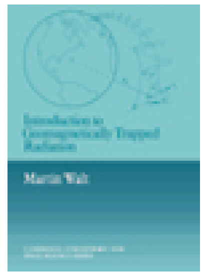

Introduction to Geomagnetically Trapped Radiation

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3

The geomagnetic field

Earth's magnetic field is produced by a number of current systems. By far the most important from the standpoint of trapped radiation is the interior current system of the Earth's dynamo. Deep in the Earth the convection of hot, conducting material forms a system of moving conductors. The motion of these conductors across the geomagnetic field induces electric currents, which in turn reinforce the magnetic field. Thus, convection driven by heat acts as a self-exciting dynamo to produce the main part of the geomagnetic field. While this field is steady on a time scale of less than a year, secular changes do occur and have been measured directly for several centuries. Systematic variations in the shape of the field are taking place and the overall geomagnetic field is becoming weaker at a rate which, if continued, will cause the Earth's field to disappear in about 2000 years. However, the present downward trend may only be a temporary fluctuation and could change at any time.

There is clear geological evidence that the polarity of the geomagnetic field has reversed at irregular intervals of about one million years. The reason for these reversals is not known, but explanations proposed include internally driven oscillations similar to those causing the solar sunspot cycles. Another mechanism suggested invokes meteoritic bombardment which would disrupt the existing polarity and allow a new polarity to develop in a random direction.

Most planets have magnetic fields. With the exception of Venus, Mars and possibly Pluto, all planets have strong fields. The Sun also has an overall magnetic field in addition to the more localized magnetic patches near sunspots and active regions. Thus, one finds that large, rotating bodies with conducting liquid or gaseous interiors generally have magnetic fields produced by internal current systems.

While a theory capable of predicting planetary fields has not been forthcoming, a curious relationship between planetary magnetic fields and planetary parameters has been noted. Known as the magnetic Bode's law, it proposes a linear relationship between the logarithm of the magnetic moment and the logarithm of the angular momentum of a planet. Like the original Bode's law relating planetary orbits, this relationship is entirely empirical. There is, of course, some reason to expect that the larger the planet and the more rapid the rotation, the larger will be the internally generated magnetic field.

Other sources of the geomagnetic field are crystal rocks, which retain permanent magnetism, and currents in the ionosphere and magnetosphere. The magnetism of geological deposits is generally weak and localized and has little influence on the geomagnetic field at high altitude. However, rock magnetism is quite important as a tracer of the Earth's magnetic history and as a clue to mineral deposits. The ionospheric current systems are strongest in the polar regions. They, too, have little influence on trapped radiation, and their average values can be incorporated into the model of the core field. The other major current systems of the magnetosphere are the ring current, the magnetopause current, the tail current sheet and the field aligned currents connecting the polar ionospheres to the magnetosphere. The ring current system consists of an extended band of trapped particles circling the Earth in the magnetic equatorial plane between $3R_E$ and $5R_E$ from the Earth's center. Since particles drifting around the Earth produce a magnetic field which inside their drift orbits opposes the Earth's field, the ring current manifests itself as a decrease in the magnetic field observed on Earth. From the ground the most noticeable evidence of the ring current is the decrease in the geomagnetic field observed on Earth during magnetic storms, when the number of trapped particles in the ring current region increases. The magnetopause current system flows along the boundary between the solar plasma and the Earth's magnetic field. It is largely responsible for the overall shape of the magnetosphere boundary. Similarly, the tail current sheet, which is an east-to-west current flowing in the equatorial plane and extending from about $10R_E$ to some large distance down the tail, separates the northern and southern magnetic tail lobes. The field aligned current systems flow along magnetic field lines and connect magnetospheric plasma with the ionosphere. These currents are important in transferring energy and momentum between the magnetosphere and ionosphere and are confined to field lines entering the atmosphere at high latitudes.

Computer programs have been developed to calculate the geomagnetic field at arbitrary positions. These magnetospheric models include many or all of the above mentioned current systems and will be discussed in the following section.

Representation of the Earth's interior field

In magnetospheric research it is essential to have a convenient method of calculating values of the geomagnetic field at arbitrary locations. For proving theorems and evaluating concepts, a simple, analytic representation of the field is usually adequate. However, when more accurate and detailed knowledge is required, rather extensive numerical algorithms are used to generate field values. In this chapter some of the customary ways of describing the Earth's magnetic field will be presented.

The frequently used multipole expansion was first utilized by Gauss and is obtained as follows. One wishes to express the magnetic field by a concise formula containing many adjustable parameters, but the field must satisfy Maxwell's equations regardless of the values of the parameters.

Maxwell's equations for magnetic fields are

$$\nabla \cdot \mathbf{B} = 0 \tag{3.1}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{i} \tag{3.2}$$

In the steady state $\partial/\partial t = 0$, and if there are no currents passing through the magnetosphere in the region of interest, (3.2) becomes

$$\nabla \times \mathbf{B} = 0 \tag{3.3}$$

If the curl of a quantity is zero, that quantity can be represented as the gradient of a scalar. Therefore, for some potential field $\psi(\mathbf{r})$ equation (3.3) will hold if

$$\mathbf{B}(\mathbf{r}) = -\nabla\psi(\mathbf{r}) \tag{3.4}$$

From equation (3.1),

$$\nabla \cdot \mathbf{B} = -\nabla^2\psi = 0 \tag{3.5}$$

A choice of ψ which satisfies equation (3.5) will automatically satisfy equations (3.1) and (3.3). The solution to equation (3.5) (Laplace's equation) in spherical coordinates is found by assuming ψ to be separable into a product of functions of the three coordinates

$$\psi = R(r)\,\Theta(\theta)\,\Phi(\phi) \tag{3.6}$$

Inserting this expression into (3.5), dividing by ψ and separating variables leads to the various functions

$$R(r) = Ar^n + \frac{B}{r^{n+1}} \tag{3.7}$$

$$\Phi(\phi) = C\cos m\phi + D\sin m\phi \tag{3.8}$$

$$\Theta(\theta) = E\,P_n^m(\cos\theta) + F\,Q_n^m\,(\cos\theta) \tag{3.9}$$

where P_n^m and Q_n^m are Legendre functions. The quantities A, B, C, D, E and F are constants of integration and n and m are separation constants. The separation constants are not arbitrary: m must be an integer if $\Phi(\phi)$ is to be single valued. The Legendre functions in equation (3.9) must have integer n or they will diverge. Similarly, $A = F = 0$ when representing the Earth's field since the functions they multiply are infinite at $r = \infty$ or at $\theta = 0°$ and 180°. Thus, the general form of (3.6) appropriate to representation of the core field is

$$\psi = \sum_{n=1}^{\infty}\sum_{m=0}^{n}\frac{1}{r^{n+1}}\,P_n^m\,(\cos\theta)\,(C_n^m\sin m\phi + D_n^m\cos m\phi) \tag{3.10}$$

The $n = 0$ term is excluded to avoid divergence of $\mathbf{B}$ at the origin.

In geomagnetism it is cusomary to write (3.10) as

$$\psi = \mathrm{R_E}\sum_{n=1}^{\infty}\left(\frac{\mathrm{R_E}}{r}\right)^{n+1}\sum_{m=0}^{n}(g_n^m\cos m\phi + h_n^m\sin m\phi)\,P_n^m\,(\cos\theta) \tag{3.11}$$

where the Legendre functions have the Schmidt normalization, namely

$$P_n^m = \left[\frac{(n-m)!(2-\delta_{0,m})}{(n+m)!}\right]^{1/2} P_{n,m}$$

where $P_{n,m}$ are the normal Legendre functions and $\delta_{0,m}$ is unity for $m = 0$ and zero otherwise. The constant factor $\mathrm{R_E}$ is included in equation (3.11) to give g_n^m and h_n^m the dimensions of a magnetic field. The coefficients g_n^m and h_n^m are adjusted to fit experimental values of the magnetic field sampled on a worldwide basis. Although the sum over n extends to infinity, the magnitude of the terms drops rapidly with increasing n. The first few coefficients for a reference field in 1985 in nT (10^{-9} tesla) are

n	m	g_n^m	h_n^m
1	0	−29 877	
1	1	−1903	5497
2	0	−2073	
2	1	3045	−2191
5	–	~200	~150

Most magnetic models include $\geqslant 48$ terms. The models also give the secular variations for the terms, namely the values of $\mathrm{d}g_n^m/\mathrm{d}t$ and $\mathrm{d}h_n^m/\mathrm{d}t$, so field calculations can be done for any time epoch.

Because of the $r^{-(n+1)}$ dependence of ψ the importance of the higher-order terms decreases rapidly with distance from the Earth. Hence, much of trapped radiation theory is developed based on the dominant $n = 1$ or dipole term. However, when comparing radiation belt measurements taken at various points, it is necessary to use a magnetic field representation which is accurate enough to specify the geomagnetic field. Computer programs exist for calculating **B** at any point in space and for tracing geomagnetic field lines. The most recent programs contain additional functions to represent the various current systems in space as well as those in the interior of the Earth. These programs usually specify current distributions or potential functions for the various current systems and add the magnetic fields produced by these systems to give an overall magnetospheric field.

The dipole field

The lowest order, but dominant, term in (3.11) is the dipole term with $n = 1$, $m = 0$. Because many of the important features of the radiation belts can be illustrated with a dipole field, some useful relations for this field will be derived. The dipole potential from (3.11) is

$$\psi = \mathrm{R_E}\left(\frac{\mathrm{R_E}}{r}\right)^2 g_1^0 \cos\theta \tag{3.12}$$

where the distance r is measured from the center of the dipole and θ is the polar angle or colatitude (see Figure 3.1). The magnetic field **B** is equal to $-\nabla\psi$. In spherical polar coordinates the components of **B** are

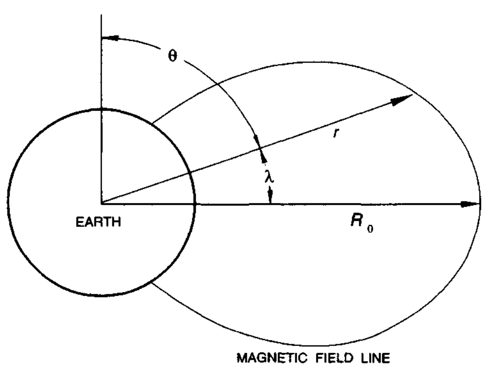


Figure 3.1. Dipole field coordinate system.

$$B_r = -\frac{\partial\psi}{\partial r} = 2\left(\frac{R_E}{r}\right)^3 g_1^0 \cos\theta = -2B_0\left(\frac{R_E}{r}\right)^3 \cos\theta \tag{3.13}$$

$$B_\theta = -\frac{1}{r}\frac{\partial\psi}{\partial\theta} = \left(\frac{R_E}{r}\right)^3 g_1^0 \sin\theta = -B_0\left(\frac{R_E}{r}\right)^3 \sin\theta \tag{3.14}$$

where B_0 is the mean value of the field on the equator at the Earth's surface. The components are negative since the direction of the field is in the minus $\hat{\mathbf{e}}_\theta$ direction and in the northern hemisphere it is in the minus $\hat{\mathbf{e}}_I$ direction. When considering only the magnitude of the field the minus signs can be omitted. For the Earth $B_0 = 3.12 \times 10^{-5}$ T. The dipole field is symmetric about its axis so that $B_\phi = 0$ everywhere.

The strength of a dipole can be characterized by the magnetic moment $\mathcal{M}$ whose units are ampere meters^2. In terms of the dipole moment the radial field component is

$$B_r = -\left(\frac{4\pi}{\mu_0}\mathcal{M}\right)\frac{2\cos\theta}{r^3}$$

Some authors designate the quantity $(4\pi/\mu_0)\mathcal{M}$ as the dipole moment so some confusion exists in published reports. The use of the equatorial surface field B_0 in (3.13) and subsequent equations avoids this ambiguity.

The intensity of the dipole field at any point in space is

$$B = \sqrt{(B_r^2 + B_\theta^2)} = B_0\left(\frac{R_E}{r}\right)^3 \sqrt{(1 + 3\cos^2\theta)} \tag{3.15}$$

The field intensity falls as r^{-3} with distance above the Earth and at constant r the intensity increases as one moves towards the poles. For a given value of r the field strength is twice as high over the poles as it is over the equator. Note that in equation (3.15) the r and θ dependences are separable. Hence, along any constant latitude line the field decreases as r^{-3}.

The equation for a geomagnetic field line in spherical coordinates is obtained by noting that the ratio of the lengths of the $\hat{\mathbf{e}}_r$ and $\hat{\mathbf{e}}_\theta$ components of the field line is

$$\frac{\mathrm{d}r}{r\mathrm{d}\theta} = \frac{B_r}{B_\theta} = \frac{2\cos\theta}{\sin\theta} \tag{3.16}$$

This equation can be integrated to give

$$r = R_0 \sin^2\theta \tag{3.17}$$

where R_0 is the value of r when $\theta = 90°$, namely the distance from the dipole to the point where the field line crosses the equatorial plane.

Expressed in terms of latitude λ,

$$r = R_0 \cos^2 \lambda \tag{3.18}$$

The distance along a field line for a dipole can be obtained analytically by integrating the equation for a distance element ds

$$ds = \sqrt{((dr)^2 + (r\, d\theta)^2)}$$

where dr and $r\,$dθ are constrained to be on a field line. Expressing dr in terms of dθ by differentiating the field line equation (3.17):

$$\begin{aligned} dr &= 2R_0 \sin\theta \cos\theta\, d\theta \\ ds &= \sqrt{(4\, R_0{}^2 \sin^2\theta \cos^2\theta + R_0{}^2 \sin^4\theta)}\, d\theta \\ &= R_0\sqrt{(1 + 3\cos^2\theta)} \sin\theta\, d\theta \end{aligned} \tag{3.19}$$

By changing the variable to $\chi = \cos\theta$ equation (3.19) can be integrated from $\chi = 0$ (equatorial plane) to some off-equator value χ giving

$$s = \int_0^{\chi} ds = \frac{R_0}{2}\left[\chi\sqrt{(1 + 3\chi^2)} + \frac{1}{\sqrt{3}} \ln\left(\sqrt{(1 + 3\chi^2)} + \sqrt{3}\,\chi\right)\right] \tag{3.20}$$

The intesity of the magnetic field along a field line passing through R_0 is obtained as a function of colatitude by inserting r from equation (3.17) into (3.15), giving

$$B(\theta) = B_0\left(\frac{\mathrm{R_E}}{R_0}\right)^3 \frac{\sqrt{(1 + 3\cos^2\theta)}}{\sin^6\theta} \tag{3.21}$$

$$= B_{\mathrm{eq}} \frac{\sqrt{(1 + 3\cos^2\theta)}}{\sin^6\theta}$$

$$= B_{\mathrm{eq}} \frac{\sqrt{(1 + 3\sin^2\lambda)}}{\cos^6\lambda} \tag{3.22}$$

B_{eq} is the value of B in the equatorial plane at distance R_0 from the dipole. From these equations it is apparent that the magnetic field along a field line increases monotonically with latitude as one moves from the equator to the poles.

If the Earth's field were a pure dipole located at the center of the Earth, contours of constant B on the Earth's surface would be lines of constant latitude. However, asymmetries in the interior current system introduce higher-order terms and the actual isointensity lines are as shown in Figure 3.2. Much of the distortion is caused by the fact that the magnetic axis is not aligned with the spin axis of the Earth and the center of the magnetic dipole is not at the center of the Earth. The poles are over northern Canada and southern Australia on the Mercator projection. Note, particularly, the large region of reduced field on the east coast of South America.

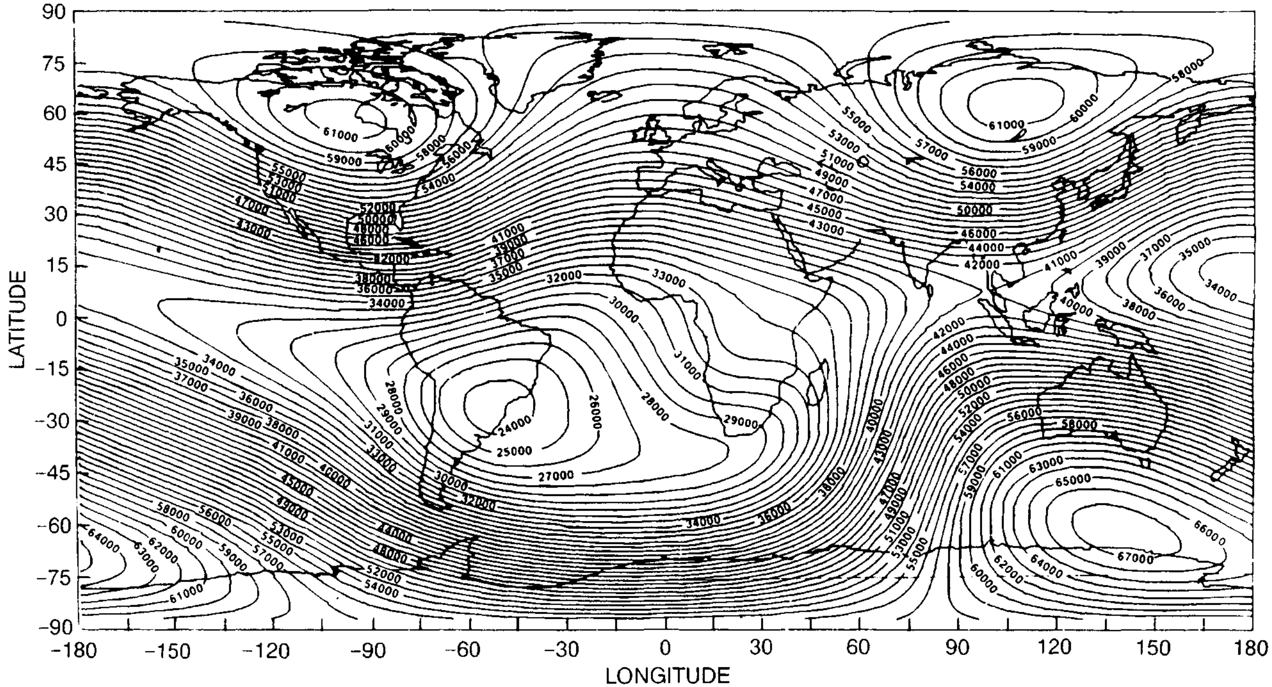


Figure 3.2. Isointensity lines for geomagnetic field at Earth's surface. Magnetic field intensity is in units of 10^{-9} tesla. In a pure centered dipole field the isointensity lines would be horizontal.

This anomaly has important consequences for the structure of the radiation belts at low altitude.

Representation of the external current systems

Although the electrical currents flowing inside the Earth are the most important currents in the production of the geomagnetic field, the external currents flowing in the magnetosphere also influence the field. These external currents are important at distances beyond about 4R and become dominant near the magnetopause or deep in the tail region. As stated earlier, the primary magnetospheric current systems are the magnetopause currents, the ring current, the tail current sheet and the field aligned currents. Although trapped particles rely on the properties of the core field for containment, the distortion of the core field by the external current systems is also relevant, and it is frequently necessary to include these systems in models of the geomagnetic field. Also, because the short-term time variations of **B** in the trapping region are caused by changes in these external currents, modeling the field fluctuations requires modeling the external field produced by these current systems.

Because the magnetopause currents are confined to the surface of the magnetosphere, the magnetic field produced by these currents can be expressed as the gradient of a scalar function for positions within the magnetosphere. The solution of Laplace's equation is again obtained by a spherical harmonic expansion as in equation (3.11). However, in this case positive powers of the radial distance are needed which give a potential field of

$$\psi_{\mathrm{ex}} = \mathrm{R_E} \sum_{n=1}^{\infty} \left(\frac{r}{\mathrm{R_E}}\right)^n \sum_{m=0}^{n} (\bar{g}_n^m \cos m\phi + \bar{h}_n^m \sin m\phi)\, P_n^m(\cos\theta) \quad (3.23)$$

With appropriate constants, this potential, when added to the core potential, represents a confined magnetosphere.

The ring current, the tail current and the field aligned current systems cannot be described by scalar potentials as they flow within the region of space under consideration and, therefore, $\nabla \times \mathbf{B}$ does not vanish. However, various analytical models have been developed to describe these magnetic fields and to allow rapid computer generation of field quantities. These analytic expressions contain parameters which are adjusted to fit magnetic field measurements. The most sophisticated models allow one to compute the external field for various conditions of geomagnetic activity, solar wind characteristics and angle between the dipole axis and the solar wind flow direction.

Problems

1. Assuming that the Earth has a centered dipole field consider the magnetic field line which passes through the equatorial plane at $2R_E$ (R_E = radius of Earth). Find
   (a) The latitude at which the field line reaches the Earth.
   (b) The latitude where $B(\lambda) = 2B_{eq}$ (B_{eq} = value of B on the field line at the equator). (Don't try to give an analytic solution. Use trial and error to obtain an approximate answer.)
   (c) The distance along the field line from the equator to the surface of the Earth.
   (d) The distance along the field line from the equator to the center of the Earth.

2. An auroral physicist wishes to aim her ground based camera upward parallel to the geomagnetic field. Fortunately, she lives on a planet where the magnetic field is a pure dipole located at the center of the planet and aligned with the geographic axis. If she works at a latitude of 67°, at what zenith angle (south of the local vertical) must she point her camera?

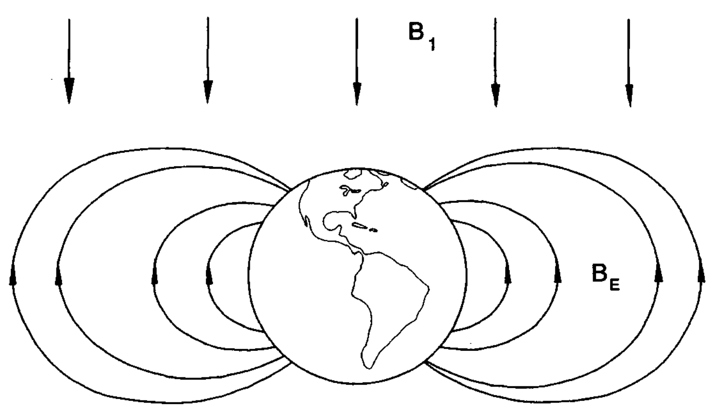
3. A 1 MeV proton with $v_\parallel = 0$ is in the equatorial plane at $r = 2R_E$ and drifts around the Earth in a time interval τ_d. Find the energy of a proton at $3R_E$, $v_\parallel = 0$ which will drift around the Earth in the same time interval.

4. The dipole field of the Earth can be expressed by the formulas for the two components:

$$B_r = -2B_0\left(\frac{R_E}{r}\right)^3 \cos\theta\,\hat{\mathbf{e}}_r$$

$$B_\theta = -B_0\left(\frac{R_E}{r}\right)^3 \sin\theta\,\hat{\mathbf{e}}_\theta$$

where θ is the polar angle or *co-latitude*; B_0 is the magnetic field intensity on



the Earth's surface at the equator. A uniform magnetic field of intensity B_1 and direction downward over the north magnetic pole is added to the Earth's field:

(a) What is the formula for B_r in the combined fields?
(b) What is the formula for B_θ in the combined fields?
(c) What is the formula for the total field strength?