UNIVERSITY OF ATHENS
Department of Economics
Course: Advanced Microeconomics
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Assignment no. 2. Answers

The answers are due on December 16

Exercise 4.1 *You observe a consumer in two situations: with an income of \$100 he buys 5 units of good 1 at a price of \$10 per unit and 10 units of good 2 at a price of \$5 per unit. With an income of \$175 he buys 3 units of good 1 at a price of \$15 per unit and 13 units of good 2 at a price of \$10 per unit. Do the actions of this consumer conform to the basic axioms of consumer behaviour?*

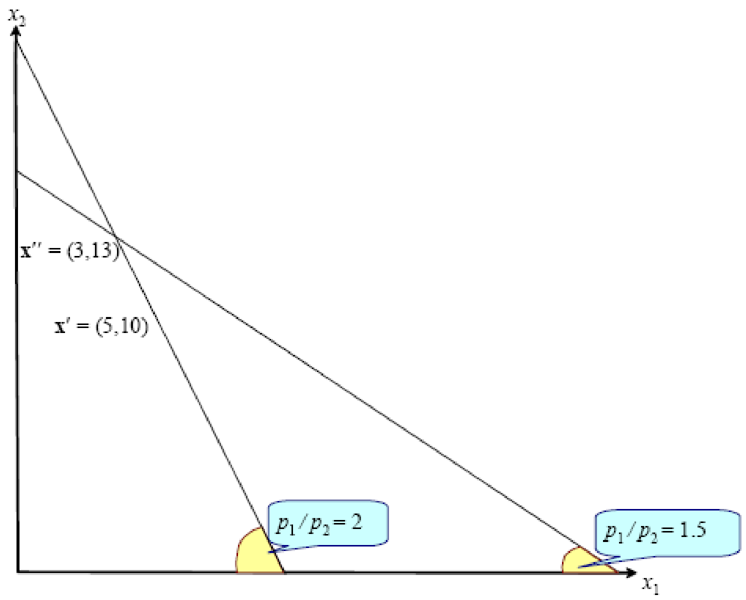

Figure 4.1: WARP violated

Outline Answer

At the original price ratio $p_1/p_2 = 2$ the choice is $\mathbf{x}' = (5, 10)$; but at those prices the and with that budget the consumer could have afforded $\mathbf{x}'' = (3, 13)$: $\mathbf{x}'$ is revealed-preferred to $\mathbf{x}''$. But at the new price ratio $p_1/p_2 = 1.5$ $\mathbf{x}''$ is chosen, although $\mathbf{x}'$ is still affordable: $\mathbf{x}''$ is revealed-preferred to $\mathbf{x}'$. This violates WARP – see Figure 4.1.

Exercise 4.3 *Suppose a person has the Cobb-Douglas utility function*

$$\sum_{i=1}^{n} a_i \log(x_i)$$

where x_i is the quantity consumed of good i, and $a_1, ..., a_n$ are non-negative parameters such that $\sum_{j=1}^{n} a_j = 1$. If he has a given income y, and faces prices $p_1, ..., p_n$, find the ordinary demand functions. What is special about the expenditure on each commodity under this set of preferences?

Outline Answer

The relevant Lagrangean is

$$\sum_{i=1}^{n} \alpha_i \log x_i + \nu \left[y - \sum_{i=1}^{n} p_i x_i \right] \tag{4.1}$$

The first-order conditions yield:

$$x_i^* = \frac{\alpha_i}{\nu^* p_i},\ i = 1, 2, ..., n. \tag{4.2}$$

$$y = \sum_{i=1}^{n} p_i x_i^* \tag{4.3}$$

From the $n + 1$ equations (4.2,4.3) we get at the optimum: $y = \sum_{i=1}^{n} \alpha_i / \nu^* = 1/\nu^*$. So the demand functions are

$$x_i^* = \frac{\alpha_i y}{p_i},\ i = 1, 2, ..., n. \tag{4.4}$$

and expenditure on each commodity i is

$$e_i := p_i x_i^* = \alpha_i y, \tag{4.5}$$

– a constant proportion of income.

Exercise 4.9 *A person has preferences represented by the utility function*

$$U(\mathbf{x}) = \sum_{i=1}^{n} \log x_i$$

where x_i is the quantity consumed of good i and $n > 3$.

1. *Assuming that the person has a fixed money income y and can buy commodity i at price p_i find the ordinary and compensated demand elasticities for good 1 with respect to p_j, $j = 1, ..., n$.*

Outline Answer

1. For the specified utility function it is clear that the indifference curves do not touch the axes for any finite x_i, so we cannot have a corner solution. The budget constraint is

$$\sum_{i=1}^{n} p_i x_i \leq y.$$

The problem of maximising utility subject to the budget constraint is equivalent to maximising the Lagrangean

$$\sum_{i=1}^{n} \log x_i + \lambda \left[y - \sum_{i=1}^{n} p_i x_i . \right]$$

The FOC are

$$\frac{1}{x_i^*} - \lambda p_i = 0, i = 1, ..., n \tag{4.23}$$

and the (binding) budget constraint. From (4.23) we get

$$n - \lambda \sum_{i=1}^{n} p_i x_i^* = 0. \tag{4.24}$$

and so, using the budget constraint, we find $\lambda = n/y$. Substituting the value of λ into (4.23) we find:

(a) The ordinary demand function for good i is

$$x_i^* = \frac{y}{np_i} \tag{4.25}$$

The indirect utility function V is given by $\upsilon = V(\mathbf{p}, y) = U(\mathbf{x}^*) = \sum_{i=1}^{n} \log x_i^*$. So, from (4.25) we have:

$$\upsilon = \log\left(\frac{y^n}{n^n p_1 p_2 p_3 ... p_n}\right) \tag{4.26}$$

Inverting the relation (4.26) the cost function C is given by

$$y = C(\mathbf{p}, \upsilon) = \left[n^n p_1 p_2 p_3 ... p_n e^{\upsilon}\right]^{\frac{1}{n}} = n\left[p_1 p_2 p_3 ... p_n e^{\upsilon}\right]^{\frac{1}{n}} \tag{4.27}$$

Differentiating (4.27) the compensated demand for good 1 is

$$x_1^* = p_1^{\frac{1-n}{n}}\left[p_2 p_3 p_4 ... p_n e^{\upsilon}\right]^{\frac{1}{n}} \tag{4.28}$$

(b) From (4.25) we have the elasticities

$$\begin{aligned} \left.\frac{\partial \log x_1^*}{\partial \log p_1}\right|_{y=\text{const}} &= -1, \\ \left.\frac{\partial \log x_1^*}{\partial \log p_j}\right|_{y=\text{const}} &= 0, \ j = 2, ..., n. \end{aligned}$$

(c) From (4.28) we have the compensated elasticities

$$\begin{aligned} \left.\frac{\partial \log x_1^*}{\partial \log p_1}\right|_{\upsilon=\text{const}} &= \frac{1-n}{n} < 0, \\ \left.\frac{\partial \log x_1^*}{\partial \log p_j}\right|_{\upsilon=\text{const}} &= \frac{1}{n} > 0, \ j = 2, ..., n \end{aligned}$$

Exercise 4.11 *Suppose an individual has Cobb-Douglas preferences given by those in Exercise 4.3.*

1. *Write down the consumer's cost function and demand functions.*

Outline Answer

1. Using the results from previous exercises we immediately get

$$C(\mathbf{p}, \upsilon) = \left[\frac{p_1}{\alpha_1}\right]^{\alpha_1} \left[\frac{p_2}{\alpha_2}\right]^{\alpha_2} ... \left[\frac{p_n}{\alpha_n}\right]^{\alpha_n}.$$

This is sufficient. However, it may be useful to see the proof from first principles. The relevant Lagrangean is

$$\sum_{i=1}^{n} p_i x_i + \lambda \left[\upsilon - \sum_{i=1}^{n} \alpha_i \log x_i\right] \tag{4.36}$$

The first-order conditions are:

$$x_i = \frac{\alpha_i \lambda}{p_i},\ i = 1, 2, ..., n. \tag{4.37}$$

$$\upsilon = \sum_{i=1}^{n} \alpha_i \log x_i \tag{4.38}$$

From the n equations (4.37) we get at the optimum:

$$\lambda^* = \frac{\sum_{i=1}^{n} p_i x_i^*}{\sum_{i=1}^{n} \alpha_i} = \sum_{i=1}^{n} p_i x_i^* = y \tag{4.39}$$

where y is the budget, or minimised cost and $\sum_{i=1}^{n} \alpha_i = 1$. From (4.38) we get, using (4.37):

$$\upsilon = \sum_{i=1}^{n} \alpha_i \log \alpha_i + \log \lambda^* \sum_{i=1}^{n} \alpha_i - \sum_{i=1}^{n} \alpha_i \log p_i \tag{4.40}$$

Using (4.39) and writing $\sum_{i=1}^{n} \alpha_i \log \alpha_i = -\log A$, equation (4.40) gives:

$$y = Ae^{\upsilon} p_1^{\alpha_1} p_2^{\alpha_2} ... p_n^{\alpha_n} = C(\mathbf{p}, \upsilon). \tag{4.41}$$

This is the required cost function. The demand functions are known from Exercise 4.2 or are obtained immediately from (4.37) and (4.39):

$$x_i^* = \frac{\alpha_i y}{p_i},\ i = 1, 2, ..., n. \tag{4.42}$$