MICROECONOMICS

Principles and Analysis

GENERAL EQUILIBRIUM: EXCESS DEMAND AND THE RÔLE OF PRICES

SOME UNSETTLED QUESTIONS

- Under what circumstances can we be sure that an equilibrium exists?
- Will the economy somehow "tend" to this equilibrium?
- And will this determine the price system for us?
- We will address these using the standard model of a general-equilibrium system
- To do this we need just one more new concept.

OVERVIEW...

General Equilibrium: Excess Demand+

- Excess Demand Functions
- Equilibrium Issues
- Prices and Decentralisation

Definition and properties

INGREDIENTS OF THE EXCESS DEMAND FUNCTION

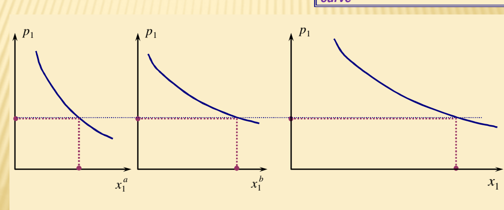
- Aggregate demands (the sum of individual households' demands).
- Aggregate net-outputs (the sum of individual firms' net outputs).
- Resources.
- Incomes determined by prices.

AGGREGATE CONSUMPTION, NET OUTPUT

- From household's demand function
  $x_i^h = D^{ih}(\mathbf{p}, y^h)$
  $= D^{ih}(\mathbf{p}, y^h(\mathbf{p}))$
  - Because incomes depend on prices
- So demands are just functions of **p**
  $x_i^h = x_i^h(\mathbf{p})$
  - $x_i^h(\bullet)$ depends on holdings of resources and shares
- If all goods are private (rival) then aggregate demands can be written:
  $x_i(\mathbf{p}) = \Sigma_h x_i^h(\mathbf{p})$
  - "Rival": extra consumers require additional resources. Same as in "consumer: aggregation"
- From firm's supply of net output
  $q_i^f = q_i^f(\mathbf{p})$
  - standard supply functions/ demand for inputs
- Aggregate:
  $q_i = \Sigma_f q_i^f(\mathbf{p})$
  - aggregation is valid if there are no externalities. Just as in "Firm and the market")

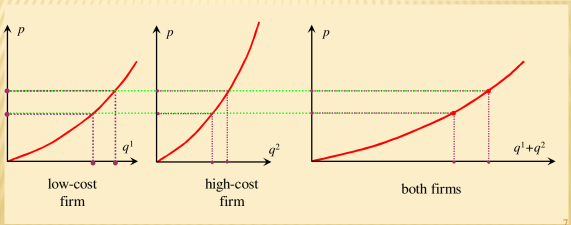
DERIVATION OF $X_I(P)$

- *Alf's demand curve for good 1.*
- *Bill's demand curve for good 1.*
- *Pick any price*
- *Sum of consumers' demand*
- *Repeat to get the market demand curve*

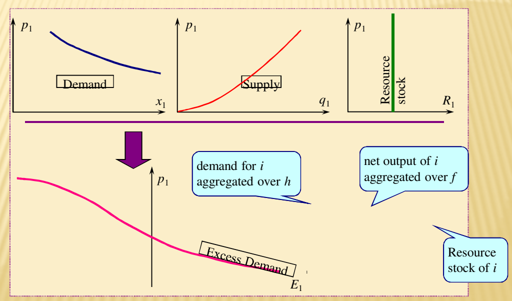


DERIVATION OF $Q_i(P)$

- *Supply curve firm 1 (from MC).*
- *Supply curve firm 2.*
- *Pick any price*
- *Sum of individual firms' supply*
- *Repeat…*
- *The market supply curve*



SUBTRACT Q AND R FROM X TO GET E:



EQUILIBRIUM IN TERMS OF EXCESS DEMAND

Equilibrium is characterised by a price vector $\mathbf{p}^* \geq \mathbf{0}$ such that:

- For every good *i*:

$$E_i(\mathbf{p}^*) \leq 0$$

The materials balance condition (dressed up a bit)

- For each good *i* that has a positive price in equilibrium (i.e. if $p_i^* > 0$):

$$E_i(\mathbf{p}^*) = 0$$

If this is violated, then somebody, somewhere isn't maximising...

You can only have excess supply of a good in equilibrium if the price of that good is 0.

USING E TO FIND THE EQUILIBRIUM

- Five steps to the equilibrium allocation
  1. From technology compute firms' net output functions and profits.
  2. From property rights compute household incomes and thus household demands.
  3. Aggregate the **x**s and **q**s and use **x**, **q**, **R** to compute **E**
  4. Find **p*** as a solution to the system of **E** functions
  5. Plug **p*** into demand functions and net output functions to get the allocation
- But this begs some questions about step 4

ISSUES IN EQUILIBRIUM ANALYSIS

- Existence
  - Is there any such **p***?
- Uniqueness
  - Is there only one p*?
- Stability
  - Will p "tend to" p*?
- For answers we use some fundamental properties of E.

TWO FUNDAMENTAL PROPERTIES...

- Walras' Law. For *any* price **p**:

$$\sum_{i=1}^{n} p_i E_i(\mathbf{p}) = 0$$

You only have to work with n-1 (rather than n) equations

Hint #1: think about the "adding-up" property of demand functions...

- Homogeneity of degree 0. For *any* price **p** and any $t > 0$:

$$E_i(t\mathbf{p}) = E_i(\mathbf{p})$$

You can normalise the prices by any positive number

Hint #2: think about the homogeneity property of demand functions...

Link to consumer demand

Can you explain why they are true?
Reminder : these hold for any competitive allocation, not just equilibrium

PRICE NORMALISATION

- We may need to convert from n numbers p_1, p_2,...p_n to $n-1$ relative prices.
- The precise method is essentially arbitrary.
- The choice of method depends on the purpose of your model.
- It can be done in a variety of ways:

You could divide by

set of prices that sum to 1

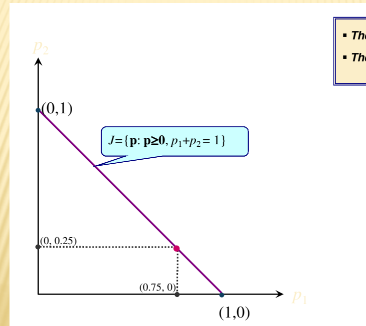
$$\sum_{i=1} p_i$$

to give a

Marx bar theory of value

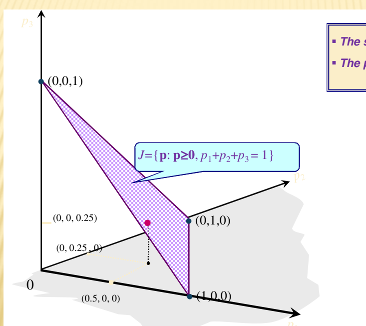
- *This method might seem <u>weird</u>*
- *But it has a nice property.*
- *The set of all normalised prices is convex and compact.*

NORMALISED PRICES, *N*=2



- *The set of normalised prices*
- *The price vector (0,75, 0.25)*

NORMALISED PRICES, *N*=3



- *The set of normalised prices*
- *The price vector (0,5, 0.25, 0.25)*

OVERVIEW...

Is there any p*?

General Equilibrium: Excess Demand+

- Excess Demand Functions
- Equilibrium Issues
  - Existence
  - Uniqueness
  - Stability
- Prices and Decentralisation

APPROACH TO THE EXISTENCE PROBLEM

- Imagine a rule that moves prices in the direction of excess demand:
  - "if E_i >0, increase p_i"
  - "if E_i <0 and p_i >0, decrease p_i"
  - An example of this under "stability" below.
- This rule uses the E-functions to map the set of prices into itself.
- An equilibrium exists if this map has a "fixed point."
  - a **p*** that is mapped into itself?
- To find the conditions for this, use normalised prices
  - **p** ∈ J.
  - J is a compact, convex set.
- We can examine this in the special case n = 2.
  - In this case normalisation implies that $p_2 \equiv 1 - p_1$.

Why?

EXISTENCE OF EQUILIBRIUM?

Why boundedness below?

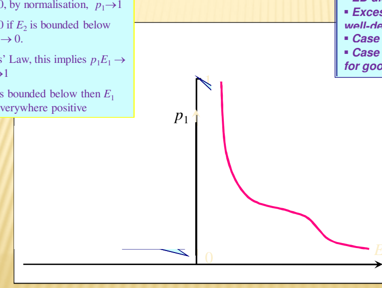
As $p_2 \to 0$, by normalisation, $p_1 \to 1$

As $p_2 \to 0$ if E_2 is bounded below then $p_2E_2 \to 0$.

By Walras' Law, this implies $p_1E_1 \to 0$ as $p_1 \to 1$

So if E_2 is bounded below then E_1 can't be everywhere positive

- *ED diagram, normalised prices*
- *Excess demand function with well-defined equilibrium price*
- *Case with discontinuous E*
- *Case where excess demand for good2 is unbounded below*



E-functions are:
- *continuous,*
- *bounded below*

- *No equilibrium price where E crosses the axis*
- *E never crosses the axis*

EXISTENCE: A BASIC RESULT

- An equilibrium price vector must exist if:
  1. excess demand functions are continuous and
  2. bounded from below.
  - ("continuity" can be weakened to "upper-hemi-continuity").
- Boundedness is no big deal.
  - Can you have infinite excess supply...?
- However continuity might be tricky.
  - Let's put it on hold.
  - We examine it under "the rôle of prices"

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OVERVIEW...

General Equilibrium: Excess Demand+

- Excess Demand Functions
- Equilibrium Issues
  - •Existence
  - •**Uniqueness**
  - •Stability
- Prices and Decentralisation

Is there just one p*?

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THE UNIQUENESS PROBLEM

- Multiple equilibria imply multiple allocations, at normalised prices...
- ...with reference to a given property distribution.
- Will not arise if the E-functions satisfy WARP.
- If WARP is not satisfied this can lead to some startling behaviour...

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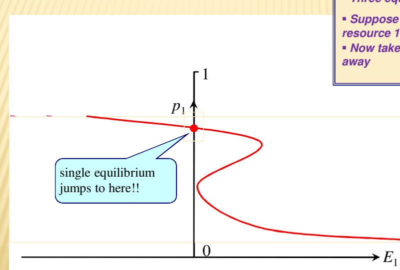
MULTIPLE EQUILIBRIA



- *Three equilibrium prices*
- *Suppose there were more of resource 1*
- *Now take some of resource 1 away*

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OVERVIEW...

General Equilibrium: Excess Demand+

- Excess Demand Functions
- Equilibrium Issues
  - •Existence
  - •Uniqueness
  - •**Stability**
- Prices and Decentralisation

Will the system tend to p*?

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STABILITY ANALYSIS

- We need...
- A definition of equilibrium
- A process
- Initial conditions

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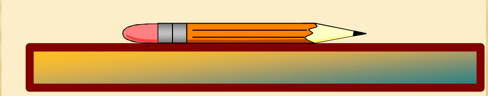
A STABLE EQUILIBRIUM



Stable:

If we apply a small shock the built-in adjustment process (gravity) restores the status quo

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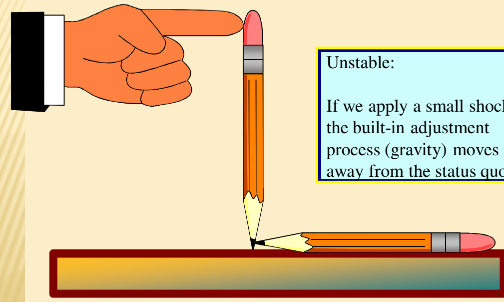
AN UNSTABLE EQUILIBRIUM



Unstable:

If we apply a small shock the built-in adjustment process (gravity) moves us away from the status quo

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“GRAVITY” IN THE CE MODEL

- Imagine there is an auctioneer to announce prices, and to adjust if necessary.
- If good *i* is in excess demand, increase its price.
- If good *i* is in excess supply, decrease its price (if it hasn't already reached zero).
- Nobody trades till the auctioneer has finished.

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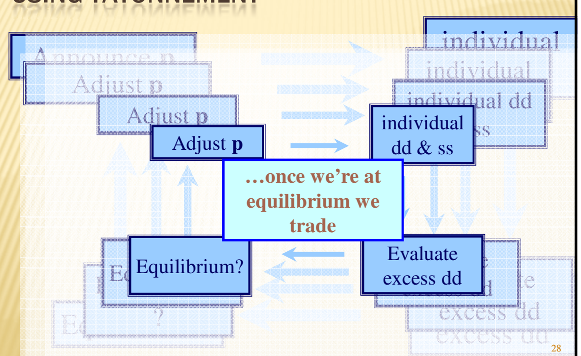
“GRAVITY” IN THE CE MODEL: THE AUCTIONEER USING TÂTONNEMENT



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ADJUSTMENT AND STABILITY

- Adjust prices according to sign of E_i:
  - If $E_i > 0$ then increase p_i
  - If $E_i < 0$ and $p_i > 0$ then decrease p_i
- A linear tâtonnement adjustment mechanism:

$$\frac{dp_i(t)}{dt} = \begin{cases} \alpha_i E_i(\mathbf{p}(t)) & \text{if } p_i(t) \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

- Define distance between $\mathbf{p}(t)$ and equilibrium $\mathbf{p}^*$.
- Given WARP, then under tâtonnement distance must fall with t.

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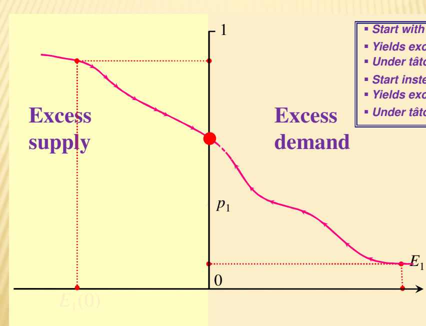
GLOBALLY STABLE…



- *Start with a very high price*
- *Yields excess supply*
- *Under tâtonnement price falls*
- *Start instead with a low price*
- *Yields excess demand*
- *Under tâtonnement price rises*

- *If E satisfies WARP then the system must converge…*

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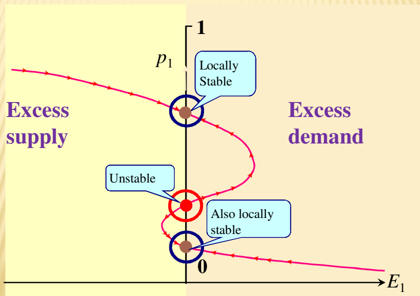
NOT GLOBALLY STABLE...



OVERVIEW...

The separation theorem and the role of large numbers

- General Equilibrium: Excess Demand+
  - Excess Demand Functions
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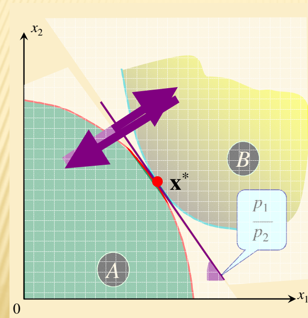
DECENTRALISATION

- Recall the important result on decentralisation that we discussed in the case of Crusoe's island.
- The counterpart is true for this multi-person world.
- Requires assumptions about convexity of two sets, defined at the aggregate level:
  - the "attainable set": $A := \{\mathbf{x}: \mathbf{x} \le \mathbf{q}+\mathbf{R},\ \Phi(\mathbf{q}) \le 0\}$
  - the "better-than" set: $B(\mathbf{x}^*) := \{\Sigma_h \mathbf{x}^h: U^h(\mathbf{x}^h) \ge U^h(\mathbf{x}^{*h})\}$
- To see the power of the result we can appeal to an "averaging" result we used in lecture for the firm

Link to Crusoe:
Link to Firm and market

DECENTRALISATION AGAIN



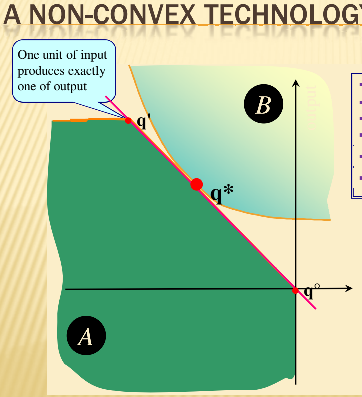
- The attainable set
- The "Better-than-x^*" set
- The price line
- Decentralisation

- $A = \{\mathbf{x}: \mathbf{x} \le \mathbf{q}+\mathbf{R},\ \Phi(\mathbf{q}) \le 0\}$
- $B = \{\Sigma_h \mathbf{x}^h: U^h(\mathbf{x}^h) \ge U^h(\mathbf{x}^{*h})\}$
- *x^* maximises income over A*
- *x^* minimises expenditure over B*

PROBLEMS WITH PRICES

- Either non-convex technology (increasing returns or other indivisibilities) for some firms, or...
- ...non-convexity of *B*-set (non-concave-contoured preferences) for some households...
- ...may imply discontinuous excess demand function and so...
- ...absence of equilibrium.
- But if there are are large numbers of agents everything may be OK.

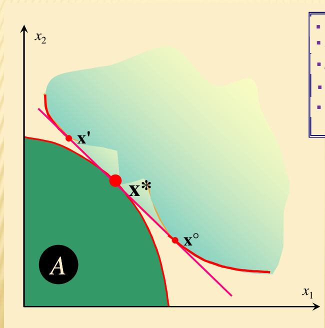
A NON-CONVEX TECHNOLOGY



- The case with 1 firm
- Rescaled case of 2 firms,
- ... , 8 , 16
- Limit of the averaging process
- The "Better-than" set
- "separating" prices and equilibrium

- *Limiting attainable set is convex*
- *Equilibrium $\mathbf{q}^*$ is sustained by a mixture of firms at $\mathbf{q}°$ and $\mathbf{q}'$.*

NON-CONVEX PREFERENCES



- *The case with 1 person*
- *Rescaled case of 2 persons,*
- *A continuum of consumers*
- *The attainable set*
- *"separating" prices and equilibrium*

- *Limiting better-than set is convex*
- *Equilibrium* $\mathbf{x}^*$ *is sustained by a mixture of consumers at* $\mathbf{x}^\circ$ *and* $\mathbf{x}'$ *.*

SUMMARY

- Excess demand functions are handy tools for getting results.
- Continuity and boundedness ensure existence of equilibrium.
- WARP ensures uniqueness and stability.
- But requirements of continuity may be demanding.