University of Athens
Department of Economics
Master's program in Applied Economics and Finance

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Course: Economic Policy

Assignment no. 2: Answer all questions

Question 1.

Consider the utility function U = $\alpha \log(x_1) + \beta \log(x_2) - l$ and budget constraint $wl = q_1x_1 + q_2x_2$.
a. Show that the price elasticity of demand for both commodities is equal to -1.
b. Setting producer prices at $p_1 = p_2 = 1$, show that the inverse elasticity rule implies $t_1/t_2 = q_1/q_2$.
c. Letting $w = 100$ and $\alpha + \beta = 1$, calculate the tax rates required to achieve revenue of $R = 10$.

Question 1. Solution.

a. The consumer's demands solve

max $\alpha \log(x_1) + \beta \log(x_2) - l$ over x_1, x_2, l subject to $wl = q_1x_1 + q_2x_2$.

Or max $\alpha \log(x_1) + \beta \log(x_2) - (q_1/w)x_1 - (q_2/w)x_2$

Equating the partial derivatives to zero gives

$\alpha / x_1 = q_1 / w$, $\beta / x_2 = q_2 / w$.

The demands are therefore $\alpha / x_1 = \alpha w/q_1$, $x_2 = \beta w/q_2$

The elasticity of demand is defined by $\varepsilon_i^d = \frac{dx_i}{dq_i}\frac{q_i}{x_i}$.

Calculating this for good 1 obtains

$$\varepsilon_1^d = \frac{\alpha w}{q_1^2}\frac{q_1}{\frac{aw}{q_1}} = -1$$

The same calculation holds for good 2.

b. The inverse elasticity rule states that

$$\frac{t_i}{1+t_i} = \frac{\alpha - \lambda}{\lambda}\frac{1}{\varepsilon_i^d} \qquad , i = 1,2.$$

Hence

$$\frac{t_1}{1+t_1}\varepsilon_1^d = \frac{\alpha - \lambda}{\lambda} = \frac{t_2}{1+t_2}\varepsilon_2^d$$

But $\varepsilon_i^d = -1$ and $1 + t_i = q_i$, so $t_1/t_2 = q_1/q_2$.

c. Revenue is defined by $R = t_1x_1 + t_2x_2$.
Using the solutions for demands obtains

$$R = t_1\frac{\alpha w}{q_1} + t_2\frac{\beta w}{q_2}$$

Since $t_1 = \frac{q_1 t_2}{q_2}$, this becomes $R = \frac{t_2}{q_2}\alpha w + \frac{t_2}{q_2}\beta w$

Finally, since $\alpha + \beta = 1$, $R = 10$ and $w = 100$, the optimal tax on good 2 is $10 = (100t_2)/(1+t_2)$, which has solution $t_2 = 1/9$, and therefore $t_1 = 1/9$.

Question 2.

''If all commodities are taxed at the same rate, the distortion in prices is minimized.'' Explain why this statement does not act as a guide for setting optimal commodity taxes.

Question 2. Answer.

The statement is formally true. What drives consumer decisions are the relative prices of the commodities, since relative prices measure the rate at which one commodity can be traded for another. Taxing all commodities at the same rate does not distort relative prices. Think of an example with just two commodities.
It does not follow from this statement that such a tax system is efficient. We have shown in the lectures that, is all commodities, including labour, are taxed at the same rate, no revenue is raised. Hence, at least one commodity must be taxed differently if revenue is to be raised. The analysis has shown that one commodity has to be chosen as an untaxed numeraire. Assume that this is labour. The uniform tax on all commodities excluding labour then distorts prices relative to labour. The tax system will thus alter the trade-offs faced by consumers between labour and other commodities. In general different commodities have different relations to labour: some may be substitutes, and other may be compliments. These relations should be reflected in how the trade-off is changed by commodity taxation. Taxing all goods but one at the same rate is therefore rarely inefficient.

Question 3.

One country has a tax rate of 10% on the first 20,000 euros of taxable income, then 25% on the next 30,000 euros, then 50% on all taxable income above 50,000 euros. This country also provides a 4,000 euro exemption per family member. Mario's family has 3 members and earns 50,000 euros per year. What is the marginal and average tax rates faced by this family?

Question 3. Solution.

First, we calculate Mario's family's taxable income: before exemptions, its income is 50,000. It gets a 4,000 euro exemption for each of the three family members, for a total of 12,000 in exemptions. Hence, taxable income is 50,000 – 12,000 = 38,000. Since this is between 20,000 and 50,000, the family faces a 25% marginal tax rate. To compute the average tax rate, first compute the total tax liability. The first 20,000 of taxable income is taxed at 10%. The next 18,000 is taxed at 25%. The total tax is thus .1 × 20,000 + .25 × 18,000 = 2,000 + 4,500 = 6,500. The average tax rate is thus 100% × 6,500/50,000 = 13%.

Question 4.

Consider the utility function $U = x - l^2$, where x is consumption and l is labour.

a. For U = 10, plot the indifference curve with l on the horizontal axis and x on the vertical axis.
b. Now define $z = sl$. For s = 0.5, 1, and 2 plot the indifference curves for U = 10 with z on the horizontal axis and x on the vertical.
c. Plot the indifference curves for s = 0.5, 1, and 2 through the point x = 20, z = 2.
d. Prove that at any point (x,z) the indifference curve of a high-skill consumer is flatter than that of a low-skill.

Question 4. Solution.

a. For U = 10, solve $10 = x - l^2$ to write $x = 10 + l^2$.

b. Define $z = sl$. Then $x = U + [z/s]^2$. For U = 10, this becomes $x = 10 + 4z^2$ for $s = ½$, it is $x = 10 + z^2$ for $s = 1$, and it is $x = 10 + (z/4)^2$ for $s = 2$.

c. At the point $z = 2$, $x = 20$. So a consumer of skill s achieves a utility of $U = 20 - (4/s^2)$. For $s = ½$, $U = 4$, for $s = 1$, $U = 16$ and for $s = 2$, $U = 19$. The indifference curves from part b can then be re-plotted using these new utility values.

d. Since U = = $x - l^2$, total differentiation gives

$$dU = dx - \frac{2z}{s^2}dz$$

Hence the gradient of the indifference curve is $\frac{dx}{dz} = \frac{2z}{s^2}$. For any value of *z*, this gradient is clearly decreasing in *s*.

Question 5.

Consider an economy with two consumers who have skill levels $s_1 = 1$ and $s_2 = 2$ and utility function $U = 10x^{1/2} - l^2$. Let the government employ an income tax function that leads to the allocation $x = 4$, $z = 5$ for the consumer of skill $s = 1$ and $x = 9$, $z = 8$ for the consumer of skill $s = 2$.

a. Show that this allocation satisfies the incentive compatibility constraint that each consumer must prefer his allocation to that of the other.

b. Keeping incomes fixed, consider a transfer of 0.01 units of consumption from the high-skill to the low-skill consumer.

i. Calculate the effect on each consumer's utility.

ii. Show that the sum of utilities increases.

iii. Show that the incentive compatibility constraint is still satisfied.

iv. Use parts i through iii to prove that the initial allocation is not optimal for a utilitarian social welfare function.

Question 5. Solution.

a. To show that the allocation satisfies incentive compatibility, the utility of both consumers is evaluated at the two consumption-income pairs. For consumer 1,

$$U\left(x_1, \frac{z_1}{s_1}\right) = 10x_1^{1/2} - \left(\frac{z_1}{s_1}\right)^2 = -5,$$

$$U\left(x_2, \frac{z_2}{s_1}\right) = 10x_2^{1/2} - \left(\frac{z_2}{s_1}\right)^2 = -34,$$

So $U\left(x_1, \frac{z_1}{s_1}\right) > U\left(x_2, \frac{z_2}{s_1}\right)$.

For consumer 2, $U\left(x_2, \frac{z_2}{s_2}\right) = 10x_2^{1/2} - \left(\frac{z_2}{s_2}\right)^2 = 14,$

$$U\left(x_1, \frac{z_1}{s_2}\right) = 10x_1^{1/2} - \left(\frac{z_1}{s_2}\right)^2 = 13.75,$$

$U\left(x_2, \frac{z_2}{s_2}\right) > U\left(x_1, \frac{z_1}{s_2}\right)$. Hence the incentive compatibility constraint is satisfied.

b.i. After the transfer of consumption the utility of consumer 1 is

$$10\times(4+0.01)^{1/2}-\left(\frac{5}{1}\right)^2=-4.975$$

And the utility of consumer 2 is

$$10\times(9+0.01)^{1/2}-\left(\frac{8}{1}\right)^2=13.983.$$

The initial utility levels were -5 and 14. After the reallocation the change to -4.975 and 13.983.

b.(ii). The sum of the two utility levels is 9 before the transfer and 9.008 after the transfer. The sum has increased.

b.(iii) Incentive compatibility is clearly still satisfied for consumer 1 after the transfer, since x_1 has risen and x_2 has fallen. For consumer 2 selecting the allocation intended for consumer 1 gives

$$U\left(x_1,\frac{z_1}{s_2}\right)=10x_1^{1/2}-\left(\frac{z_1}{s_2}\right)^2=10\times(4+0.01)^{1/2}-\left(\frac{5}{2}\right)^2=13.775.$$

This is below the value (13.983) obtained from selecting the correct allocation, so incentive compatibility is maintained.

b(iv) Since the new allocation satisfies incentive compatibility and the sum of utilities has increased, the original allocation could not have been optimal for a utilitarian social welfare function.

Question 6.

A consumer has a choice between two occupations. One occupation pays a salary of €80,000 but gives no chance for tax evasion. The other pays €75,000 but does permit evasion. With the probability of detection $p = 0.3$, the tax rate $t = 0.3$, and the fine rate $F = 0.5$, which occupation will be chosen if $U = Y^{1/2}$?

Question 6. Answer.

The first step is to compute the expected utility from choosing the occupation than permits evasion. This involves choosing the optimal declaration. The choice of income declared, X, solves the optimisation

Max $EU = (1\text{-}0.1)(75000\text{-}0.3X)^{1/2}\text{+}.1((1\text{-}0.3)75000\text{-}0.5(75000\text{-}X))^{1/2}$

Differentiation with respect to X provides:

$$X = 75000 \left(\frac{(0.5)^2 - (0.7)^2 0.2}{0.5(0.7)^2 + (0.5)^2 0.3} \right) = 35625 \quad .$$

The value of expected utility is

$EU = 0.7(75000-0.3*35625)^{1/2} + 0.3((1-0.3)75000 – 0.5(75000-35625))1/2 = 231.86$.

Accepting the occupation that provides no possibility of evasion gives utility:

$U = Y^{1/2} = 80000^{½} = 282.84$.

Combining these number shows that the consumer will choose the occupation that does not permit tax evasion.

Question 7.

Are the following statements true or false?

(a) The theory of optimal commodity taxation argues that tax rates should be set equal across all commodities so as to maximize efficiency by "smoothing taxes".

(a) False. The inverse elasticity rule makes it very clear that taxes should generally be differentiated between commodities in order to minimize excess burden. In particular, a commodity with a low elasticity of demand should be tax at a higher rate than a commodity with a high elasticity.

(b) In the United States prescription drugs and CDs are taxed at the same rate of 10 percent. The Ramsey rule suggests that this is the optimal tax policy.

(b) False if we interpret prescription drugs as a necessity (low elasticity of demand) and CDs as a luxury (high elasticity). The Ramsey rule would most likely place the heavier tax burden on prescription drugs (but note all cross-effects in demand would have to be considered to completely justify this answer).

(c) Some economists have proposed replacing the income tax with a consumption tax to avoid taxing savings twice. This is a good policy both in terms of efficiency and equity.

(c) True. A consumption tax is equivalent in its effect to an income tax that exempts income from savings. A tax on the income from savings has the effect of raising the price of future consumption relative to the price of current

consumption. This is a distortion in prices that creates inefficiency. Unless there is reason for taxing future consumption more heavily than present consumption (such as a difference in the elasticity of demand or in the distribution across consumers of different income) then this distortion should be eliminated.

Question 8.

Consider two consumption tax systems: (a) one in which all goods are taxed at the same rate and (b) another in which the “necessities” are not taxed and “luxuries” are taxed at a higher rate. Compare the equity and efficiency of these two systems.

Question 8. Answer

Optimal tax theory would argue in favor of plan (a). This plan is a broad-based tax that is difficult to avoid, so it will not distort behavior significantly. Furthermore, given the tax’s broad base, the rate can be relatively low to raise the same amount of revenue. Plan (b) violates most tenets of efficient taxation: it does not tax goods for which demand is inelastic (necessities), even though the Ramsey Rule indicates that taxes on necessities will generate the least deadweight loss. Plan (b) does tax luxuries, for which demand is likely to be elastic. Thus, this tax will distort behavior and generate substantial deadweight loss. Plan (a) is clearly more efficient.
However, plan (a) is regressive: poorer taxpayers will spend a higher percentage of their income on taxes than will wealthier taxpayers. That is because poorer taxpayers cannot afford to save or invest large portions of their income; they spend it on the goods they need. By consuming most of their income, poorer taxpayers are subjecting a high proportion of their income to the consumption tax. Plan (b) is not as regressive, because the kinds of goods that lower-income taxpayers purchase are not taxed but the kinds of goods purchased by higher-income taxpayers are taxed. Plan (b) is clearly more equitable

Question 9.

Suppose that the tax rate is 30%. Suppose also that the probability of getting caught evading taxes is 10% plus an additional 2.5% for every €1,000 in tax evasion. (Hence, the probability of been caught $P = 0.1 + 0.025X$, where X is the number of euros (in thousands) of evasion.) Individuals who are caught evading taxes will be forced to pay the taxes they owe in addition to a €10,000 penalty. How much evasion will a risk-neutral taxpayer engage in? How would your answer change for a risk-averse taxpayer?

Question 9. Answer

Letting X denote the number of thousands of euros of evasion, the probability of getting caught is (0.1 + 0.025X). The cost of evasion and getting caught is €10,000. So the expected cost of evasion, in thousands of dollars, is thus 10 × (0.1 + 0.025X). The probability of not getting caught is 1 – (0.1 + 0.025X), so the expected benefit from evasion, again in thousands of euros, is X(0.90 – 0.025X).

We find the marginal costs and benefits by differentiating the costs and benefits with respect to X. The marginal cost of evasions is thus MC = 0.25 and the marginal benefit is MB = 0.90 – 0.05X. Setting them equal and solving yields X = 13 or about €13,000.

Question 10.

1. Marmara, Inc., is a monopolist whose cost of production is given by $10Q + Q^2$. Demand for Marmara's products is $Q = 200 - 2P$.

a. What price will the monopolist charge and what profits will the monopolist earn? What will consumer surplus be?

First we calculate the profit-maximizing quantity by setting marginal cost equal to marginal revenue. Marginal cost is 10 + 2Q. Marginal revenue can be found by solving for the inverse demand curve,

P = 100 – ½Q and noting that the marginal revenue curve has the same P-axis intercept and is twice as steeply sloped. Hence, marginal revenue is 100 – Q. Setting MR = MC and solving for Q, 10 + 2Q = 100 – Q, or 3Q = 90, or Q = 30. Therefore, the profit-maximizing quantity is 30, and the profit-maximizing price can be found from the inverse demand curve: P = 100 – ½ (30) = €85. Profits are computed as the difference between total revenue and total cost, or €85 (30) – 10(30) – 302 = 2,550 – 1,200 = 1,350. Consumer surplus can be computed as the area of the triangle with width Q = 30 and height 100 – 85 = 15 (the difference between the P-intercept of demand and the price paid). Computing, consumer surplus = ½ (30)(15) = 225.

b. How will the monopolist's price and profits change if a tax of €15 per unit is imposed on the buyers of the product?

Imposing a €15 tax on buyers will change their demand curve to Q = 200 – 2(P+15), or Q = 170 – 2P, where P is the pretax ("sticker") price. The new inverse demand is P = 85 – ½ Q, and the new marginal revenue is P = 85 – Q. Setting equal to marginal cost and solving gives

10 + 2Q = 85 – Q, or 3Q = 75, or Q = 25.

The profit-maximizing price is thus P = 85 – ½ Q = €72.50. Profits are given by €72.50 (25) – 10(25) – 252 = 1,812 – 875 = 937.5.

c. What is the excess burden of the tax?

To compute the excess burden of the tax, we look at the change in total surplus (including tax revenue as surplus). The after-tax consumer surplus can be computed from the new demand curve:

½ (25)(12.50) = 156.25, where 25 is the quantity purchased and 12.50 = 85 – 72.5 is the difference between the P-intercept of demand and the price paid. The tax revenue is 25(15) = 375. Hence, the excess burden of the tax is (1,350 + 225) – (937.5 + 156.25 + 375) = 1575 – 1468.75 = 106.25.

Question 11.

In an effort to reduce alcohol consumption, the government is considering a €1 tax on each litre of liquor sold (the tax is levied on producers). Suppose that the demand curve is Q^D = 500,000 -20,000*P* (where Q^D is the number of litres of liquor demanded and *P* is the price per litre), and the supply curve for liquor is Q^S = 30,000*P* (where Q^S is the number of litres supplied).

a. Compute how the tax affects the price paid by consumers and the price received by producers.

b. How much revenue does the tax raise for the government? How much of the revenue comes from consumers, and how much from producers?

c. Suppose that the demand for liquor is more elastic for younger drinkers than for older drinkers. Will the liquor tax be more, less, or equally effective at reducing liquor consumption among young drinkers? Explain.

Question 11. Answer

a. Before-tax equilibrium: P = €10 and Q = 300,000

After-tax equilibrium: P = €10.60 and Q = 288,000.

Consumers pay €10.60 and producers receive €9.60.

b. Revenue = €288,000. Consumers bear 60 percent of the tax burden and producers bear 40 percent. So, €172,800 comes from consumers and €115,200 from producers.

c. With a more elastic demand curve, quantity consumed will decrease even more as a result of the tax, so the liquor tax will be more effective at reducing consumption among young drinkers.

Question 12.

A good is traded in a competitive market. The demand function is given by X = 75 — 5P and supply is perfectly elastic at the price P = 10.

a. A specific tax of value t = 2 is introduced. Determine the tax incidence.

b. An ad valorem tax at a rate of t = 0.2 is introduced. Determine the tax incidence.

c. How do the incidence of the specific tax and the ad valorem tax differ if supply is given by Y = 2.5P

Question 12. Answer

First, we need to calculate the equilibrium price and quantity without tax. In the first case, with the perfectly elastic supply, the equilibrium price is $P^0 = 10$, and the equilibrium quantity is $X^0 = 75 - 5 \times 10 = 25$. In the second case, with the supply given by Y = 2.5P, the equilibrium price is determined by

$$75 - 5P = 2.5P,$$

$$P = 10,$$

and the equilibrium quantity is X = 25, i.e. the same as in the first case.

a. Since the supply is perfectly elastic, the price received by the seller does not change, P^S = 10. The price paid by the buyer, P^B, satisfies

$$X = 75 - 5P^B,$$

$$(P^B - P^S = 2.$$

This gives P^B = 12 and X = 15. The tax revenue is given by

$$TR = 2 \times 15 = 30,$$

and the tax incidence is fully on the buyer.

b. As in part (a), the price received by the seller is P^S = 10. The price paid by the buyer, P^B, satisfies

$$X = 75 - 5P^B,$$

$$P^B = 1.2P^S.$$

This gives $P^B = 12$ and $X = 15$. The tax revenue is the same as in part (a), and the tax incidence, again, is fully on the buyer.

c. **For the specific tax, the seller's price and the buyer's price solve the following system of equations:**

$$X = 75 - 5P^B\ ,$$

$$Y = 2.5P^S,$$

$$X = Y,$$

$$P^B - P^S = 2.$$

This solves to give $P^B = 10(2/3)$, $P^S = 8(2/3)$, and $X = 21(2/3)$. The tax revenue is $TR = 43(1/3)$, of which the tax incidence on the buyer and on the seller are given, respectively, by

$TR^B = (P^B – P^0)\ X = [10(2/3) - 10]\ (21(2/3) = 130/9 = 14(4/9)$,

$TR^S = (P^0 – P^S)\ X = [10 – 8(2/3)]\ (21(2/3) = 260/9 = 28(8/9)$,

That is, the tax incidence on the seller is twice the tax incidence on the buyer. For the ad valorem tax we have

$$X = 75 - 5P^B,$$

$$Y = 2.5P^S,$$

$$X = Y,$$

$$P^B = 1.2P^S,$$

And the solution is

$$P^B = 10(10/7), \quad P^S = 8(14/17), \quad X = 22(1/16)$$

The tax revenue is $TR = 38(268/289)$.

The tax incidence on the buyer and the seller are given by

$TR^B = (P^B – P^0)\ X = [10(10/17) - 10]\ (22(1/17) = 3780/289 = 13(23/289)$,

$TR^S = (P^0 – P^S)\ X = [10 – 8(14/17)]\ (22(1/17) = 11340/289 = 39(69/289)$,

In this case, the tax incidence on the seller is three times the tax incidence on the buyer.

Question 13

Is tax evasion just a gamble?

Question 13, Answer

The basic model of tax evasion portrays the choice problem as a gamble: the tax evader takes a chance that evasion will be successful. This model provides comparative statics predictions which are in line with the data. However the sufficient condition for evasion suggests that if the model is correct much more tax evasion should be observed. In addition, the prediction that evasion is reduced as the tax rate increases conflicts with a priori expectations and with some of the empirical evidence. Empirical and experimental evidence also highlights the importance of social interaction in the evasion decision. Tax evasion has elements of a gamble but the evidence suggests that it is more than just a gamble.

Question 14.

Tax evasion is particularly common for workers in professions such as waiting tables and bartending, where tips make up a substantial fraction of compensation. Use economic theory to explain why this is the case.

Question 14. Answer

Tips are often paid in cash. It is quite easy to hide this income by underreporting cash tips, and it is very difficult to verify small amounts of underreporting. When the likelihood of being caught for tax evasion is lower, economic theory tells us that individuals are more likely to evade taxes.

Question 15.

What is the difference between tax evasion and tax avoidance? How would you empirically distinguish the two phenomena?

Question 15. Answer

Tax evasion is illegal: it is the failure to pay tax that is owed. Tax avoidance is legal: taxpayers are allowed to seek out and take advantage of provisions of the tax code that reduce their tax liability. Some taxpayers are more creative at tax avoidance than others, but if they stay within the provisions of the tax code and the judicial interpretations of that code, then what they do is technically legal and thus is avoidance rather than evasion.

The difference between evasion and avoidance is not always clear, as shown by differences in opinion among government tax auditors. Two tax auditors can review the same tax return and one can determine it to be compliant with the law, while the other finds evasion. That is why taxpayers resort to courts in order to overturn decisions of the tax authorities, on the basis that they avoided but not evaded their taxes.