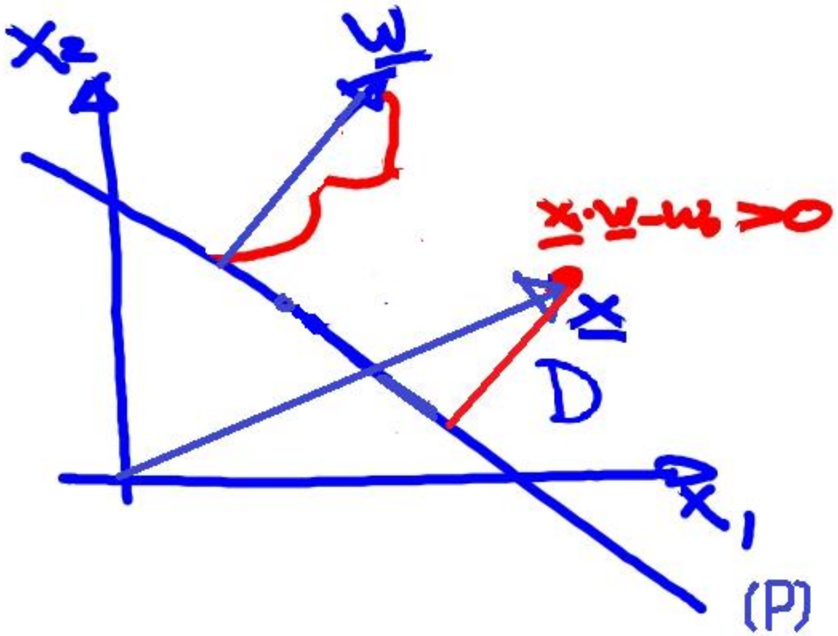
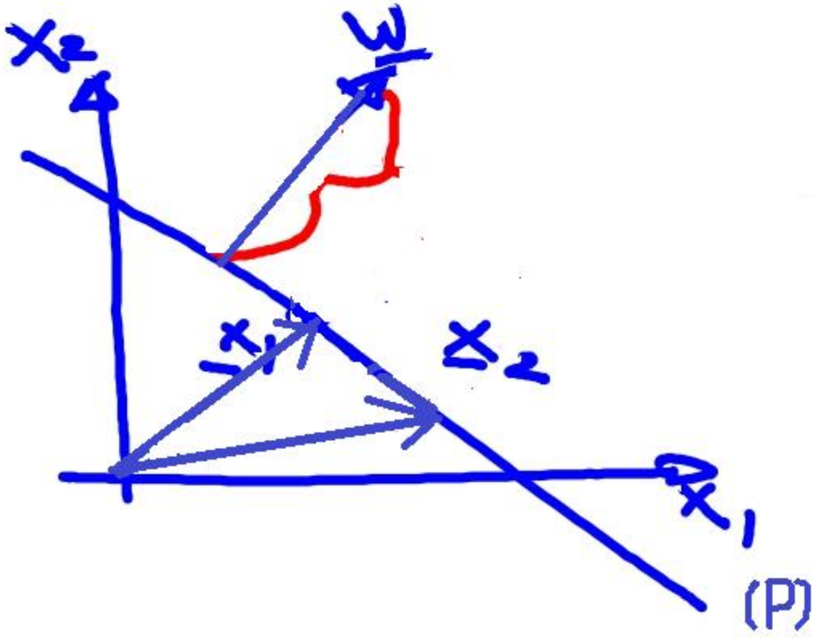


1) Weight vector $\perp$ separating hyperplane (P)

2) Weight vector directed toward "(+) semispace".

3) Distance of $\underline{x}$ from sep. hyperplane:

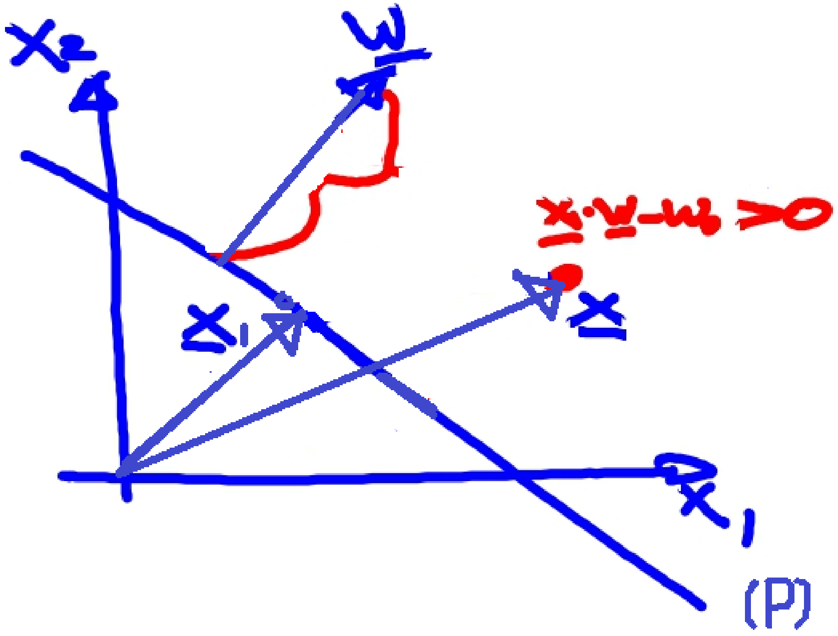
$$D = \frac{\underline{x}\cdot\underline{w} - w_0}{\|\underline{w}\|}$$



1) Weight vector $\perp$ separating hyperplane (P)

$\underline{x}_1, \underline{x}_2 \in (P)$

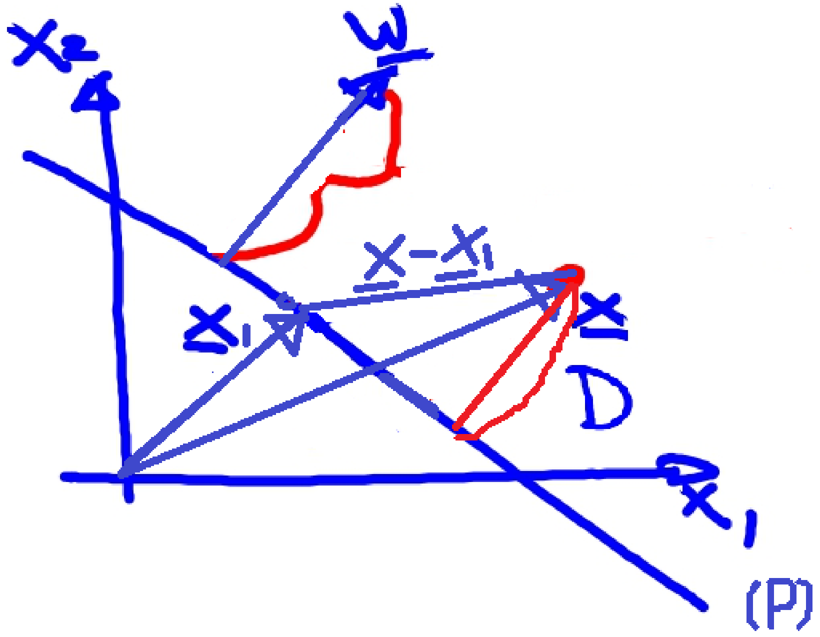
$$\left.\begin{array}{l}\underline{w}\cdot\underline{x}_1 - w_0 = 0\\ \underline{w}\cdot\underline{x}_2 - w_0 = 0\end{array}\right\} \Rightarrow \underline{w}\cdot(\underline{x}_1 - \underline{x}_2) = 0.$$



2) Weight vector directed toward "(+) semispace".

Consider: $\underline{x}$: $\underline{w}\cdot\underline{x}-w_0>0.$
$\underline{x}_1 \in (P)$: $\underline{w}\cdot\underline{x}_1-w_0=0.$ $\Big\} \Rightarrow$

$$\underline{w}\cdot(\underline{x}-\underline{x}_1)>0.$$



3) Distance of $\underline{x}$ from sep. hyperplane:

$$D = (\underline{x} - \underline{x}_1) \cdot \frac{\underline{w}}{\|\underline{w}\|} =$$

$$= \frac{(\underline{x} \cdot \underline{w} - w_0) - \cancel{(\underline{x}_1 \cdot \underline{w} - w_0)}^{\,0}}{\|\underline{w}\|}$$

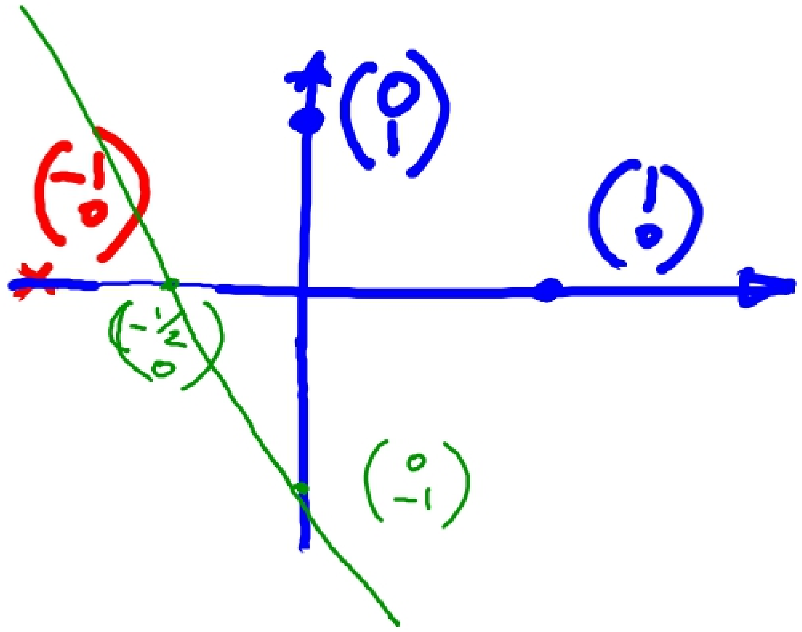
$$= \frac{\underline{x} \cdot \underline{w} - w_0}{\|\underline{w}\|}$$

$$D = \|\underline{x} - \underline{x}_1\| \cos\theta$$

$$(\underline{x} - \underline{x}_1) \cdot \underline{w} = \|\underline{x} - \underline{x}_1\| \, \|\underline{w}\| \cos\theta$$

$$\Rightarrow D = \|\underline{x} - \underline{x}_1\| \cos\theta = \frac{(\underline{x} - \underline{x}_1) \cdot \underline{w}}{\|\underline{w}\|}$$

perceptron: Example

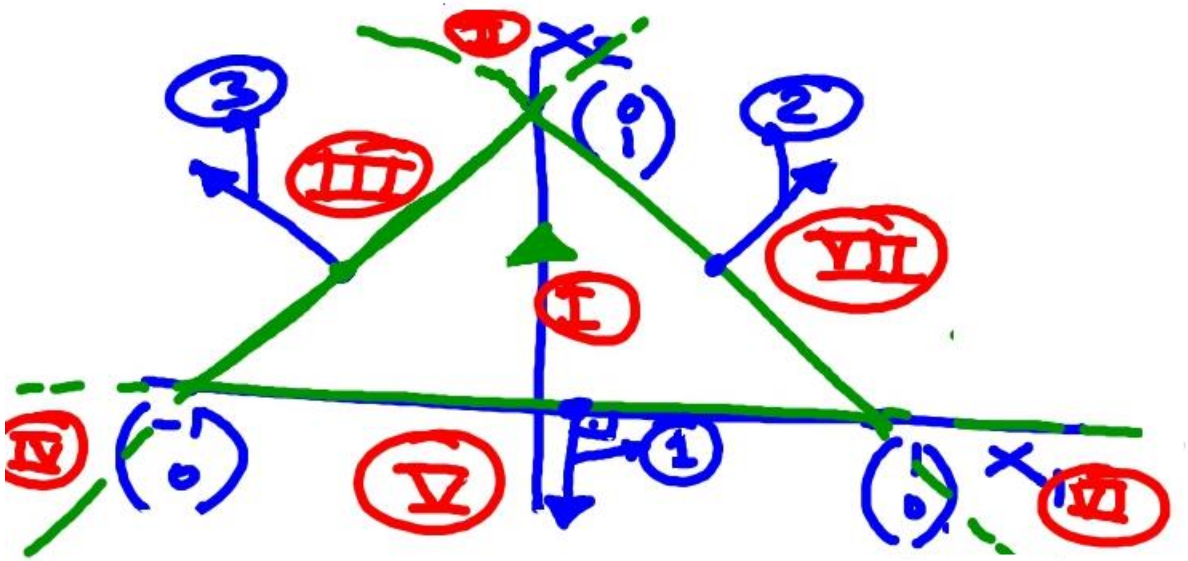


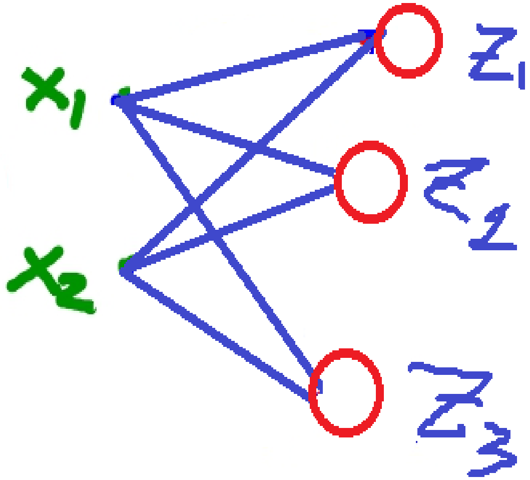
patterns		targets
1	0	1
0	1	1
-1	0	-1

$\varepsilon = 1$

w_1	w_2	w_0	tx_1	tx_2	tx_0	$t\underline{w}\cdot\underline{x}$	update Y/N	$\Delta \underline{w}$
0	0	0	1	0	-1	0	Y	1 0 -1
1	0	-1	0	1	-1	1	N	0 0 0
1	0	-1	1	0	1	0	Y	1 0 1
2	0	0	1	0	-1	2	N	0 0 0
2	0	0	0	1	-1	0	Y	0 1 -1
2	1	-1	1	0	1	1	N	0 0 0
2	1	-1	1	0	-1	3	N	0 0 0
2	1	-1	0	1	-1	2	N	0 0 0
2	1	-1	1	0	1	1	N	0 0 0

2-layered perceptron: Example.





$$\boxed{-x_2 = 0}$$

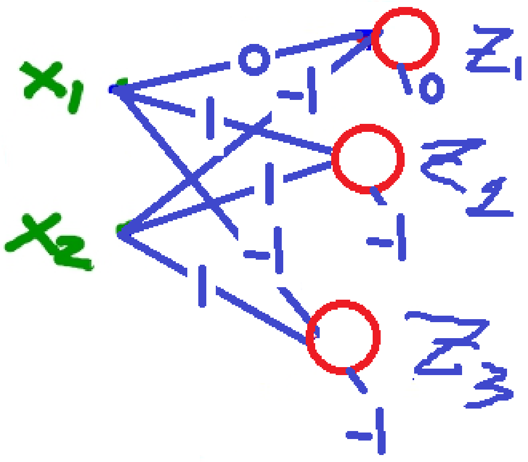
$$x_1 + x_2 - 1 = 0$$

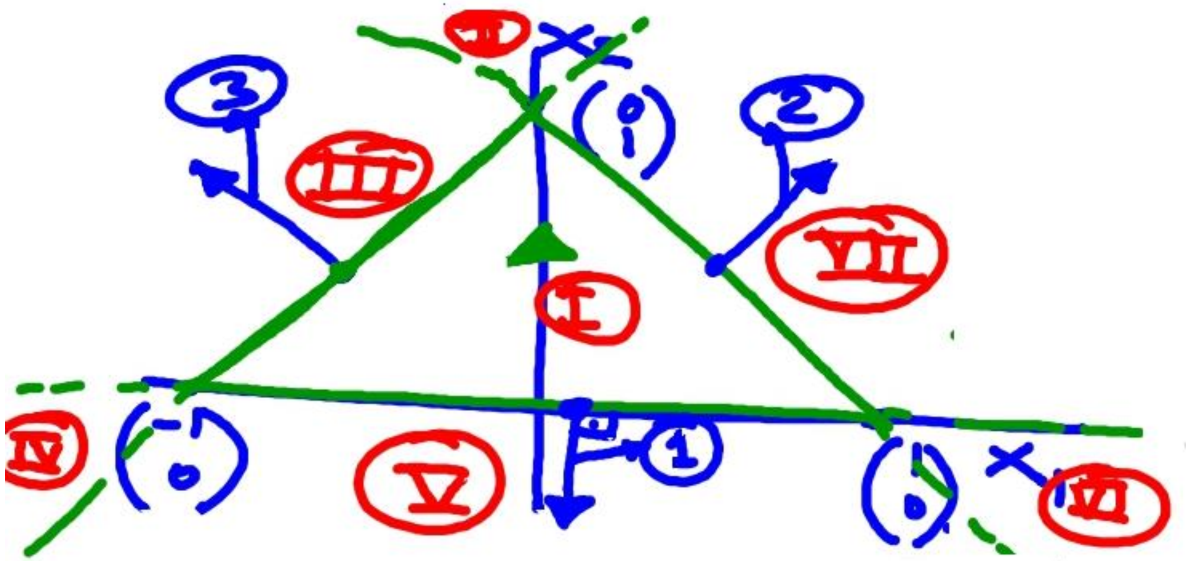
$$\boxed{-x_1 + x_2 - 1 = 0}$$

$$\boxed{-x_2 = 0}$$

$$x_1 + x_2 - 1 = 0$$

$$\boxed{-x_1 + x_2 - 1 = 0}$$



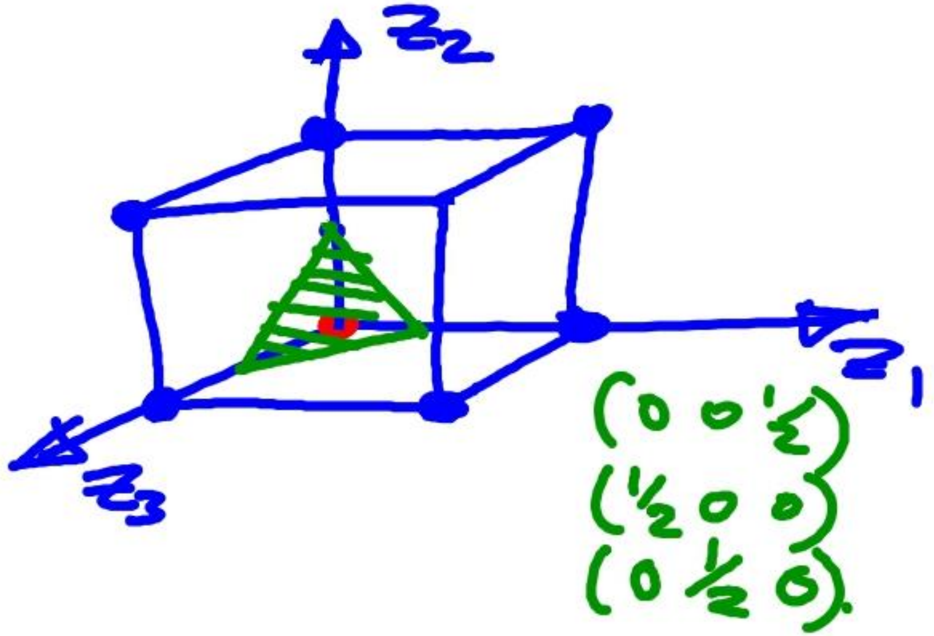


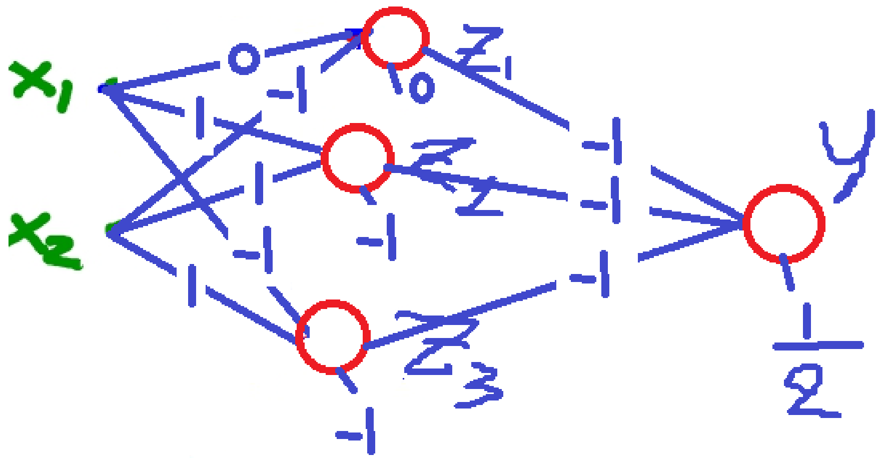
Regions:	z_1	z_2	z_3
I	0	0	0
II	0	1	1
III	0	0	1
IV	1	0	1
V	1	0	0
VI	1	1	0
VII	0	1	0

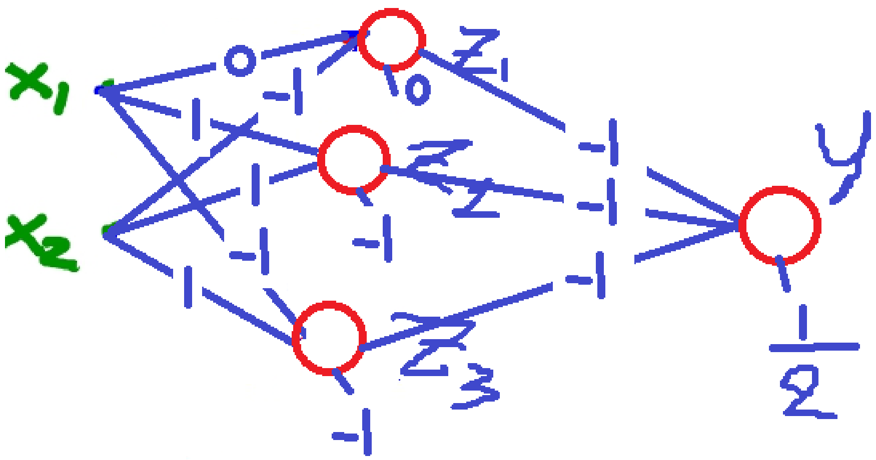
$$V_1 z_1 + V_2 z_2 + V_3 z_3 + V_0 = 0$$

$$\left.\begin{array}{l} \frac{1}{2} V_3 + V_0 = 0 \\ \frac{1}{2} V_2 + V_0 = 0 \\ \frac{1}{2} V_1 + V_0 = 0 \end{array}\right\} \Rightarrow V_1 = V_2 = V_3 = -2V_0$$

$$\boxed{-z_1 - z_2 - z_3 + \frac{1}{2} = 0}$$







$$\underline{\underline{x_1 = 0 \quad x_2 = \tfrac{1}{2}}}$$

$$z_1 = f\left(0 - \tfrac{1}{2}\right) = 0.$$

$$z_2 = f\left(1 \cdot 0 + 1 \cdot \tfrac{1}{2} - 1\right) = 0.$$

$$z_3 = f\left(-1 \cdot 0 + 1 \cdot \tfrac{1}{2} - 1\right) = 0.$$

$$y = f\left(\tfrac{1}{2}\right) = \underline{1}.$$