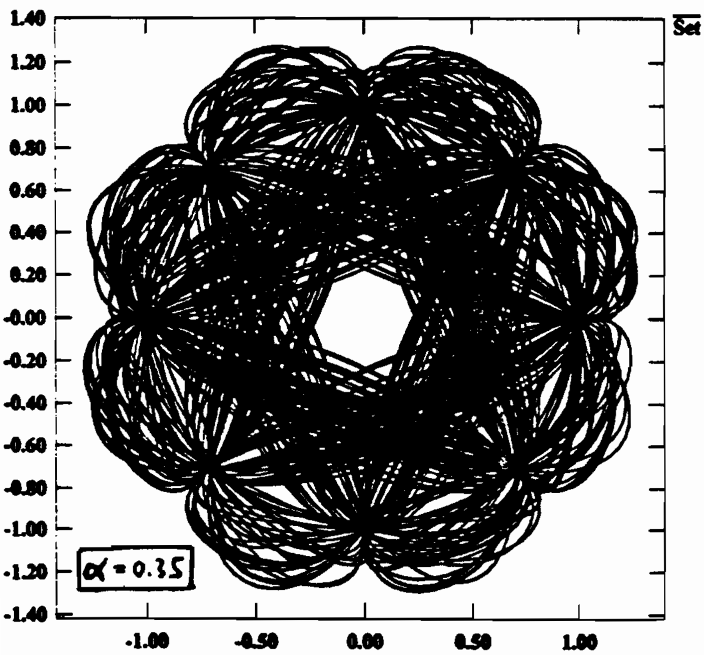
$$\Delta\theta \in \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$$





$$\theta_i = \theta_{i-1} + \Delta\theta_i$$

Tx-phase



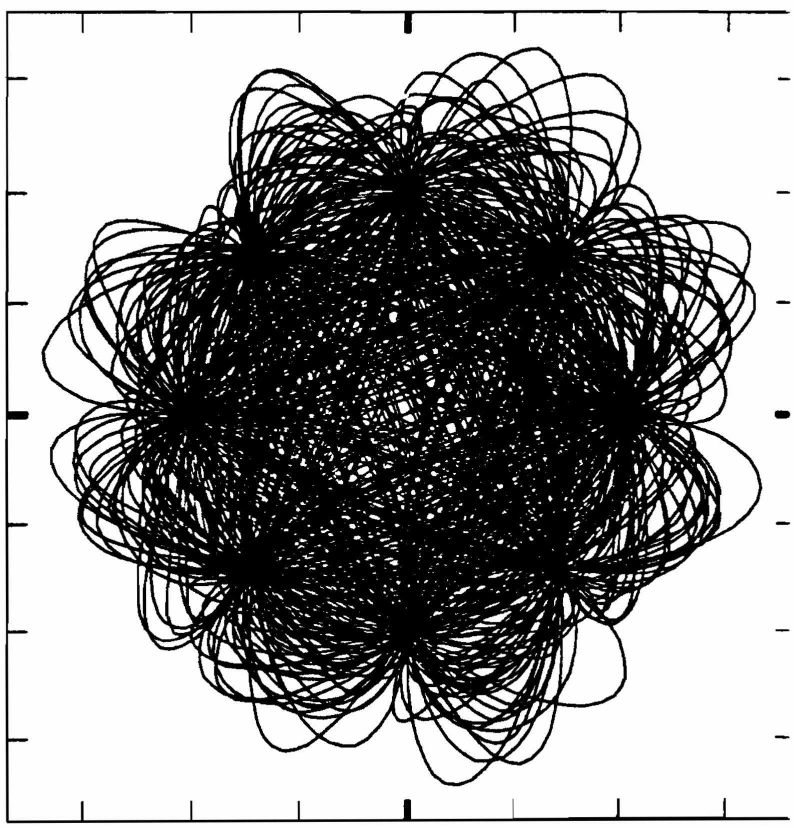
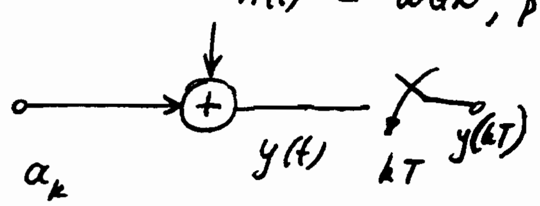


Figure 10 State-space diagram of a computer-simulated π/4–QPSK system employing raised cosine filters with an excess bandwidth of 35% and which is operated in the presence of CCI (C/I= 55 dB) and Gaussian noise (C/N=60 dB). The number of interferers is equal to 4.

ADDITIVE WHITE GAUSSIAN NOISE (AWGN): FILTER CONSIDERATIONS

Example 1

(No filter) $n(t)$ – WGN, psd: $\frac{N_0}{2}$



$(1/T)$

$$y_k = \hat{a}_k + n_k$$

$$[\, y_k = y(kT) \; ; \; n_k = n(kT) \,]$$

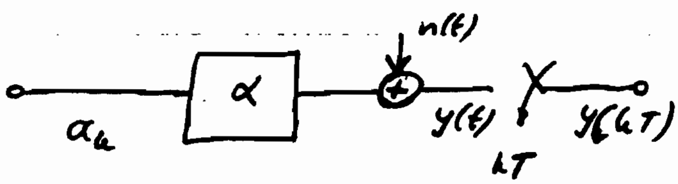
① No filtering $\Rightarrow$ $\hat{a}_k = a_k$

② Power of $n_k = P_n = E\{|n_k|^2\} = \int_{-\infty}^{\infty} \frac{N_0}{2}\, df \rightarrow \infty$

$\therefore$ a_k is totally hidden in noise!!

Example 2

(Nyquist filter at the Tx)

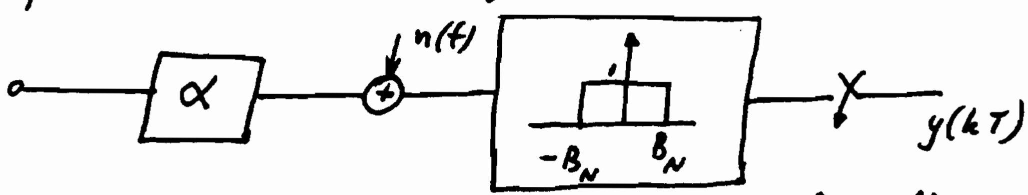


No ISI $\longrightarrow$ $\hat{a}_k = a_k$

(but does not help!)

Example 3

(Rx filter to reduce noise)



$$P_n = \frac{N_0}{2} \times 2B_N = N_0 B_N$$

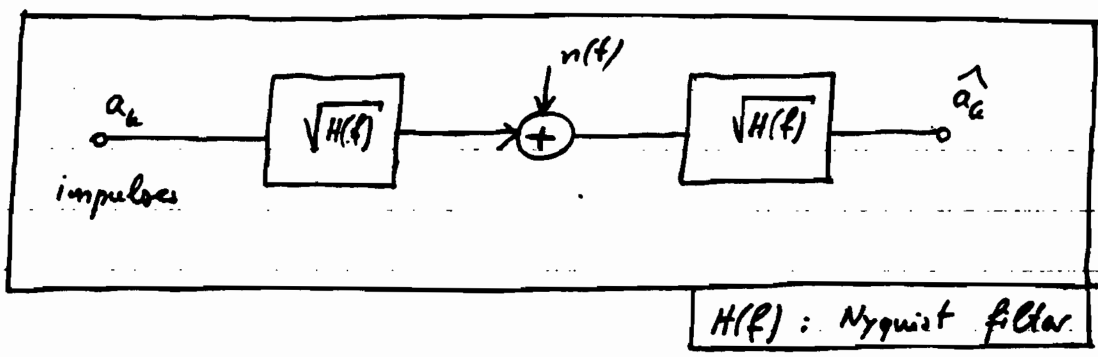
$\therefore$ The noise power which corrupts the received sample depends upon B_N.

Rx filter reduces the power of the noise
BUT at the same time it should not distort (in the Nyquist sense) i.e., should not create any ISI at the sampling instant.

Increase Rx Bandwidth → (reduces ISI)

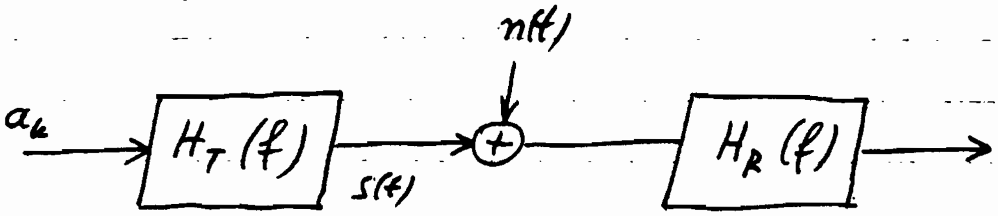
Decrease Rx Bandwidth ← (reduces noise power)

OPTIMAL



(Optimal: in the sense that the signal-to-noise radio is maximized.)

Proof



$$h_T(t) \longleftrightarrow H_T(f)$$

$$h_R(t) \longleftrightarrow H_R(f)$$

$$\frac{1}{T} = \text{symbol rate}$$

$$H(f) = H_T(f)\, H_R(f) = \begin{cases} \text{Nyquist} \\ \text{Filter} \end{cases}$$

$$s(t) = \sum_k a_k\, h_T(t - kT)$$

$$P_s = \left\langle \lim_{K\to\infty} \frac{1}{2KT} \int_{-KT}^{KT} \left[\sum_{k=-K}^{K} a_k\, h_T(t-kT)\right]^2 dt \right\rangle$$

$$= \lim_{K\to\infty} \frac{1}{2KT} \sum_n \sum_m \langle a_n a_m \rangle \int_{-KT}^{KT} h_T(t-nT)\, h_T(t-mT)\, dt$$

a_n, a_m are uncorrelated for $n \neq m$, i.e.,

$$\langle a_n a_m \rangle = \begin{cases} 0 & n \neq m \\ \overline{a^2} & n = m \end{cases}$$

$$\therefore \quad P_s = \lim_{K\to\infty} \frac{\overline{a^2}}{2KT} \sum_{k=-K}^{K} \int_{-KT}^{KT} h_T^2(t-kT)\,dt$$

$$= \frac{\overline{a^2}}{T} \int_{-\infty}^{\infty} h_T^2(t)\,dt$$

Parseval's theorem

$$\int_{-\infty}^{\infty} h^2(t)\,dt = \int_{-\infty}^{\infty} |H(f)|^2\,df$$

$$\therefore \quad \boxed{P_s = \frac{\overline{a^2}}{T} \int_{-\infty}^{\infty} |H_T(f)|^2\,df}$$

Since $H(f) = H_T(f)\, H_R(f)$ has to satisfy Nyquist $\Longrightarrow$ the signal power at the output of the Rx is <u>invariant</u> under changes of $H_R(f)$.

$\therefore$ The signal-to-noise ratio is maximized by minimizing the noise power at the output of the Rx filter.

$$P_N = \int_{-\infty}^{\infty} N(f)\, |H_R(f)|^2\, df$$

under the conditions that

1. $$P_S = \frac{\overline{a^2}}{T} \int_{-\infty}^{\infty} |H_T(f)|^2\, df$$

2. $$P_R = P_S \int |H_R(f)|^2\, df = \text{constant}$$

Using techniques of the variational analysis we can form the following functional V

$$V = \lambda P_s + P_N$$

λ = Lagrange multipler.

$$V = \int_{-\infty}^{\infty} \left[\lambda \frac{\overline{a^2}}{T} \frac{|H(f)|^2}{|H_R(f)|^2} + N(f)\, |H_R(f)|^2 \right] df$$

Minimization of V is achieved on the stationary point of the above equation, i.e.,

$$H_R(f) = \frac{\sqrt{H(f)}}{N^{1/4}(f)}$$

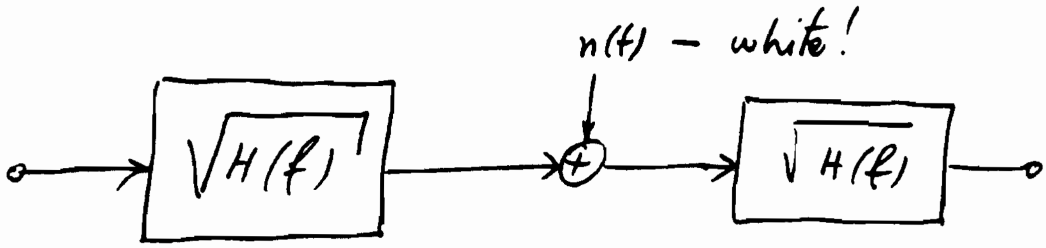
so that

$$H_T(f) = \frac{H(f)\, N^{1/4}(f)}{\sqrt{H(f)}}$$ (real filters).

For flat noise (const. ,) $\Rightarrow N^{1/4}(f) = 1$

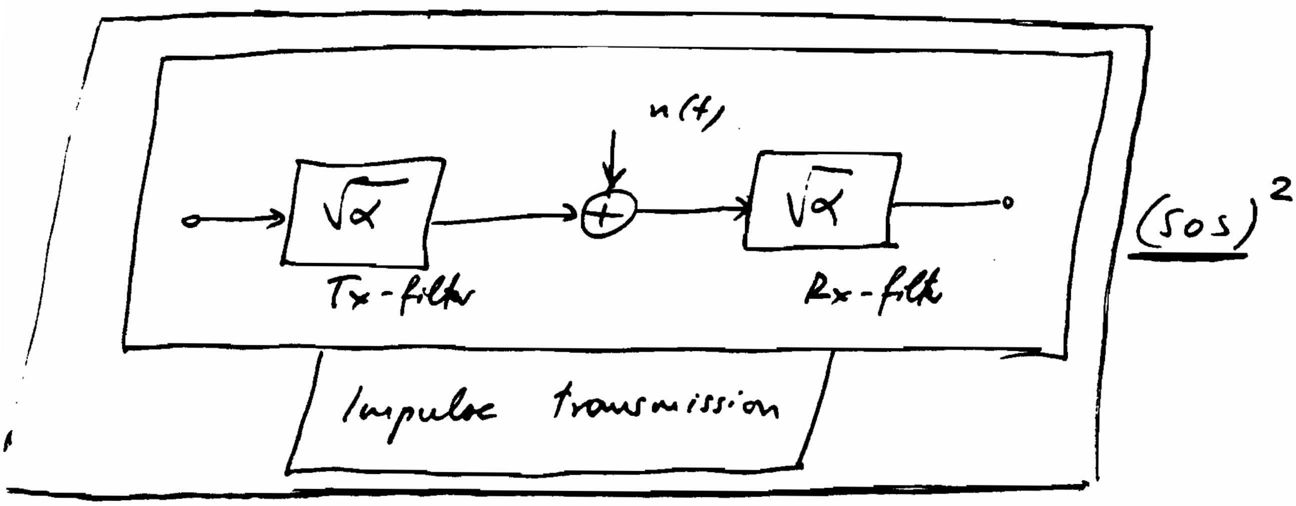
$$\boxed{H_R(f) = H_T(f) = \sqrt{H(f)}}$$

$\therefore$



$H(f)$: Nyquist filter

In practice we raised cosine filter α.



For pulse transmission an extra $\frac{f}{\sin f} = H_E(f)$ filter is needed.

ORTHOGONAL SIGNALS, MATCHED FILTER & CORRELATION RECEIVERS AND ERROR PROBABILITY PERFORMANCE IN AN AWGN CHANNEL

- ORTHOGONAL SIGNALS

Consider N orthonormal functions $\phi_q(t)$, $q = 1, 2, \ldots, N$ over the interval $(0, T]$

$$\int_0^T \phi_i(t)\,\phi_k(t)\,dt = \delta_{ik} = \begin{cases} 1 & \text{for } i = k \\ 0 & \text{"} \ i \neq k. \end{cases}$$

These functions can be used to define a set of M finite energy signals

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\,\phi_j(t) \qquad \begin{array}{l} i = 1, 2, \ldots, M \\ 0 < t \leq T \end{array}$$

with $s_{ij} = \int_0^T s_i(t)\,\phi_j(t)\,dt$

$\therefore$ $s_i(t)$ is completely specified by $\overline{s_i} = (s_{i1}, s_{i2}, \ldots, s_{iN})$ - assuming $\{\phi_j(t)\}$ are known.

$$\text{Energy} = E_i = \int_0^T s_i^2(t)\,dt = \int_0^T \sum_{j=1}^{N} \sum_{k=1}^{N} s_{ij}\,s_{ik}\,\phi_j(t)\,\phi_k(t)\,dt =$$

$$= \sum_{j=1}^{N} \sum_{k=1}^{N} s_{ij}\,s_{ik}\,\delta_{jk} = \sum_{j=1}^{N} s_{ij}^2$$

Alternatively $E_i = \bar{s}_i \cdot \bar{s}_i = |\bar{s}_i|^2$

- $\hat{=}$ dot product ; $\bar{s}_i \cdot \bar{s}_j \triangleq |\bar{s}_i|\,|\bar{s}_j| \cos\theta_{ij}$

Correlation coefficient between the i-th and the j-th signal

$$\rho_{ij} \triangleq \frac{\int_0^T s_i(t)\, s_j(t)\, dt}{\sqrt{E_i E_j}} = \frac{\sum_{k=1}^{N} s_{ik}\, s_{jk}}{\sqrt{E_i E_j}} = \frac{\bar{s}_i \cdot \bar{s}_j}{\sqrt{E_i E_j}}$$

Alternatively: $\rho_{ij} = \cos\theta_{ij}$

θ_{ij} is the angle between the vectors $\bar{s}_i$ and $\bar{s}_j$.

Euclidean distance

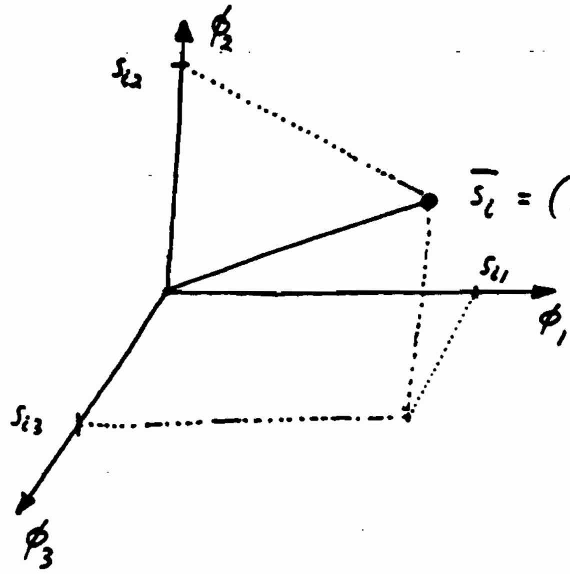
$$d_{ij}^2 \doteq \int_0^T [s_i(t) - s_j(t)]^2\, dt = \sum_{k=1}^{N} s_{ik}^2 + \sum_{k=1}^{N} s_{jk}^2 - 2\sum_{k=1}^{N} s_{ik}\, s_{jk}$$

$$= E_i + E_j - 2\rho_{ij}\sqrt{E_i E_j}$$

Examples

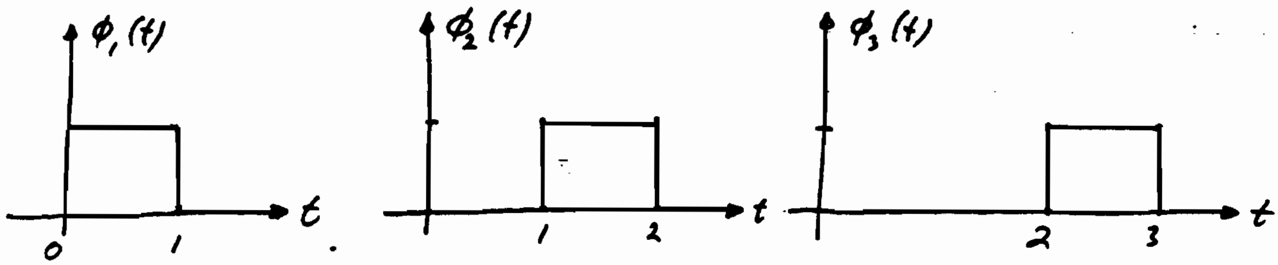
$\boxed{N = 3}$

(a)



3-dimensional space

(b)



$\boxed{N = 2}$ → 2-dimensions

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$$

$0 \le t \le T$

2-dim. → practically any signal representation (PSK, QAM, QPRS ...).

$$s_i(t) = s_{i1}\sqrt{\frac{2}{T}}\cos(2\pi f_c t) - s_{i2}\sqrt{\frac{2}{T}}\sin(2\pi f_c t)$$

$$= \sqrt{\frac{2E}{T}}\cos(2\pi f_c t + \theta_i) \qquad E = s_{i1}^2 + s_{i2}^2$$

$$\theta_i = \arctan\frac{s_{i2}}{s_{i1}}$$

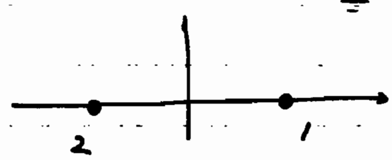
$$\rho_{ij} = \frac{1}{E}\int_0^T s_i(t)s_j(t)\,dt = \ldots = \cos(\theta_i - \theta_j)$$ (assuming that $f_c = k\frac{1}{T}$)

For M-ary PSK systems $\Rightarrow \theta_i = \frac{2\pi}{M}i \Rightarrow \rho_{ij} = \cos\left[\frac{2\pi(i-j)}{M}\right]$

Examples

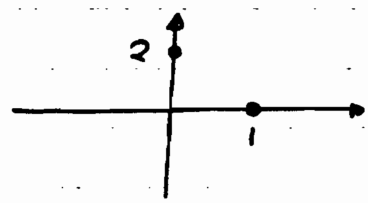
$\boxed{M=2}$

$\rho_{12} = -1$ → (antipodal)



(BPSK)

$\rho_{12} = 0$ → (orthogonal)

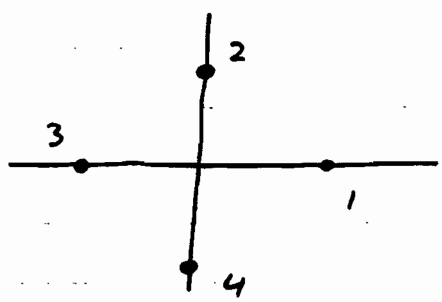


(FSK)

$\boxed{M=4}$

$\rho_{12} = \rho_{23} = \rho_{34} = \rho_{41} = 0$

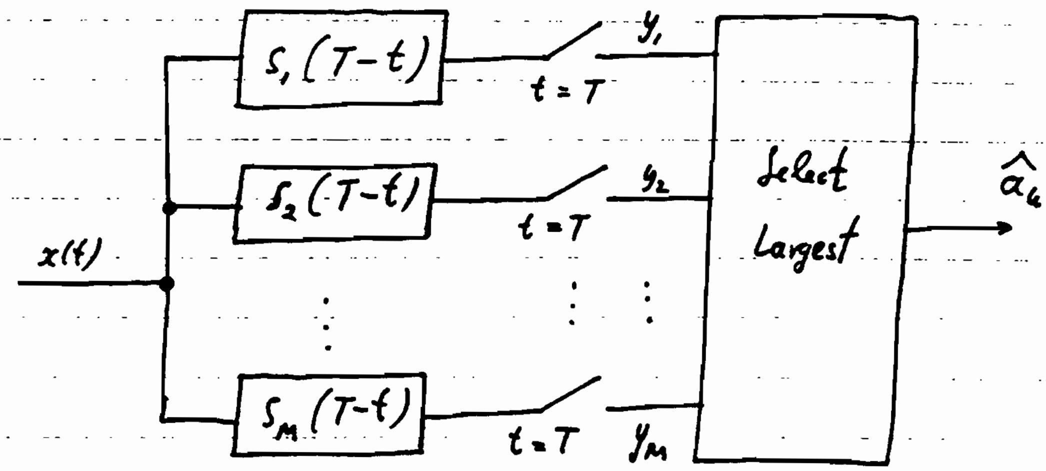
$\rho_{13} = \rho_{24} = -1$



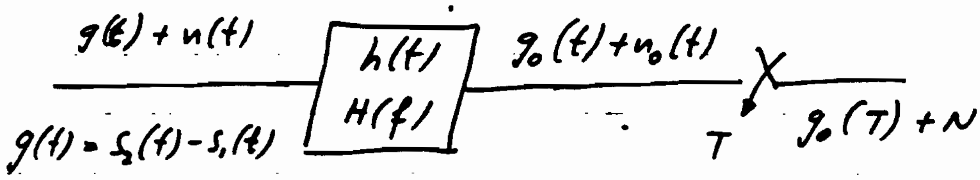
(QPSK)

• MATCHED FILTER RECEIVERS

- AWGN channel
- M-ary signals → $s_1(t), s_2(t), \ldots, s_M(t)$ (equiprobable)
- It can be shown that the following receiver structure minimizes the error probability.



(For $M = 2$ we have



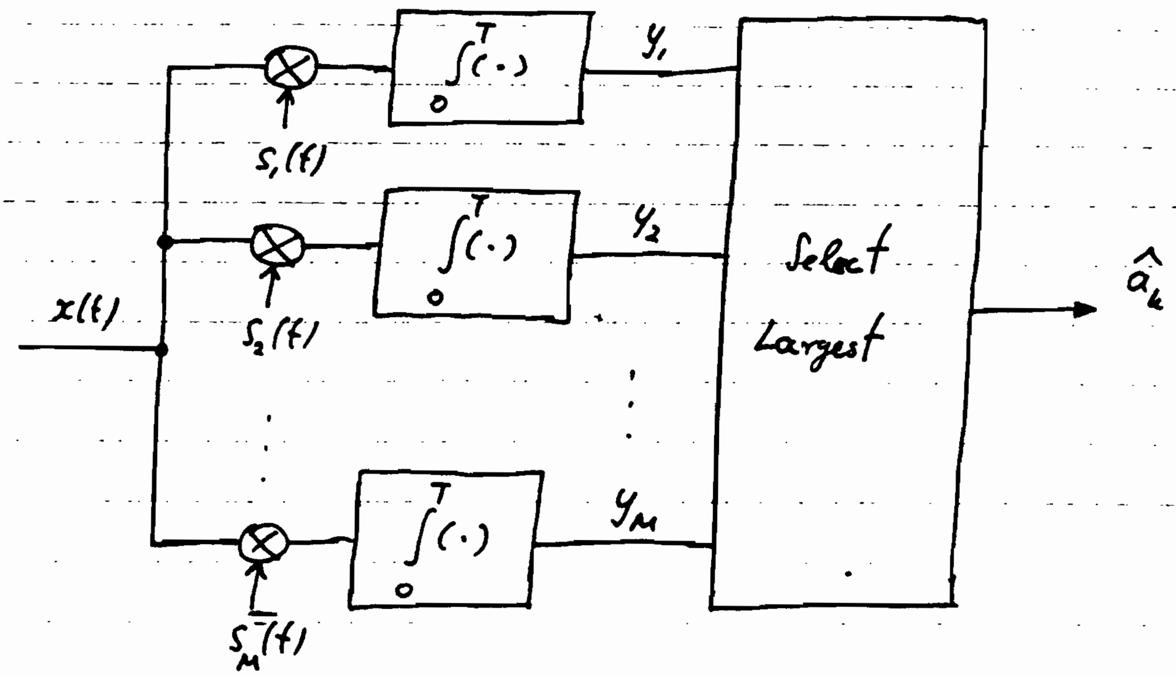
Find $h(t)$ which maximizes $J = \dfrac{s_{02}(T) - s_{01}(T)}{\sigma_0}$)

Proof

Hw

• CORRELATION RECEIVERS

It is easy to show that an equivalent to the "Matched Filter" receiver is the following Correlation Receiver



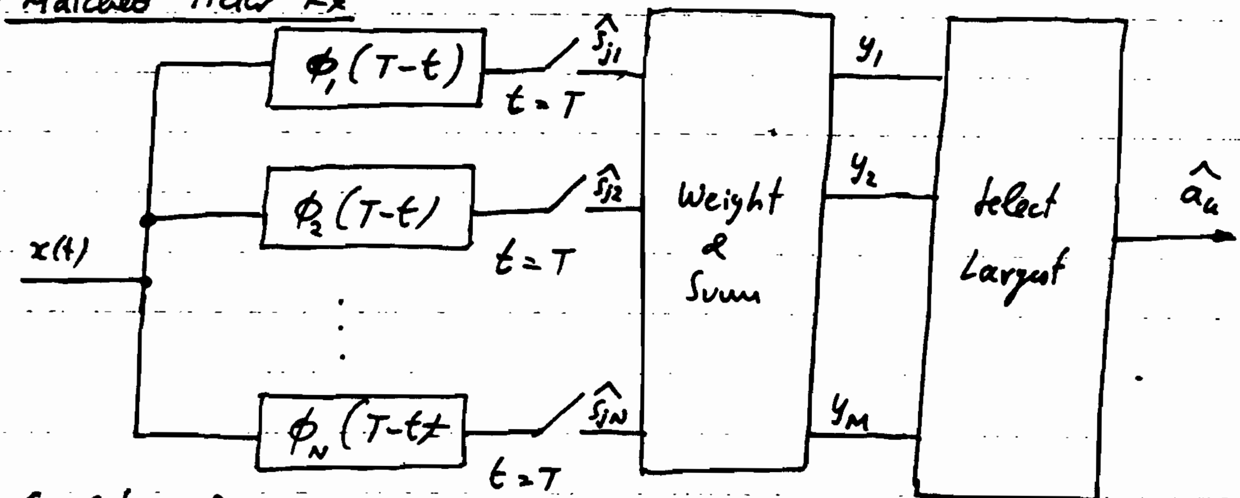
Although its the "same thing", it might have implementation advantages.

- For almost any "practical" digital communication system the transmitted signals can be represented by two orthonormal functions e.g., $\phi_1(t)$ & $\phi_2(t)$.

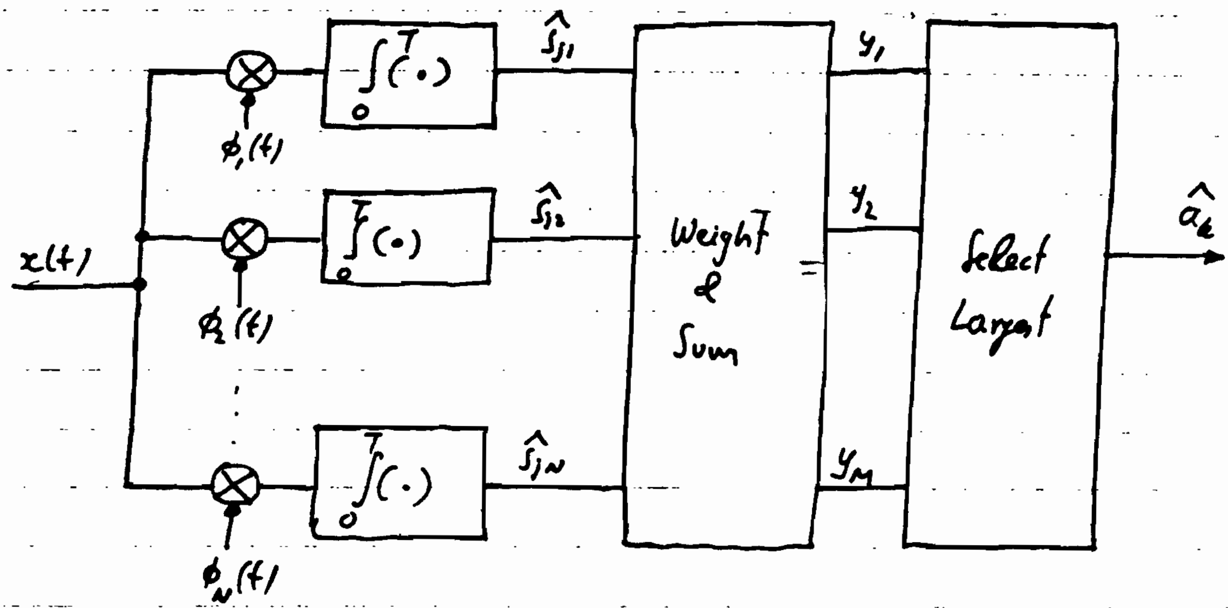
$\Rightarrow$ The previous receivers can be very much simplified by using these two orthonormal functions.

In more general terms: Assume N orthonormal functions.

- Matched Filter Rx



- Correlation Rx



$$s_i(t) = \sum_{j=1}^{N} s_{ij}\,\phi_j(t)$$

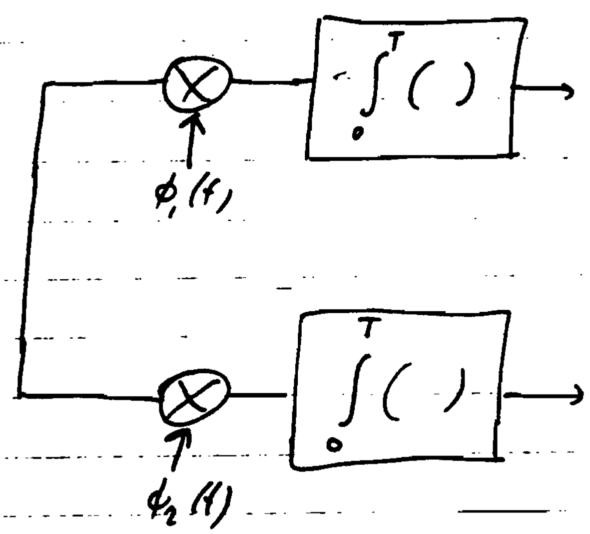
N correlation receivers which produce a set of N coefficients

$$\hat{s}_{jk} \equiv \text{estimate of } s_{jk} \text{ when } s_j(t) \text{ is transmitted}$$
$$= \int_0^T x(t)\,\phi_k(t)\,dt$$

In order to obtain the signal samples y_i, the following mathematical operations are needed.

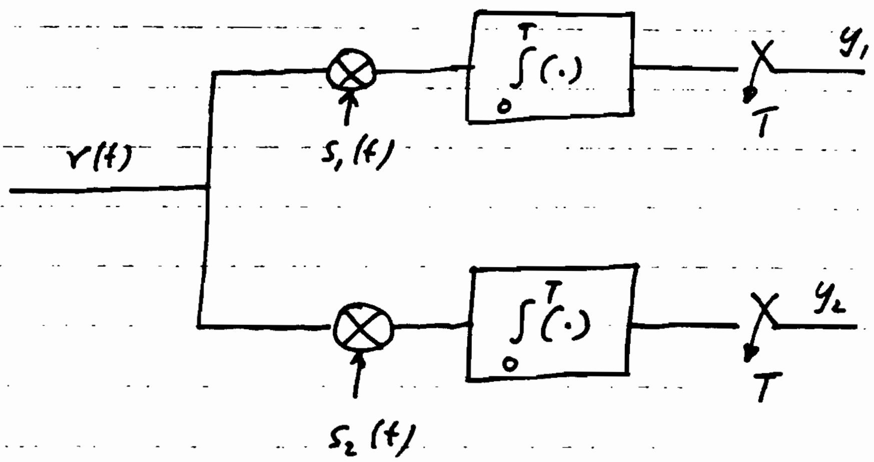
$$y_i = \int_0^T x(t)\,s_i(t)\,dt = \int_0^T x(t) \sum_{k=1}^{N} s_{ik}\,\phi_k(t)\,dt$$
$$= \sum_{k=1}^{N} s_{ik} \int_0^T x(t)\,\phi_k(t)\,dt = \sum_{k=1}^{N} s_{ik}\,\hat{s}_{jk} \qquad i = 1, 2, \ldots, M$$

For QAM – only 2 functions → cos & sin !!



PROBABILITY PERFORMANCE IN AN AWGN CHANNEL

Binary system : $s_i(t)$ $\begin{cases} i = 1, 2 \\ 0 \le t \le T \\ \text{equiprobable} \end{cases}$; $E = \int_0^T s_i^2(t)\,dt$



$$y_1 \underset{s_2(t)}{\overset{s_1(t)}{\gtrless}} y_2$$

$$\Updownarrow$$

$$z = y_1 - y_2 \underset{s_2(t)}{\overset{s_1(t)}{\gtrless}} 0$$

$$r(t) = s_i(t) + n(t)$$

$n(t) \hookrightarrow$ WGN $\rightarrow R(\tau) = \frac{N_0}{2}\delta(\tau)$

Assume that $s_1(t)$ is transmitted

$$y_1 = \int_0^T r(t)s_1(t)\,dt = \int_0^T s_1^2(t)\,dt + \int_0^T s_1(t)\,n(t)\,dt$$

$$y_2 = \int_0^T r(t)\,s_2(t)\,dt = \int_0^T s_1(t)s_2(t)\,dt + \int_0^T s_2(t)\,n(t)\,dt$$

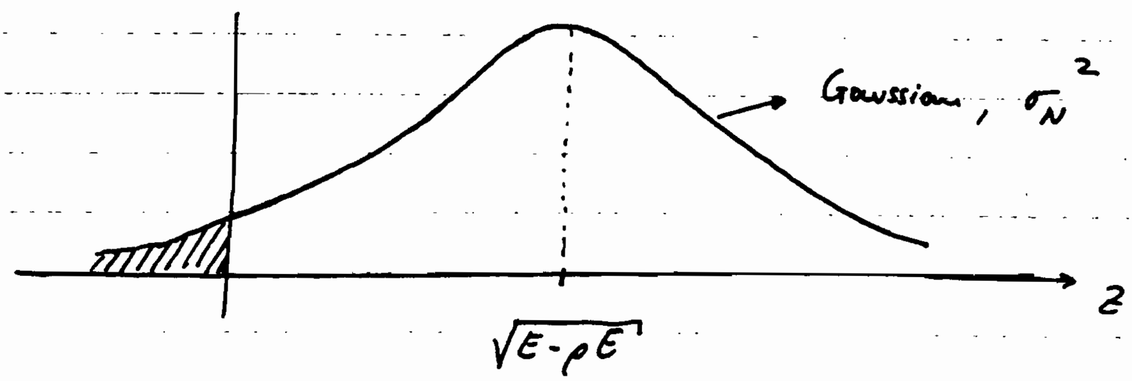
$$\therefore\quad z = y_1 - y_2 = \int_0^T s_1^2(t)\,dt - \int_0^T s_1(t)s_2(t)\,dt + \int_0^T n(t)[s_1(t) - s_2(t)]\,dt$$

$$= E - \rho E + N$$

$$\rho \triangleq \text{correlation coefficient} = \frac{1}{E}\int_0^T s_1(t)s_2(t)\,dt$$

$N = \int_0^T n(t)\left[s_1(t) - s_2(t)\right]dt$ is a G.r.v. with zero mean and variance $\sigma_N = E(N^2) = \cdots = E(1-\rho)N_0$

$\therefore$

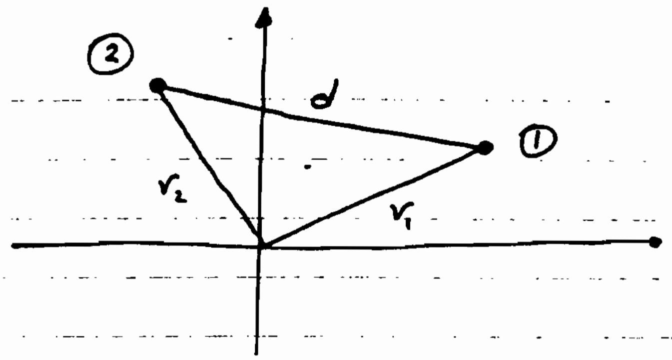


Prob. of an error = Prob that $z<0$

$$\therefore \quad P_e = \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}\,\sigma_N} \exp\left[\frac{z-\sqrt{E-\rho E}}{2\sigma_N^2}\right] dz.$$

$$\boxed{P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E(1-\rho)}{2N_0}}\right)}$$

A Geometrical Interpretation



signal ① has energy E_1
" ② " " E_2

$$r_1 = \sqrt{E_1}$$
$$r_2 = \sqrt{E_2}$$

$$d^2 = E_1 + E_2 - 2\rho\sqrt{E_1 E_2}$$

For equal energy signals, i.e., $E_1 = E_2 = E$

$$d^2 = 2E(1-\rho) \Rightarrow d = \sqrt{2}\sqrt{E(1-\rho)}$$

It can be easily shown that for equiprobable ①, ②

$$P_e = Q\left(\frac{d/2}{\sigma}\right) \qquad \left[\mathrm{erfc}(x) = 2Q(\sqrt{2}\,x)\right]$$

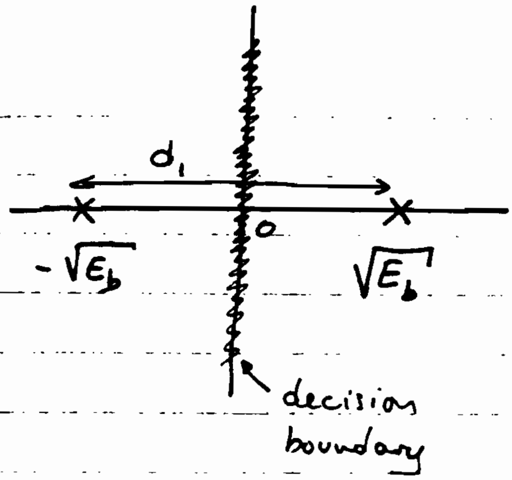
$\frac{d}{2}$ is half the Euclidean distance between ①, ② $\rightarrow$ { decision boundary

σ^2 is the noise variance $= \frac{N_0}{2}$

$$\therefore \quad P_e = Q\left(\frac{d}{2\sigma}\right) = Q\left(\frac{\sqrt{2}\sqrt{E(1-\rho)}}{2\sqrt{N_0/2}}\right) = Q\left(\sqrt{\frac{E(1-\rho)}{N_0}}\right)$$

Examples

(a)



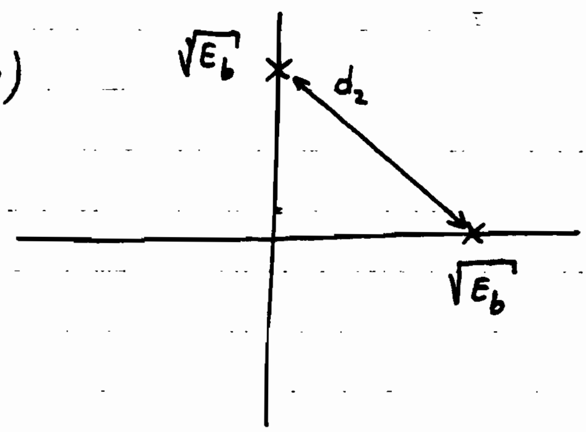
$\rho = -1 \Rightarrow$ antipodal (BPSK)

$$\therefore \; P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

In terms of the Q-function: $d_1 = 2\sqrt{E_b}$

$$\therefore \; P_e = Q\left(\frac{d_1/2}{\sigma}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

(b)



$\rho = 0 \Rightarrow$ orthogonal (FSK)

$$\therefore \; P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

Q-function approach: $d_2 = \sqrt{2}\sqrt{E_b}$

$$\therefore \; P_e = Q\left(\frac{d_2/2}{\sigma}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

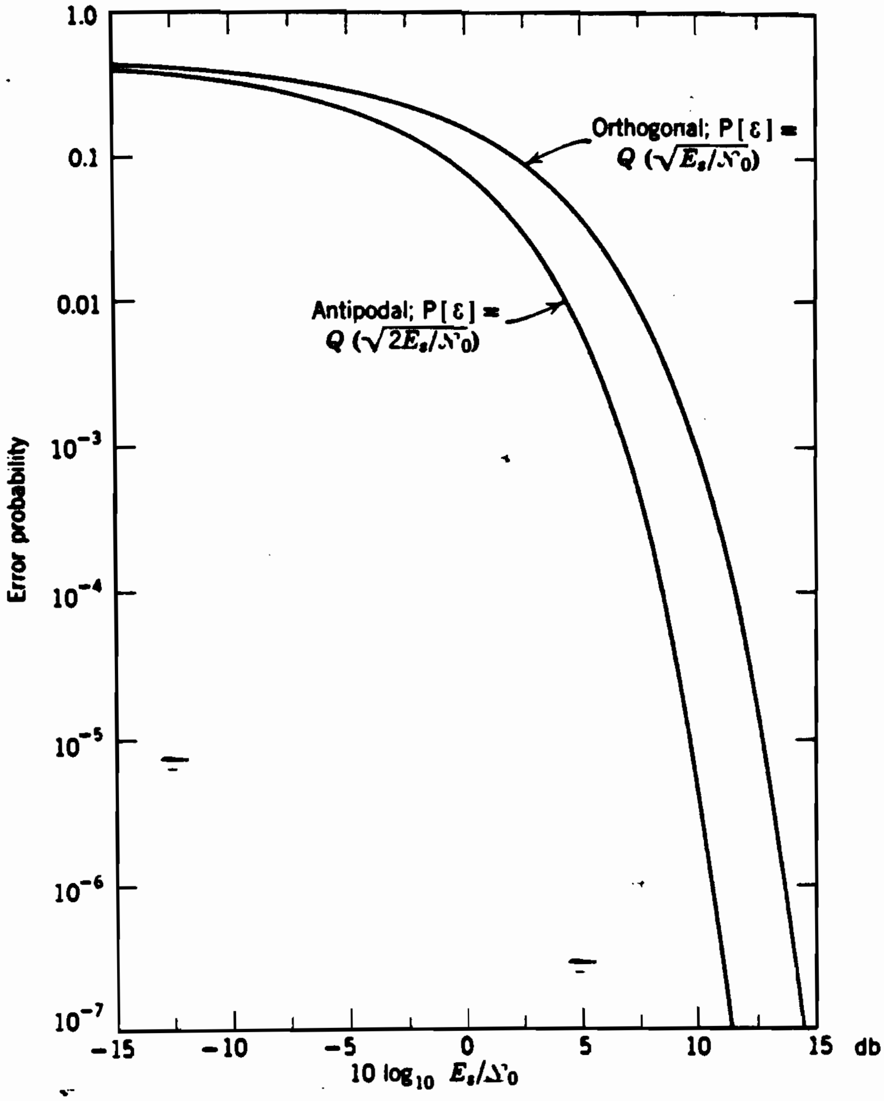


Figure 4.31 Probability of error for binary antipodal and binary orthogonal signaling with equally likely messages.

Ref. Wozencraft and Jacobs, Principles of Communication Engineering., J. Wiley, 1965

$$\frac{1}{\sqrt{2\pi}\alpha} e^{-\alpha^2/2}\left(1 - \frac{1}{\alpha^2}\right) < Q(\alpha) < \frac{1}{\sqrt{2\pi}\alpha} e^{-\alpha^2/2}; \qquad \alpha > 0. \qquad (2.121)$$

These two bounds are plotted together with $Q(\alpha)$ in Fig. 2.36.

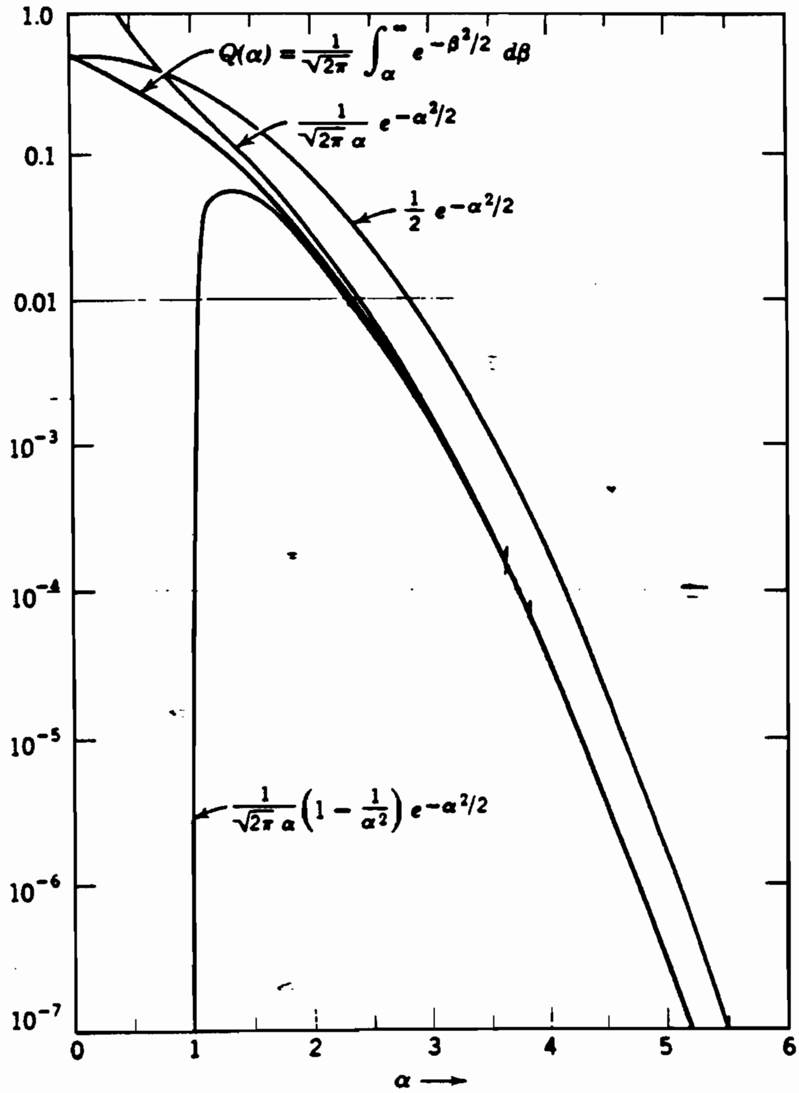
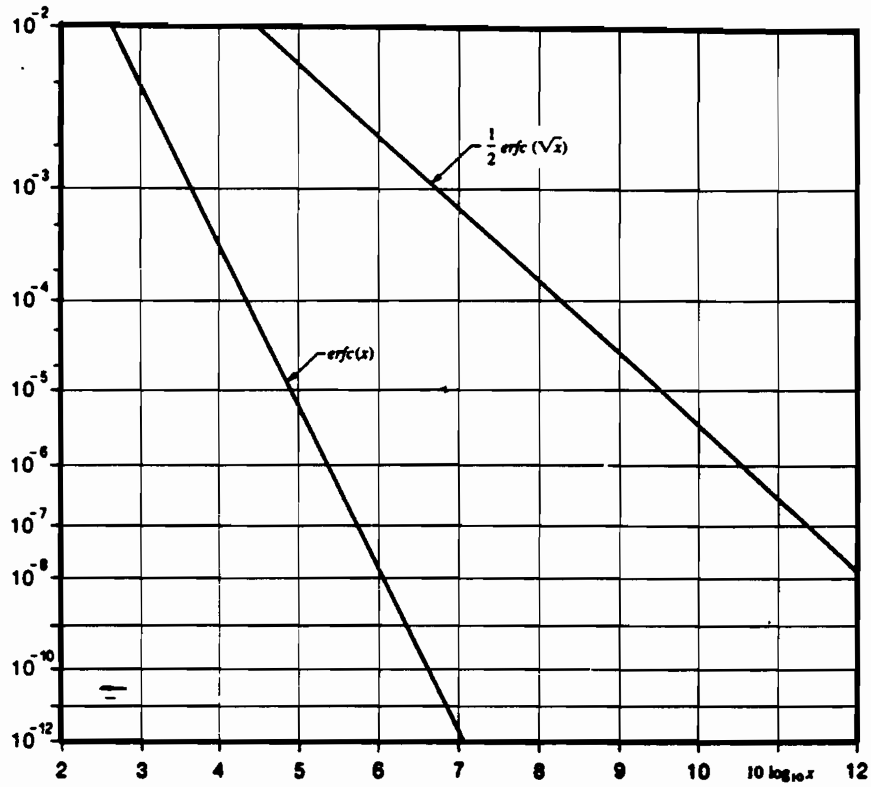


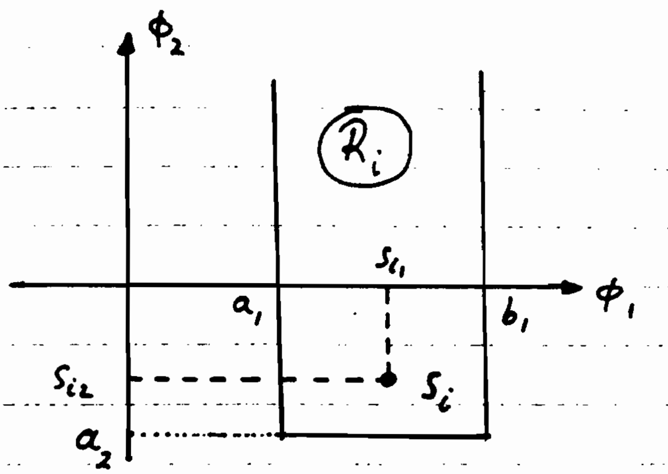
Figure 2.36 The function $Q(\alpha)$ and three bounds.

Ref. W. + J.



10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-10}
10^{-12}
2
3
4
5
6
7
8
9
10
$10\log_{10}x$
12
$\frac{1}{2}\,erfc\,(\sqrt{x})$
$erfc(x)$

Rectangular Signal Sets



Its always more efficient to find the Probability of having a correct decision P_c and then take $1-P_c$ to find P_e.

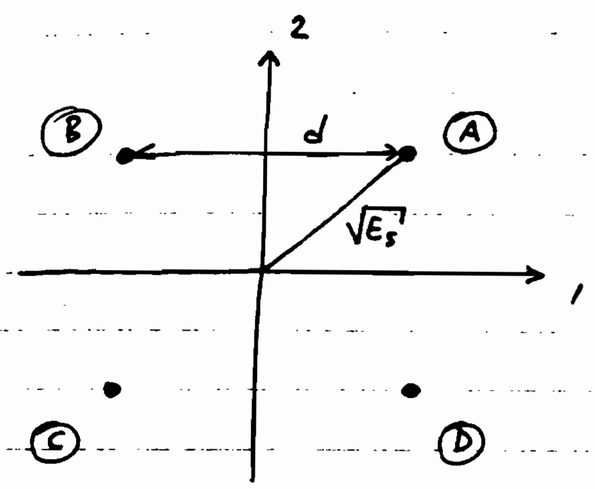
$$Pr(c/s_i) = Pr\{r \in R_i / s_i\} = Pr\{a_1 \le r_1 \le b_1,\ a_2 \le r_2 < \infty / s_i\}$$

$$= Pr\{a_1 - s_{i_1} \le n_1 \le b_1 - s_{i_1}\}\ Pr\{a_2 - s_{i_2} \le n_2 < \infty\}$$

n_1, n_2 independent G.r.v., zero mean, with variance σ^2

$$\therefore \quad Pr(c/s_i) = \int_{a_1 - s_{i_1}}^{b_1 - s_{i_1}} f_n(\alpha)\,d\alpha \int_{a_2 - s_{i_2}}^{\infty} f_n(\alpha)\,d\alpha \quad =$$

QPSK



All four points equiprobable, having the same energy E_s

Symmetry $\Rightarrow$ only one point is needed to be examined

$$d^2 = 2E_s \Rightarrow d = \sqrt{2E_s}$$

$$\Pr(\text{correct}/A) = \Pr\left\{ \frac{d}{2} < r_1 < \infty \;\&\; -\infty < r_2 < -\frac{d}{2} \right\}$$

$$= \Pr\left\{ n_1 < \frac{d}{2} \;\&\; n_2 > -\frac{d}{2} \right\} = \underbrace{\Pr\left\{n_1 < \frac{d}{2}\right\}}_{1-p} \underbrace{\Pr\left\{n_2 > -\frac{d}{2}\right\}}_{1-p}$$

$$p = Q\left(\frac{d/2}{\sigma}\right) = Q\left(\sqrt{\frac{E_s}{N_0}}\right) \underset{E_s = 2E_b}{=} Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$\left[\text{or } p = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)\right]$$

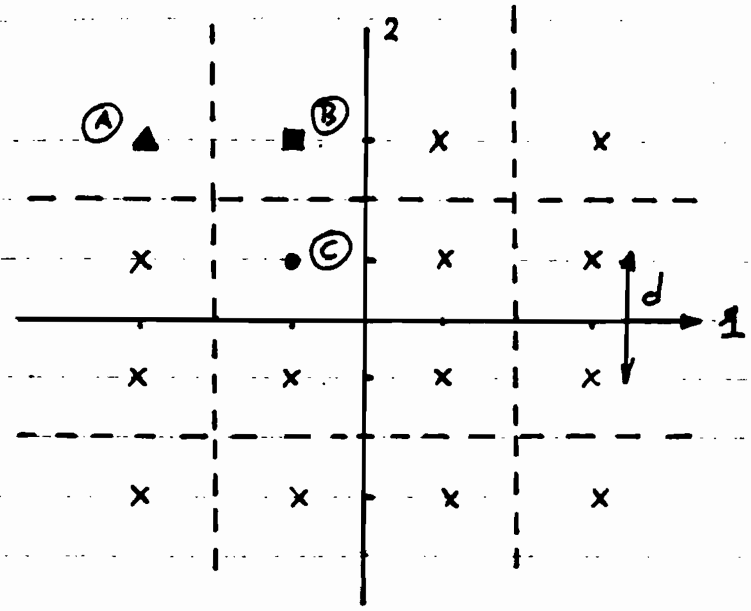
$\therefore$

$$P_e = 1 - \Pr(\text{correct}/A) = 1 - (1-p)^2 = 2p - p^2 \simeq 2p$$

(for high $p > 10^{-2}$)

Performance of QPSK (2b/s/Hz) $\equiv$ Performance of BPSK 1 b/s/Hz

16-QAM



$P_A = Pr(c/A)$

$P_B = Pr(c/B)$

$P_C = Pr(c/C)$

$P_A = (1-p)^2$

$P_B = (1-2p)(1-p)$

$P_C = (1-2p)^2$

$$P_e = 1 - P_c = 1 - \frac{1}{M}\sum_{i=1}^{M} P(c/s_i)$$

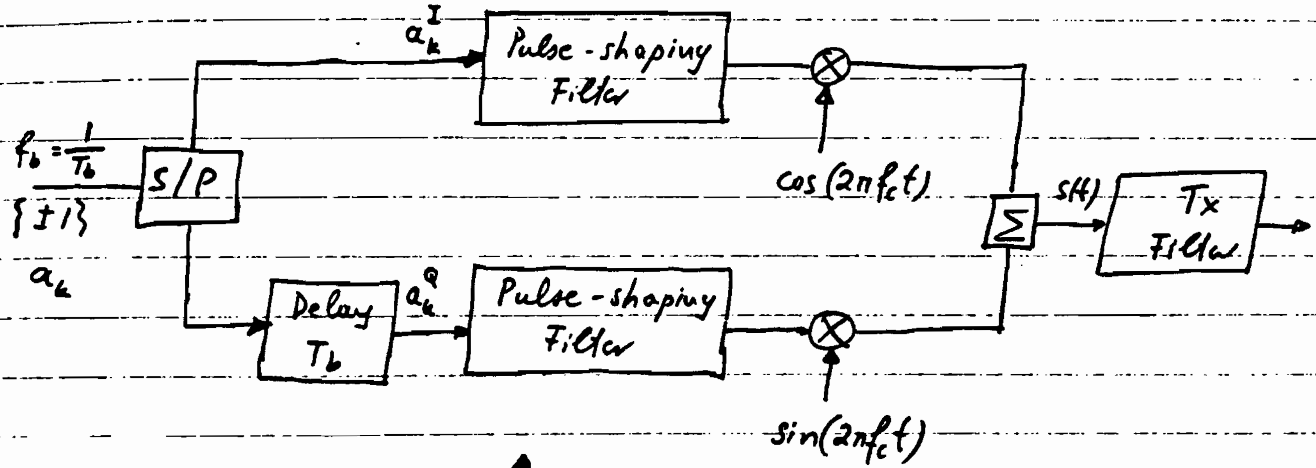
$$\therefore \quad P_e = 1 - \frac{1}{16}\left[4P_A + 8P_B + 4P_C\right] = \cdots = 1 - \frac{1}{4}(2-3p)^2$$

$$= 3p - \frac{9}{4}p^2$$

where $p = Q\left(\frac{d/2}{\sigma}\right)$

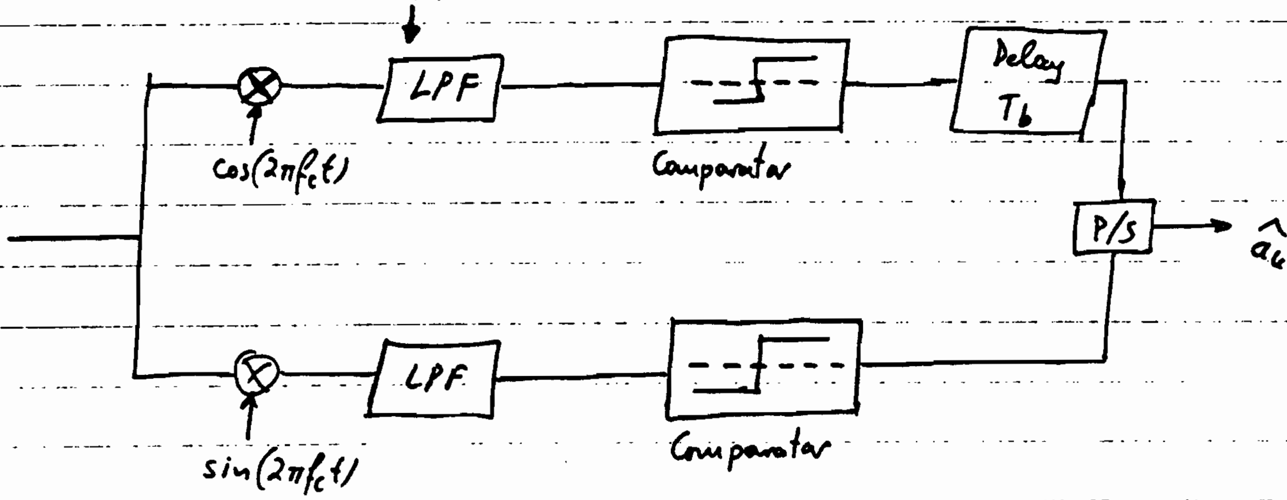
Offset - QPSK (OQPSK)

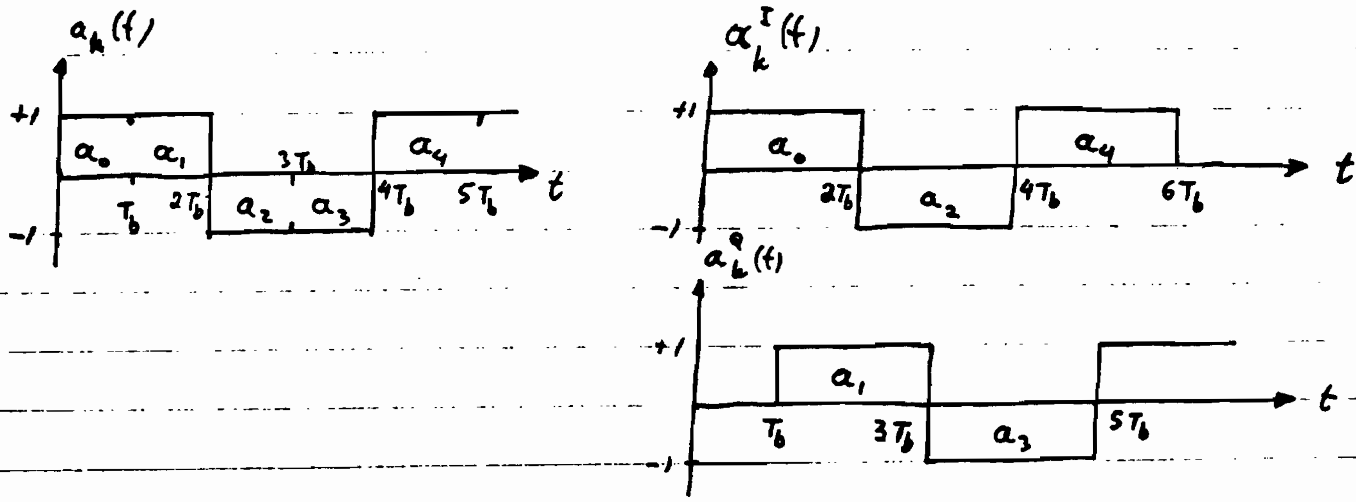
The I- and Q-channel are delayed by one bit duration



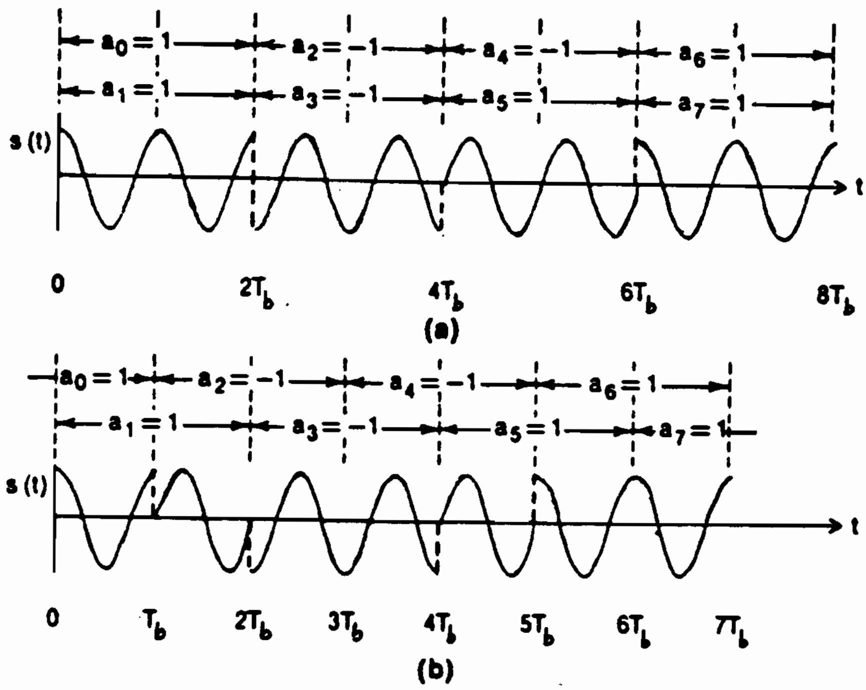
Transmitter

Receiver





Assuming no pulse-shaping filter, s(t) is given by:

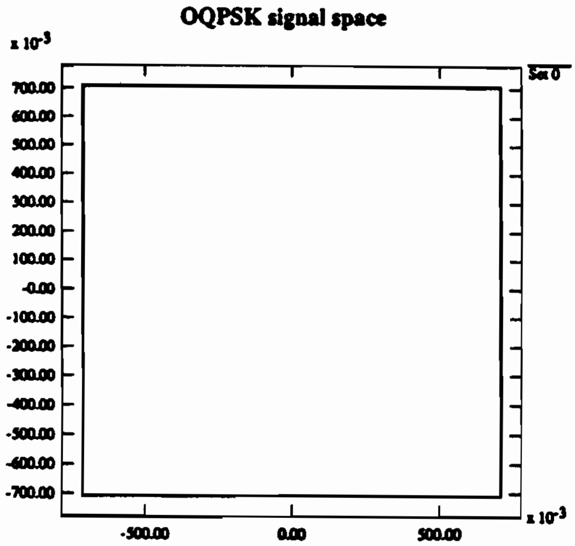


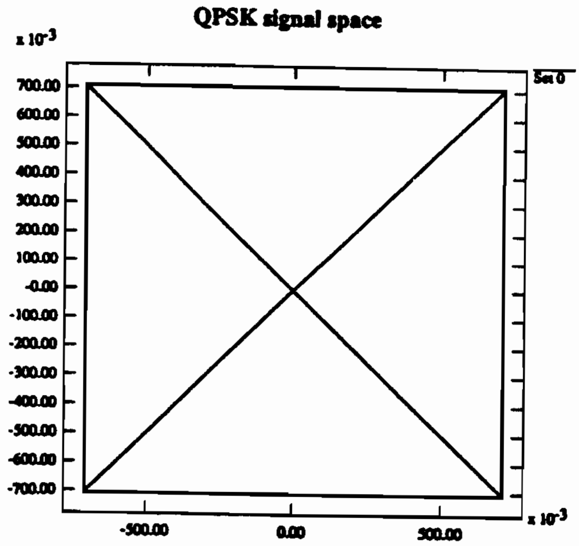
(a) QPSK waveform; (b) OPQSK waveform.

$\Delta\phi \in \{0, \pm 90^\circ, 18$

$\Delta\phi \in \{0^\circ, \pm 90^\circ\}$

State - Space Diagrams





Comments

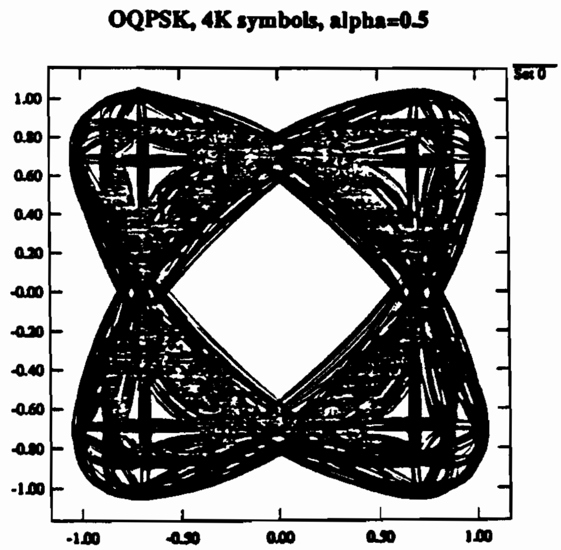
- Since the staggering does not change the orthogonality of the carriers ⇒ in an AWGN channel OQPSK has the same performance as QPSK.

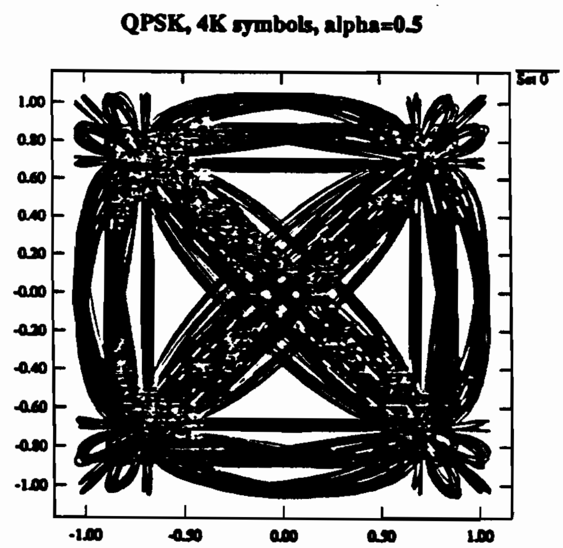
- The staggering does not change the power spectral density ⇒ QPSK & OQPSK have the same spectrum (in a linear channel)

- For OQPSK there are no transition through the origin. ⇒ Reduced envelope fluctuation ⇒ it has better spectral characteristics after bandlimiting & nonlinear amplification.

Effects of bandlimiting

- State space diagrams



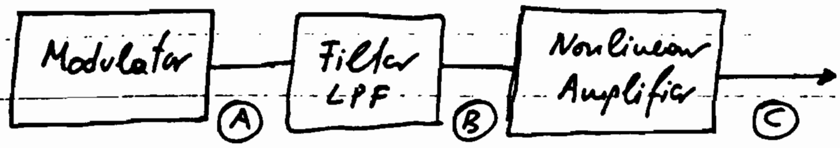


- In an AWGN channel the BER performance is "almost" the sam

- The delay T_b has been shown to be optimal in terms of phase jitter immunity in an AWGN channel.

- When bandlimited OQPSK goes through a nonlinear amplifier (e.g., hardlimiter), the relatively small envelope fluctuations will be removed by the limiter. Furthermore, the absence of rapid phase shifts (≡ high frequency content) means that this limiting will <u>not</u> regenerate high frequency components which originally were removed by the bandlimiting filter ⇒ out-of-band spectral regrowth (interference) will be small.

Typical system configuration (Tx):



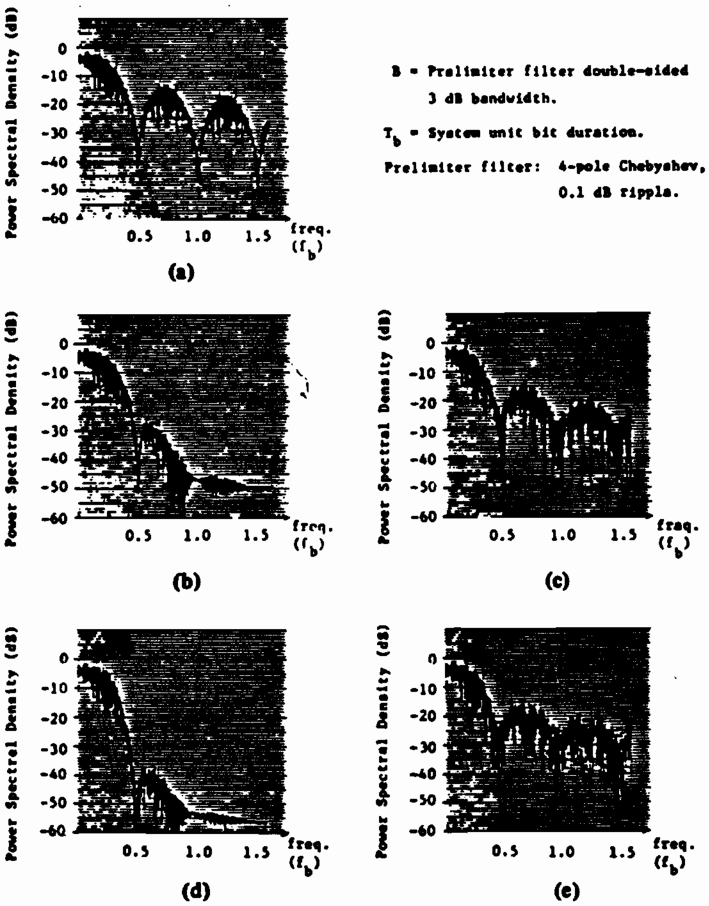


Fig. 2. Computed power spectral densities for QPSK. (a) Spectral density at pt. *A*, Fig. 1. Unfiltered, $BT_b = \infty$. (b) Spectral density at pt. *B*, Fig. 1. Filtered, $BT_b = 1$. (c) Spectral density at pt. *C*, Fig. 1. Filtered, $BT_b = 1$, then limited. (d) Spectral density at pt. *B*, Fig. 1. Filtered, $BT_b = 0.75$. (e) Spectral density at pt. *C*, Fig. 1. Filtered, $BT_b = 0.75$, then limited.

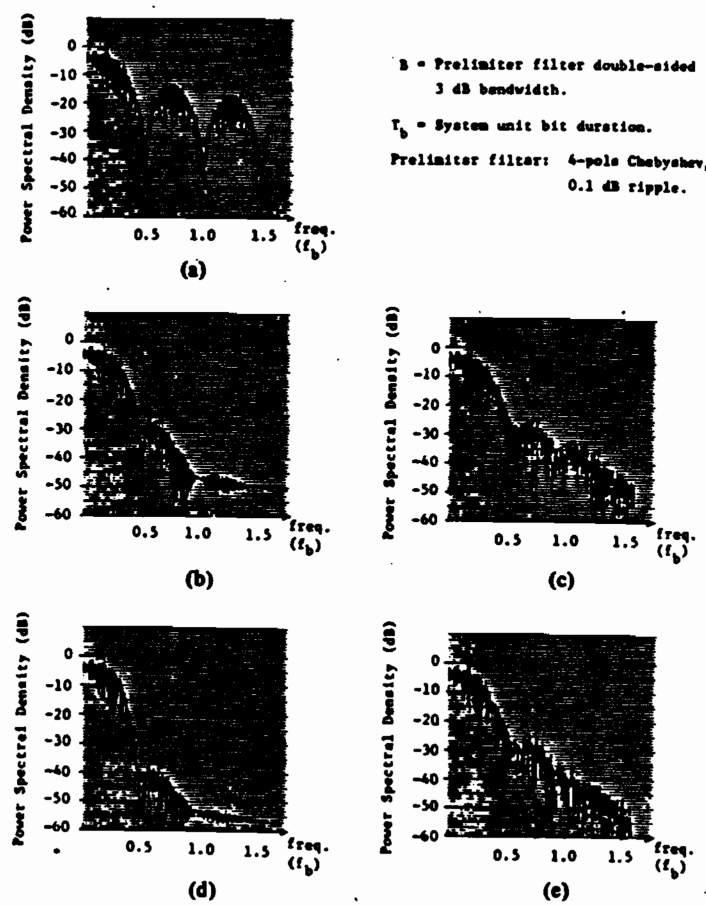


Fig. 3. Computed power spectral densities for offset QPSK. (a) Spectral density at pt. *A*, Fig. 1. Unfiltered, $BT_b = \infty$. (b) Spectral density at pt. *B*, Fig. 1. Filtered, $BT_b = 1$. (c) Spectral density at pt. *C*, Fig. 1. Filtered, $BT_b = 1$, then limited. (d) Spectral density at pt. *B*, Fig. 1. Filtered, $BT_b = 0.75$. (e) Spectral density at pt. *C*, Fig. 1. Filtered, $BT_b = 0.75$, then limited.

LPF: 4 pole Chebyshev filter
0.1 dB ripple

Conclusions

- OQPSK is a much better scheme than QPSK
  - Almost same BER performance
  - Significantly better spectrum
  - Same cost!

- These advantages are due to the fact that $\Delta\phi \in \{+90°, -90°, 0°\}$ and not $\pm 180°$!!

- However, it is <u>not</u> a constant envelope (amplitude) scheme, such as:

$$s(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + \phi(t, \underline{\alpha})\right]$$

A large class of signals with constant envelope are the Continuous Phase Modulation (CPM) signals.

Perhaps the most simple signals in the CPM is the Minimum Shift Keying (MSK)

MINIMUM SHIFT KEYING (MSK)

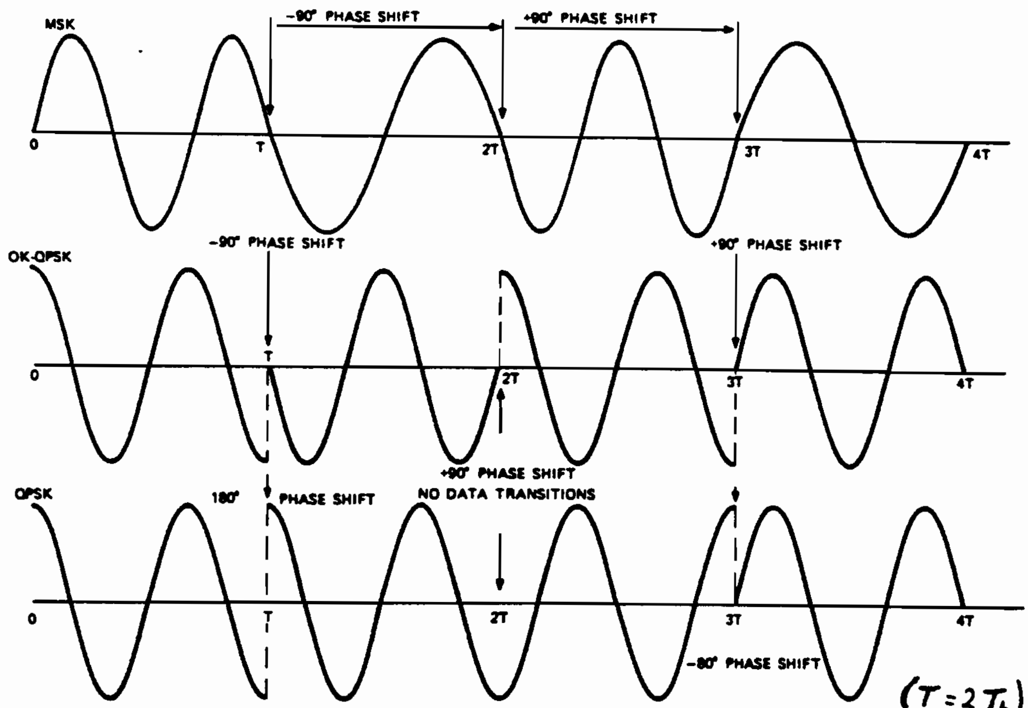


Fig. 3. Typical waveforms for MSK, offset QPSK, and conventional QPSK.

There are two different ways of "seeing" MSK:

i) As a special case of OQPSK (with sinusoidal pulse shape)

ii) As a continuous phase FSK (with a frequency separation of $\frac{1}{2T_b}$, i.e., $\frac{1}{2}$ the bit rate)

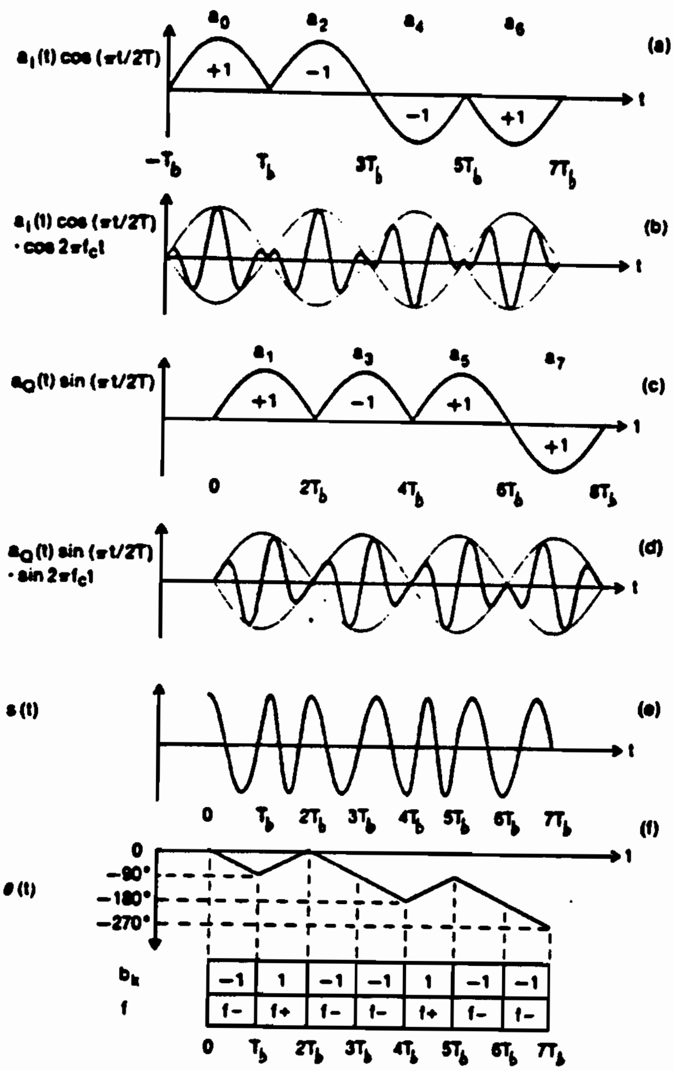


Fig. 5. MSK waveforms.

$$s(t) = a_I(t)\cos\left(\frac{\pi t}{2T_b}\right)\cos(2\pi f_c t) + a_Q(t)\sin\left(\frac{\pi t}{2T_b}\right)\sin(2\pi f_c t)$$

$$= \cos\left[2\pi f_c t + b_k(t)\frac{\pi t}{2T_b} + \phi_k\right]; \quad b_k(t) = -a_I(t)a_Q(t) \;; \quad \phi_k = \begin{cases} 0 \;; & a_I = 1 \\ \pi \;; & a_Q = -1 \end{cases}$$

$$s(t) = \cos\left[2\pi f_c t + \underbrace{\frac{1}{2}\, b_k(t)\frac{2\pi t}{2T_b}}_{} + \phi_k\right]$$

$\theta(t) \triangleq$ excess phase

$h \triangleq$ modulation index ($= \frac{1}{2}$ for MSK type of signals)

Comments

- $s(t)$ is a constant envelope signal
- There is a phase continuity in the carrier at the bit transition instants
- $s(t)$ can be also viewed as an FSK signal with signalling frequencies

$$f_+ = f_c + \frac{1}{4T_b} \; ; \; f_- = f_c - \frac{1}{4T_b}$$

∴ Frequency deviation $\Delta f = f_+ - f_- = \frac{1}{2T_b}$

Notice that this Δf is the minimum frequency which allows two FSK signals to be orthogonal to each other. In this respect, it is called Fast FSK. (FFSK).

- Excess phase $\theta(t) = b_k(t)\,\frac{\pi t}{2T_b} = \pm \frac{\pi t}{2T_b}$

∴ Linear change (increase or decrease) during each bit period. T_b.

$b_k = +1 \Rightarrow$ increase carrier phase shift by $90°\ (\frac{\pi}{2}) \longleftrightarrow$ FSK; f_+
$b_k = -1 \Rightarrow$ decrease - - - - $90°\ (\frac{\pi}{2}) \longleftrightarrow$ FSK; f_-

See also Figure of previous page.

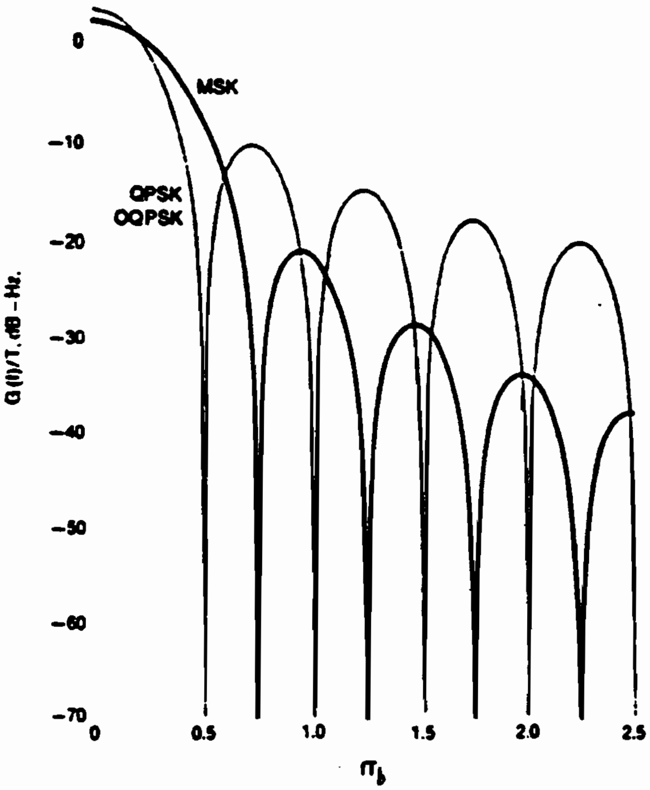


Fig. 6. Spectral density of QPSK, OPQSK, and MSK.

BW $\triangleq$ Bandwidth which contains 99% of the total power

MSK $\longrightarrow$ BW $\simeq \frac{1.2}{T_b}$; QPSK & OQPSK $\longrightarrow$ BW $\simeq \frac{8}{T_b}$

However MSK has wider mainlobe $\Rightarrow$ in narrowband channels MSK may not be the preferred mod. scheme.

With nonlinearities & ACI things are more complicated.
($\beta \triangleq$ channel spacing $\times$ bit duration)

- MSK is better than QPSK for $\beta > 1.8$
- " " " " OQPSK " $\beta > 2.3$
- OQPSK " " " QPSK " $\beta < 1.4$

i) Approximation
ii) Small differenc

Many other schemes which use pulse shaping to improve spectrum

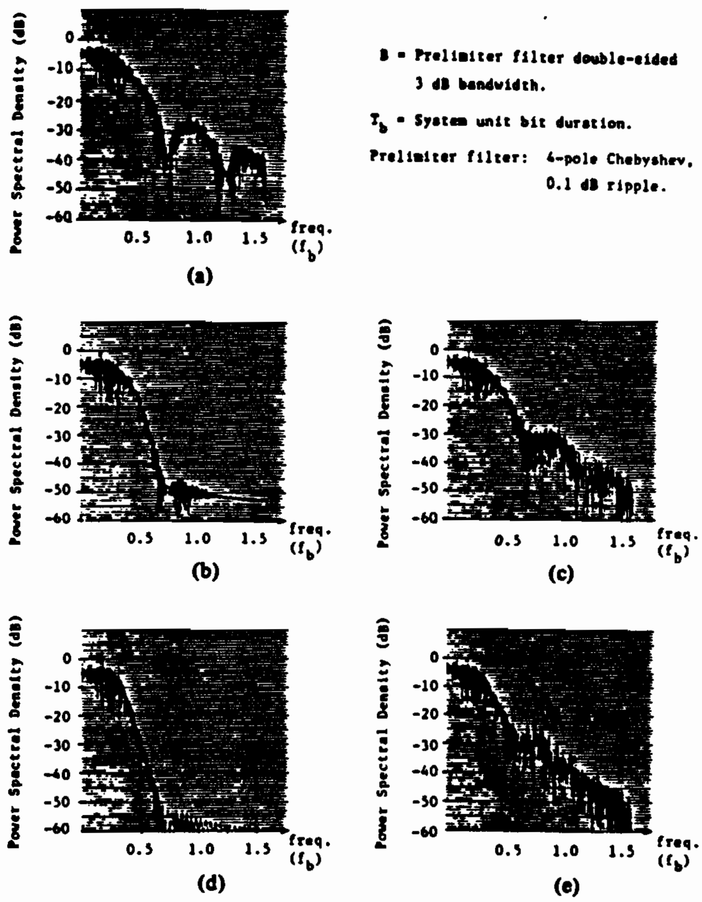


Fig. 4. Computed power spectral densities for MSK. (a) Spectral density at pt. *A*, Fig. 1. Unfiltered, $BT_b = \infty$. (b) Spectral density at pt. *B*, Fig. 1. Filtered, $BT_b = 1$. (c) Spectral density at pt. *C*, Fig. 1. Filtered, $BT_b = 1$, then limited. (d) Spectral density at pt. *B*, Fig. 1. Filtered, $BT_b = 0.75$. (e) Spectral density at pt. *C*, Fig. 1. Filtered, $BT_b = 0.75$, then limited.

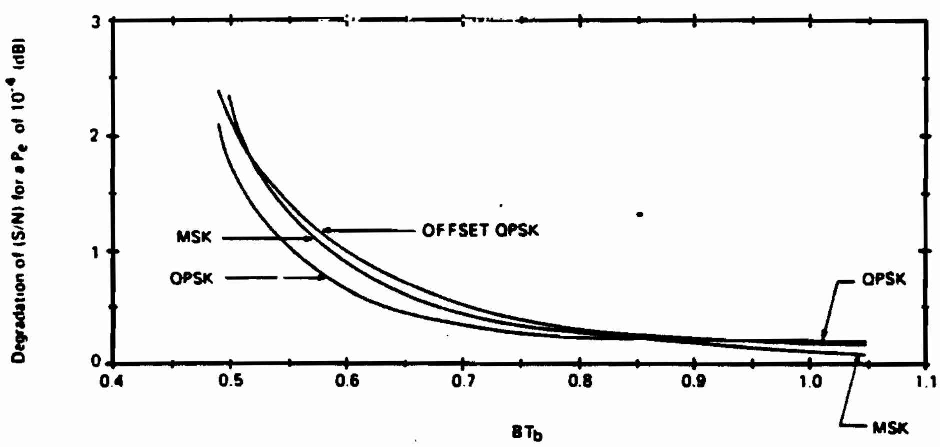
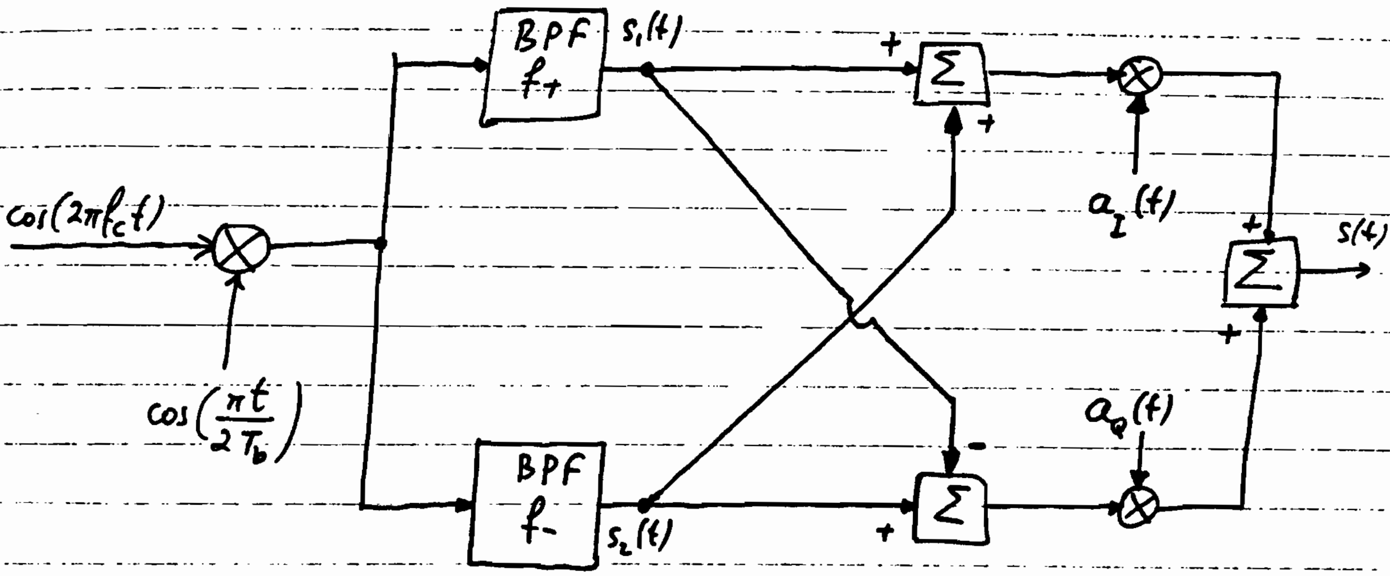


Fig. 9. Degradation, due to filtering, then limiting, of $(S/N_b)_{rcvr.}$ input versus normalized prelimited filter bandwidth.

Ref. Morrais & Feher, "The effects of filtering and limiting on the performance of QPSK, OQPSK and MSK systems," IEEE Trans. Commun., vol. COM-28, pp. 1994-2009, Dec. 1980.

MSK transmitter implementation

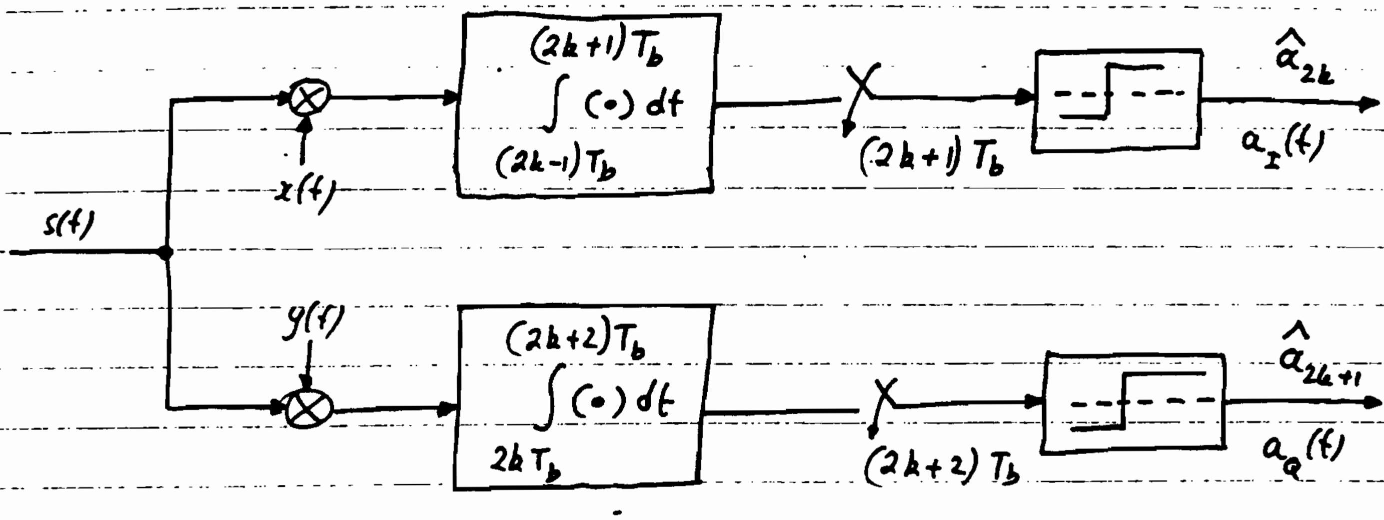


$$f_+ = f_c + \frac{1}{4T_b} \quad ; \quad f_- = f_c - \frac{1}{4T_b}$$

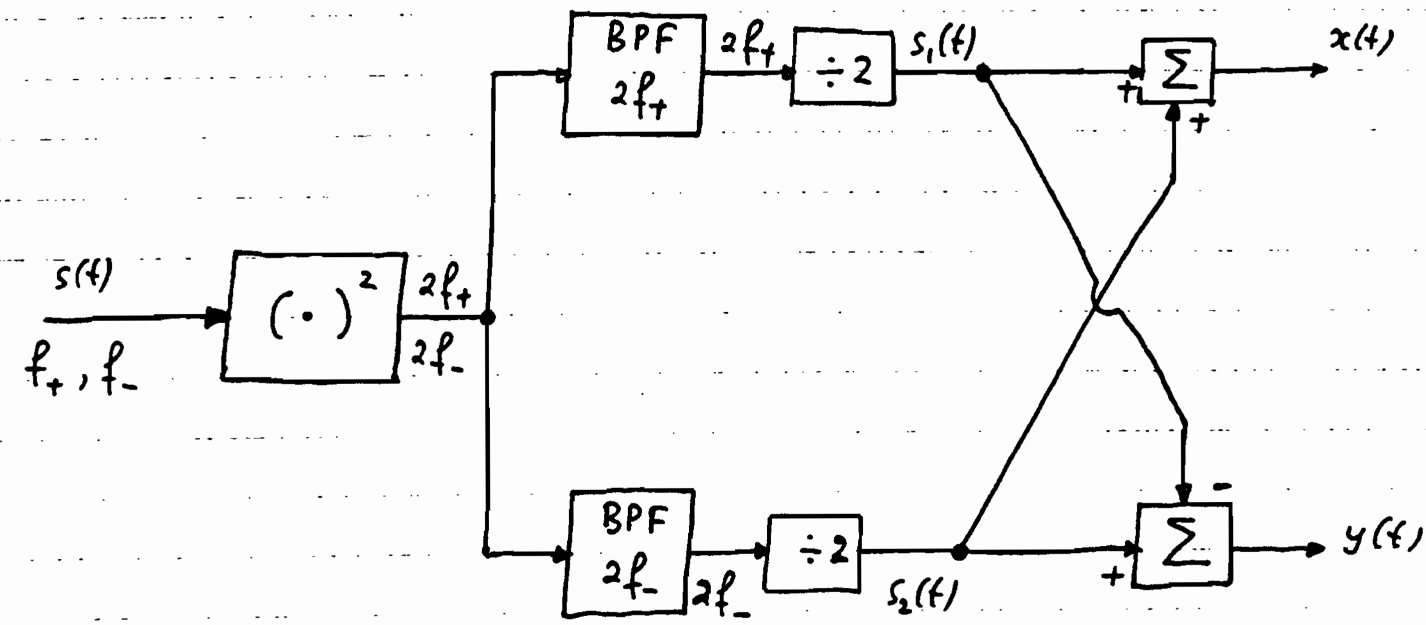
$$s_1(t) = \frac{1}{2}\cos\left(2\pi f_c t + \frac{\pi t}{2T_b}\right)$$

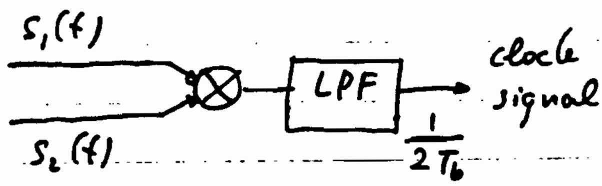
$$s_2(t) = \frac{1}{2}\cos\left(2\pi f_c t - \frac{\pi t}{2T_b}\right)$$

MSK receiver implementation



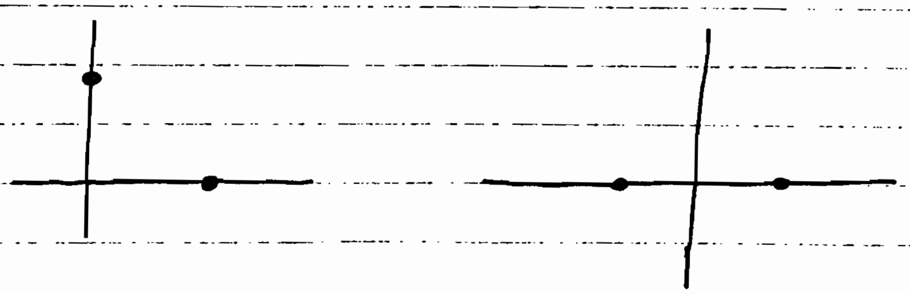
Synchronization subsystems





Final comments

- For MSK an observation interval of $2T_b$ will result performance equal to QPSK, OQPSK & BPSK.
- If MSK is detected as an FSK signal, i.e., observation over T_b $\Rightarrow$ 3 dB degradation



- For other modulation indexes, larger observation intervals are necessary for optimal performance.

e.g. $h = \frac{2}{3} \rightarrow$ gain of 0.8 dB for observation over $4T_b$.