Introduction to Polymake: Polytopes and convex hulls

Computational Geometry cource 2025 - Vissarion Fisikopoulos

A polytope is the convex hull of finitely many points in some Euclidean space. Equivalently, a polytope is the bounded intersection of finitely many affine halfspaces. `polymake` can deal with polytopes in both representations and provides numerous tools for analysis.

This tutorial shows basic ways of defining a polytope and how to compute some basic properties and visualization.

Installation

Ensure Polymake is installed on your system. If not, install it via:

```
sudo apt install polymake  # On Debian-based systems
```

Start the polymake client by running:

```
polymake
```

For more information https://polymake.org/doku.php/download/start

Defining a Polytope

A polytope can be defined using either vertices (V-polytope) or inequalities (H-polytope). Below is an example of defining a polytope using vertices:

```
use application 'polytope';

$VP = new Polytope(POINTS=>[[1, 0, 0], [1, 1, 0], [1, 0, 1], [1, 1, 1]]);
print $VP->VERTICES;
```

Note: the first coordinate is always `1`;

Then we can define a polytope by a set of linear inequalities.

```
$HP = new Polytope(INEQUALITIES=>[[1,1,1,0],[1,1,0,1],[1,1,-1,0],
    [1,1,0,-1],[1,17,1,1]]);
```

Computing the Facets or Vertices

To obtain the facets of a V-polytope, we need a convex hull algorithm. Available algorithms in `polymake` are double description (`cdd` or `ppl`), reverse search (`lrs`), and beneath beyond (`beneath_beyond`). It is also possible to specify explicitly which method to use by using the prefer_now command.

```
prefer_now "lrs";
print $VP->FACETS;
```

or print the inequalities

```
print_constraints($VP->FACETS);
```

Known polytopes

Polymake can be used to compute well known polytopes such as the cube and the simplex.

```
$cube = cube(3);
$simplex = simplex(4);
```

Or more complicates polytopes

```
$perm = permutahedron(3);
print $perm->DIM;
print_constraints($perm->FACETS);
print $perm->VERTICES;
```

Checking properties

```
print $cube->SIMPLE;
print $cube->SIMPLICIAL;
print $cube->F_VECTOR;
```

Visualizing the Polytope

We can visualize polytopes up to dimension 4.

```
$cube->VISUAL;
$simplex->VISUAL;
```

Polar dual polytopes

```
$p_primal = new Polytope(POINTS=>[[1, -1, 1], [1, 2, 1], [1, -1, -2], [1, 0, -2], [1, 2, 0]]);
print $p_primal->VERTICES;
print_constraints($p_primal->FACETS);
$p_polar = $p_primal->POLAR;
print $p_polar->VERTICES;
```

Further Exploration

- Explore more applications with `help("polytope")` in Polymake.
- Check the official documentation.