MICROECONOMICS

Principles and Analysis

CONSUMER: WELFARE

1

USING CONSUMER THEORY

- Consumer analysis is not just a matter of consumers' reactions to prices.
- We pick up the effect of prices on incomes on attainable utility - consumer's welfare.
- This is useful in the design of economic policy, for example.
  - The tax structure?
- We can use a number of tools that have become standard in applied microeconomics
  - price indices?

2

OVERVIEW...

Interpreting the outcome of the optimisation in problem in welfare terms

- Consumer welfare
  - **Utility and income**
  - CV and EV
  - Consumer's surplus

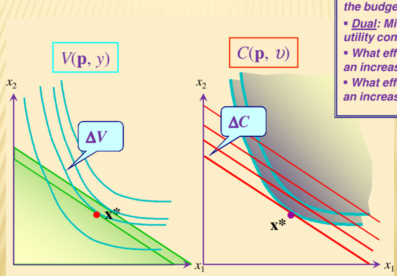
3

HOW TO MEASURE A PERSON'S "WELFARE"?

- We could use some concepts that we already have.
- Assume that people know what's best for them...
- ...So that the preference map can be used as a guide.
- We need to look more closely at the concept of "maximised utility"...
- ...the indirect utility function again.

4

THE TWO ASPECTS OF THE PROBLEM



- ***Primal**: Max utility subject to the budget constraint*
- ***Dual**: Min cost subject to a utility constraint*
- *What effect on max-utility of an increase in budget?*
- *What effect on min-cost of an increase in target utility?*

Interpretation of Lagrange multipliers

5

INTERPRETING THE LAGRANGE MULTIPLIER (1)

- The solution fu[...]

  $V(\mathbf{p}, y) = U(\mathbf{x}^*)$

  $= U(\mathbf{x}^*) + \mu^* [y - \sum_i p_i x_i^*]$

- Differentiate with respect to y:

  $V_y(\mathbf{p}, y) = \sum_i U_i(\mathbf{x}^*) D^i_y(\mathbf{p}, y)$

  $+ \mu^* [1 - \sum$ [...]

- Rearrange:

  $V_y(\mathbf{p}, y) = \sum_i [U_i(\mathbf{x}^*) - \mu^* p_i] D^i_y(\mathbf{p}, y) + \mu^*$

  $V_y(\mathbf{p}, y) = \mu^*$

Optimal value of demands

Optimal va[...] Lagrange [...]

All summations are from 1 to n.

Vanishes because of FOC $U_i(\mathbf{x}^*) = \mu^* p_i$

[...]cond line follows because, at the optimum, either the constraint binds or the Lagrange multiplier is zero

We've just used the demand functions $x_i^* = D^i(\mathbf{p}, y)$)

The Lagrange multiplier in the primal is just the marginal utility of money!

And (with little surprise) we will find that the same trick can be worked with the solution to the dual…

6

INTERPRETING THE LAGRANGE MULTIPLIER (2)

- The solution function for the dual:

$$C(\mathbf{p}, \upsilon) = \Sigma_i p_i x_i^*$$
$$= \Sigma_i p_i x_i^* - \lambda^* [U(\mathbf{x}^*) - \upsilon]$$

- Differentiate with respect to υ:

$$C_\upsilon(\mathbf{p}, \upsilon) = \Sigma_i p_i H^i_\upsilon(\mathbf{p}, \upsilon)$$
$$- \lambda^* [\Sigma_i U_i(\mathbf{x}^*)]$$

Vanishes because of FOC $\lambda^* U_i(\mathbf{x}^*) = p_i$

- Rearrange:

$$C_\upsilon(\mathbf{p}, \upsilon) = \Sigma_i [p_i - \lambda^* U_i(\mathbf{x}^*)] H^i_u(\mathbf{p}, \upsilon) + \lambda^*$$

$$C_\upsilon(\mathbf{p}, \upsilon) = \lambda^*$$

Once again, at the optimum, either the constraint binds or the Lagrange multiplier is zero

(Make use of the conditional demand functions $x_i^* = H^i(\mathbf{p}, \upsilon)$)

Lagrange multiplier in the dual is the marginal cost of utility

Again we have an application of the general envelope theorem.

A USEFUL CONNECTION

- the u... be written ... way...

y...

- the o... solution this way.

$$\upsilon = V(\mathbf{p}, y)$$

- Putting the two parts together...

$$y = C(\mathbf{p}, V(\mathbf{p}, y))$$

- Differentiate ... y:

$$1 = C_\upsilon(\mathbf{p}, \ldots$$

$$C_\upsilon(\mathbf{p}, \upsilon) = \frac{1}{V_y(\mathbf{p}, y)}$$

Minimised budget in the dual

Constraint income in the primal

Constraint utility in the dual

Maximised utility in the primal

marginal cost (in terms of utility) of a dollar of the budget = λ^*

marginal cost of utility in terms of money = μ^*

Mapping utility into income

Mapping income into utility

We can get fundamental results on the person's welfare...

A relationship between the ... of C and V.

UTILITY AND INCOME: SUMMARY

- This gives us a framework for the evaluation of marginal changes of income...
- ...and an interpretation of the Lagrange multipliers
- The Lagrange multiplier on the income constraint (primal problem) is the marginal utility of income.
- The Lagrange multiplier on the utility constraint (dual problem) is the marginal cost of utility.
- But does this give us all we need?

UTILITY AND INCOME: LIMITATIONS

- This gives us some useful insights but is limited:

1. We have focused only on marginal effects
   - infinitesimal income changes.
2. We have dealt only with income
   - not the effect of changes in prices

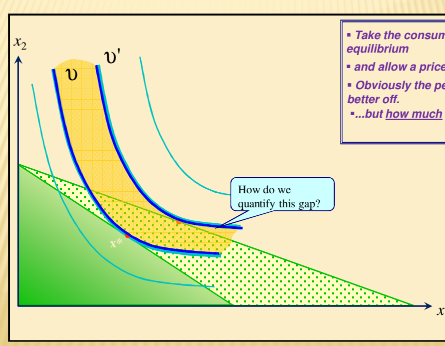
- We need a general method of characterising the impact of budget changes:
  - valid for arbitrary price changes
  - easily interpretable
- For the essence of the problem re-examine the basic diagram.

OVERVIEW...

Exact money measures of welfare

Consumer welfare
- Utility and income
- CV and EV
- Consumer's surplus

THE PROBLEM...



- *Take the consumer's equilibrium*
- *and allow a price to fall...*
- *Obviously the person is better off.*
- *...but how much better off?*

APPROACHES TO VALUING UTILITY CHANGE

- Three t… are not much use:
  1. $\upsilon' -$ … — depends on the **units** of the U function
  2. υ … — depends on the **origin** of the U function
  3. $d(\upsilon', \upsilon)$ — depends on the **cardinalisation** of the U function

Utility differences

Utility ratios

some distance function

- A more productive idea:
  1. *Use income not utility as a measuring rod*
  2. *To do the transformation we use the V function*
  3. *We can do this in (at least) two ways...*

STORY NUMBER 1

- Suppose **p** is the original price vector and **p'** is vector after good 1 becomes cheaper.
- This causes utility to rise from υ to υ'.
  - $\upsilon = V(\mathbf{p}, y)$
  - $\upsilon' = V(\mathbf{p}', y)$
- Express this rise in money terms?
  - What hypothetical change in income would bring the person back to the starting point?
  - (and is this the right question to ask...?)
- Gives us a standard definition....

IN THIS VERSION OF THE STORY WE GET THE *COMPENSATING VARIATION*

$$\upsilon = V(\mathbf{p}, y)$$

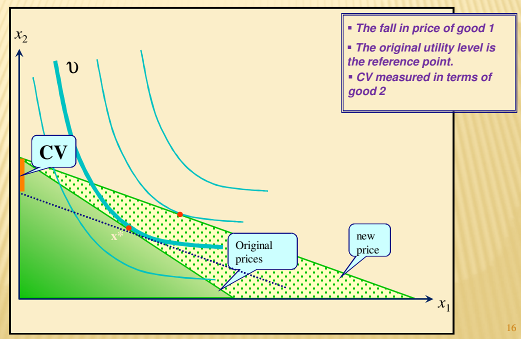
the original utility level at prices **p** and income y

$$\upsilon = V(\mathbf{p}', y - \mathrm{CV})$$

the original utility level restored at new prices **p'**

- *The amount CV is just sufficient to "undo" the effect of going from **p** to **p'**.*

THE COMPENSATING VARIATION



CV – ASSESSMENT

- The CV gives us a clear and interpretable measure of welfare change.
- It values the change in terms of money (or goods).
- But the approach is based on one specific reference point.
- The assumption that the "right" thing to do is to use the original utility level.
- There are alternative assumptions we might reasonably make. For instance...

HERE'S STORY NUMBER 2

- Again suppose:
  - **p** is the original price vector
  - **p'** is the price vector after good 1 becomes cheaper.
- This again causes utility to rise from υ to υ'.
- But now, ask ourselves a different question:
  - Suppose the price fall had never happened
  - What hypothetical change in income would have been needed ...
  - ...to bring the person to the *new* utility level?

IN THIS VERSION OF THE STORY WE GET THE *EQUIVALENT VARIATION*

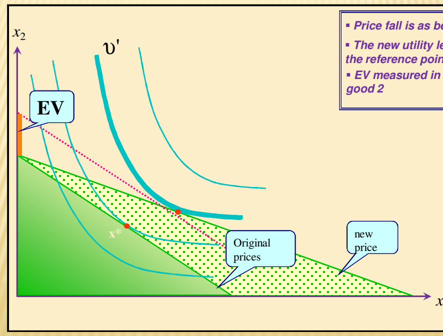
$\upsilon' = V(\mathbf{p}', y)$ — the utility level at new prices **p'** and income *y*

$\upsilon' = V(\mathbf{p}, y + \mathrm{EV})$ — the new utility level reached at original prices **p**

- The amount EV is just sufficient to "mimic" the effect of going from **p** to **p'**.

19

THE EQUIVALENT VARIATION



- *Price fall is as before.*
- *The new utility level is now the reference point*
- *EV measured in terms of good 2*

20

CV AND EV...

- Both definitions have used the indirect utility function.
  - But this may not be the most intuitive approach
  - So look for another standard tool..
- As we have seen there is a close relationship between the functions *V* and *C*.
- So we can reinterpret CV and EV using *C*.
- The result will be a welfare measure
  - the change in cost of hitting a welfare level.

remember: cost decreases mean welfare increases.

21

WELFARE CHANGE AS – Δ(COST)

- Compensa[Prices before]riation as –Δ([Reference utility level]

$\mathrm{CV}(\mathbf{p}\to\mathbf{p}') = C(\mathbf{p}, \upsilon) - C(\mathbf{p}', \upsilon)$

(–) change in cost of hitting utility level υ. If positive we have a welfare *increase*.

- Equivalent Variation as –Δ(cost):

$\mathrm{EV}(\mathbf{p}\to\mathbf{p}') = C(\mathbf{p}, \upsilon') - C(\mathbf{p}', \upsilon')$

(–) change in cost of hitting utility level υ'. If positive we have a welfare *increase*.

- Using the above definitions we also have

$\mathrm{CV}(\mathbf{p}'\to\mathbf{p}) = C(\mathbf{p}', \upsilon') - C(\mathbf{p}, \upsilon')$

$= -\,\mathrm{EV}(\mathbf{p}\to\mathbf{p}')$

Looking at welfare change in the reverse direction, starting at **p'** and moving to **p**.

22

WELFARE MEASURES APPLIED...

- The concepts we have developed are regularly put to work in practice.
- Applied to issues such as:
  - Consumer welfare indices
  - Price indices
  - Cost-Benefit Analysis
- Often this is done using some (acceptable?) approximations...

Example of cost-of-living index

23

COST-OF-LIVING INDICES

- An index based on CV:

$\dfrac{C(\mathbf{p}', \upsilon)}{\ldots\,\upsilon)}$

What's the change in cost of hitting the base welfare level υ?

All summations are from 1 to *n*.

- An approximation:

$I_{\mathrm{L}} = \dfrac{\Sigma_i p'_i x_i}{\Sigma_i p_i x_i}$ [$\geq C(\mathbf{p}', \upsilon)$; $= C(\mathbf{p}, \upsilon)$]

$\geq I_{\mathrm{CV}}$.

What's the change in cost of buying the base consumption bundle **x**? This is the *Laspeyres* index – the basis for the Retail Price Index and other similar indices.

- An index based on EV:

$I_{\mathrm{EV}} = \dfrac{C(\mathbf{p}', \upsilon')}{C(\mathbf{p}, \upsilon')}$

What's the change in cost of hitting the new welfare level υ' ?

- An approximation:

$I_{\mathrm{P}} = \dfrac{\Sigma_i p'_i x'_i}{\Sigma_i p_i x'_i}$ [$= C(\mathbf{p}', \upsilon')$; $\geq C(\mathbf{p}, \upsilon')$]

$\leq I_{\mathrm{EV}}$.

What's the change in cost of buying the new consumption bundle **x′**? This is the *Paasche* index

24

OVERVIEW…

- Consumer welfare
  - Utility and income
  - CV and EV
  - **Consumer's surplus**

A simple, practical approach?

ANOTHER [illegible] RM FOR CV

- Use the cost-differ[illegible]ce defini[illegible]
  CV(**p**→**p'**) = C(**p**, υ) – C(**p'**, υ)

(–) change in cost of hitting utility level υ. If positive we have a welfare *increase*.

Prices before

Pr[illegible] after

Reference utility level

- Assume that the price of good 1 changes from p_1 to $p_1{}'$ while other prices remain unchanged. Then we can rewrite the above as:

$$CV(\mathbf{p}\to\mathbf{p}') = \int_{p_1'}^{p_1} C_1(\mathbf{p}, \upsilon)\, dp_1$$

(Just using the definition of a definite integral)

- Further rewrite as:

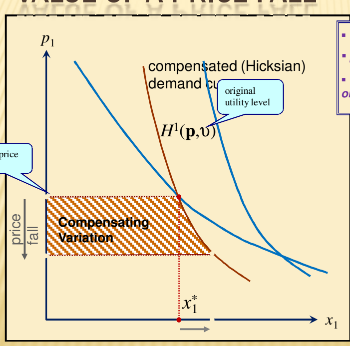
$$CV(\mathbf{p}\to\mathbf{p}') = \int_{p_1'}^{p_1} H^1(\mathbf{p}, \upsilon)\, dp_1$$

Hicksian (compensated) demand for good 1

You're right. It's using Shephard's lemma again

So CV can be seen as an area under the compensated demand curve

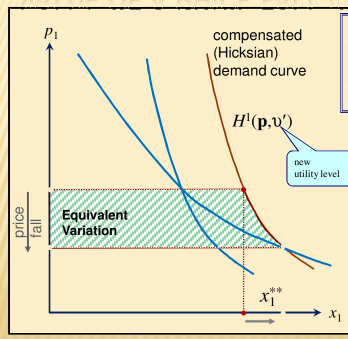
COMPENSATED DEMAND AND THE VALUE OF A PRICE FALL



- *The initial equilibrium*
- *price fall: (welfare increase)*
- *value of price fall, relative to original utility level*

- *The CV provides an exact welfare measure.*
- *But it's not the only approach*

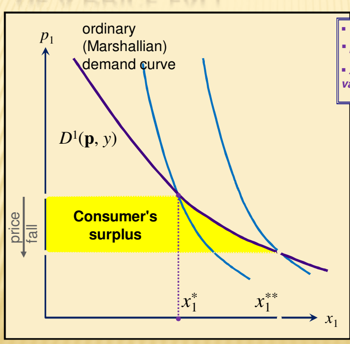
COMPENSATED DEMAND AND THE VALUE OF A PRICE FALL (2)



- *As before but use new utility level as a reference point*
- *price fall: (welfare increase)*
- *value of price fall, relative to new utility level*

- *The EV provides another exact welfare measure.*
- *But based on a different reference point*
- *Other possibilities…*

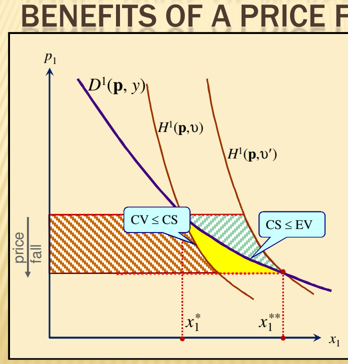
ORDINARY DEMAND AND THE VALUE OF A PRICE FALL



- *The initial equilibrium*
- *price fall: (welfare increase)*
- *An alternative method of valuing the price fall?*

- *CS provides an __approximate__ welfare measure.*

THREE WAYS OF MEASURING THE BENEFITS OF A PRICE FALL



- *Summary of the three approaches.*
- *Conditions for __normal__ goods*
- *So, for normal goods:* **CV ≤ CS ≤ EV**
- *For inferior goods:* **CV > CS > EV**

SUMMARY: KEY CONCEPTS

- Interpretation of Lagrange multiplier
- Compensating variation
- Equivalent variation
  - CV and EV are measured in monetary units.
  - In all cases: CV($\mathbf{p} \to \mathbf{p}'$) = – EV($\mathbf{p}' \to \mathbf{p}$).
- Consumer's surplus
  - The CS is a convenient approximation
  - For normal goods: CV $\leq$ CS $\leq$ EV.
  - For inferior goods: CV > CS > EV.

31