Computational Geometry Introduction

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What is geomerty?

The study of the shape of the physical world.



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Oxyrhynchus Papyri, 3rd century BC

Computational Geometry

human vs machine

- High dimensional geometry
- Complex numerical computations
- Visualization (molecules, objects, tissues)

Big data



Polygons and polyhedra



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A shadow (projection) of a polyhedron is a polygon.

Polytopes



fullerene C_{60} (truncated icosahedron); made of twenty hexagons and twelve pentagons

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Triangulations



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Subdivisions, triangulations and (sums of) polytopes



The graph of (regular) subdivisions can be embedded as a polytope.

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Voronoi diagrams



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Image processing



e.g. Triangulated Mesh Approximation, Image Segmentation

Structural biology



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protein docking: cavities in nuclease and ligands (red)

Visualization and mesh generation







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Non Euclidean geometry



 L_∞ Voronoi Diagrams and Applications to VLSI Layout and Manufacturing

(L_{∞} : maximum between the horizontal and the vertical distance)

Non linear geometry



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Modeling objects (e.g. CAD & medical imaging)



(images taken from CGAL website)

Maps and navigation





Data managment



Octrees: www.youtube.com/watch?v=F_c0feEWQEM [Psalidas'10]

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Spatial data managment



ACM SIGSPATIAL GIS Cup 2017: range queries under Fréchet distance

Massive data



NLP and LLM rely on Approximate Nearest Neighbor (ANN) search to efficiently retrieve similar word embeddings or document representations from high-dimensional vector spaces.

Biogeography & Ecology



Studying the biodiversity of bird spieces. For each cell in the map the volume of the intersection of two polytopes is computed.

Autonomous systems



Reachability analysis study the set of all possible states ensuring safety verification of autonomous systems.

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Sampling

truncated distributions using geometric random walks



Rows: random walks; columns: walk steps & convergence to uniform distribution

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Finance



Sampling from the (non-convex) intersection of an ellipse with a simplex to study anomaly detection in stock markets.

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Metabolic networks



Flux quantifies the steady-state flow of mass and energy in the network. Sampling from convex polytopes that represent metabolic networks to study flux distributions.