### LECTURE 1b

#### **TOOLS OF NORMATIVE ANALYSIS**

- What is to be produced and in what quantities?
- How is the desired output to be produced?
- How is the desired output to be distributed?
- How does the economy provide for cyclical stability?
- How does the economy sustain economic growth overtime?

- What is to be produced and in what quantities?
- How is the desired output to be produced?

# RESOURCE ALLOCATION QUESTIONS

• How is the desired output to be distributed?



**DISTRIBUTION QUESTION** 

- How does the economy provide for cyclical stability?
- How does the economy sustain economic growth overtime?

## STABILIZATION QUESTIONS

#### **MARKET SYSTEM**

- A price system is a social economic organization based on individual choices and property rights.
- Understanding the price system is important because:
- 1 The market is the alternative to government intervention and control.
- 2 Tax and expenditure policies impact decisions in the private markets.
- 3 The concept of economic efficiency needs to be defined more specifically.

#### **OVERVIEW OF THE PRICE SYSTEM**

#### **GOODS MARKET**

Demand for commodities
Supply of commodities
Equilibrium price and quantity of goods

#### LABOR MARKET

Demand for
labor
Supply of
labor
Equilibrium
wage and
quantity of
labor

#### CAPITAL MARKET

Demand for
capital
Supply of
capital
Equilibrium interest
rate and quantity
of capital

#### **OVERVIEW OF THE PRICE SYSTEM**

#### **GOODS MARKET**

Demand for commodities
Supply of commodities
Equilibrium price
and quantity of goods

#### LABOR MARKET

Demand for labor Supply of labor Equilibrium wage and quantity of labor

#### **CAPITAL MARKET**

Demand for capital
Supply of capital
Equilibrium interest rate
and quantity of capital

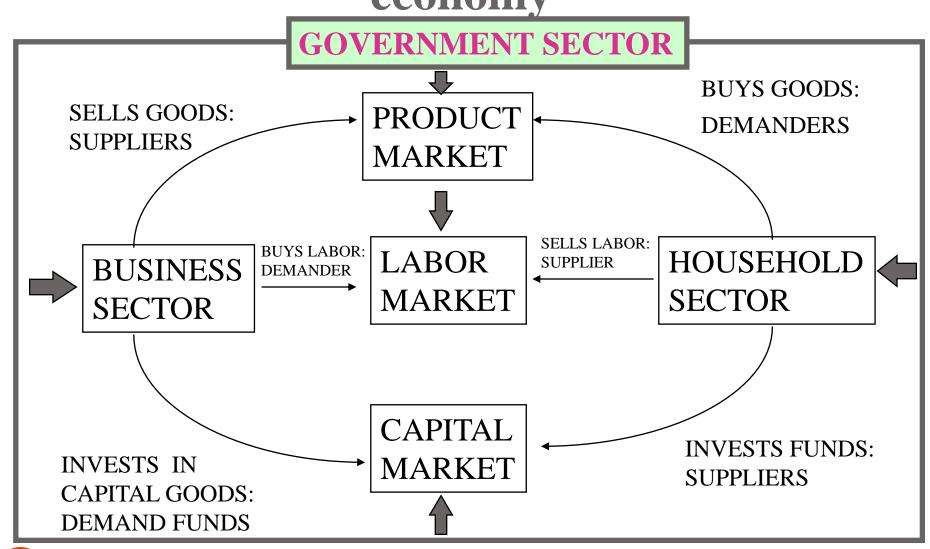
Excise Taxes
Pollution Taxes

Income Taxes
Minimum Wage

Social Security
Capital Gains Taxes

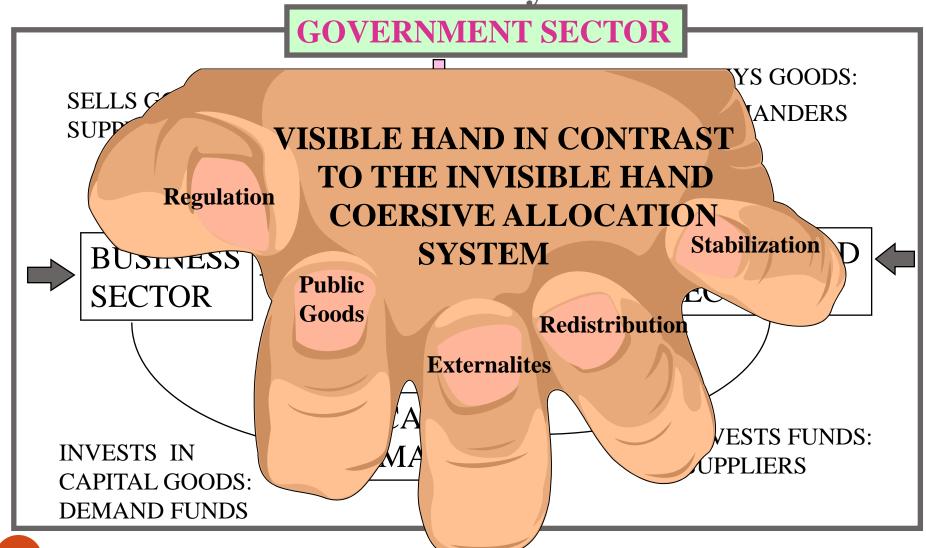
**GOVERNMENT** 

# Simple characterization of a mixed economy



#### Simple characterization of a mixed economy External Force **GOVERNMENT SECTOR BUYS GOODS: SELLS GOODS: PRODUCT DEMANDERS SUPPLIERS MARKET SELLS LABOR: BUYS LABOR:** HOUSEHOLD **LABOR BUSINESS SUPPLIER DEMANDER MARKET SECTOR** SECTOR **CAPITAL INVESTS FUNDS:** INVESTS IN **MARKET SUPPLIERS CAPITAL GOODS: DEMAND FUNDS** 10

# Simple characterization of a mixed economy



### **Efficiency criterion**

- <u>Positive Economics</u> is the scientific view of economic events. It tries to find cause and effect, predictive relationships.
- Normative Economics is based on value judgments. It tries to formulate recommendations as to what should be.
- The *efficiency criterion* is satisfied when resources are used over a given period of time in such away as to make it impossible to increase the well-being of any one person without reducing the well-being of any other person. This situation is referred to as a *Pareto Optimum* state

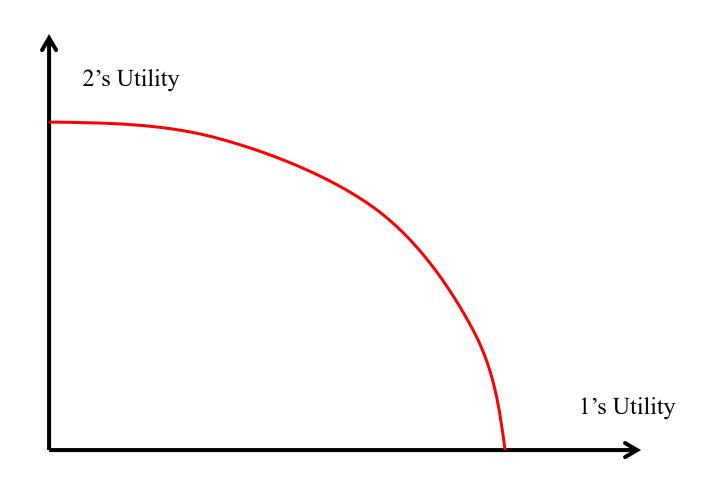
## Efficiency criterion

Definition: An *allocation* of resources is *Pareto Efficient* if it is not possible to reallocate resources to make everyone better off.

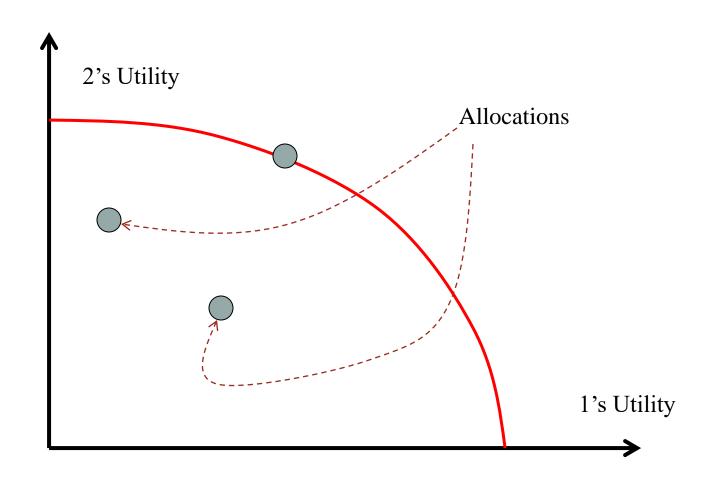
How do we measure better off?

We use *Utility* to measure welfare/happiness.

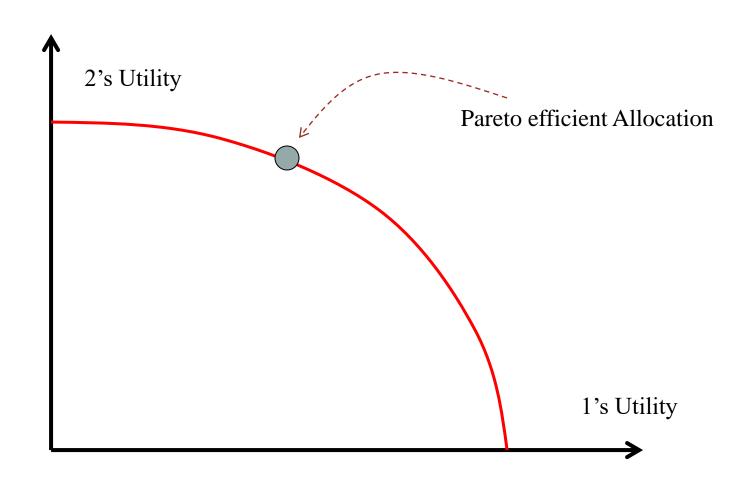
## Utility Possibilities: What is Feasible



### Utility Possibilities: What is Feasible



### Pareto efficiency: There is no waste



- Marginal Condition for Exchange.
- To attain a *Pareto Maximum*, the marginal rate of substitution (MRS) between any pair of goods must be the same for all individuals who consumer both goods.

- Marginal Condition for Factor Substitution.
- To attain *Pareto Maximum*, the marginal rate of technical substitution (MRTS) between any pair of inputs must be the same for all producers who use both inputs.

- Marginal Condition for Product Substitution.
- To attain a *Pareto Maximum*, the marginal rate of transformation (MRT) in production must equal the marginal rate of substitution in consumption for every pair of commodities and for every individual who consumes both.

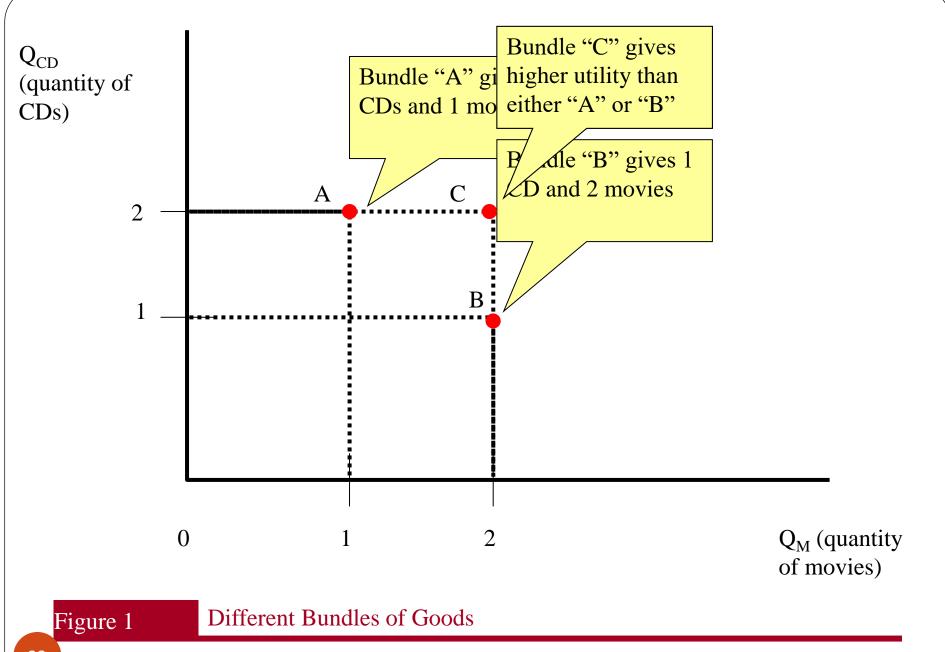
- Corollary Proposition.
- If the political organization of a society is such to accord paramount importance to its individual members -- mechanistic approach to government -- social welfare will be maximized if every consumer, every firm, and every input *market* is perfectly competition.

## CONSTRAINED UTILITY MAXIMIZATION

- Constrained utility maximization means that all decisions are made in order to maximize the well-being of the individual, subject to his available resources.
- Utility maximization involves *preferences* and a *budget constraint*.
- One of the key assumptions about preferences is *non-satiation*—that "more is preferred to less."

## Constrained Utility Maximization Preferences and indifference curves

- Figure 1 illustrates some preferences over movies (on the x-axis) and CDs (on the y-axis).
- Because of non-satiation, bundles *A* and *B* are both inferior to bundle *C*.

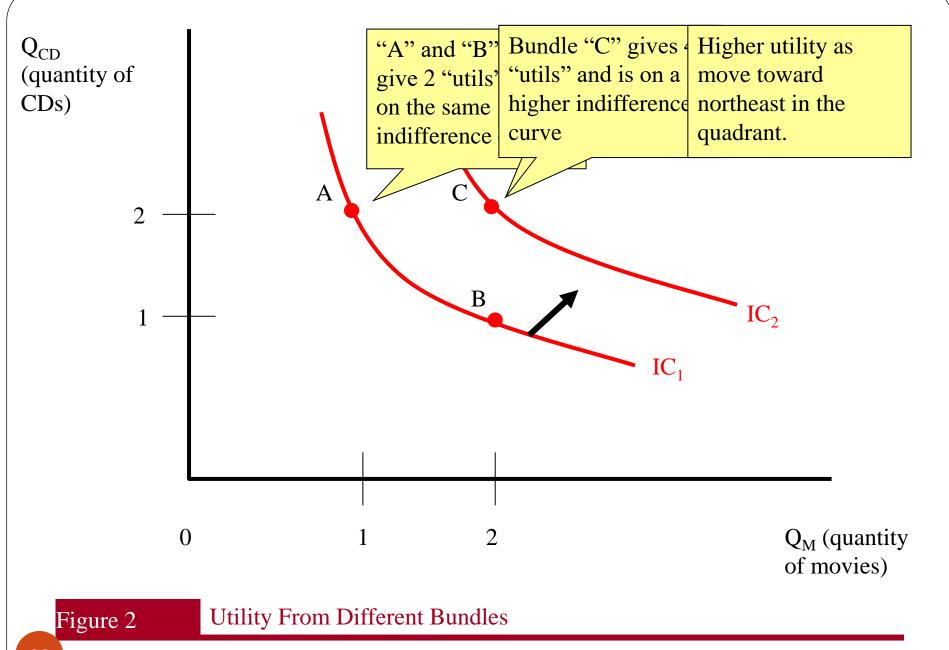


## Constrained Utility Maximization: Preferences and indifference curves

- A *utility function* is a mathematical representation
- $U = f(X_1, X_2, X_3, ...)$ 
  - Where  $X_1, X_2, X_3$  and so on are the *goods* consumed by the individual,
  - And  $f(\bullet)$  is some mathematical function.

## Constrained Utility Maximization: Preferences and indifference curves

- One formulation of a utility function is  $U(Q_M,Q_C) = Q_MQ_C$ , where  $Q_M$  = quantity of movies and  $Q_C$  = quantity of CDs.
- The combinations {1, 2} (bundle A) and {2,1} (bundle B) both give 2 "utils."
- The combination {2, 2} (bundle *C*) gives 4 "utils."
- With these preferences, *indifferent* to *A* or *B*.
- Figure 2 illustrates this.



### Constrained Utility Maximization: Utility mapping of preferences

- How are indifference curves derived?
- Set utility equal to a constant level and figure out the bundles of goods that get that utility level.
- For  $U = Q_M Q_C$ , how would we find the bundles for the indifference curve associated with 25 utils?
  - Set  $25 = Q_M Q_C$ ,
  - Yields  $Q_C = 25/Q_M$ ,
  - Or bundles like {1,25}, {1.25,20}, {5,5}, etc.

# Constrained Utility Maximization: Marginal utility

- *Marginal utility* is the additional increment to utility from consuming an additional unit of a good.
- *Diminishing marginal utility* means each additional unit makes the individual less happy than the previous unit.

# Constrained Utility Maximization: Marginal utility

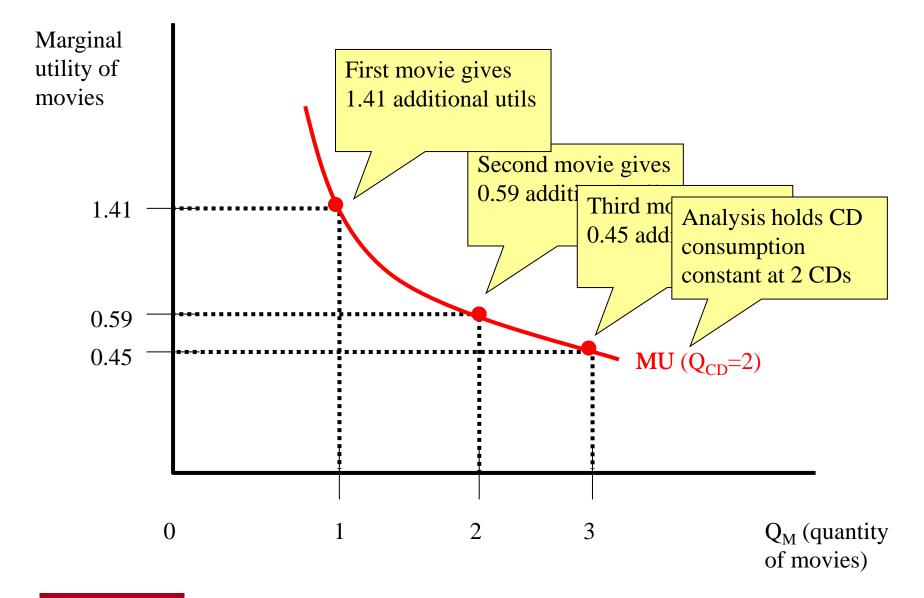
• With the utility function given before,  $U = Q_M Q_C$ , the marginal utility is:

$$MU_{Q_M} = \frac{\partial U}{\partial Q_M} = Q_C$$

• Take the partial derivative of the utility function with respect to  $Q_M$  to get the marginal utility of movies.

# Constrained Utility Maximization: Marginal utility

- Evaluating the utility function  $U = (Q_M Q_C)^{1/2}$ , at  $Q_C = 2$  allows us to plot a relationship between marginal utility and movies consumed.
- Figure 3 illustrates this.



Declining Marginal Utility From Movies

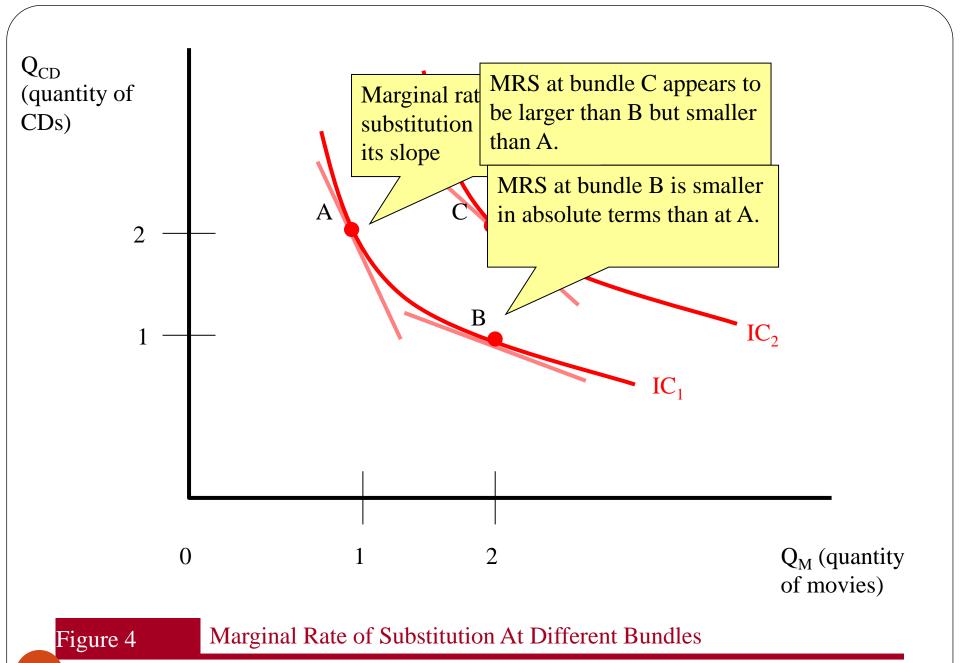
Figure 3

# Constrained Utility Maximization: Marginal utility

- Why does diminishing marginal utility make sense?
  - Most consumers order consumption of the goods with the highest utility first.

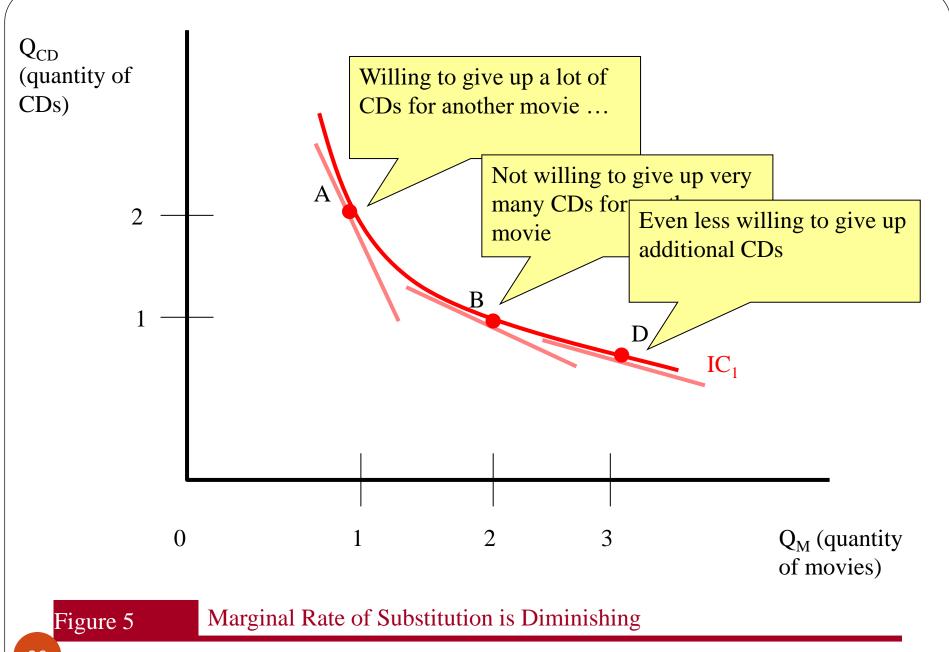
### Constrained Utility Maximization: Marginal rate of substitution

- *Marginal rate of substitution*—slope of the indifference curve is called the *MRS*, and is the rate at which consumer is willing to trade off the two goods.
- Returning to the (CDs, movies) example.
- Figure 4 illustrates this.



### Constrained Utility Maximization: Marginal rate of substitution

- *MRS* is diminishing (in absolute terms) as we move along an indifference curve.
- This means that Andrea is willing to give up fewer CD's to get more movies when she has more movies (bundle *B*) than when she has less movies (bundle *A*).
- Figure 5 illustrates this.



## Constrained Utility Maximization Marginal rate of substitution

• Direct relationship between MRS and marginal utility.

$$MRS = -\frac{MU_{M}}{MU_{C}}$$

- *MRS* shows how the relative marginal utilities evolve over the indifference curve.
- Straightforward to derive this relationship graphically, as well.
- Consider the movement from bundle A to bundle B. Figure 6 illustrates this.

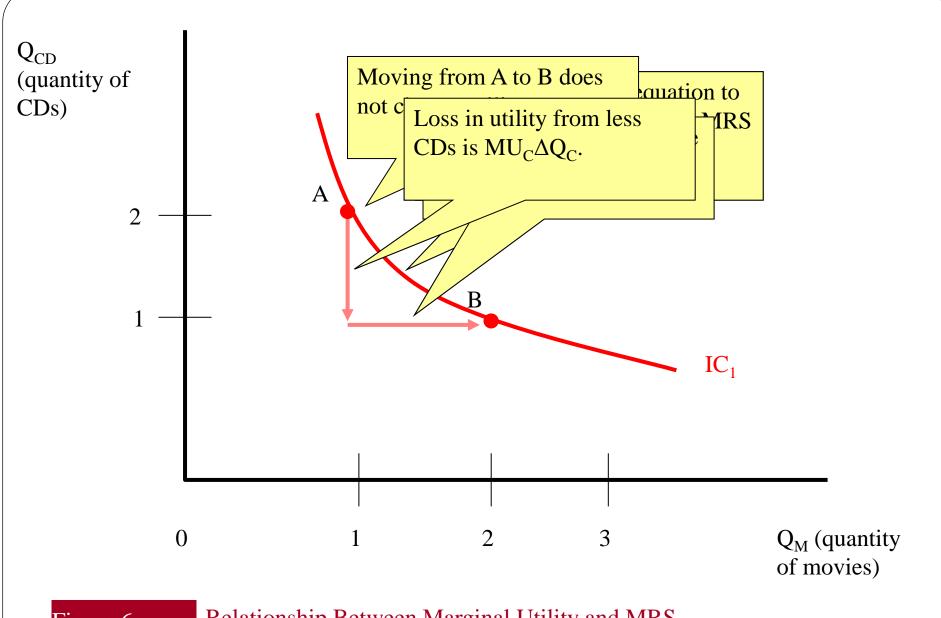


Figure 6 Relationship Between Marginal Utility and MRS

- The *budget constraint* is a mathematical representation of the combination of goods the consumer can afford to buy with a given income.
- Assume there is no saving or borrowing.
- In the example, denote:
  - Y = Income level
  - $P_M$  = Price of one movie
  - $P_C$  = Price of one CD

• The expenditure on movies is:

$$P_{M}Q_{M}$$

• While the expenditure on CDs is:

$$P_{C}Q_{C}$$

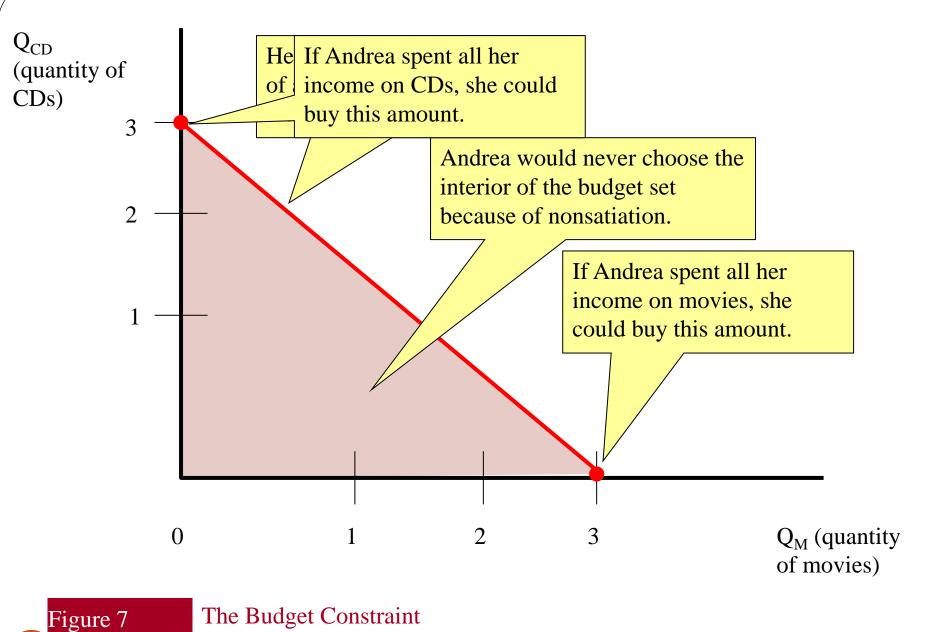
• Thus, the total amount spent is:

$$P_M Q_M + P_C Q_C$$

• This must equal income, because of no saving or borrowing.

$$Y = P_M Q_M + P_C Q_C$$

- This budget constraint is illustrated in the next figure.
- Figure 7 illustrates this.

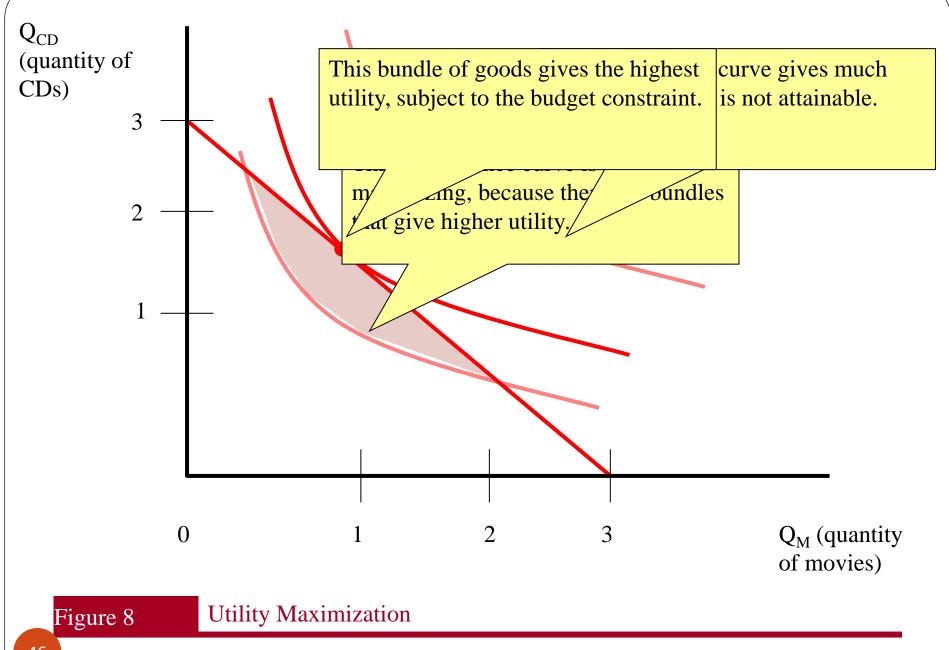


• The slope of the budget constraint is:

$$-\frac{P_{\scriptscriptstyle M}}{P_{\scriptscriptstyle C}}$$

• It is thought that government actions can change a consumer's budget constraint, but that a consumer's preferences are fixed.

- What is the highest indifference curve that an individual can reach, given a budget constraint?
- Preferences tells us what a consumer wants, and the budget constraint tells us what a consumer can actually purchase.
- This leads to utility maximization, shown graphically, in **Figure 8**.

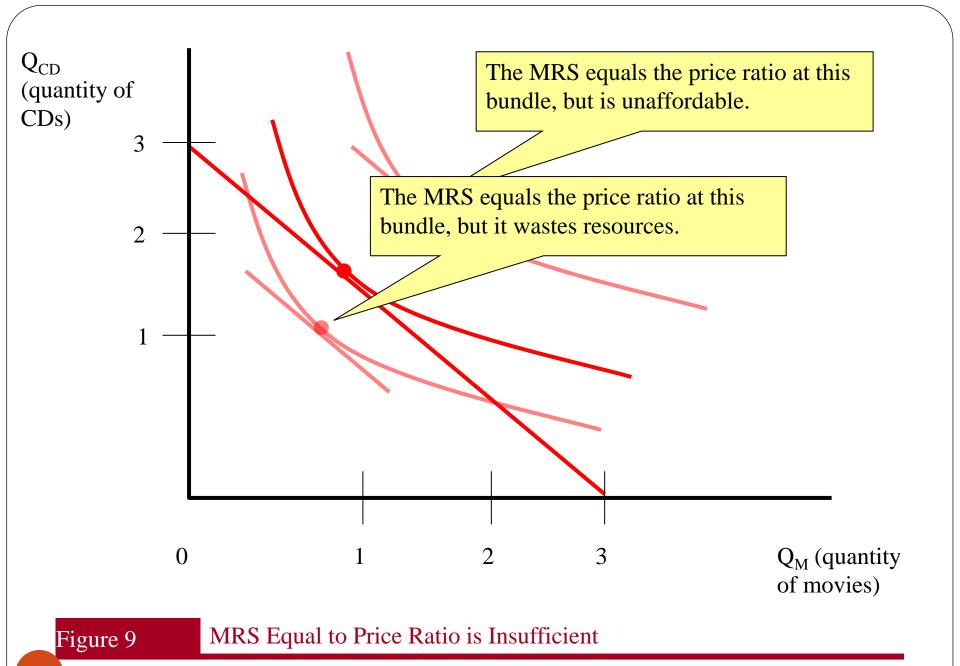


- In this figure, the utility maximizing choice occurs where the indifference curve is *tangent* to the budget constraint.
- This implies that the slope of the indifference curve equals the slope of the budget constraint.

• Thus, the marginal rate of substitution equals the ratio of prices:

$$MRS = -\frac{MU_{M}}{MU_{C}} = -\frac{P_{M}}{P_{C}}$$

- At the optimum, the ratio of the marginal utilities equals the ratio of prices. But this is not the only condition for utility maximization.
- Figure 9 illustrates this.



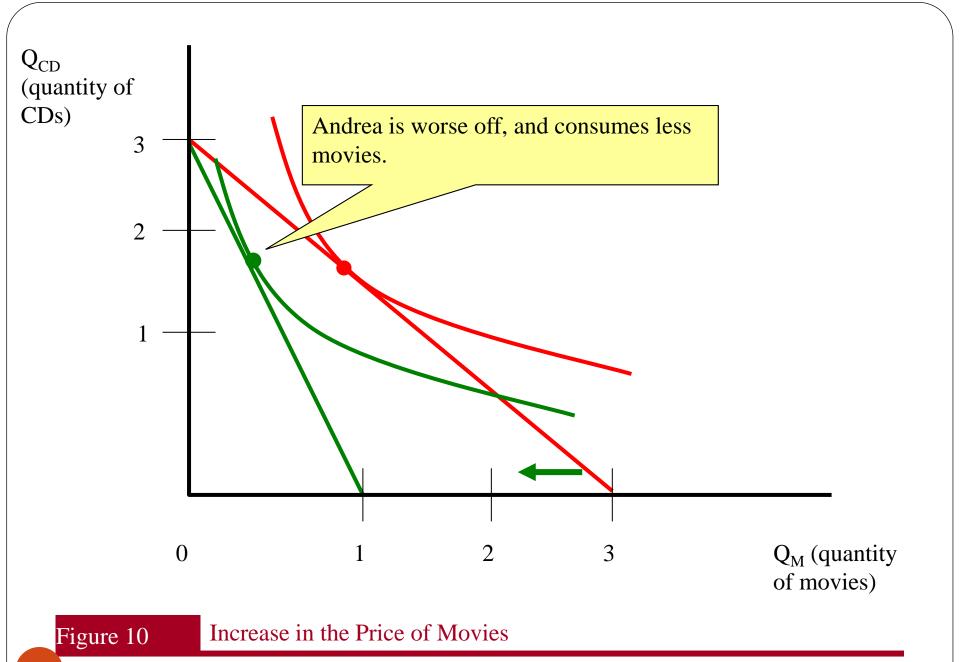
• Thus, the second condition is that all of the consumer's money is spent:

$$Y = P_M Q_M + P_C Q_C$$

• These two conditions are used for utility maximization.

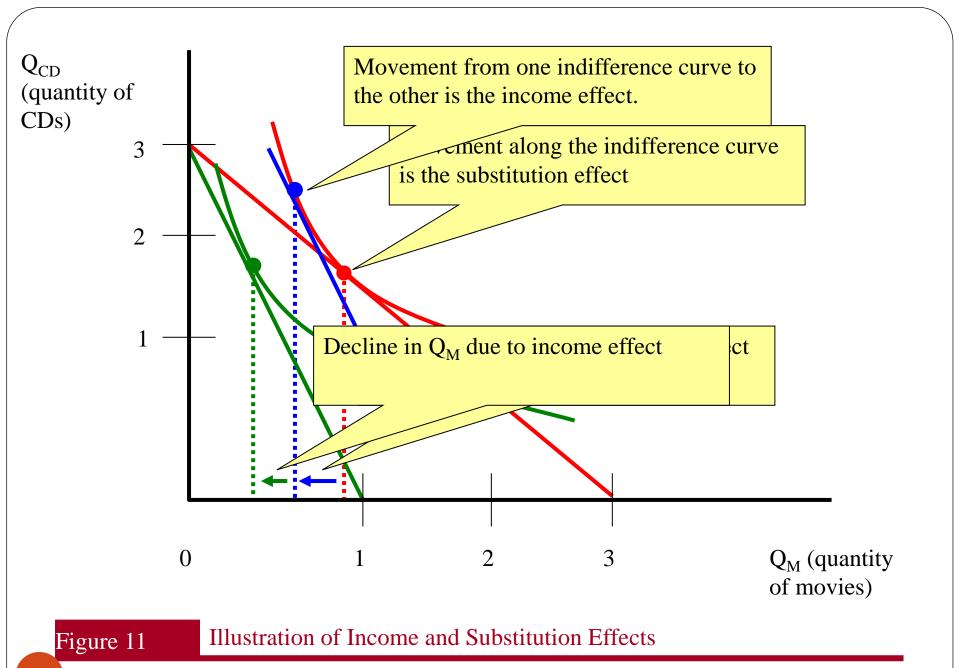
### The Effects of Price Changes: Substitution and income effects

- Consider a typical price change in our framework:
- Increase the price of movies,  $P_M$ .
- This rotates the budget constraint inward along the x-axis.
- Figure 10 illustrates this.



#### The Effects of Price Changes: Substitution and income effects

- A change in price consists of two effects:
- **Substitution effect**—change in consumption due to change in relative prices, *holding utility constant*.
- *Income effect*—change in consumption due to feeling "poorer" after price increase.
- Figure 11 illustrates this.



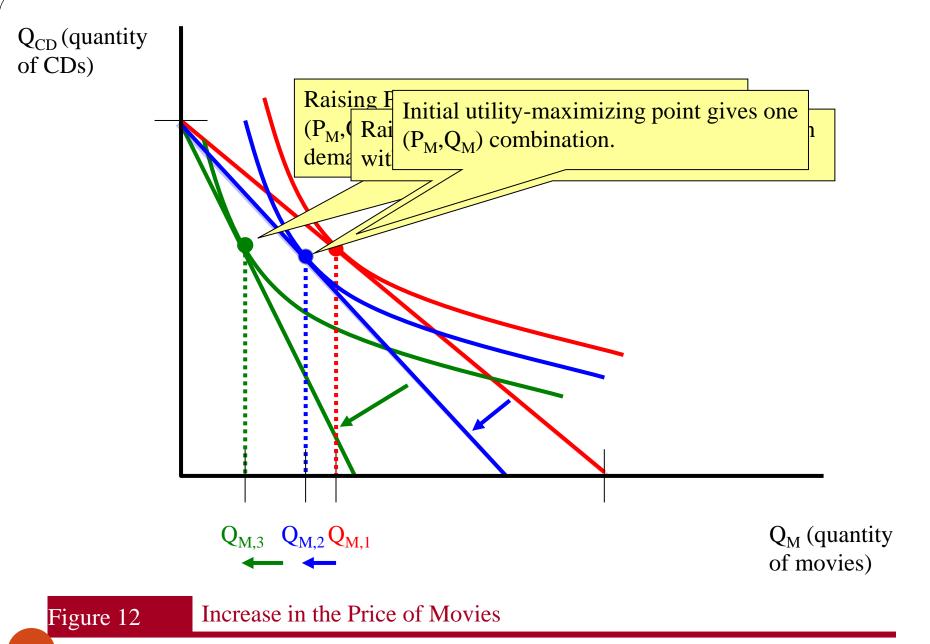
#### EQUILIBRIUM AND SOCIAL WELFARE

- Welfare economics is the study of the determinants of well-being, or welfare, in society.

  It depends on:
- Determinants of social efficiency, or size of the economic "pie."
- Redistribution.

### EQUILIBRIUM AND SOCIAL WELFARE Demand curves

- **Demand curve** is the relationship between the price of a good and the quantity demanded.
- Derive demand curve from utility maximization problem, as shown in **Figure 12**.



## EQUILIBRIUM AND SOCIAL WELFARE Demand curves

- This gives various  $(P_M, Q_M)$  combinations that can be mapped into price/quantity space.
- This gives us the demand curve for movies.
- Figure 13 illustrates this.

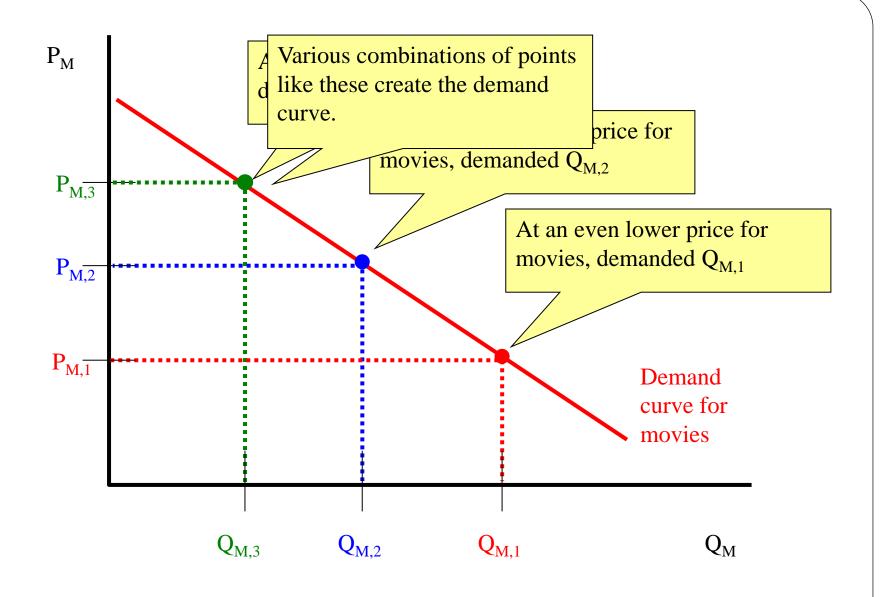


Figure 13 Deriving the Demand Curve for Movies

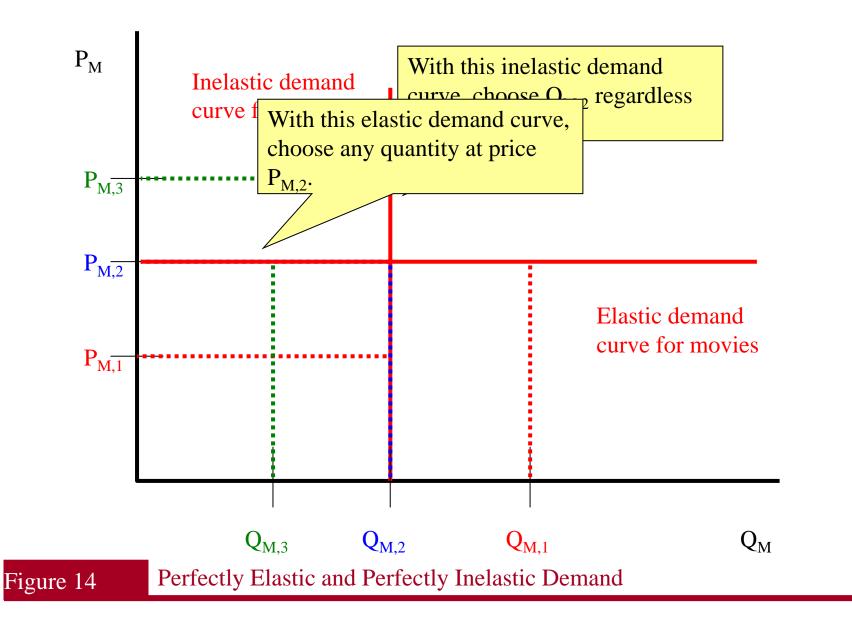
• A key feature of demand analysis is the *elasticity of demand*. It is defined as:

$$\varepsilon_{D} = \frac{\Delta Q_{D}}{\Delta P/P}$$

• That is, the percent change in quantity demanded divided by the percent change in price.

- For example, an increase in the price of movies from € 8 to €12 is a 50% rise in price.
- If the number of movies purchased fell from 6 to 4, there is an associated 33% reduction in quantity demanded.
  - The demand elasticity is therefore -0.67.
- Demand elasticities features:
  - Typically negative number.
  - Not constant along the demand curve (for a linear demand curve).

- For a vertical demand curve
  - Elasticity of demand is zero—quantity does not change as price goes up or down.
  - Perfectly inelastic
- For a horizontal demand curve
  - Elasticity of demand is negative infinity—quantity changes infinitely for even a small change in price.
  - Perfectly elastic
- Figure 14 illustrates this.



• More generally, an *elasticity* divides the percent change in a dependent variable by the percent change in an independent variable:

$$\varepsilon = \frac{\Delta Y/Y}{\Delta X/X}$$

• For example, *Y* is often the quantity demanded or supplied, while *X* might be own-price, cross-price, or income.

- Supply curve is the relationship between the price of a good and the quantity supplied.
  - Derive supply curve from profit maximization problem.
- The firm's *production function* measures the impact of a firm's input use on output levels.

• Assume two inputs, labor (*L*) and capital (*K*). Firm's production function for movies is, in general:

$$Q_M = f(L_M, K_M)$$

- That is, the quantity of movies produced is related to the amount of labor and capital devoted to movie production.
- Similarly, there would be a production function for CDs.

• One specific production function is:

$$Q_M = \sqrt{L_M K_M}$$

• From a production function like this, we can figure out the *marginal productivity* of an input by taking the derivative with respect to it.

# Equilibrium and Social Welfare: Supply curves

• For example, the marginal productivity of labor is:

$$\frac{\partial Q_M}{\partial L_M} = \frac{1}{2} \sqrt{\frac{K_M}{L_M}} > 0$$

• This is the partial derivative of Q with respect to L. The marginal product is positive.

# Equilibrium and Social Welfare: Supply curves

• Taking the second derivative yields:

$$\frac{\partial^2 Q_M}{\partial L_M^2} = -\frac{1}{4} \sqrt{\frac{K_M}{L_M^3}} < 0$$

• This second derivative is negative, meaning that the production function features *diminishing marginal productivity*.

• *Diminishing marginal productivity* means that holding all other inputs constant, increasing the level of one input (such as labor) yields less and less additional output.

• The total costs of production are given by:

$$TC = rK + wL$$

• In this case, r and w are the input prices of capital and labor, respectively.

• If we assume capital is fixed in the short-run, the cost function becomes:

$$TC = r\overline{K} + wL$$

• Thus, only labor can be varied in the short run. The *marginal cost* is the incremental cost of producing one more unit of *Q*, or the product of the wage rate and amount of labor used to produce that unit.

# EQUILIBRIUM AND SOCIAL WELFARE Supply curves

- Diminishing marginal productivity implies rising marginal costs.
- Since each additional unit, Q, means calling forth less and less productive labor at the same wage rate, costs of production rise.

# EQUILIBRIUM AND SOCIAL WELFARE Supply curves

- *Profit maximization* means maximizing the difference between total revenue and total costs.
- This occurs at the quantity where *marginal revenue* equals *marginal costs*.

## EQUILIBRIUM AND SOCIAL WELFARE Equilibrium

- In a perfectly competitive market, the marginal revenue is the market price. Thus, the firm produces until:
  - $\bullet$  P = MC.
- Thus, the *MC* curve is the supply curve.

# EQUILIBRIUM AND SOCIAL WELFARE Equilibrium

- In equilibrium, we *horizontally sum* individual demand curves to get aggregate demand.
- We also *horizontally sum* individual supply curves to get aggregate supply.
- Competitive equilibrium represents the point at which both consumers and suppliers are satisfied with the price/quantity combination.
- Figure 15 illustrates this.

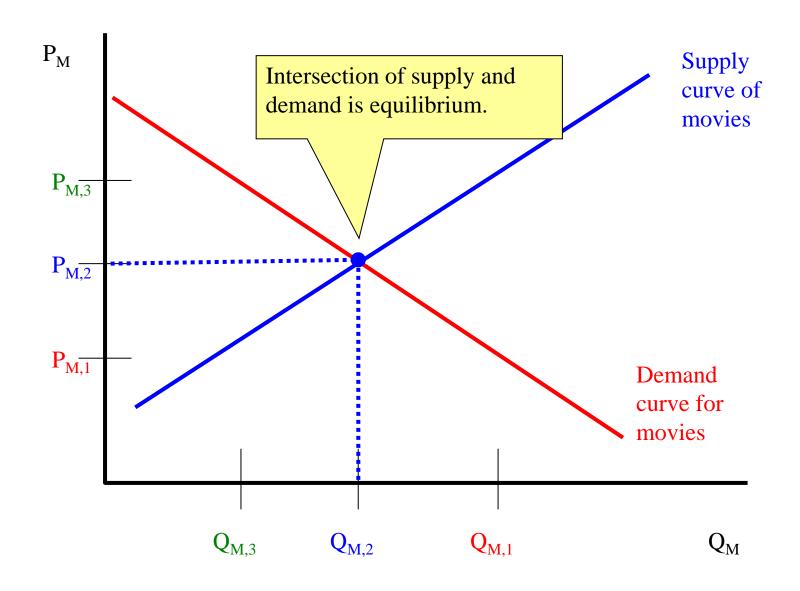


Figure 15 Equilibrium with Supply and Demand

• Measuring *social efficiency* is computing the potential size of the economic pie. It represents the net gain from trade to consumers and producers.

- *Consumer surplus* is the benefit that consumers derive from a good, beyond what they paid for it.
- Each point on the demand curve represents a "willingness-to-pay" for that quantity.
- Figure 16 illustrates this.

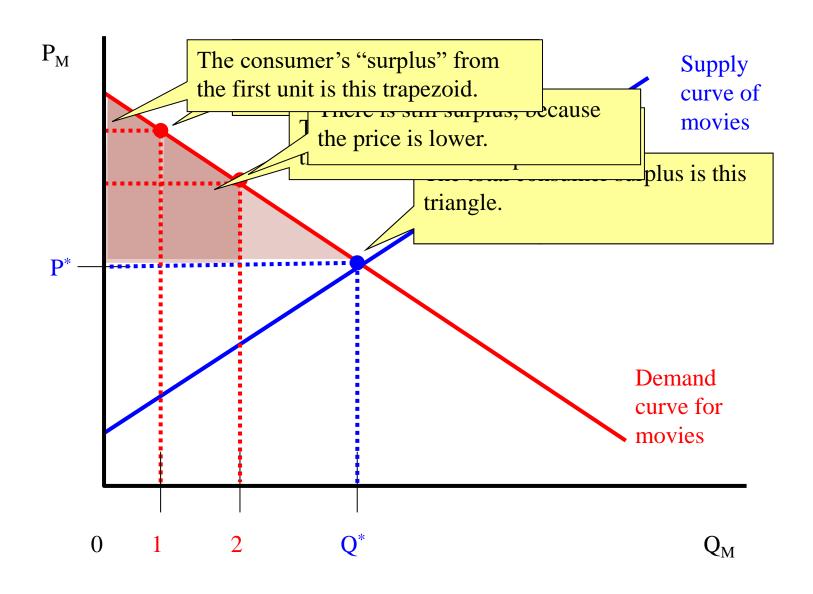


Figure 16 Deriving Consumer Surplus

- Consumer surplus is determined by market price and the elasticity of demand:
  - With inelastic demand, demand curve is more vertical, so surplus is higher.
  - With elastic demand, surplus is lower.
- Figure 17 illustrates this.

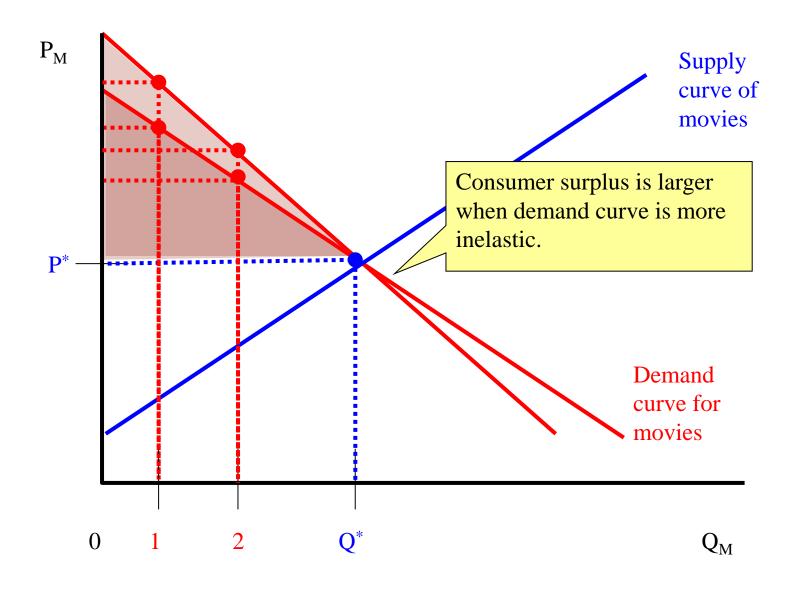


Figure 17 Consumer Surplus and Inelastic Demand

- *Producer surplus* is the benefit derived by producers from the sale of a unit above and beyond their cost of producing it.
- Each point on the supply curve represents the marginal cost of producing it.
- Figure 18 illustrates this.

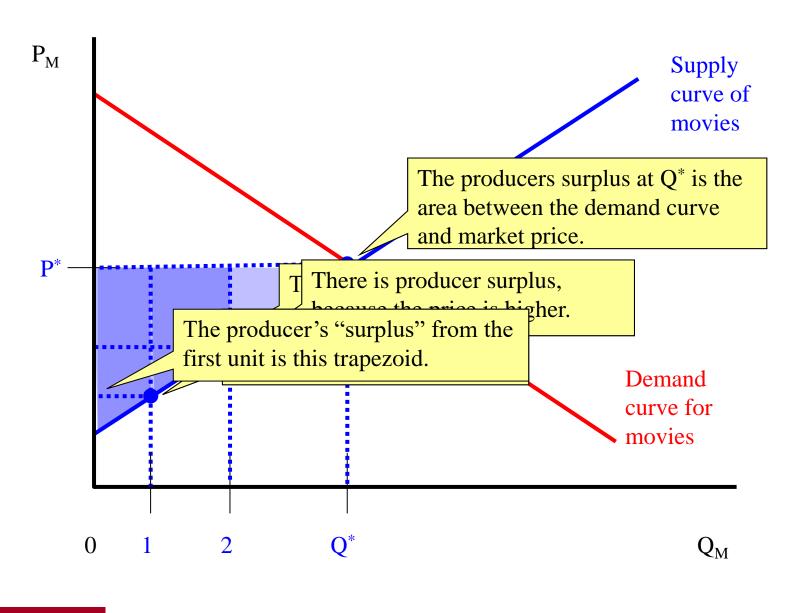


Figure 18

**Producer Surplus** 

- Similar to consumer surplus, producer surplus is determined by market price and the elasticity of supply:
  - With inelastic supply, supply curve is more vertical, so producer surplus is higher.
  - With elastic supply, producer surplus is lower.

- The *total social surplus*, also known as "social efficiency," is the sum of the consumer's and producer's surplus.
- Figure 19 illustrates this.

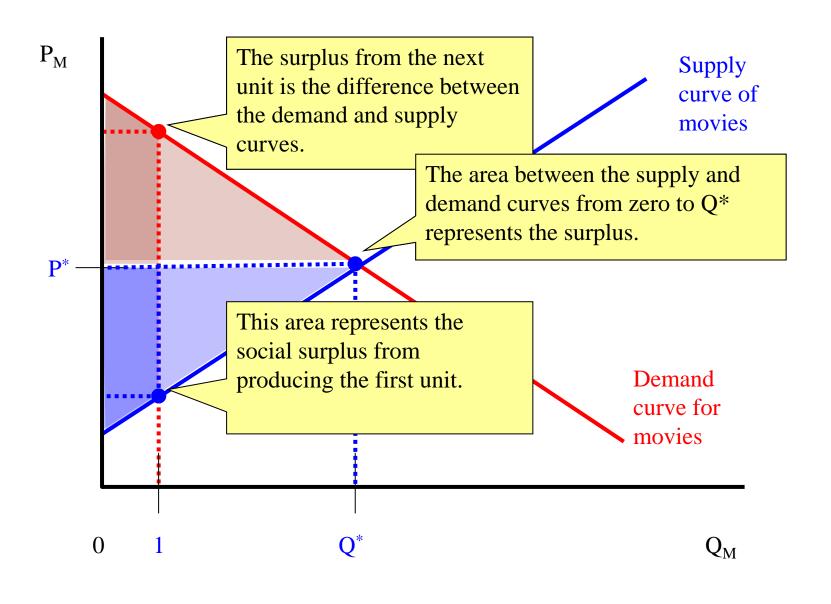


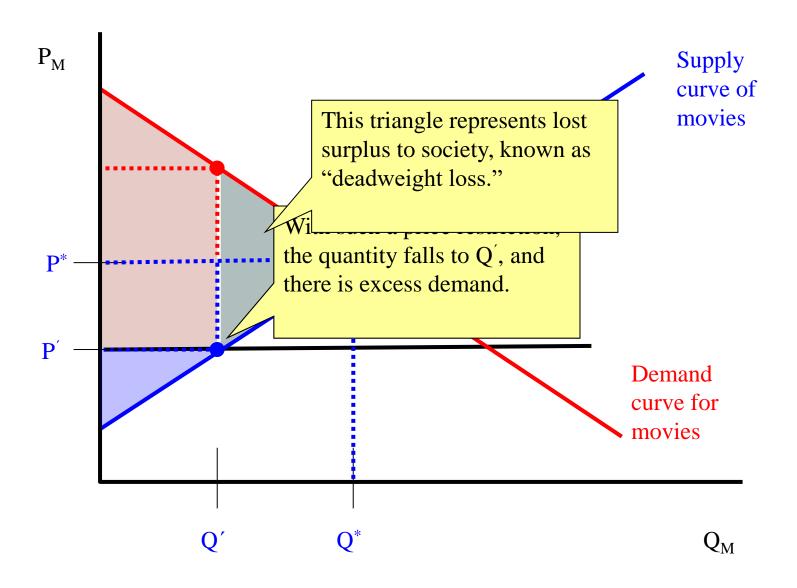
Figure 19

Social Surplus

### EQUILIBRIUM AND SOCIAL WELFARE

Competitive equilibrium maximizes social efficiency

- The *First Fundamental Theorem of Welfare Economics* states that the competitive equilibrium, where supply equals demand, maximizes social efficiency.
- Any quantity other than  $Q^*$  reduces social efficiency, or the size of the "economic pie."
- Consider restricting the price of the good to  $P' < P^*$ .
- Figure 20 illustrates this.



Deadweight Loss from a Price Floor

### EQUILIBRIUM AND SOCIAL WELFARE

Competitive equilibrium maximizes social efficiency

• A policy like price controls creates *deadweight loss*, the reduction in social efficiency by restricting quantity below the competitive equilibrium.

- Societies usually care not only about how much surplus there is, but also about how it is distributed among the population.
- Social welfare is determined by both criteria.
- The *Second Fundamental Theorem of Welfare Economics* states that society can attain any efficient outcome by a suitable redistribution of resources and free trade.
- In reality, society often faces an equity-efficiency tradeoff.

- Society's tradeoffs of equity and efficiency are models with a *Social Welfare Function*.
- This maps individual utilities into an overall social utility function.

• The *utilitarian* social welfare function is:

$$SWF = \sum_{i} U_{i}$$

- The utilities of all individuals are given equal weight.
- Implies that government should transfer from person 1 to person 2 as long as person 2's gain is bigger than person 1's loss in utility.

- Utilitarian SWF is defined in terms of utility, not euros.
- Society not indifferent between giving €1 of income to rich and poor; rather indifferent between one *util* to rich and one util to poor.

• Utilitarian SWF is maximized when the marginal utilities of everyone are equal:

$$MU_1 = MU_2 = ... = MU_i$$

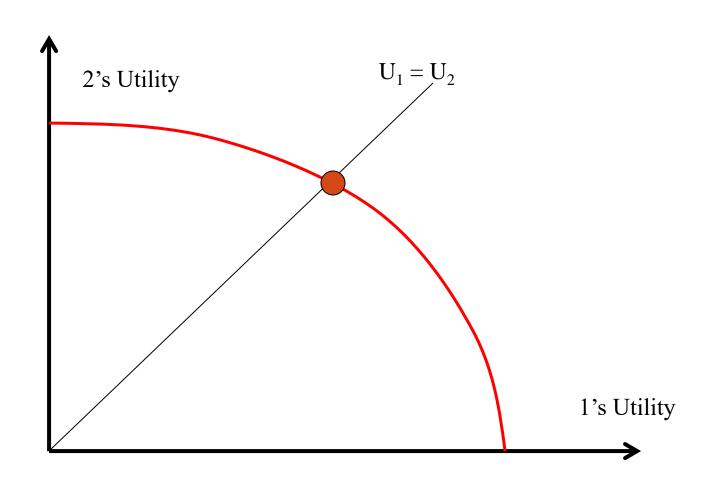
• Thus, society should redistribute from rich to poor if the marginal utility of the next euro is higher to the poor person than to the rich person.

• The *Rawlsian* social welfare function is:

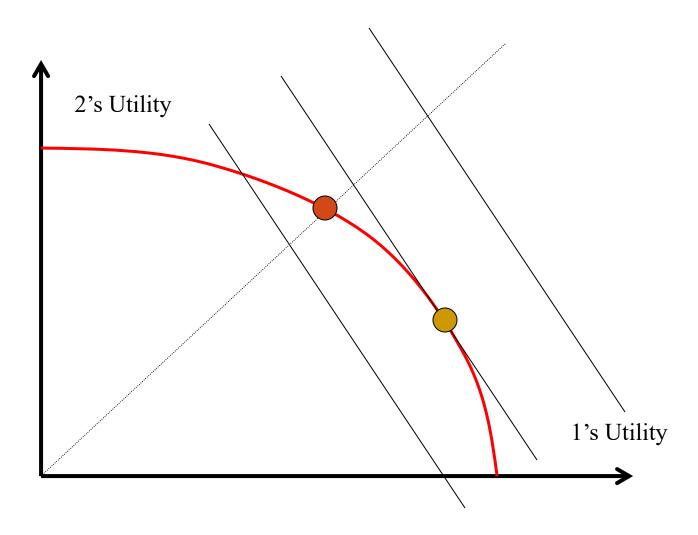
$$SWF = \min(U_1, U_2, \dots, U_N)$$

- Societal welfare is maximized by maximizing the wellbeing of the worst-off person in society.
- Generally suggests more redistribution than the utilitarian SWF.

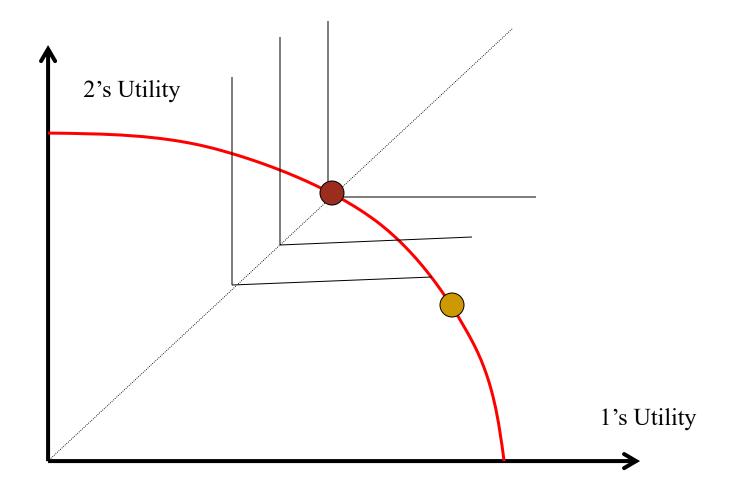
### Equity: equal shares



### Utilitarianism: Maximize U(1)+U(2)



### Rawls: Maximize $min\{U(1),U(2)\}$



### Recap of Theoretical Tools

- Utility maximization
- Labor supply example
- Efficiency
- Social welfare functions

### Welfare maximization in general equilibrium

- In the preceding analysis we analyzed the efficiency conditions in a partial equilibrium framework.
- In other words, we assumed that in the market there was one good, and one factor of production.
- Reality, however is different. We live in a world with many goods and many factors of production.
- In a general equilibrium framework, with many goods and factors of production the conditions for welfare maximization are presented below.

# Requirements for welfare maximization

• *Marginal rate of substitution* between every pair of goods must be the same for all consumers. In a pure market setting, this occurs when consumers equate the MRS's to the common market determined output ratio.

### Requirements for welfare maximization

• Marginal rate of technical substitution\_between every pair of inputs must be the same for all producers . in a pure market setting, this occurs when producers maximize profit by equating MRTS's to the common market determined input price ratio.

### Requirements for welfare maximization

• *Marginal rate of transformation* must be equal to the marginal rate of substitution in consumption for each pair of goods. In a pure market setting, this condition occurs when producers set marginal cost (MC) equal to the output price.

$$MRS_{XY}^{A} = MRS_{XY}^{B} = MRT_{XY}$$

$$MRT_{XY} = \frac{P_X}{P_Y} = \frac{MC_X}{MC_Y}$$

$$MRT_{XY} = \frac{P_X}{P_Y} = MRS_{XY}$$

### Competitive equilibrium

Maximisation of consumer welfare

$$MRS \frac{A}{XY} = \frac{P_X}{P_Y}$$

$$MRS \frac{B}{XY} = \frac{P_X}{P_Y}$$

implies

Pareto efficiency

Efficiency in exchange

 $MRS \frac{A}{XY} = MRS \frac{B}{XY}$ 

### Competitive equilibrium

Cost minimization

$$MRTS \frac{X}{LK} = \frac{w}{r}$$

$$MRTS \frac{Y}{LK} = \frac{w}{r}$$

**Implies** 

Pareto Efficiency

**Efficiency in production** 

$$MRTS_{KL}^{X} = MRTS_{KL}^{Y}$$

### Competitive equilibrium

**Profit maximisation** 

$$P_X = MC_X$$

$$P_Y = MC_Y$$

### Implies

#### Pareto Efficiency

**Overall efficiency** 

$$MRT_{XY} = \frac{MC_X}{MC_Y}$$
$$= \frac{P_X}{P_Y} = MRS_{XY}$$

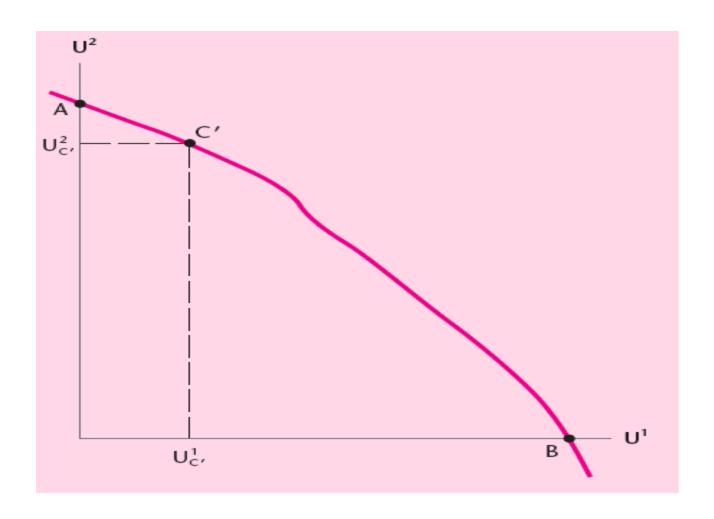
# The First Fundamental Theorem of Welfare Economics

# A competitive economy can achieve a Pareto optimal allocation of resources

Necessary conditions for a Pareto optimum:

- 1. Consumption: Marginal rates of substitution between X & Y must be equal for 1 & 2
- 2. **Production**: Marginal rates of technical substitution between K & L must be equal for production of X & Y
- 3. Consumption-production: Marginal rates of substitution between X & Y must also equal Marginal rates of transformation between X & Y

#### Every point on the Utility possibilities frontier is Pareto efficient



# Efficiency and equity

- In the above diagram the distribution of utility is very unequal.
- If society is interested in a more equal distribution of utility can this be achieved through the free markets mechanism?
- The answer is given by the second fundamental theorem of welfare economics

# The Second Fundamental Theorem of Welfare Economics

- Second welfare theorem says that a new Paretooptimal outcome can be achieved given existing resources, without government intervention.
- Any point on the UPF can be achieved through the functioning of decentralized markets, by an appropriate initial distribution of resources.

# **Review Questions**

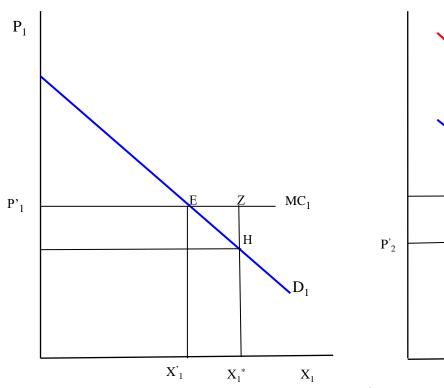
- What will happen in our two goods, two-person world if prices do not reflect true marginal benefits and all increment costs to society are not included in marginal costs?
- The market will still generate an equilibrium but it will not be Pareto optimal.
- True marginal benefits will not equal marginal costs or vice versa.
- When the market or price system gets the wrong signals we say that there has been a **market failure**.

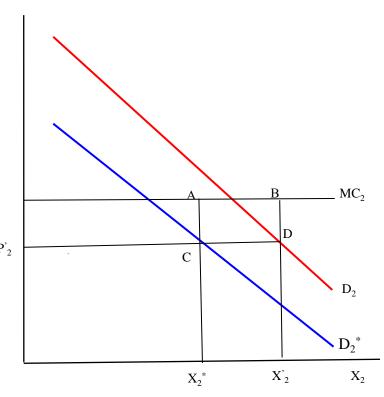
## Market failures

- Imperfect competition
- Public goods
- Externalities
- Incomplete markets
- Imperfect information
- Unemployment, inflation and other macroeconomic disturbances

- Basic question:
- What happens to Pareto optimality when one efficiency condition is violated? Should we continue sticking to the rest of the efficiency conditions?
- Generally, the answer is No.
- Consider an economy with three goods,  $X_1$ ,  $X_2$ ,  $X_3$ .  $X_1$  is controlled by the government, in  $X_2$  there is a distortion (i.e. P  $\neq$ MC), and  $X_3$ , is a composite good that includes all other goods, with price=MC.

- Suppose an economy with 3 goods: : Good X<sub>1</sub> is produced by a state company, good X<sub>2</sub> in which there is a distortion and the price is not equal to marginal cost, and a good X<sub>3</sub>, a composite good, which includes all other goods with a price equal to marginal cost.
- Good X<sub>3</sub> is the numeraire, which implies that its price is equal to 1.





Good  $X_2$  could be the urban transportation system buses , and  $X_1$  is metro. The price of  $X_2$  is  $P'_2$  and is lower than the marginal cost because the government thinks that this lower price encourages people to take the buses instead of their cars, ad thus reduces congestion and pollution. So, the buses will be used above their optimal level.

This is depicted in the above diagram where we assume constant marginal costs.

The demand curves  $D_1$   $\kappa\alpha\iota D_2$  reflect the demand that would exist when the price of the metro was  $P'_1=MC_1$  for  $X_1$   $\kappa\alpha\iota P'_2 < MC_2$  for  $X_2$  and the quantities demanded would be  $X'_1$   $\kappa\alpha\iota X'_2$ .

Suppose that the distortion between  $P_2$  and  $MC_2$  is fixed. We also assume that all other prices do not change and incomes remain unchanged.

- As we said earlier in good 2 there is overemployment of resources when in sector 1 price is equal to marginal cost. T is therefore, possible to improve social welfare by moving resources from sector 2 to other sectors.
- In our example  $X_1$  and  $X_2$  are substitutes and a reduction in the price of  $X_1$  to  $P_1^* < MC_1$ , will shift curve  $D_2$  to the left, to  $D_2^*$ . With the distortion between  $P_2$  and  $MC_2$  constant and constant marginal costs,  $P_2$  does not change and the demand for  $X_2$  is reduced from  $X_2'$  to  $X_2^*$ . Also in sector 1 the quantity demanded increases from  $X_1'$  to  $X_1^*$ .
- From these changes there is a change in welfare that can be measured as follows:

- In sector 1 the increase in the cost of resources because of the reduction in the price is X'<sub>1</sub>EZX\*<sub>1</sub>, while the increase in welfare, i.e. of the consumer surplus is X'<sub>1</sub>EHX\*<sub>1</sub>.
- Hence, we have a reduction in welfare in sector 1 equal to triangle EZH.
- In sector 2 we have the following changes. With the fall in demand the cost is reduced my the area X'<sub>2</sub>ABX\*<sub>2</sub>. Also the total benefit is reduced by the area X'<sub>2</sub>CDX\*<sub>2</sub>. Thus, the net benefit from sector 2 is the area ABDC. If ABDC larger than EZH, the reduction in price leads to an increase in social welfare. The second best price is the one that maximizes the difference between ABDC και EZH.

#### A mathematical treatment

The social welfare function is

$$U=U(X_1, X_2, X_3)$$

Differentiation yields

$$dU = \frac{\partial U_1}{\partial X_1} dX_1 + \frac{\partial U_2}{\partial X_2} dX_2 + \frac{\partial U_3}{\partial X_3} dX_3$$

$$dU = MU_1 dX_1 + MU_2 dX_2 + MU_3 dX_3$$

•  $X_3$  is the numeraire, and its price is set equal to one

$$q_3 = 1$$

#### A mathematical treatment

• With q being the consumer price of the good, we have

$$\frac{MU_1}{MU_2} = \frac{q_1}{q_2}$$

• Dividing by we  $U_3$  get

$$dW = \frac{dU}{U_3} = q_1 dX_1 + q_2 dX_2 + q_3 dX_3$$

#### A mathematical treatment

• Suppose now that in there is a fixed per unit distortion of  $d_2$  on good  $X_2$ , and as a result

$$q_2 = p_2 + d_2$$

- Where  $p_2$  is the producer price without distortion and is equal to marginal cost  $MC_2$
- Since the government controls  $X_1$ , it can give a subsidy or impose a tax  $t_1$ , so that

$$q_1 = p_1 + t_1$$

#### A mathematical treatment

With

$$q_3 = p_3$$

We have

$$dW = \sum_{1}^{3} p_{i} dX_{i} + d_{2} dX_{2} + t_{1} dX_{1}$$

On the production side we have the transformation function

$$F(X_1, X_2, X_3) = 0$$

Total differentiation yields

$$\frac{\partial F_1}{\partial X_1} dX_1 + \frac{\partial F_2}{\partial X_2} dX_2 + \frac{\partial F_3}{\partial X_3} dX_3 = 0$$

#### A mathematical treatment

• With  $\partial F/\partial X = p$  = marginal cost, we get

$$p_1 dX_1 + p_2 dX_2 + p_3 dX_3 = 0$$

and

$$dW = t_1 dX_1 + d_2 dX_2$$

• Since we have assumed that is  $d_2$  fixed, there will be a change in welfare, if government changes  $t_1$ .

$$dW = \left[t_1\left(\frac{\partial X_1}{\partial t_1}\right) + d_2\left(\frac{\partial X_2}{\partial t_1}\right)\right]dt_1$$

#### A mathematical treatment

• Maximization of welfare requires that  $dW/dt_1 = 0$ . Thus

$$t^* = -d_2 \frac{\frac{\partial X_2}{\partial t_1}}{\frac{\partial X_1}{\partial t_1}}$$

$$dp = 0$$
  $\cot dq_1 = dt_1$ 

In the case of constant marginal costs,

$$t^* = -d_2 \frac{\frac{\partial X_2}{\partial q_1}}{\frac{\partial X_1}{\partial q_1}}$$

#### A mathematical treatment

- With  $d_2>0$ , which implies that  $q_2>p_2$  in the distorted sector, then  $t^*>0$ , if  $(\partial X_2/\partial q_i)<0$ , that is  $X_1$  and  $X_2$  are substitutes.
- On the contrary  $t^*<0$  when  $(\partial X_2/\partial q_l)>0$ , that is  $X_1$  and  $X_2$  are complements.
- The above results change when  $d_2 < 0$ .
- The preceding analysis can be generalised for N goods.

#### A mathematical treatment

- When in the distorted sector P<MC, then the theory of second best suggests that the price in the controlled sector is higher than the MC, if the goods are complementary, and smaller than MC if the goods are substitutes.
- When in the distorted sector P>MC, then the theory of second best suggests that the price in the controlled sector is smaller than the MC if the goods are complementary, and greater than MC if the goods are substitutes.
- If the two goods are not related with each other then the price must be equal to MC.