Measuring welfare changes

Compensating variation, Equivalent variation, Consumer's Surplus

This presentation, to a large extent, has been borrowed from Varian's Intermediate Microeconomics 1

1

Monetary Measures of Gains-to-Trade

- You can buy as much gasoline as you wish at €1 per litre once you enter the gasoline market.
- Q: What is the most you would pay to enter the market?

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Monetary Measures of Gains-to-Trade

- A: You would pay up to the euro value of the gains-to-trade you would enjoy once in the market.
- How can such gains-to-trade be measured?

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Monetary Measures of Gains-to-Trade

- Three such measures are:
  - Consumer's Surplus
  - Equivalent Variation, and
  - Compensating Variation.
- Only in one special circumstance do these three measures coincide.

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€ Equivalent Utility Gains

- Suppose gasoline can be bought only in lumps of one litre.
- Use r_1 to denote the most a single consumer would pay for a 1st litre -- call this her ***reservation price*** for the 1st litre.
- r_1 is the euro equivalent of the marginal utility of the 1st litre.

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€ Equivalent Utility Gains

- Now that she has one litre, use r_2 to denote the most she would pay for a 2nd litre -- this is her reservation price for the 2nd litre.
- r_2 is the euro equivalent of the marginal utility of the 2nd litre.

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€ Equivalent Utility Gains

- Generally, if she already has n-1 litres of gasoline then r_n denotes the most she will pay for an nth litre.
- r_n is the euro equivalent of the marginal utility of the nth litre.

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€ Equivalent Utility Gains

- $r_1 + \ldots + r_n$ will therefore be the euro equivalent of the total change to utility from acquiring n litres of gasoline at a price of €0.
- So $r_1 + \ldots + r_n - p_L n$ will be the euro equivalent of the total change to utility from acquiring n litres of gasoline at a price of €p_L each.

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€ Equivalent Utility Gains

- A plot of $r_1, r_2, \ldots, r_n, \ldots$ against n is a reservation-price curve. This is not quite the same as the consumer's demand curve for gasoline.

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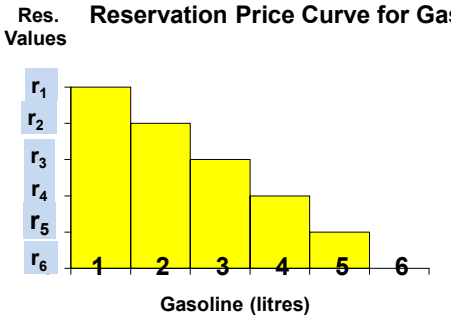
€ Equivalent Utility Gains



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€ Equivalent Utility Gains

- What is the monetary value of our consumer's gain-to-trading in the gasoline market at a price of €p_L?

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€ Equivalent Utility Gains

- The euro equivalent net utility gain for the 1st litre is €$(r_1 - p_L)$
- and is €$(r_2 - p_L)$ for the 2nd litre,
- and so on, so the euro value of the gain-to-trade is

  €$(r_1 - p_L)$ + €$(r_2 - p_L)$ + …

  for as long as $r_n - p_L > 0$.

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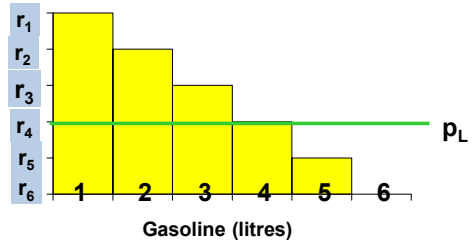
€ Equivalent Utility Gains

Res. Values

Reservation Price Curve for Gasoline



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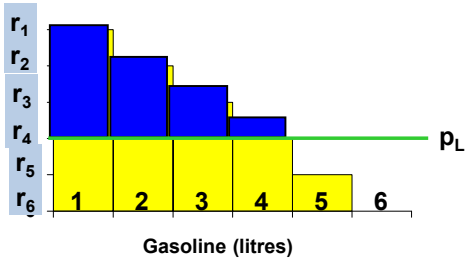
€ Equivalent Utility Gains

Res. Values

Reservation Price Curve for Gasoline



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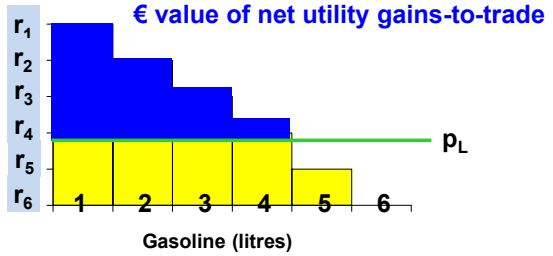
€ Equivalent Utility Gains

Res. Values

Reservation Price Curve for Gasoline

€ value of net utility gains-to-trade



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€ Equivalent Utility Gains

- Now suppose that gasoline is sold in half-litre units.
- $r_1, r_2, \ldots, r_n, \ldots$ denote the consumer's reservation prices for successive half-litres of gasoline.
- Our consumer's new reservation price curve is

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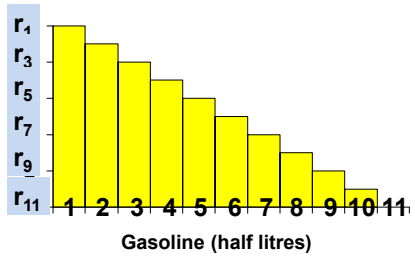
€ Equivalent Utility Gains

Res. Values

Reservation Price Curve for Gasoline



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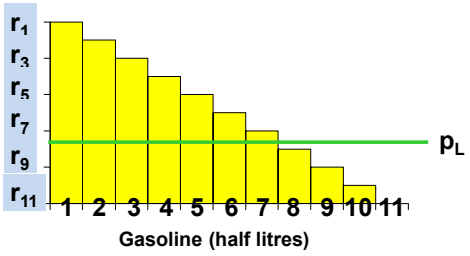
€ Equivalent Utility Gains

Res. Values

Reservation Price Curve for Gasoline



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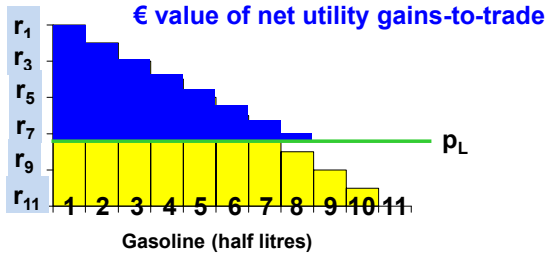
€ Equivalent Utility Gains

Res. Values

Reservation Price Curve for Gasoline



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€ Equivalent Utility Gains

- And if gasoline is available in one-quarter litre units ...

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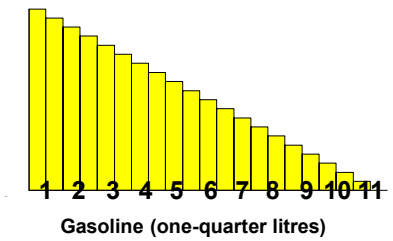
€ Equivalent Utility Gains

Res. Values

Reservation Price Curve for Gasoline



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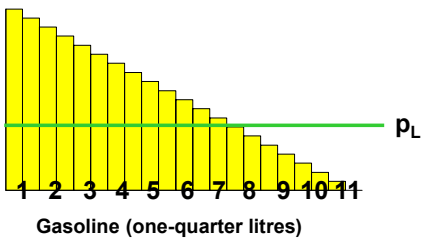
€ Equivalent Utility Gains

Res. Values

Reservation Price Curve for Gasoline



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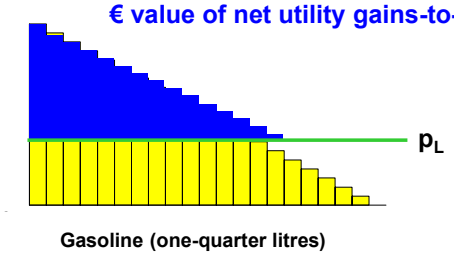
€ Equivalent Utility Gains

Res. Values

Reservation Price Curve for Gasoline

€ value of net utility gains-to-trade



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€ Equivalent Utility Gains

- Finally, if gasoline can be purchased in any quantity then ...

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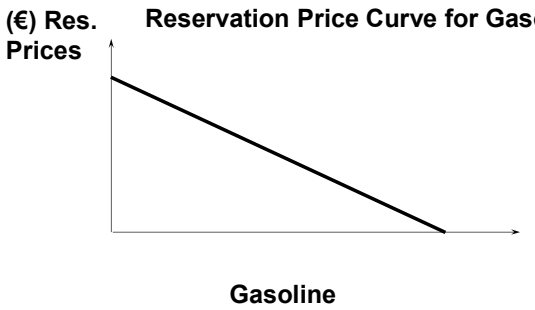
€ Equivalent Utility Gains



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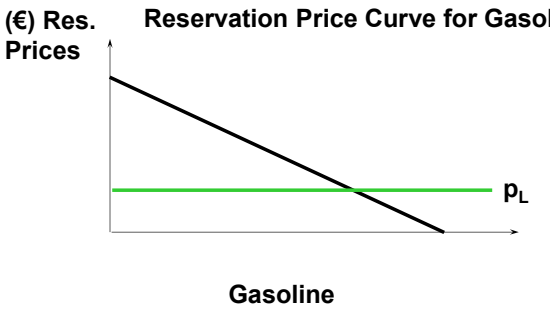
€ Equivalent Utility Gains



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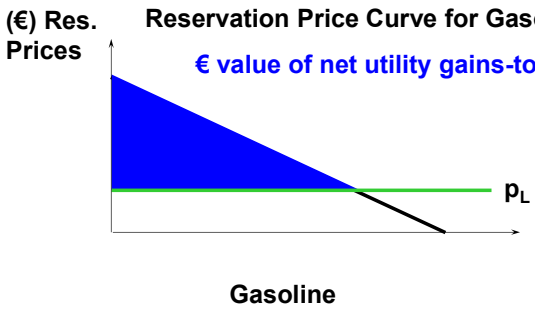
€ Equivalent Utility Gains



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€ Equivalent Utility Gains

- Unfortunately, estimating a consumer's reservation-price curve is difficult,
- so, as an approximation, the reservation-price curve is replaced with the consumer's ordinary demand curve.

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Consumer's Surplus

- A consumer's reservation-price curve is not quite the same as her ordinary demand curve. Why not?
- A reservation-price curve describes ***sequentially*** the values of successive single units of a commodity.
- An ordinary demand curve describes the most that would be paid for q units of a commodity purchased ***simultaneously.***

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Consumer's Surplus

- Approximating the net utility gain area under the reservation-price curve by the corresponding area under the ordinary demand curve gives the ***Consumer's Surplus measure of net utility gain***.

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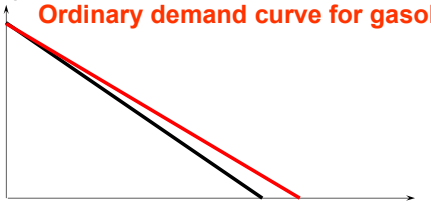
Consumer's Surplus

(€) Reservation price curve for gasoline
Ordinary demand curve for gasoline



Gasoline

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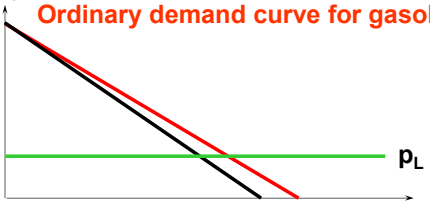
Consumer's Surplus

(€) Reservation price curve for gasoline
Ordinary demand curve for gasoline



Gasoline

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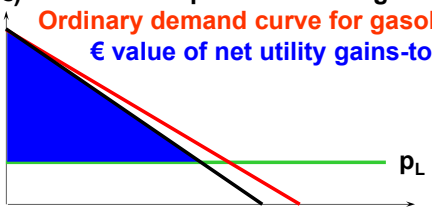
Consumer's Surplus

(€) Reservation price curve for gasoline
Ordinary demand curve for gasoline
€ value of net utility gains-to-trade



Gasoline

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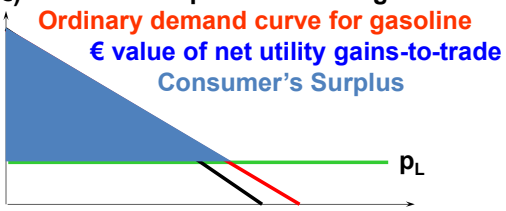
Consumer's Surplus

(€) Reservation price curve for gasoline
Ordinary demand curve for gasoline
€ value of net utility gains-to-trade
Consumer's Surplus



Gasoline

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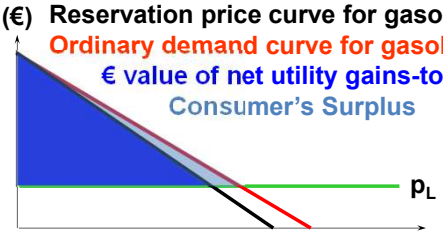
Consumer's Surplus

(€) Reservation price curve for gasoline
Ordinary demand curve for gasoline
€ value of net utility gains-to-trade
Consumer's Surplus



Gasoline

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Consumer's Surplus

- The difference between the consumer's reservation-price and ordinary demand curves is due to income effects.
- But, if the consumer's utility function is quasilinear in income then there are no income effects and Consumer's Surplus is an exact € measure of gains-to-trade.

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Consumer's Surplus

The consumer's utility function is quasilinear in x_2.

$$U(x_1,x_2) = v(x_1) + x_2$$

Take $p_2 = 1$. Then the consumer's choice problem is to maximize

$$U(x_1,x_2) = v(x_1) + x_2$$

subject to

$$p_1x_1 + x_2 = m.$$

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Consumer's Surplus

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$$p_1x_1 + x_2 = m.$$

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Consumer's Surplus

That is, choose x_1 to maximize

$$v(x_1) + m - p_1x_1.$$

The first-order condition is

$$v'(x_1) - p_1 = 0$$

That is, $p_1 = v'(x_1)$.

This is the equation of the consumer's ordinary demand for commodity 1.

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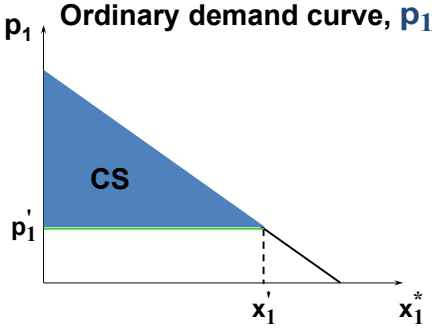
Consumer's Surplus

Ordinary demand curve, $p_1 = v'(x_1)$



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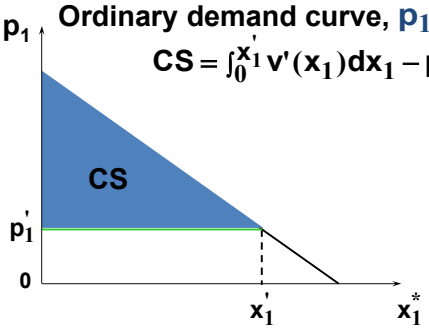
Consumer's Surplus

Ordinary demand curve, $p_1 = v'(x_1)$

$$CS = \int_0^{x_1'} v'(x_1)dx_1 - p_1'x_1'$$



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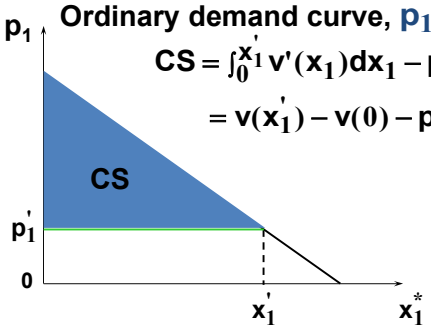
Consumer's Surplus

Ordinary demand curve, $p_1 = v'(x_1)$

$$CS = \int_0^{x_1'} v'(x_1)dx_1 - p_1'x_1'$$

$$= v(x_1') - v(0) - p_1'x_1'$$



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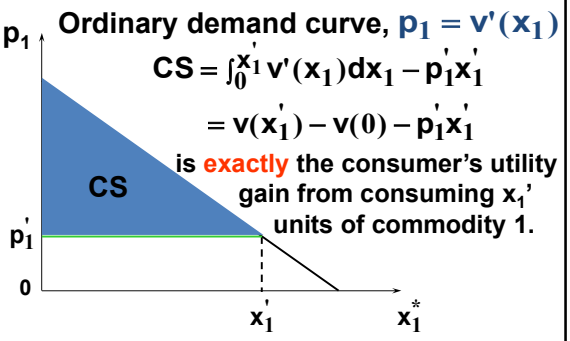
Consumer's Surplus



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Consumer's Surplus

- Consumer's Surplus is an exact euro measure of utility gained from consuming commodity 1 when the consumer's utility function is quasilinear in commodity 2.
- Otherwise Consumer's Surplus is an approximation.

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Consumer's Surplus

- The change to a consumer's total utility due to a change to p_1 is approximately the change in her Consumer's Surplus.

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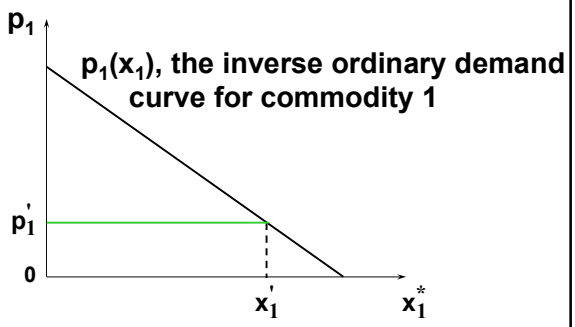
Consumer's Surplus



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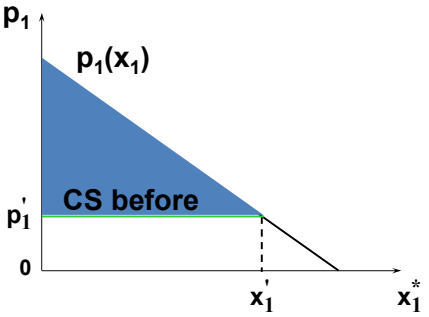
Consumer's Surplus



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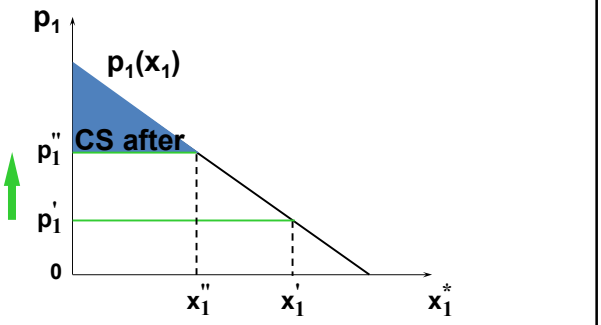
Consumer's Surplus



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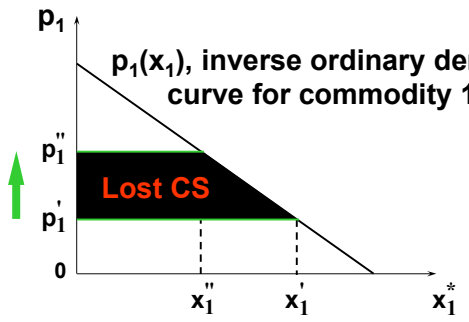
Consumer's Surplus



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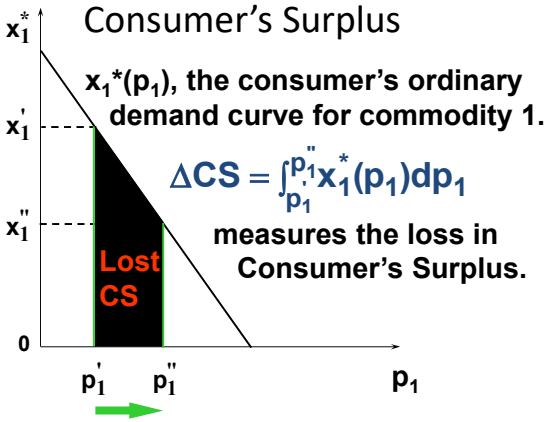
Consumer's Surplus



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Compensating Variation and Equivalent Variation

- Two additional euro measures of the total utility change caused by a price change are ***Compensating Variation*** and ***Equivalent Variation***.

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Compensating Variation

- p_1 rises.
- Q: What is the least extra income that, at the **new prices**, just restores the consumer's original utility level?

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Compensating Variation

- p_1 rises.
- Q: What is the least extra income that, at the **new prices**, just restores the consumer's original utility level?
- A: The Compensating Variation.

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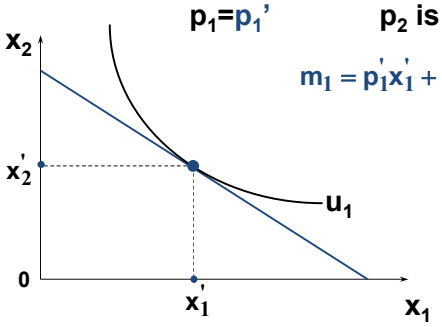
Compensating Variation

$p_1 = p_1'$ p_2 is fixed.

$m_1 = p_1'x_1' + p_2x_2'$



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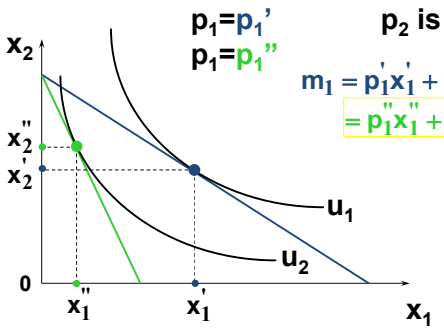
Compensating Variation



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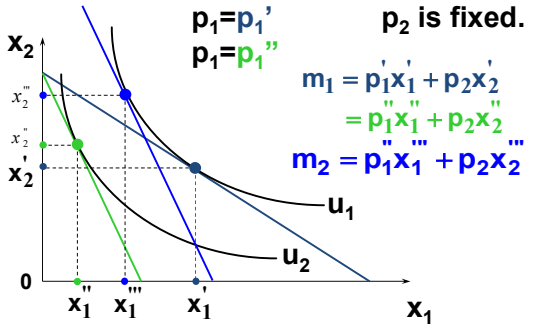
Compensating Variation



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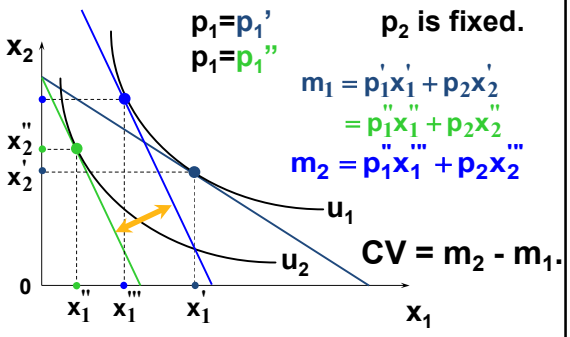
Compensating Variation



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Equivalent Variation

- p_1 rises.
- Q: What is the least extra income that, at the **original prices**, just restores the consumer's original utility level?
- A: The Equivalent Variation.

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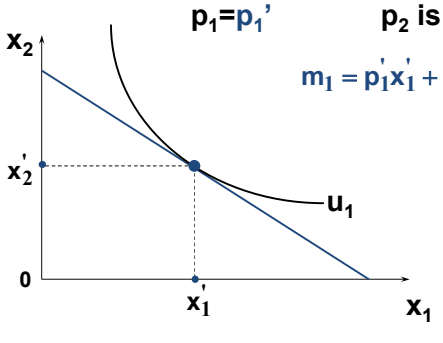
Equivalent Variation



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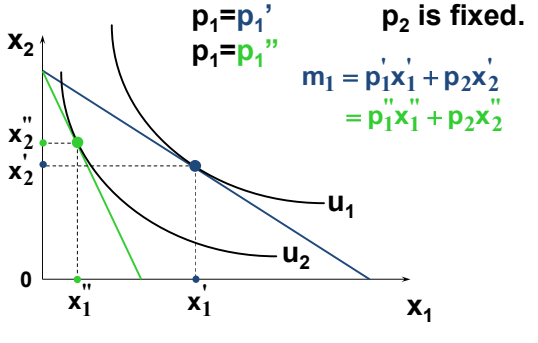
Equivalent Variation



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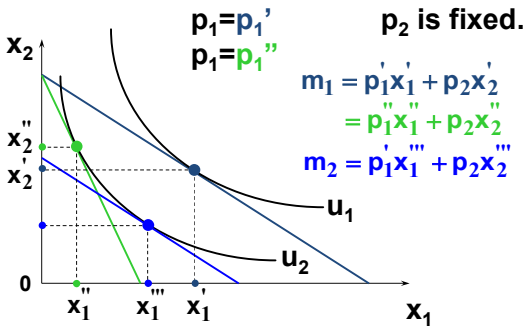
Equivalent Variation



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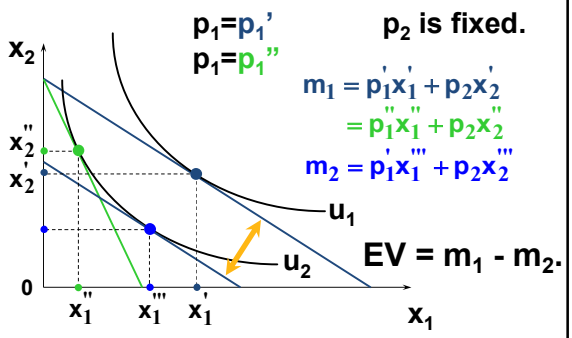
Equivalent Variation



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Consumer's Surplus, Compensating Variation and Equivalent Variation

- Relationship 1: When the consumer's preferences are quasilinear, all three measures are the same.

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Consumer's Surplus, Compensating Variation and Equivalent Variation

- Consider first the change in Consumer's Surplus when p_1 rises from p_1' to p_1''.

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Consumer's Surplus, Compensating Variation and Equivalent Variation

If $U(x_1,x_2) = v(x_1) + x_2$ then

$$CS(p_1') = v(x_1') - v(0) - p_1'x_1'$$

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Consumer's Surplus, Compensating Variation and Equivalent Variation

If $U(x_1,x_2) = v(x_1) + x_2$ then

$$CS(p_1') = v(x_1') - v(0) - p_1'x_1'$$

and so the change in CS when p_1 rises from p_1' to p_1'' is

$$\Delta CS = CS(p_1') - CS(p_1'')$$

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Consumer's Surplus, Compensating Variation and Equivalent Variation

If $U(x_1,x_2) = v(x_1) + x_2$ then

$$CS(p_1') = v(x_1') - v(0) - p_1'x_1'$$

and so the change in CS when p_1 rises from p_1' to p_1'' is

$$\Delta CS = CS(p_1') - CS(p_1'')$$
$$= v(x_1') - v(0) - p_1'x_1' - \left[v(x_1'') - v(0) - p_1''x_1''\right]$$

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Consumer's Surplus, Compensating Variation and Equivalent Variation

If $U(x_1,x_2) = v(x_1) + x_2$ then

$$CS(p_1') = v(x_1') - v(0) - p_1'x_1'$$

and so the change in CS when p_1 rises from p_1' to p_1'' is

$$\Delta CS = CS(p_1') - CS(p_1'')$$
$$= v(x_1') - v(0) - p_1'x_1' - \left[v(x_1'') - v(0) - p_1''x_1''\right]$$
$$= v(x_1') - v(x_1'') - (p_1'x_1' - p_1''x_1'').$$

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Consumer's Surplus, Compensating Variation and Equivalent Variation

- Now consider the change in CV when p_1 rises from p_1' to p_1''.
- The consumer's utility for given p_1 is

$$v(x_1^*(p_1)) + m - p_1x_1^*(p_1)$$

and CV is the extra income which, at the new prices, makes the consumer's utility the same as at the old prices. That is, ...

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Consumer's Surplus, Compensating Variation and Equivalent Variation

$$v(x_1') + m - p_1'x_1'$$
$$= v(x_1'') + m + CV - p_1''x_1''.$$

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Consumer's Surplus, Compensating Variation and Equivalent Variation

$$v(x_1') + m - p_1'x_1'$$
$$= v(x_1'') + m + CV - p_1''x_1''.$$

So

$$CV = v(x_1') - v(x_1'') - (p_1'x_1' - p_1''x_1'')$$
$$= \Delta CS.$$

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Consumer's Surplus, Compensating Variation and Equivalent Variation

- Now consider the change in EV when p_1 rises from p_1' to p_1''.
- The consumer's utility for given p_1 is

$$v(x_1^*(p_1)) + m - p_1x_1^*(p_1)$$

and EV is the extra income which, at the old prices, makes the consumer's utility the same as at the new prices. That is, ...

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Consumer's Surplus, Compensating Variation and Equivalent Variation

$$v(x_1') + m - p_1'x_1' = v(x_1'') + m + EV - p_1''x_1''.$$

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Consumer's Surplus, Compensating Variation and Equivalent Variation

$$v(x_1') + m - p_1'x_1' = v(x_1'') + m + EV - p_1''x_1''.$$

That is,

$$EV = v(x_1') - v(x_1'') - (p_1'x_1' - p_1''x_1'') = \Delta CS.$$

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Consumer's Surplus, Compensating Variation and Equivalent Variation

So when the consumer has quasilinear utility,

$$CV = EV = \Delta CS.$$

But, otherwise, we have:

Relationship 2: In size, $EV < \Delta CS < CV$.

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Producer's Surplus

- Changes in a firm's welfare can be measured in euros much as for a consumer.

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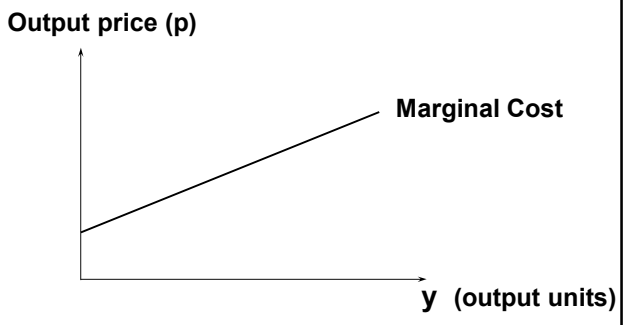
Producer's Surplus



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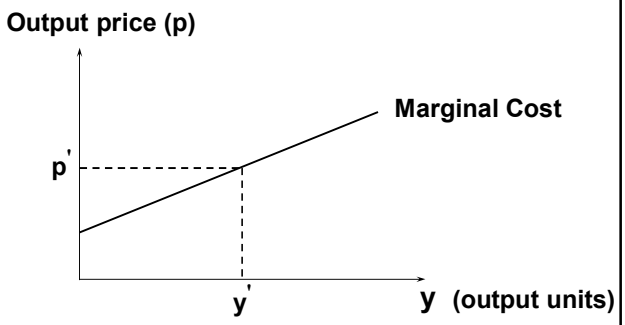
Producer's Surplus



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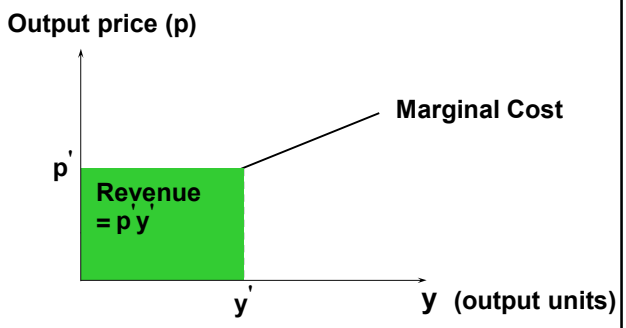
Producer's Surplus



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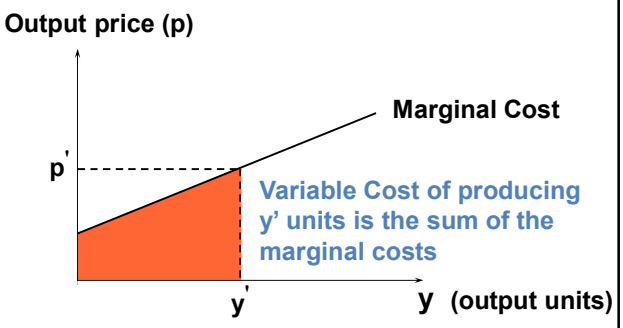
Producer's Surplus



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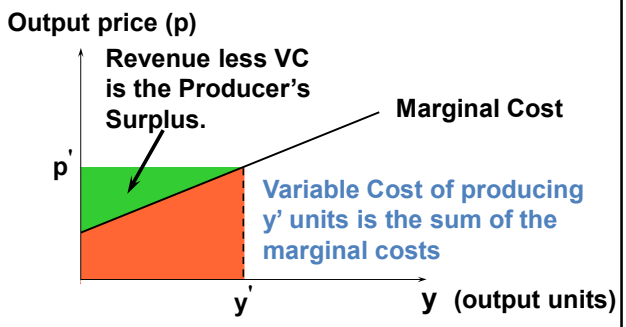
Producer's Surplus



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Benefit-Cost Analysis

- Can we measure in money units the net gain, or loss, caused by a market intervention; *e.g.,* the imposition or the removal of a market regulation?
- Yes, by using measures such as the Consumer's Surplus and the Producer's Surplus.

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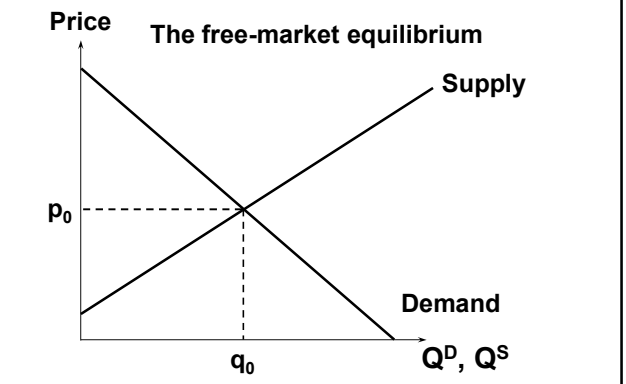
Benefit-Cost Analysis



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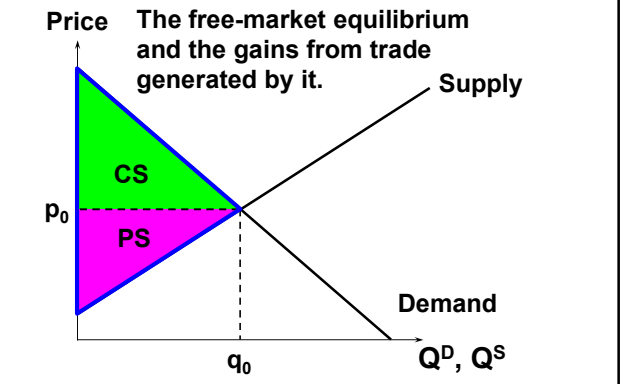
Benefit-Cost Analysis



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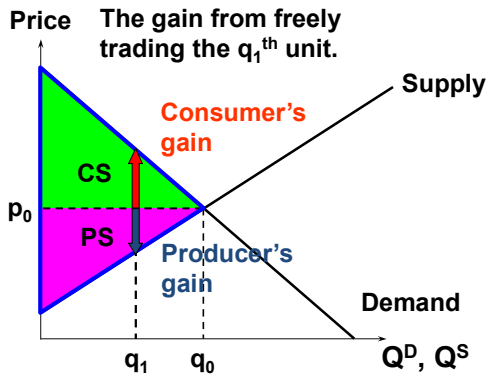
Benefit-Cost Analysis



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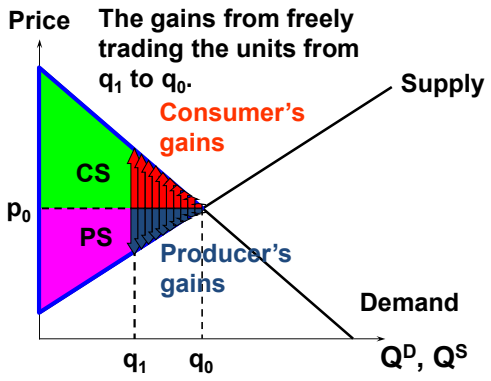
Benefit-Cost Analysis



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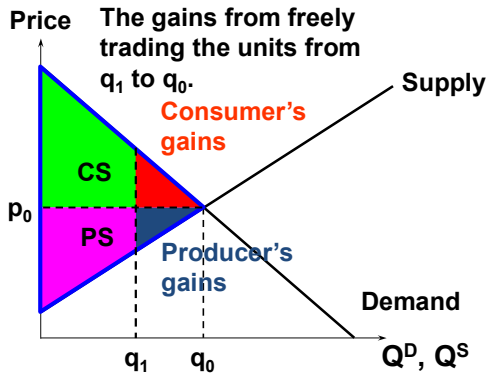
Benefit-Cost Analysis



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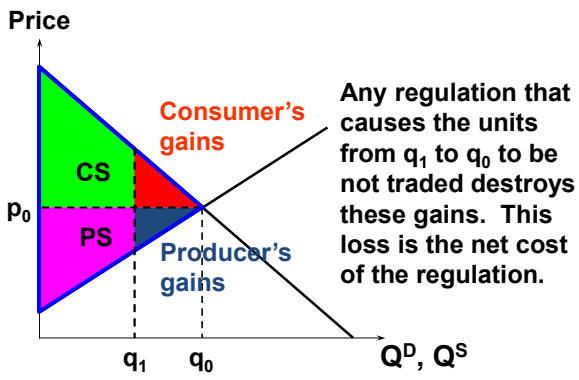
Benefit-Cost Analysis



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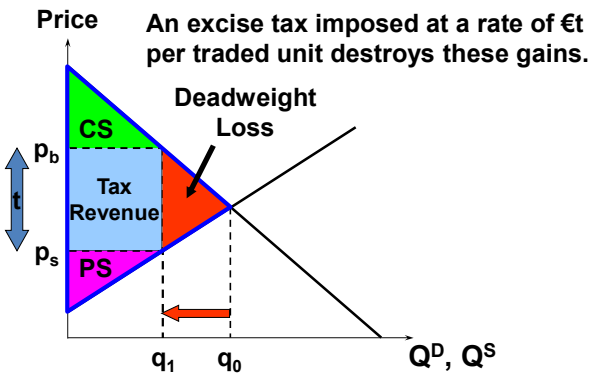
Benefit-Cost Analysis



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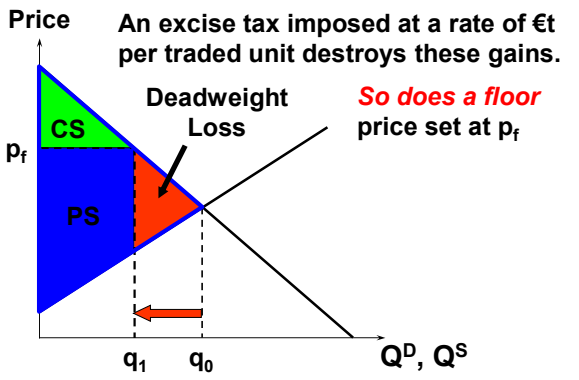
Benefit-Cost Analysis



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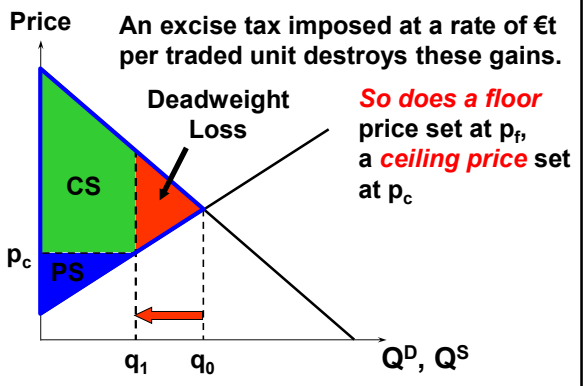

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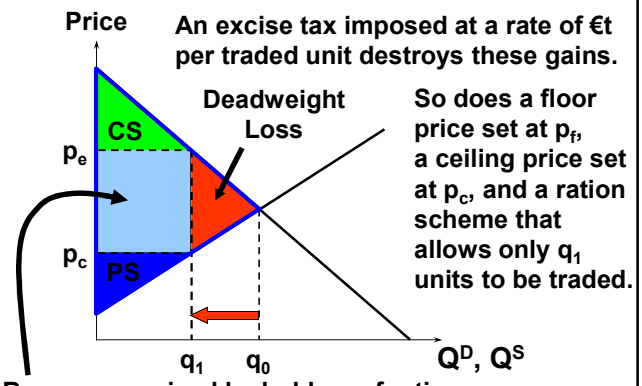

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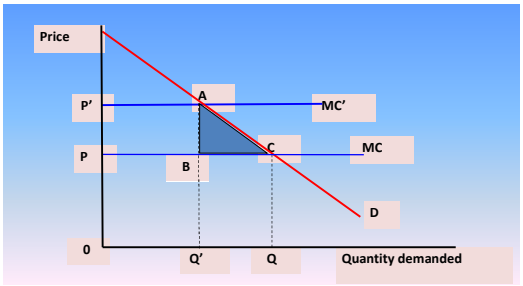
Measuring deadweight loss



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Measuring deadweight loss

$$DWL = \frac{1}{2}(\Delta P)x(\Delta Q)$$

$$e = \frac{\Delta Q}{\Delta P}\frac{P}{Q}$$

$$EB = \frac{1}{2}(\Delta P)x(e\Delta P\frac{Q}{P})$$

$$EB = \frac{1}{2}e\frac{Q}{P}t^2$$

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