# Geometric data analysis

## 1b. Voronoi diagram and Delaunay triangulation

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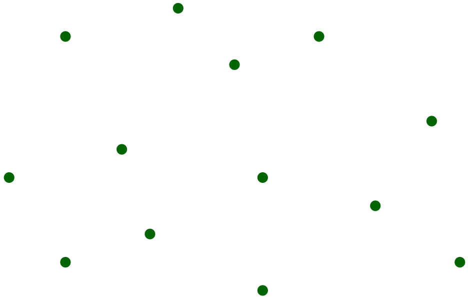
# Voronoi diagram

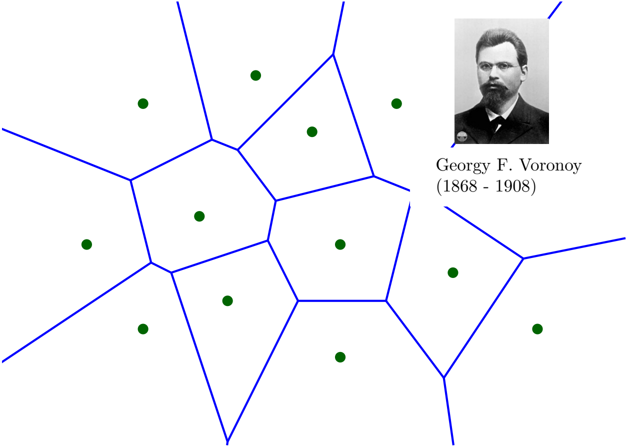
Sites: $P := \{p_1, \ldots, p_n\} \subset \mathbb{R}^2$



# Voronoi diagram

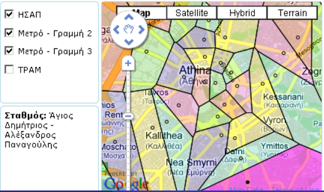
Sites: $P := \{p_1, \ldots, p_n\} \subset \mathbb{R}^2$

Voronoi cell$(p_i) = \{\, q : \mathrm{NS}(q) = p_i \,\}$, $\mathrm{NS}(\cdot)$ = Nearest site.

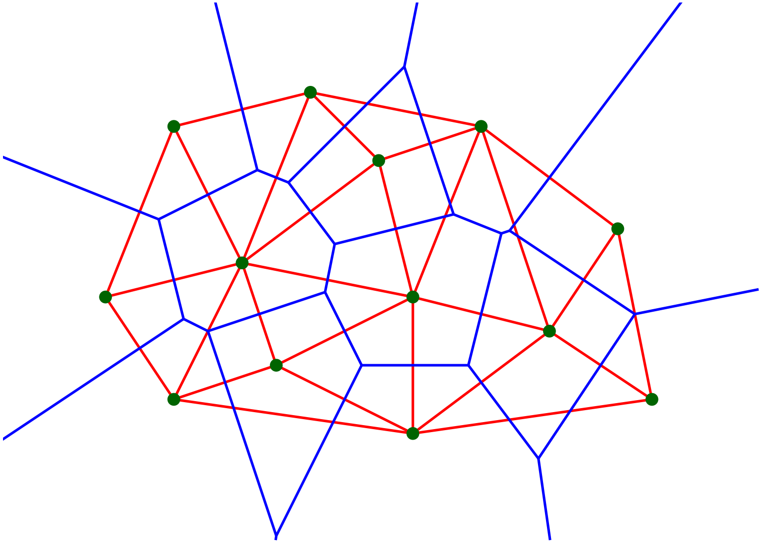


Georgy F. Voronoy
(1868 - 1908)

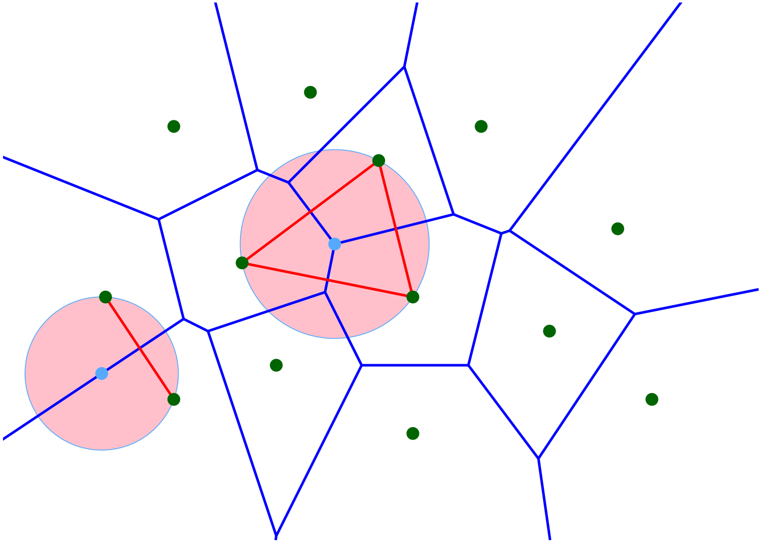
# Nature


ΗΣΑΠ
Μετρό - Γραμμή 2
Μετρό - Γραμμή 3
ΤΡΑΜ
Σταθμός: Άγιος Δημήτριος - Αλέξανδρος Παναγούλης
Satellite
Hybrid
Terrain
Athina
Kessariani
Kallithea
Vyron
Ymittos
Nea Smyrni
Dafni

# Delaunay Triangulation: dual of Voronoi diagram

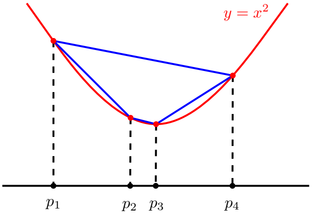


# Main Delaunay property: empty circle/sphere

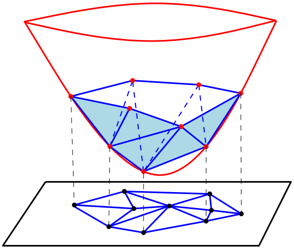


# Delaunay triangulation: projection from parabola

- Lift $p = (x) \in \mathbb{R}$ to $\widehat{p} = (x, x^2) \in \mathbb{R}^2$
- Compute Convex Hull of $\widehat{p}$'s
- Project lower hull to $\mathbb{R}$

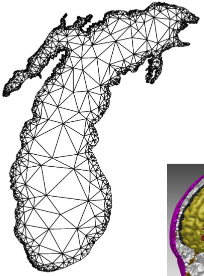



- Lift $p = (x, y) \in \mathbb{R}^2$ to $\widehat{p} = (x, y, x^2 + y^2) \in \mathbb{R}^3$
- Compute Convex hull of $\widehat{p}$'s
- Project lower hull, triangulate lower facets that are not triangles



# Complexity and applications

Delaunay triangulation in $\mathbb{R}^d \simeq$ convex hull in $\mathbb{R}^{d+1}$.
Hence Complexity in general $d$: $d$-Del $=$ $d$-Vor $= \Theta(n \log n + n^{\lceil d/2 \rceil})$



## Applications

Nearest Neighbors
Reconstruction
Meshing