

-1- Μεθοδος Givens για εύρεση ιδιοτιμών

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 2 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ 2 & 1 & 1 & 1 \end{bmatrix}$$

$$\alpha_{rq} = \alpha_{23} = -1$$

$$\begin{matrix} r=2 \\ q=3 \end{matrix}$$

$$\begin{bmatrix} a_{rr} & a_{rq} \\ a_{qr} & a_{qq} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\alpha = \frac{1}{\sqrt{a_{r-1,r}^2 + a_{r-1,q}^2}} = \frac{1}{\sqrt{a_{12}^2 + a_{13}^2}} = \frac{1}{\sqrt{2^2 + 1^2}} = \frac{1}{\sqrt{5}}$$

$$\sin\theta = -\alpha a_{r-1,q} = -\alpha a_{13} = -\frac{1}{\sqrt{5}} \cdot 1 = -\frac{1}{\sqrt{5}}$$

$$\cos\theta = \alpha a_{r-1,r} = \alpha a_{12} = \frac{1}{\sqrt{5}} \cdot 2 = \frac{2}{\sqrt{5}}$$

$$A_2 = P_1^T A P_1 =$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 2 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ 2 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2\cos\theta - \sin\theta & 2\cos\theta + \sin\theta & -\cos\theta - \sin\theta & \cos\theta - \sin\theta \\ 2\sin\theta + \cos\theta & 2\sin\theta - \cos\theta & -\sin\theta + \cos\theta & \sin\theta + \cos\theta \\ 2 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2\cos\theta - \sin\theta & 2\sin\theta + \cos\theta & 2 \\ 2\cos\theta - \sin\theta & 2\cos^2\theta + \sin^2\theta\cos\theta + \sin\theta\cos\theta + \sin^2\theta & 2\cos\theta\sin\theta + \sin^2\theta - \cos^2\theta - \sin\theta\cos\theta & \cos\theta - \sin\theta \\ 2\sin\theta + \cos\theta & 2\sin\theta\cos\theta - \cos^2\theta + \sin^2\theta - \sin\theta\cos\theta & 2\sin^2\theta - \sin\theta\cos\theta - \sin\theta\cos\theta + \cos^2\theta & \sin\theta + \cos\theta \\ 2 & \cos\theta - \sin\theta & \sin\theta + \cos\theta & 1 \end{bmatrix}$$

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δ_{10}	$b_{ir} = b_{12}$ $2 \cos \theta - \sin \theta$	b_{11} and $b_{r-1,q}$ $2 \sin \theta + \cos \theta$	δ_{10}
1	$b_{rr} = b_{22}$ $2 \cos^2 \theta + \sin^2 \theta + 2 \sin \theta \cos \theta$	$b_{rq} = b_{23}$ $2 \sin \theta \cos \theta - \cos^2 \theta + \sin^2 \theta$	2
$b_{ri} = b_{21}$ $2 \cos \theta - \sin \theta$	$b_{qr} = b_{32}$ $\sin \theta \cos \theta - \cos^2 \theta + \sin^2 \theta$	$b_{qq} = b_{33}$ $2 \sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta$	b_{24} $\cos \theta - \sin \theta$
b_{31} $2 \sin \theta + \cos \theta$	$b_{42} = b_{ir}$ $\cos \theta - \sin \theta$	$b_{43} = b_{iq}$ $\sin \theta + \cos \theta$	b_{34} $\sin \theta + \cos \theta$
2	$a_{42} \cos \theta - a_{43} \sin \theta$	$a_{42} \sin \theta + a_{43} \cos \theta$	1
δ_{10}			δ_{10}

$$= \begin{pmatrix} 1 & 5/\sqrt{5} & 0 & 2 \\ 5/\sqrt{5} & 1 & -1 & 3/\sqrt{5} \\ 0 & -1 & 2 & 1/\sqrt{5} \\ 2 & 3/\sqrt{5} & 1/\sqrt{5} & 1 \end{pmatrix} = A_2$$

$$P_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & 0 & \sin \theta \\ 0 & 0 & 1 & 0 \\ 0 & -\sin \theta & 0 & \cos \theta \end{bmatrix}$$

γιατί ισχύουν
τα ακόλουθα.

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Οδυσό στοιχείο το $a_{rq} = a_{24} = \frac{3}{\sqrt{5}}$ οπότε $r=2, q=4$.

$$\alpha = \frac{1}{\sqrt{a_{r-1,r}^2 + a_{r-1,q}^2}}$$

$$\hookrightarrow \begin{bmatrix} a_{22} & a_{24} \\ a_{42} & a_{44} \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$= \frac{1}{\sqrt{a_{12}^2 + a_{14}^2}} = \frac{1}{\sqrt{\left(\frac{5}{\sqrt{5}}\right)^2 + 2^2}} = \frac{1}{\sqrt{5+4}} = \frac{1}{3}$$

$$\sin \theta = -\alpha a_{r-1,q} = -\frac{1}{3} a_{14} = -\frac{1}{3} 2 = -\frac{2}{3}$$

$$\cos \theta = +\alpha a_{r-1,r} = +\frac{1}{3} a_{12} = \frac{1}{3} \frac{5}{\sqrt{5}} = \frac{5}{3\sqrt{5}}$$

$$A_3 = P_2^T A_2 P_2 =$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & 0 & -\sin \theta \\ 0 & 0 & 1 & 0 \\ 0 & \sin \theta & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} 1 & \frac{5}{\sqrt{5}} & 0 & 2 \\ \frac{5}{\sqrt{5}} & 1 & -1 & \frac{3}{\sqrt{5}} \\ 0 & -1 & 2 & \frac{1}{\sqrt{5}} \\ 2 & \frac{3}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & 0 & \sin \theta \\ 0 & 0 & 1 & 0 \\ 0 & -\sin \theta & 0 & \cos \theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ \frac{5}{\sqrt{5}} \cos \theta - 2 \sin \theta \\ 0 \\ \frac{5}{\sqrt{5}} \sin \theta + 2 \cos \theta \end{bmatrix} \begin{bmatrix} \frac{5}{\sqrt{5}} \\ \cos \theta - \frac{3}{\sqrt{5}} \sin \theta \\ -1 \\ \sin \theta + \frac{3}{\sqrt{5}} \cos \theta \end{bmatrix} \begin{bmatrix} 0 \\ -\cos \theta + \frac{1}{\sqrt{5}} \sin \theta \\ 2 \\ -\sin \theta + \frac{1}{\sqrt{5}} \cos \theta \end{bmatrix} \begin{bmatrix} 2 \\ \frac{3}{\sqrt{5}} \cos \theta - \sin \theta \\ \frac{1}{\sqrt{5}} \\ \frac{3}{\sqrt{5}} \sin \theta + \cos \theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & 0 & \sin \theta \\ 0 & 0 & 1 & 0 \\ 0 & -\sin \theta & 0 & \cos \theta \end{bmatrix}$$

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$$= \begin{bmatrix} 1 & \frac{5}{\sqrt{5}} \cos \theta - 2 \sin \theta & 0 & \frac{5}{\sqrt{5}} \sin \theta + 2 \cos \theta \\ \frac{5}{\sqrt{5}} \cos \theta - 2 \sin \theta & \cos^2 \theta - \frac{3}{\sqrt{5}} \sin \theta \cos \theta & -\cos \theta + \frac{1}{\sqrt{5}} \sin \theta & \cos \theta \sin \theta - \frac{3}{\sqrt{5}} \sin^2 \theta + \frac{3}{\sqrt{5}} \cos^2 \theta - \sin \theta \cos \theta \\ 0 & -\cos \theta - \frac{1}{\sqrt{5}} \sin \theta & 2 & -\sin \theta + \frac{1}{\sqrt{5}} \cos \theta \\ \frac{5}{\sqrt{5}} \sin \theta + 2 \cos \theta & \sin \theta \cos \theta + \frac{3}{\sqrt{5}} \cos^2 \theta - \frac{3}{\sqrt{5}} \sin^2 \theta - \sin \theta \cos \theta & -\sin \theta + \frac{1}{\sqrt{5}} \cos \theta & \sin^2 \theta + \frac{3}{\sqrt{5}} \sin \theta \cos \theta + \frac{3}{\sqrt{5}} \sin \theta \cos \theta + \cos^2 \theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{5}{\sqrt{5}} \cos \theta - 2 \sin \theta & 0 & \frac{5}{\sqrt{5}} \sin \theta + 2 \cos \theta \\ \frac{5}{\sqrt{5}} \cos \theta - 2 \sin \theta & 1 - \frac{3}{\sqrt{5}} \cdot 2 \sin \theta \cos \theta & -\cos \theta - \frac{1}{\sqrt{5}} \sin \theta & \frac{3}{\sqrt{5}} (\cos^2 \theta - \sin^2 \theta) \\ 0 & -\cos \theta - \frac{1}{\sqrt{5}} \sin \theta & 2 & -\sin \theta + \frac{1}{\sqrt{5}} \cos \theta \\ \frac{5}{\sqrt{5}} \sin \theta + 2 \cos \theta & \frac{3}{\sqrt{5}} (\cos^2 \theta - \sin^2 \theta) & -\sin \theta + \frac{1}{\sqrt{5}} \cos \theta & 1 + \frac{3}{\sqrt{5}} 2 \sin \theta \cos \theta \end{bmatrix} \begin{matrix} \leftarrow r \\ \leftarrow q \end{matrix}$$

$$b_{r-1,q} = b_{q,r-1} = a_{r-1,r} \sin \theta + a_{r-1,q} \cos \theta (=0)$$

$$b_{14} = b_{41} = a_{12} \sin \theta + a_{14} \cos \theta = \frac{5}{\sqrt{5}} \cdot \left(-\frac{2}{3}\right) + 2 \cdot \frac{5}{3\sqrt{5}} = 0. \quad (1)$$

$$b_{ir} = b_{ri} = a_{ir} \cos \theta - a_{iq} \sin \theta \xrightarrow{i=1} \Rightarrow b_{12} = b_{21} = a_{12} \cos \theta - a_{14} \sin \theta = \frac{5}{\sqrt{5}} \cdot \frac{5}{3\sqrt{5}} - 2 \cdot \left(-\frac{2}{3}\right) = \frac{5}{3} + \frac{4}{3} = \frac{9}{3} = 3 \quad (2)$$

$$b_{rr} = a_{rr} \cos^2 \theta + a_{qq} \sin^2 \theta - 2a_{rq} \sin \theta \cos \theta$$

$$b_{22} = a_{22} \cos^2 \theta + a_{44} \sin^2 \theta - 2a_{24} \sin \theta \cos \theta = \cos^2 \theta + \sin^2 \theta - \frac{3}{\sqrt{5}} \cdot 2 \sin \theta \cos \theta =$$

$$= 1 \cdot \left(\frac{5}{3\sqrt{5}}\right)^2 + 1 \cdot \left(-\frac{2}{3}\right)^2 - 2 \cdot \frac{3}{\sqrt{5}} \cdot \left(-\frac{2}{3}\right) \cdot \frac{5}{3\sqrt{5}} = \frac{5}{9} + \frac{4}{9} + \frac{12}{9} = \frac{21}{9} = \frac{7}{3} \quad (3)$$

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$$\begin{aligned}
 b_{qq} = b_{44} &= a_{rr} \sin^2 \theta + a_{qq} \cos^2 \theta + 2a_{rq} \sin \theta \cos \theta \\
 &= a_{22} \sin^2 \theta + a_{44} \cos^2 \theta + 2a_{24} \sin \theta \cos \theta \\
 &= 1 \sin^2 \theta + 1 \cos^2 \theta + 2 \cdot \frac{3}{\sqrt{5}} \sin \theta \cos \theta \\
 &= 1 + \frac{2 \cdot 3}{\sqrt{5}} \left(-\frac{2}{3} \right) \frac{5}{3\sqrt{5}} = 1 - \frac{4}{3} = -\frac{1}{3} \quad (4)
 \end{aligned}$$

$$\begin{aligned}
 b_{rq} = b_{24} = b_{42} &= \frac{1}{2} (a_{rr} - a_{qq}) \sin 2\theta + a_{rq} \cos 2\theta \\
 &= \frac{1}{2} (a_{22} - a_{44}) \sin 2\theta + a_{24} \cos 2\theta \\
 &= \frac{1}{2} \left(1 - 1 \right) \sin 2\theta + \frac{3}{\sqrt{5}} (\cos^2 \theta - \sin^2 \theta) = \frac{3}{\sqrt{5}} \left(\left(\frac{5}{3\sqrt{5}} \right)^2 - \left(-\frac{2}{3} \right)^2 \right) = \\
 &= \frac{3}{\sqrt{5}} \left[\frac{5}{9} - \frac{4}{9} \right] = \frac{3}{\sqrt{5}} \frac{1}{9} = \frac{1}{3\sqrt{5}} \quad (5)
 \end{aligned}$$

$$\begin{aligned}
 b_{ir} &: i=3 \\
 b_{23} = b_{32} &= a_{ir} \cos \theta - a_{iq} \sin \theta = a_{32} \cos \theta - a_{34} \sin \theta = \\
 &= -1 \cos \theta - \frac{1}{\sqrt{5}} \sin \theta = -\frac{5}{3\sqrt{5}} - \frac{1}{\sqrt{5}} \left(-\frac{2}{3} \right) = \frac{-5+2}{3\sqrt{5}} = -\frac{1}{\sqrt{5}} \quad (6)
 \end{aligned}$$

$$\begin{aligned}
 b_{iq} &: i=3 \\
 b_{34} = b_{43} &= a_{ir} \sin \theta + a_{iq} \cos \theta \\
 &= a_{32} \sin \theta + a_{34} \cos \theta = -\sin \theta + \frac{1}{\sqrt{5}} \cos \theta = -\left(-\frac{2}{3} \right) + \frac{1}{\sqrt{5}} \frac{5}{3\sqrt{5}} \\
 &= \frac{2}{3} + \frac{1}{3} = 1 \quad (7)
 \end{aligned}$$

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Ονόρε φόρμ των (1)-(7) έχουμε:

$$A_3 = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 7/3 & -1/\sqrt{5} & 1/3\sqrt{5} \\ 0 & -1/\sqrt{5} & 2 & 1 \\ 0 & 1/3\sqrt{5} & 1 & -1/3 \end{bmatrix}$$

Στη συνέχεια ορίζουμε στοιχείο είναι το

$$\alpha_{rq} = a_{34} = 1, \text{ συν } r=3, q=4 \text{ ονόρε}$$

$$\begin{bmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

συν

$$P_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$\sin\theta = -\alpha a_{r-1,q}$$

$$= -\frac{3}{\sqrt{2}} \cdot a_{24} = -\frac{3}{\sqrt{2}} \cdot \frac{1}{3\sqrt{5}} = -\frac{1}{\sqrt{10}}$$

$$\cos\theta = \alpha a_{r-1,r} = \frac{3}{\sqrt{2}} a_{23}$$

$$= \frac{3}{\sqrt{2}} \left(-\frac{1}{\sqrt{5}} \right) = -\frac{3}{\sqrt{10}}$$

Αρ2

$$\alpha = \frac{1}{\sqrt{a_{r-1,r}^2 + a_{r-1,q}^2}} = \frac{1}{\sqrt{a_{23}^2 + a_{24}^2}}$$

$$= \frac{1}{\sqrt{\left(\frac{1}{\sqrt{5}}\right)^2 + \left(\frac{1}{3\sqrt{5}}\right)^2}} = \frac{1}{\sqrt{\frac{1}{5} + \frac{1}{9 \cdot 5}}} = \frac{1}{\sqrt{\frac{1}{5} \cdot \frac{10}{9}}} = \frac{1}{\sqrt{\frac{2}{9}}} = \sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}}$$

$$A_4 = P_3^T A_3 P_3 =$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta & -\sin\theta \\ 0 & 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 7/3 & -1/\sqrt{5} & 1/3\sqrt{5} \\ 0 & -1/\sqrt{5} & 2 & 1 \\ 0 & 1/3\sqrt{5} & 1 & -1/3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 7/3 & -1/\sqrt{5} & 1/3\sqrt{5} \\ 0 & -1/\sqrt{5} & 2\cos\theta - \sin\theta & \cos\theta + \frac{1}{3}\sin\theta \\ 0 & 1/3\sqrt{5} & 2\sin\theta + \cos\theta & \sin\theta - \frac{1}{3}\cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 7/3 & -\frac{1}{\sqrt{5}}\cos\theta - \frac{1}{3\sqrt{5}}\sin\theta & -\frac{1}{\sqrt{5}}\sin\theta + \frac{1}{3\sqrt{5}}\cos\theta \\ 0 & -\frac{1}{\sqrt{5}}\cos\theta - \frac{1}{3\sqrt{5}}\sin\theta & 2\cos^2\theta - \sin^2\theta - \sin\theta\cos\theta + \frac{1}{3}\sin^2\theta & 2\cos\theta\sin\theta - \sin^2\theta + \cos^2\theta + \frac{1}{3}\sin\theta\cos\theta \\ 0 & -\frac{1}{\sqrt{5}}\sin\theta + \frac{1}{3\sqrt{5}}\cos\theta & 2\sin\theta\cos\theta + \cos^2\theta - \sin^2\theta + \frac{1}{3}\sin\theta\cos\theta & 2\sin^2\theta + \sin\theta\cos\theta + \sin\theta\cos\theta + \frac{1}{3}\cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 7/3 & -\frac{1}{\sqrt{5}}\cos\theta - \frac{1}{3\sqrt{5}}\sin\theta & -\frac{1}{\sqrt{5}}\sin\theta + \frac{1}{3\sqrt{5}}\cos\theta \\ 0 & -\frac{1}{\sqrt{5}}\cos\theta - \frac{1}{3\sqrt{5}}\sin\theta & 2\cos^2\theta + \frac{1}{3}\sin^2\theta - 2\sin\theta\cos\theta & (2 + \frac{1}{3})\sin\theta\cos\theta + \cos^2\theta - \sin^2\theta \\ 0 & -\frac{1}{\sqrt{5}}\sin\theta + \frac{1}{3\sqrt{5}}\cos\theta & (2 + \frac{1}{3})\sin\theta\cos\theta + \cos^2\theta - \sin^2\theta & 2\sin\theta\cos\theta + 2\sin^2\theta + \frac{1}{3}\cos^2\theta \end{bmatrix}$$

$$r=3, q=4$$

$$b_{r-1,q} = b_{q,r-1} = a_{r-1,r} \sin \theta + a_{r-1,q} \cos \theta (=0)$$

$$b_{24} = b_{42} = a_{23} \sin \theta + a_{24} \cos \theta = -\frac{1}{\sqrt{5}} \cdot \left(-\frac{1}{\sqrt{10}}\right) + \frac{1}{3\sqrt{5}} \left(-\frac{3}{\sqrt{10}}\right) \\ = \frac{1}{5\sqrt{2}} - \frac{1}{5\sqrt{2}} = 0.$$

$$b_{ir} = b_{ri} = a_{ir} \cos \theta - a_{iq} \sin \theta \quad \left| \begin{array}{l} b_{1q} = b_{qi} = a_{1r} \sin \theta + a_{1q} \cos \theta \\ \Rightarrow b_{14} = b_{41} = 0. \end{array} \right.$$

$$\stackrel{i=1}{\Rightarrow} b_{13} = a_{13} \cos \theta - a_{14} \sin \theta = 0$$

$$b_{rr} = a_{rr} \cos^2 \theta + a_{qq} \sin^2 \theta - 2a_{rq} \sin \theta \cos \theta$$

$$b_{33} = a_{33} \cos^2 \theta + a_{44} \sin^2 \theta - 2a_{34} \sin \theta \cos \theta = 2 \cdot \left(-\frac{3}{\sqrt{10}}\right)^2 + \frac{1}{3} \left(-\frac{1}{\sqrt{10}}\right)^2 - 2 \cdot 1 \cdot \left(-\frac{1}{\sqrt{10}}\right) \left(-\frac{3}{\sqrt{10}}\right) = \\ = 2 \cdot \frac{9}{10} - \frac{1}{3} \frac{1}{10} - \frac{6}{10} = \frac{18}{10} - \frac{1}{30} - \frac{6}{10} = \frac{12}{10} - \frac{1}{30} = \frac{36-1}{30} = \frac{35}{30}.$$

$$b_{qq} = a_{rr} \sin^2 \theta + a_{qq} \cos^2 \theta + 2a_{rq} \sin \theta \cos \theta$$

$$b_{44} = a_{33} \sin^2 \theta + a_{44} \cos^2 \theta + 2a_{34} \sin \theta \cos \theta = 2 \left(-\frac{1}{\sqrt{10}}\right)^2 - \frac{1}{3} \left(-\frac{3}{\sqrt{10}}\right)^2 + 2 \cdot 1 \cdot \left(-\frac{1}{\sqrt{10}}\right) \left(-\frac{3}{\sqrt{10}}\right) = \\ = \frac{2}{10} - \frac{1}{3} \frac{9}{10} - \frac{6}{10} = \frac{-7}{10}.$$

$$b_{rq} = b_{qr} = \frac{1}{2} (a_{rr} - a_{qq}) \sin 2\theta + a_{rq} \cos 2\theta$$

$$b_{34} = b_{43} = \frac{1}{2} (a_{33} - a_{44}) 2 \sin \theta \cos \theta + 1 (\cos^2 \theta - \sin^2 \theta) = \\ = \frac{1}{2} \left(2 + \frac{1}{3}\right) 2 \left(-\frac{1}{\sqrt{10}}\right) \left(-\frac{3}{\sqrt{10}}\right) + \left[\left(-\frac{3}{\sqrt{10}}\right)^2 - \left(-\frac{1}{\sqrt{10}}\right)^2\right] \\ = \frac{7}{3} \frac{3}{10} + \frac{9}{10} - \frac{1}{10} = \frac{15}{10}.$$

$$b_{ir} = b_{ri} = a_{ir} \cos \theta - a_{iq} \sin \theta$$

$$b_{32} = b_{23} \stackrel{i=2}{=} a_{23} \cos \theta - a_{24} \sin \theta$$

$$= -\frac{1}{\sqrt{5}} \cdot \left(-\frac{3}{\sqrt{10}}\right) - \frac{1}{3\sqrt{5}} \cdot \left(-\frac{1}{\sqrt{10}}\right) =$$

$$= \frac{3}{5\sqrt{2}} + \frac{1}{3} \frac{1}{5\sqrt{2}} = \left(3 + \frac{1}{3}\right) \frac{1}{5\sqrt{2}} = \frac{10}{3} \frac{1}{5\sqrt{2}} = \frac{2}{3\sqrt{2}} = \frac{\sqrt{2}}{3}$$

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