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Introduction to Geomagnetically Trapped Radiation

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Chapter

Appendix B - Gyration, bounce and drift frequencies in a dipole field

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Appendix B

Gyration, bounce and drift frequencies in a dipole field

The gyration frequencies v_g and periods τ_g of low-energy electrons and protons in an idealized, symmetric dipole field are shown in Figure B.1 for altitudes up to 10⁴ km. For relativistic particles these values should be divided by γ .

The bounce and drift periods of equatorial particles in a dipole field are given as a function of particle energy in Figure B.2 for electrons and in Figure B.3 for protons. Since the bounce period is inversely proportional to particle velocity (see equation (4.26)), the period becomes almost independent of energy as the electrons become relativistic.

For particles mirroring at any latitude, the bounce period can be expressed as

$$\tau_{\rm b}(\rm s) = 6.37 \times 10^6 Lg(\lambda_m)/v \tag{B.1}$$

where v is the particle velocity (meters s⁻¹) and $g(\lambda_m)$ is a geometrical quantity which depends on the mirroring latitude (or the equatorial pitch angle). A plot of $g(\lambda_m)$ is given in Figure B.4 as a function of both λ_m and α_E .

The azimuthal drift period for off-equatorial particles is expressed by the approximate formula (4.47). A graphical representation of the geometrical factor $\mathcal{D}(\lambda_m)$ as a function of mirroring latitude is given in Figure B.5. The drift period is

$$\tau_{\rm d}(s) = \frac{\mathcal{D}(\lambda_{\rm m})}{L(E+m_0c^2)(v/c)^2}$$

where the kinetic energy of the particle E and the rest mass energy m_0c^2 are expressed in MeV. The ratio $(v/c)^2$ for any kinetic energy T (in rest mass units) is given by $(v/c)^2 = \beta^2 = T(T+2)/(T+1)^2$. For example, a 1 MeV electron has T = 1.957, $\beta^2 = 0.886$, and $E + m_0c^2 = 1.511$ MeV. If it has equatorial pitch angle of 90° and is at L = 2, $\tau_d = 5300/2(1.511)(0.885) = 1.981 \times 10^3$ s, in agreement with Figure B.2.

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Figure B.1. Gyration frequencies and periods of trapped electrons and protons in a centered, dipole magnetic field.



Figure B.2. Bounce (τ_b) and drift (τ_d) periods of equatorially trapped electrons.



Figure B.3. Bounce (τ_b) and drift (τ_d) periods of equatorially trapped protons.



Figure B.4. Geometric factor for computing bounce periods of off-equatorial particles.



Figure B.5. Geometric factor for computing drift periods of off-equatorial particles.