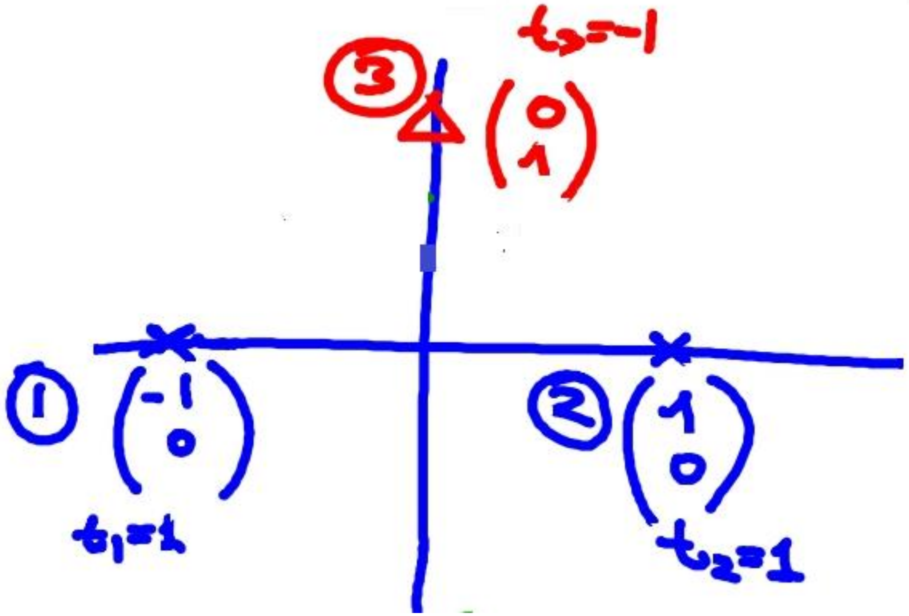
SVM classification, non-linear

Example:



$$K = (1 + \underline{x}_i \cdot \underline{x}_j)^2$$

$$Q_{11} = \left[1 + (-1\ 0)\begin{pmatrix}-1\\0\end{pmatrix}\right]^2 = 4$$

$$Q_{12} = \left[1 + (-1\ 0)\begin{pmatrix}1\\0\end{pmatrix}\right]^2 = 0$$

$$Q_{23} = \left[1 + (1\ 0)\begin{pmatrix}0\\1\end{pmatrix}\right]^2 = 1$$

$$Q = \begin{bmatrix} 4 & 0 & 1 \\ 0 & 4 & 1 \\ 1 & 1 & 4 \end{bmatrix} \qquad \mathcal{L}_D = \lambda_1 + \lambda_2 + \lambda_3 - \frac{4}{2}\lambda_1^2 - \frac{4}{2}\lambda_2^2 - \frac{4}{2}\lambda_3^2$$

$$- 0 \cdot \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 =$$

$$= \lambda_1 + \lambda_2 + \lambda_3 - 2\lambda_1^2 - 2\lambda_2^2 - 2\lambda_3^2$$

$$+ \lambda_2\lambda_3 + \lambda_3\lambda_1$$

<u>Constraints</u>: $\lambda_1 + \lambda_2 - \lambda_3 = 0, \; \lambda_i \geq 0$

<u>Symmetry</u>: $\lambda_2 = \lambda_1$

<u>Constraint</u>: $2\lambda_1 - \lambda_3 = 0 \Rightarrow \lambda_3 = 2\lambda_1 \qquad \mathcal{L}_D = 4\lambda_1 - 8\lambda_1^2$

$$\frac{\partial L_D}{\partial \lambda_1} = 0 \Rightarrow 4 - 16\lambda_1 = 0 \Rightarrow \lambda_1 = \frac{1}{4}$$

$$\lambda_1 = \frac{1}{4}, \quad \lambda_2 = \frac{1}{4}, \quad \lambda_3 = \frac{1}{2}$$

Find W_0: $\quad \sum_i \lambda_i t_i K(\underline{x}_1, \underline{x}_i) + W_0 = 1$

$$\Rightarrow \frac{1}{4} Q_{11} + \frac{1}{4} Q_{12} - \frac{1}{2} Q_{13} + W_0 = 1$$

$$\Rightarrow 1 - \frac{1}{2} + W_0 = 1 \Rightarrow \boxed{W_0 = +\frac{1}{2}}$$

Decision curve:

$$\lambda_1 t_1 K(\underline{x},\underline{x}_1) + \lambda_2 t_2 K(\underline{x},\underline{x}_2) + \lambda_3 t_3 K(\underline{x},\underline{x}_3) + \frac{1}{2} = 0$$

$$K(\underline{x},\underline{x}_i) = (1 + \underline{x}\cdot\underline{x}_i)^2 \qquad \underline{x} = \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$\frac{1}{4}(1-X)^2 + \frac{1}{4}(1+X)^2 - \frac{1}{2}(1+Y)^2 + \frac{1}{2} = 0$$

$$\Rightarrow \boxed{X^2 - Y^2 - 2Y + 1 = 0}$$