Measuring welfare changes

Compensating variation, Equivalent variation, Consumer's Surplus

Monetary Measures of Gains-to-Trade

- You can buy as much gasoline as you wish at €1 per litre once you enter the gasoline market.
- Q: What is the most you would pay to enter the market?

Monetary Measures of Gains-to-Trade

- A: You would pay up to the euro value of the gains-to-trade you would enjoy once in the market.
- How can such gains-to-trade be measured?

Monetary Measures of Gains-to-Trade

- Three such measures are:
  - Consumer's Surplus
  - Equivalent Variation, and
  - Compensating Variation.
- Only in one special circumstance do these three measures coincide.

€ Equivalent Utility Gains

- Suppose gasoline can be bought only in lumps of one litre.
- Use r_1 to denote the most a single consumer would pay for a 1st litre -- call this her ***reservation price*** for the 1st litre.
- r_1 is the euro equivalent of the marginal utility of the 1st litre.

€ Equivalent Utility Gains

- Now that she has one litre, use r_2 to denote the most she would pay for a 2nd litre -- this is her reservation price for the 2nd litre.
- r_2 is the euro equivalent of the marginal utility of the 2nd litre.

€ Equivalent Utility Gains

- Generally, if she already has n-1 litres of gasoline then r_n denotes the most she will pay for an nth litre.
- r_n is the euro equivalent of the marginal utility of the nth litre.

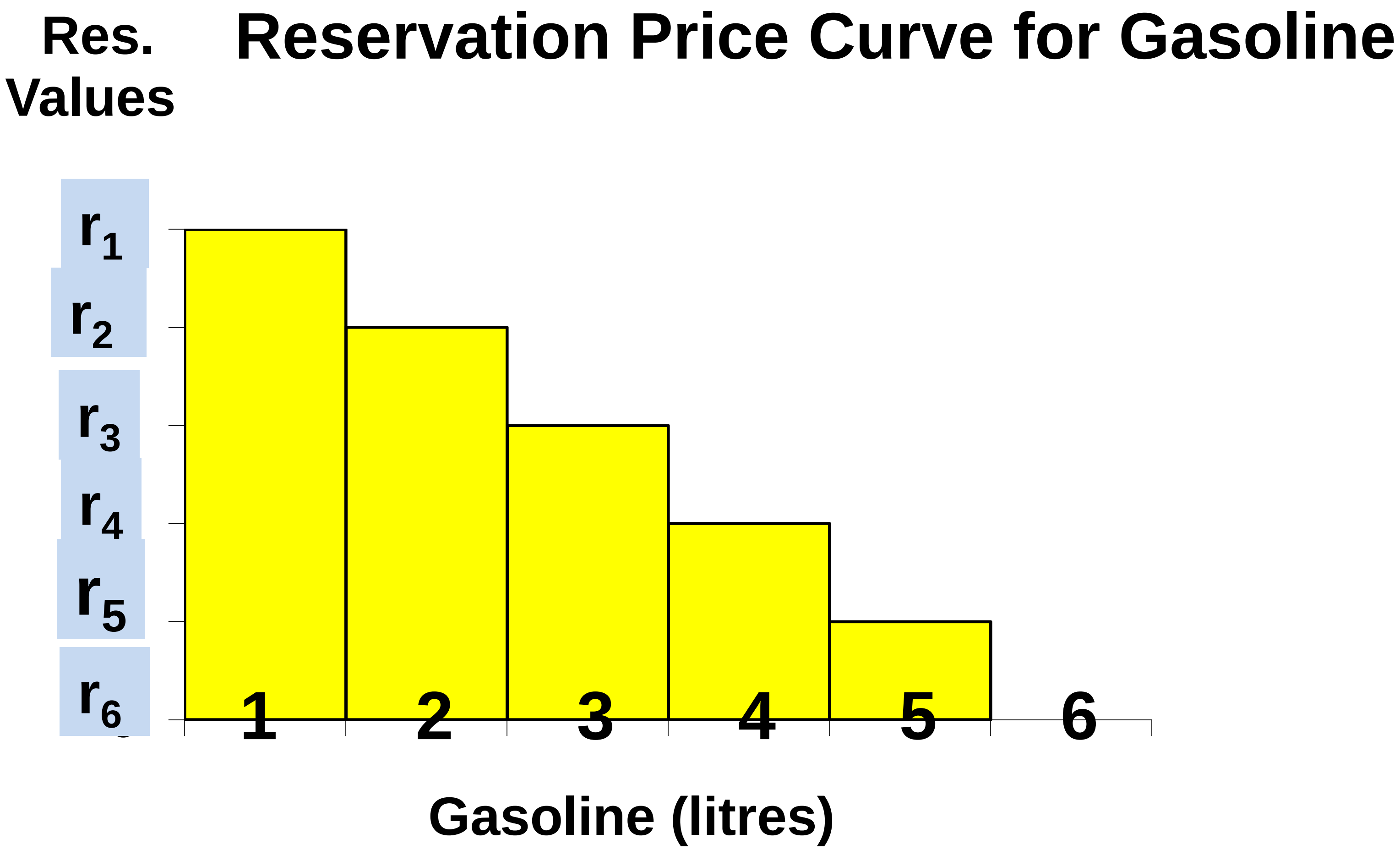
€ Equivalent Utility Gains

- $r_1 + \dots + r_n$ will therefore be the euro equivalent of the total change to utility from acquiring n litres of gasoline at a price of €0.
- So $r_1 + \dots + r_n - p_L n$ will be the euro equivalent of the total change to utility from acquiring n litres of gasoline at a price of €p_L each.

€ Equivalent Utility Gains

- A plot of r_1, r_2, … , r_n, … against n is a reservation-price curve. This is not quite the same as the consumer's demand curve for gasoline.

€ Equivalent Utility Gains



€ Equivalent Utility Gains

- What is the monetary value of our consumer's gain-to-trading in the gasoline market at a price of €p_L?

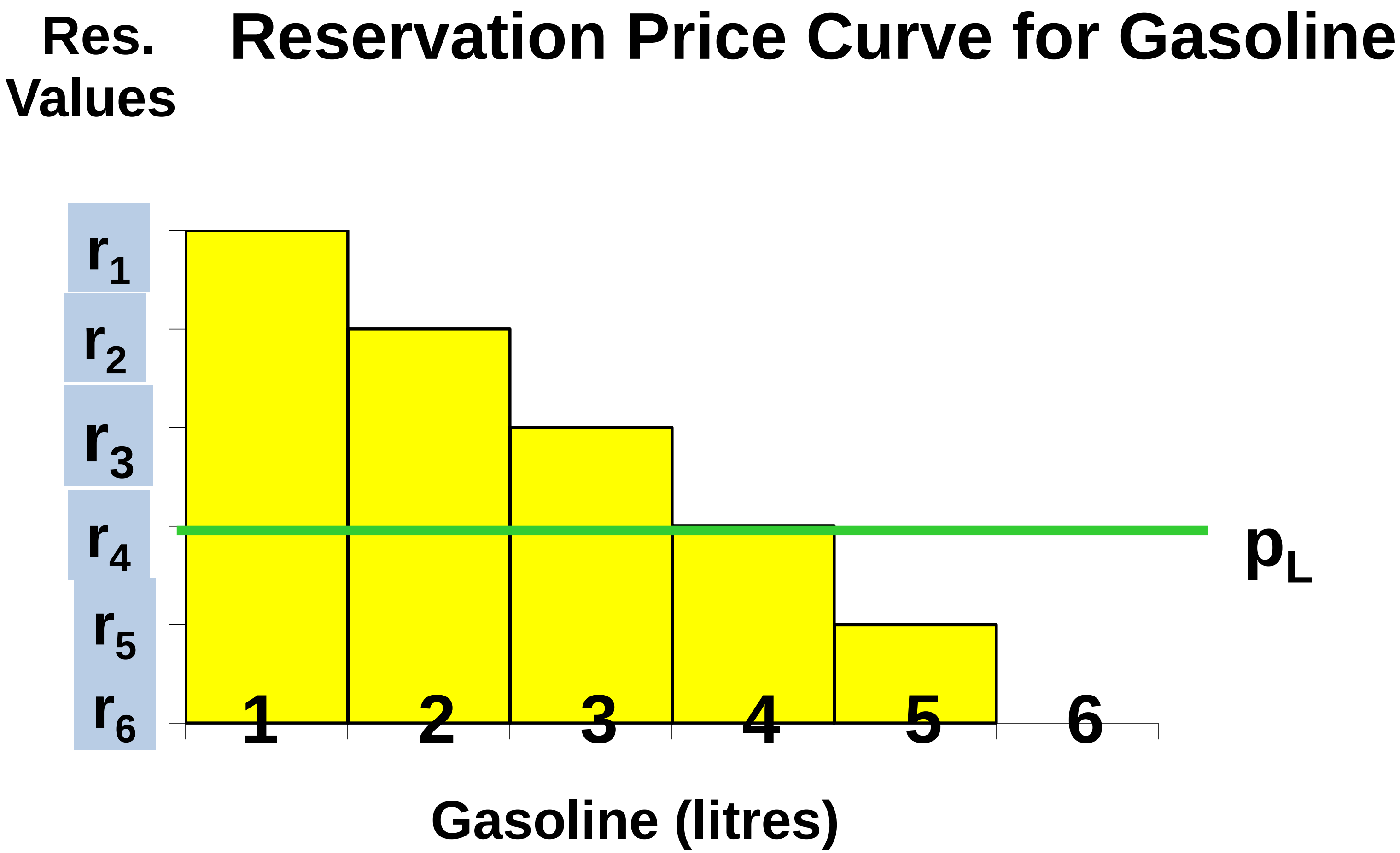
€ Equivalent Utility Gains

- The euro equivalent net utility gain for the 1st litre is €$(r_1 - p_L)$
- and is €$(r_2 - p_L)$ for the 2nd litre,
- and so on, so the euro value of the gain-to-trade is

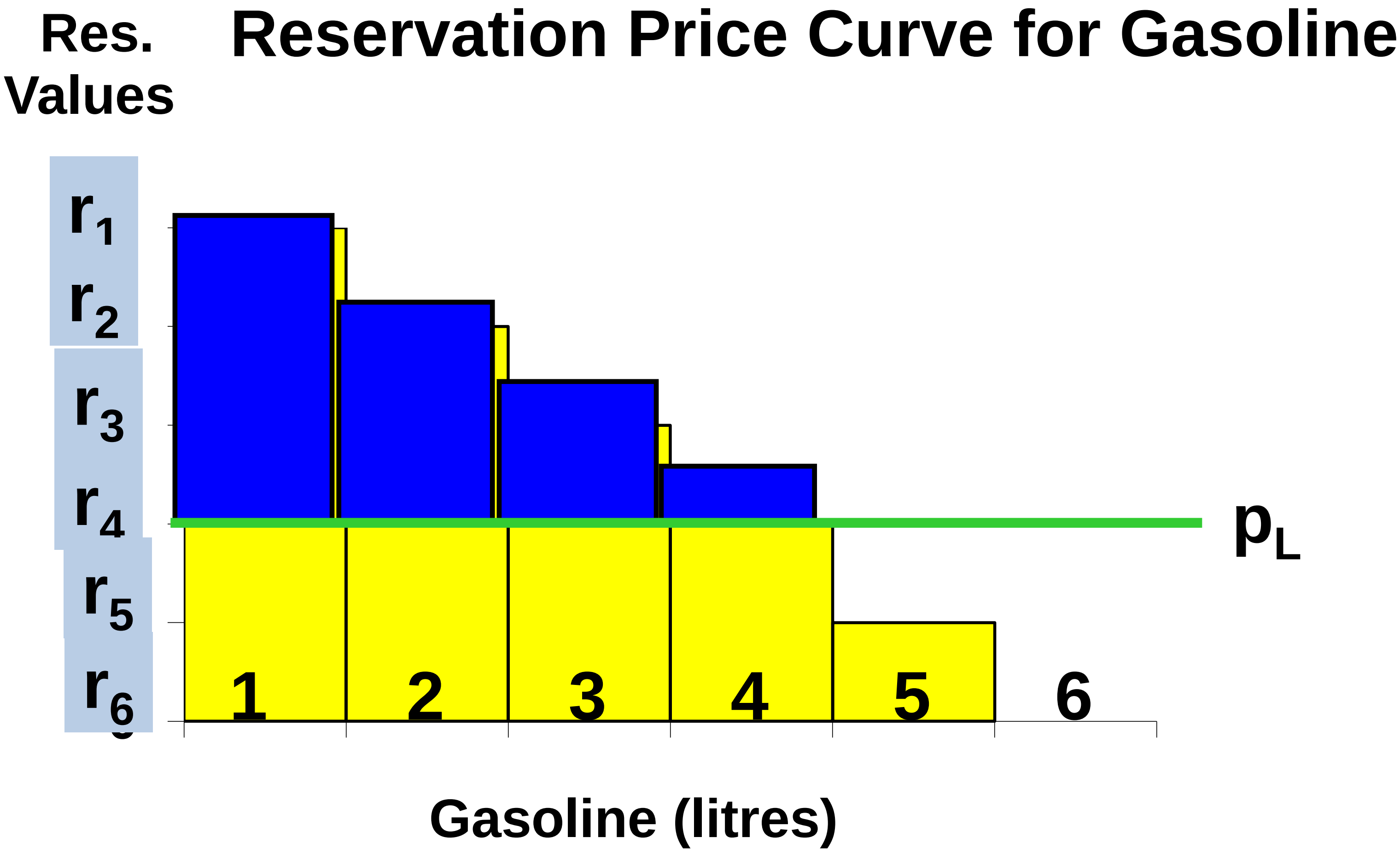
  €$(r_1 - p_L)$ + €$(r_2 - p_L)$ + ...

  for as long as $r_n - p_L > 0$.

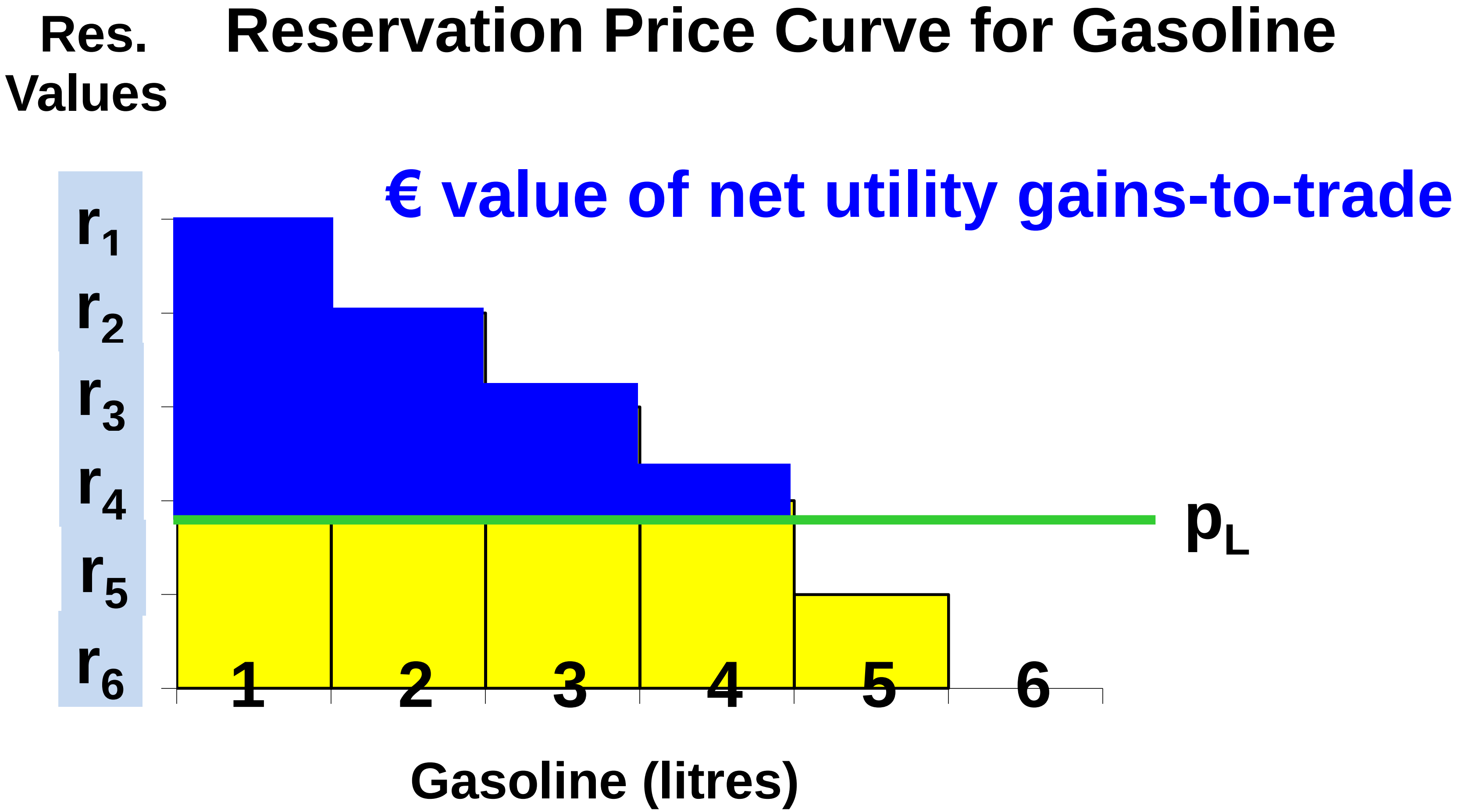
€ Equivalent Utility Gains



€ Equivalent Utility Gains



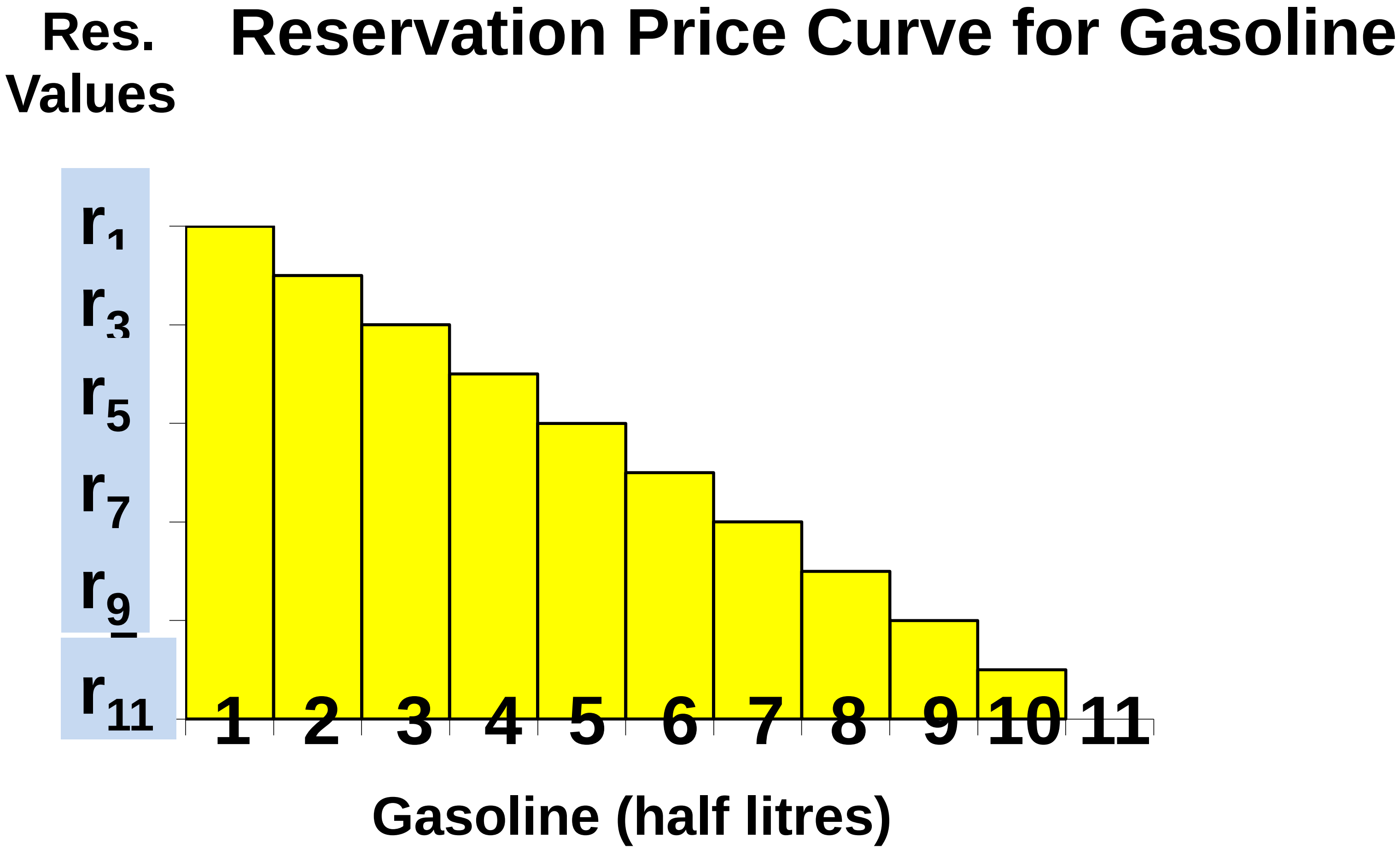
€ Equivalent Utility Gains



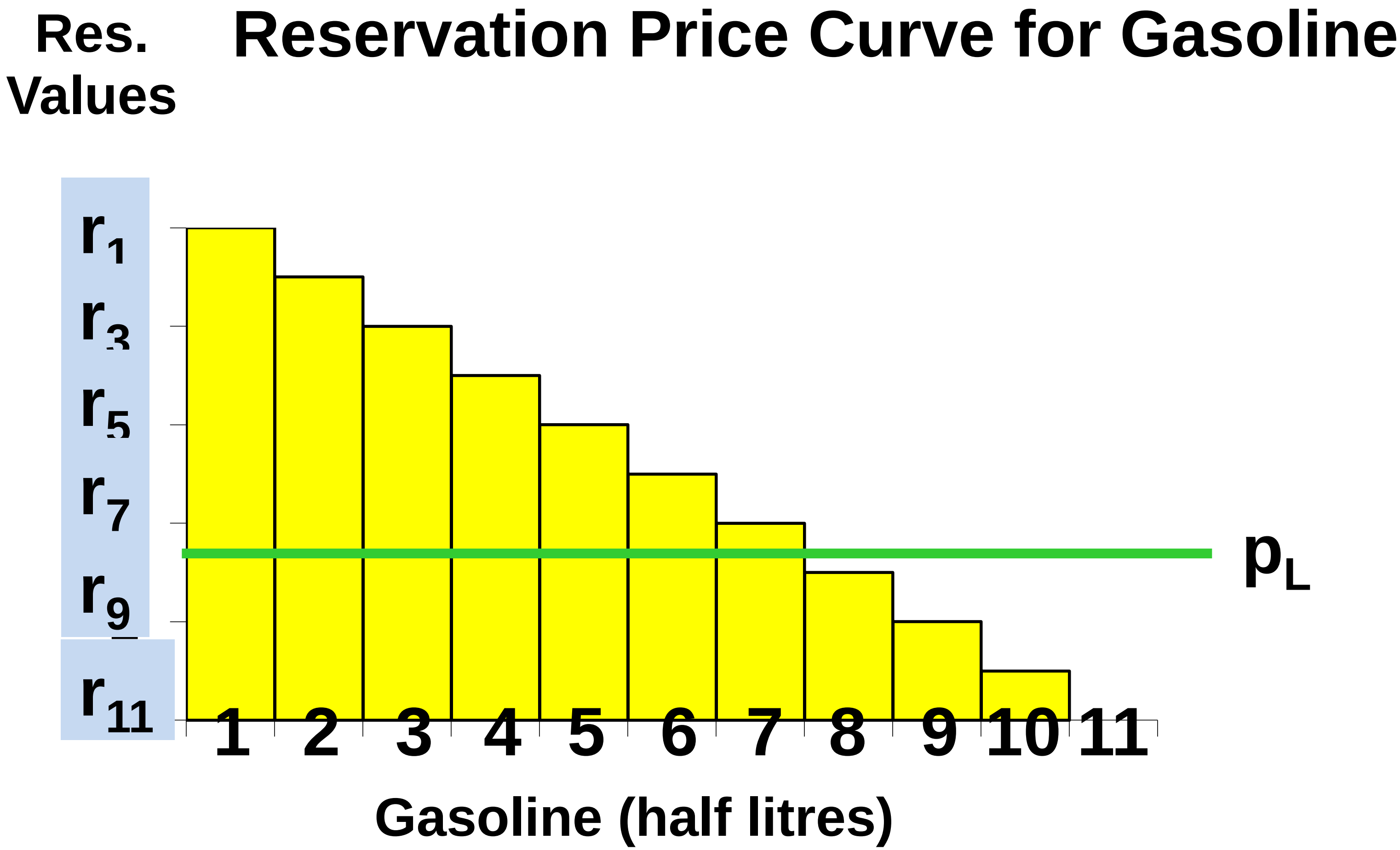
€ Equivalent Utility Gains

- Now suppose that gasoline is sold in half-litre units.
- r_1, r_2, … , r_n, … denote the consumer’s reservation prices for successive half-litres of gasoline.
- Our consumer’s new reservation price curve is

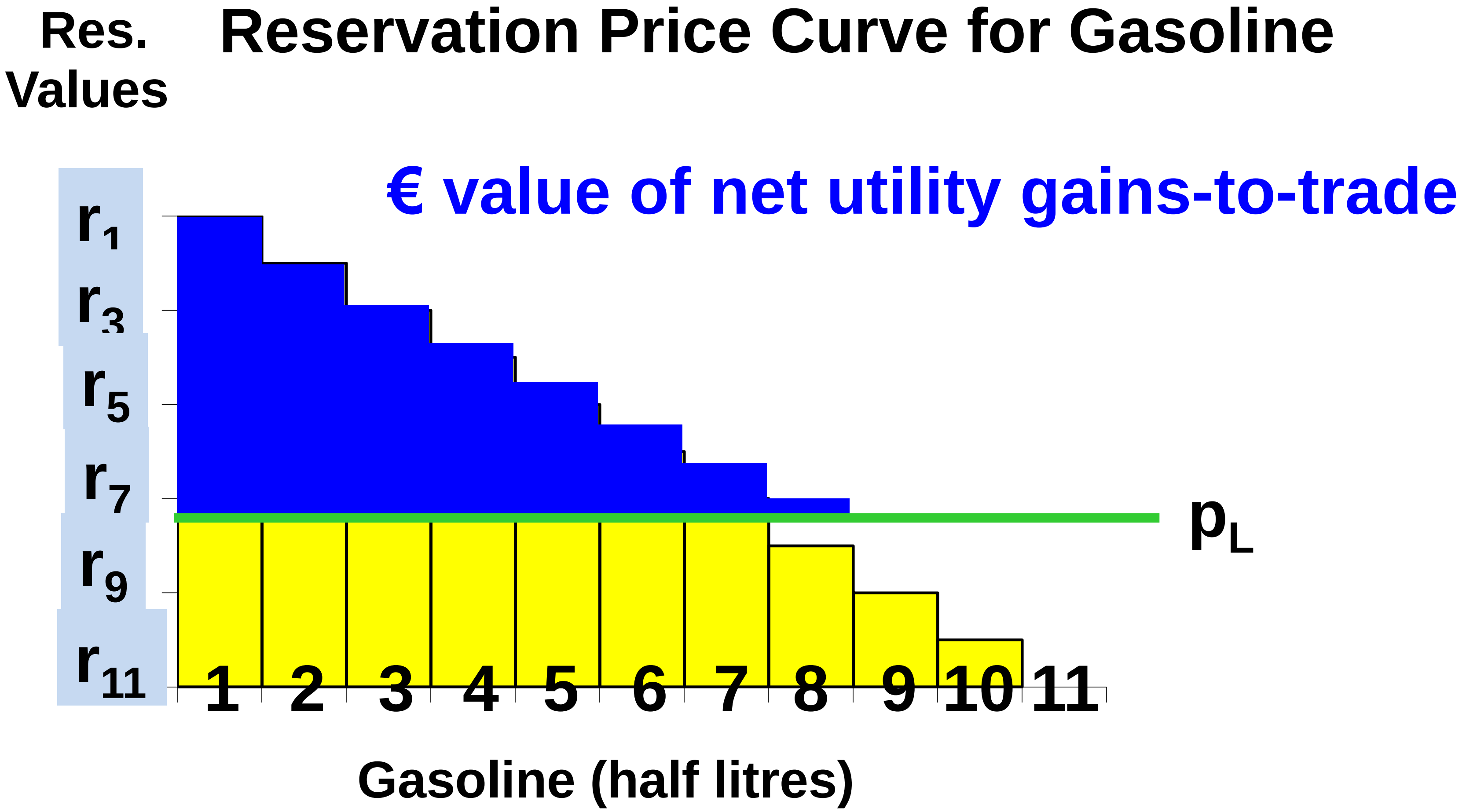
€ Equivalent Utility Gains



€ Equivalent Utility Gains



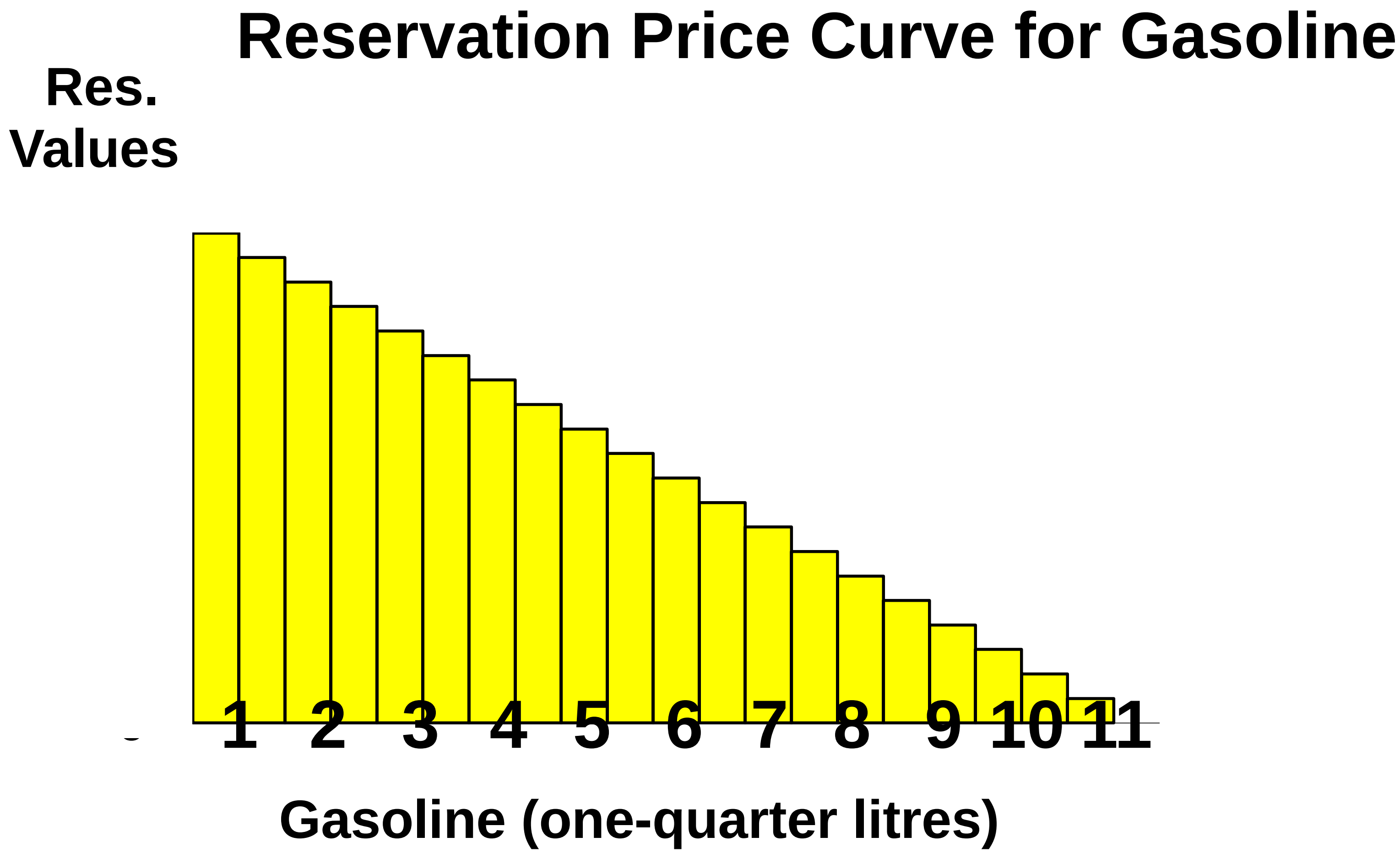
€ Equivalent Utility Gains



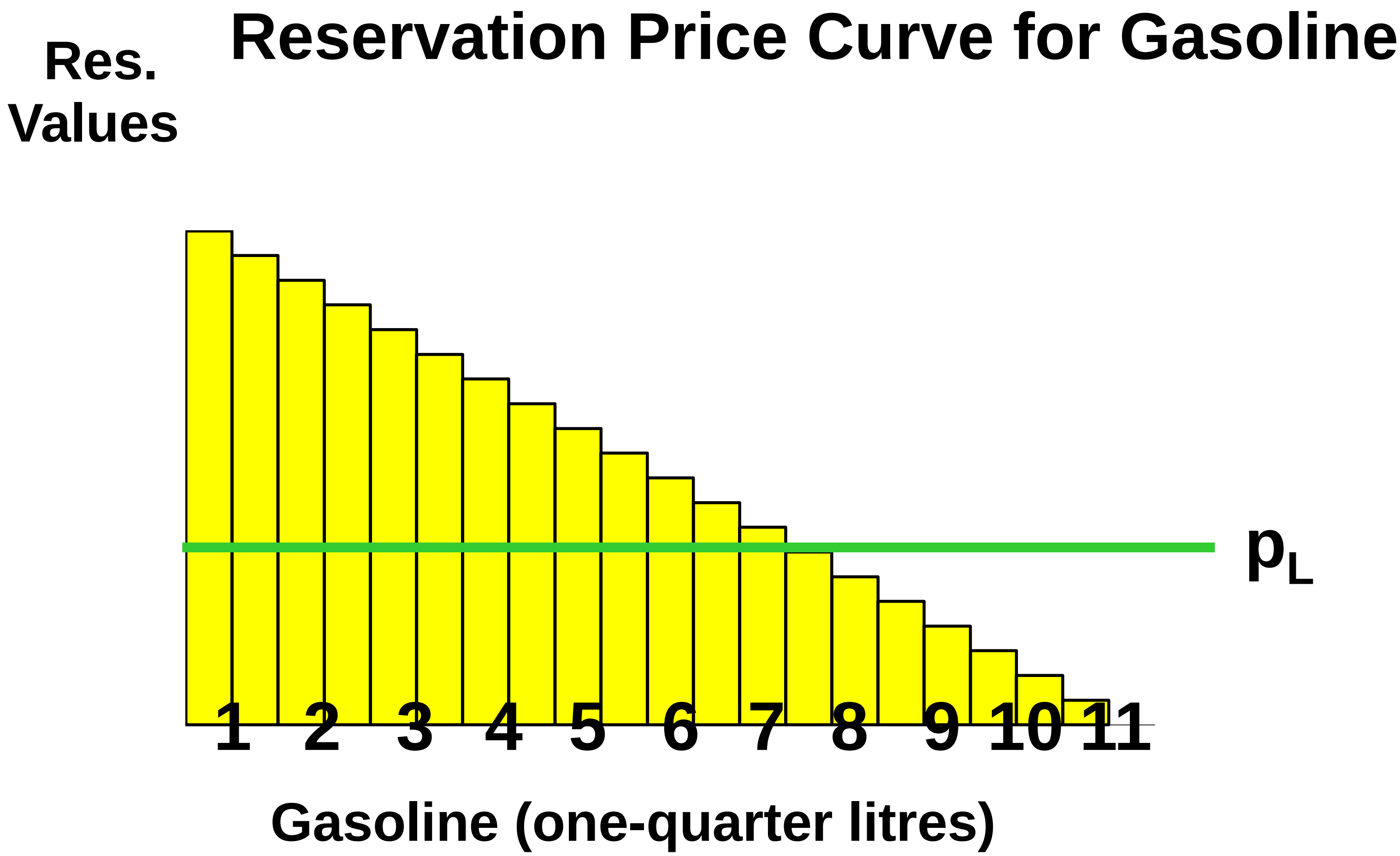
€ Equivalent Utility Gains

- And if gasoline is available in one-quarter litre units ...

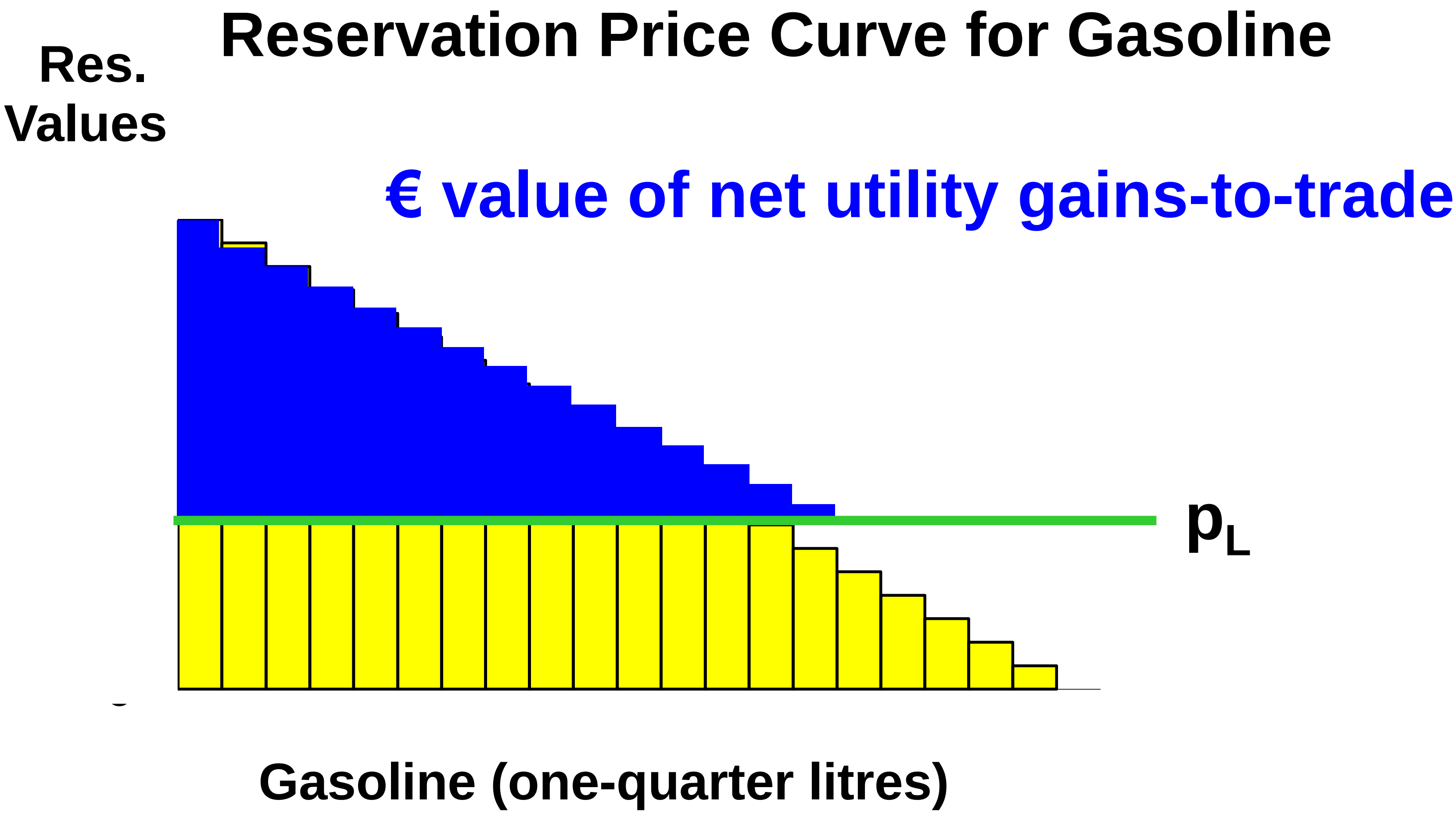
€ Equivalent Utility Gains



€ Equivalent Utility Gains



€ Equivalent Utility Gains



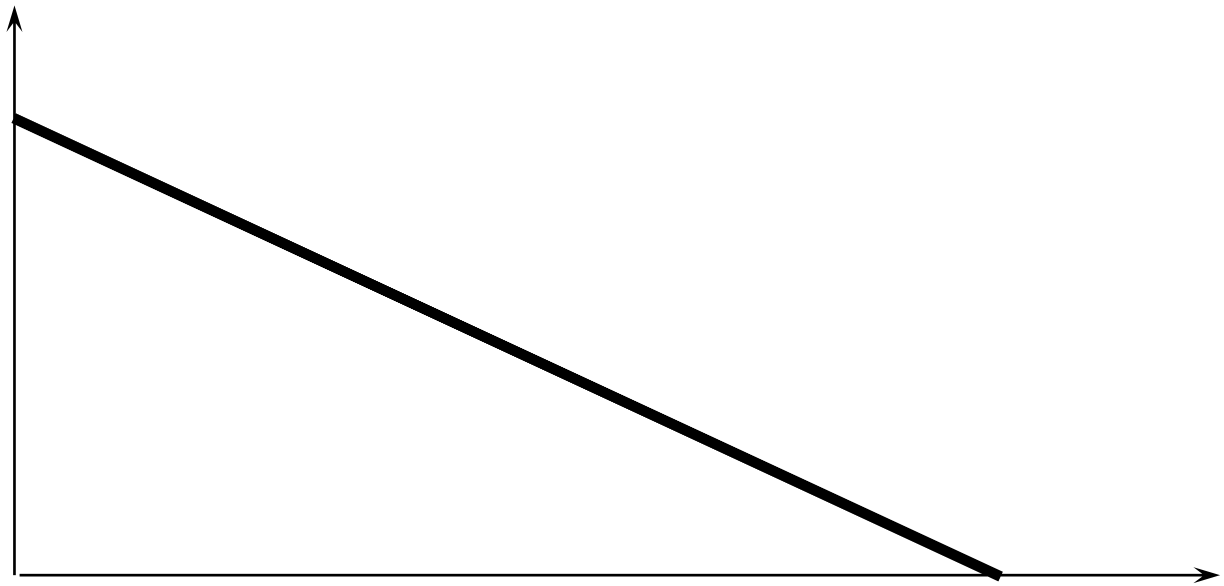
€ Equivalent Utility Gains

- Finally, if gasoline can be purchased in any quantity then ...

€ Equivalent Utility Gains

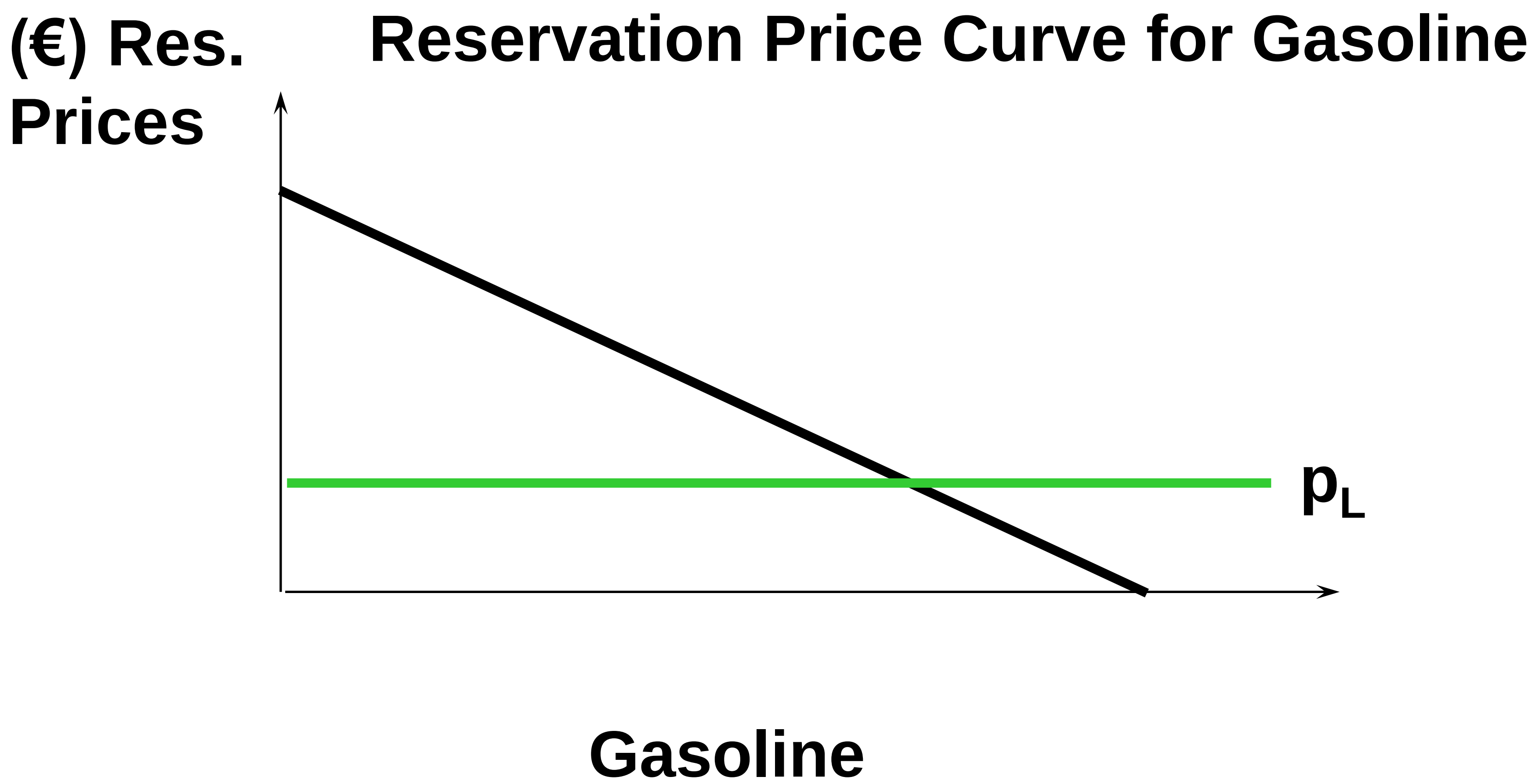
(€) Res. Prices

Reservation Price Curve for Gasoline

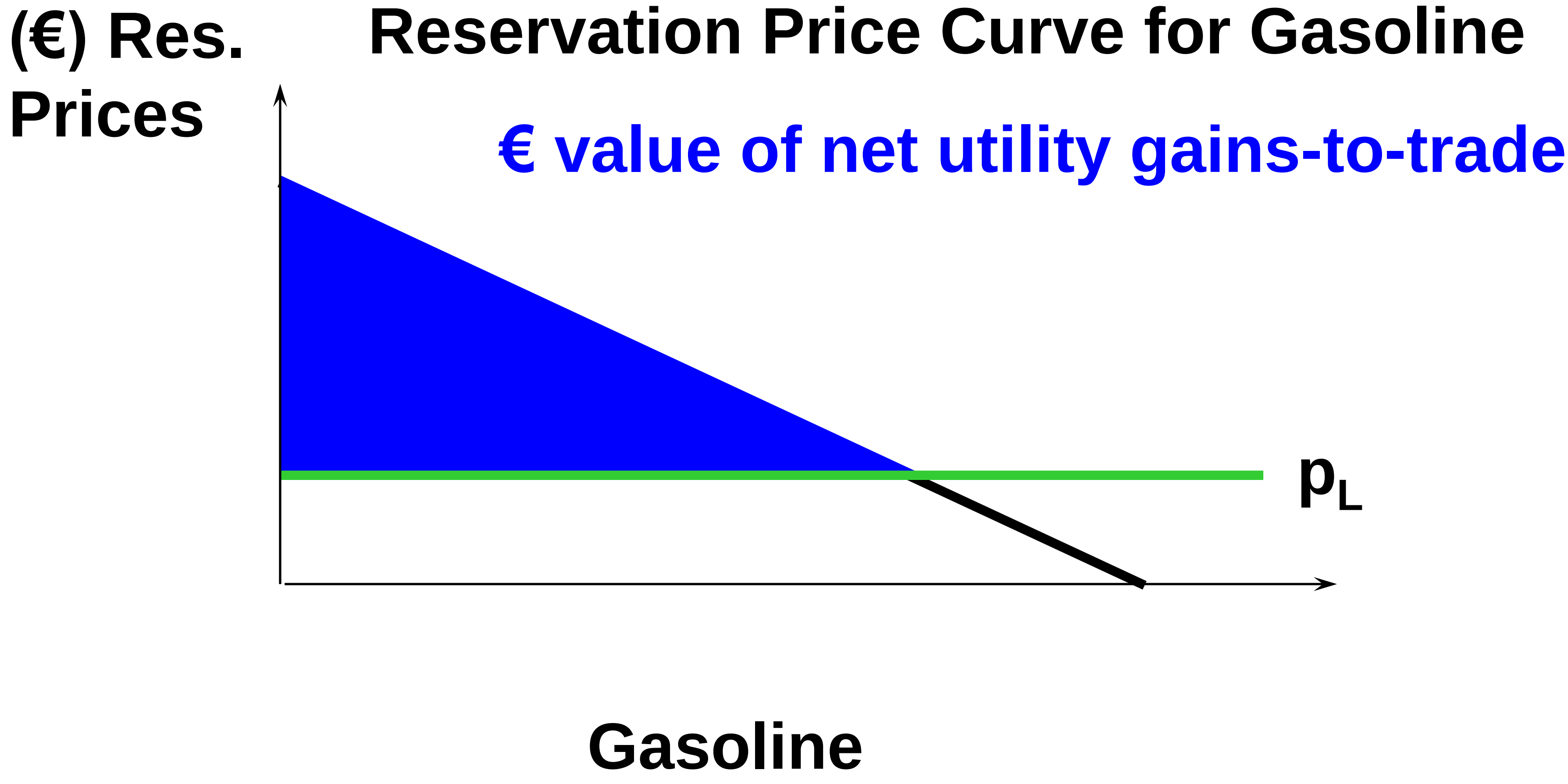


Gasoline

€ Equivalent Utility Gains



€ Equivalent Utility Gains



€ Equivalent Utility Gains

- Unfortunately, estimating a consumer's reservation-price curve is difficult,
- so, as an approximation, the reservation-price curve is replaced with the consumer's ordinary demand curve.

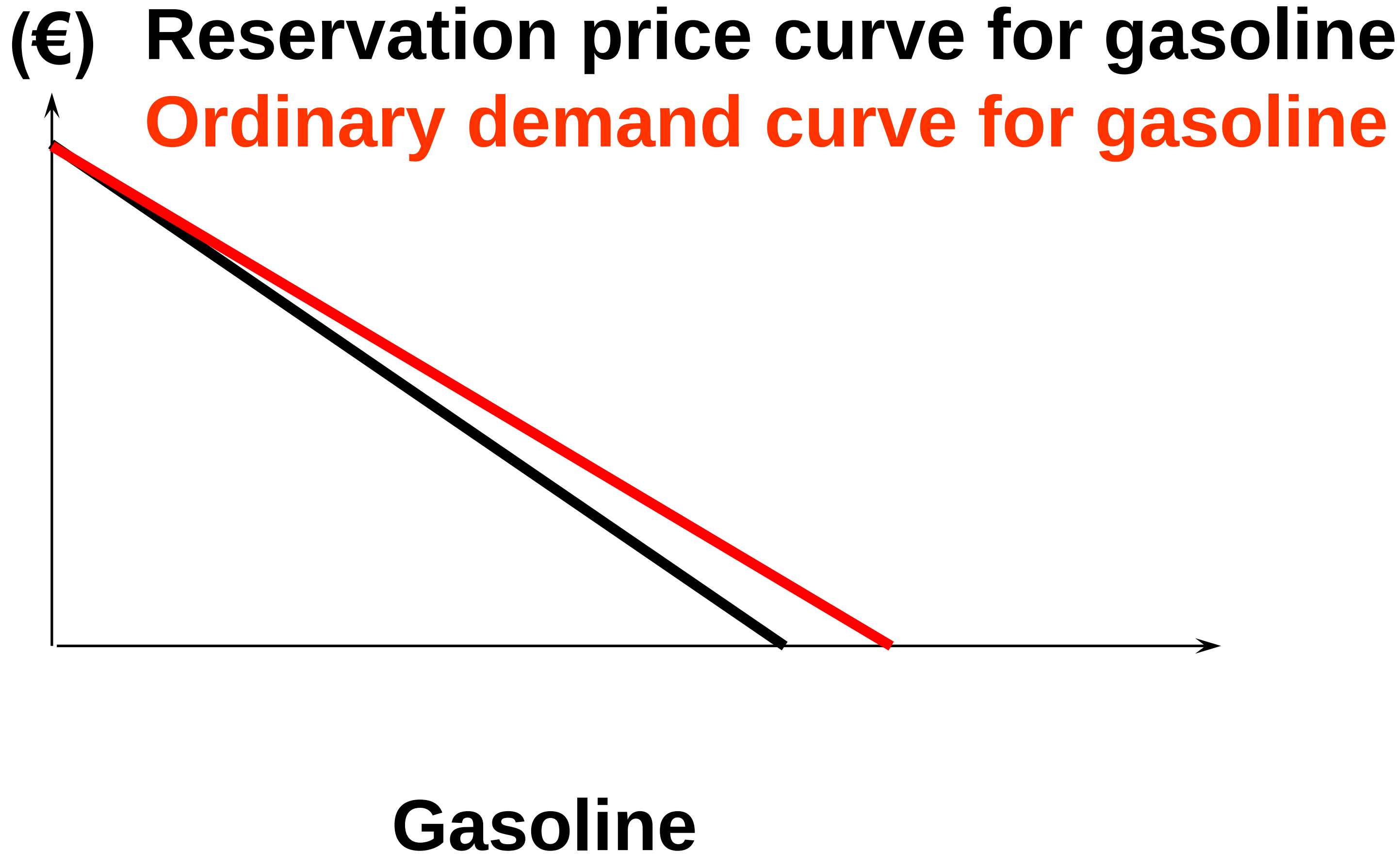
Consumer’s Surplus

- A consumer’s reservation-price curve is not quite the same as her ordinary demand curve. Why not?
- A reservation-price curve describes ***sequentially*** the values of successive single units of a commodity.
- An ordinary demand curve describes the most that would be paid for q units of a commodity purchased ***simultaneously.***

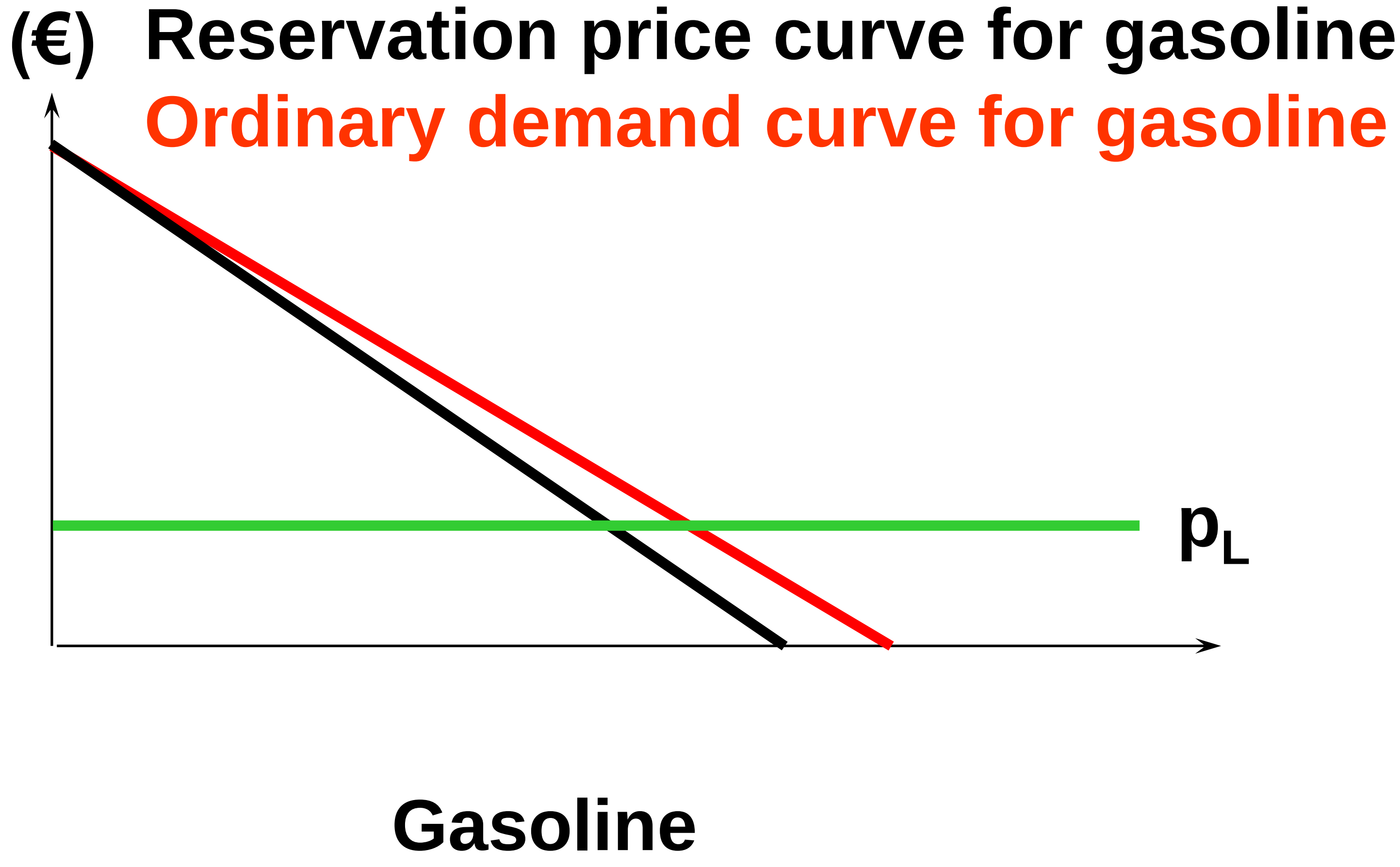
Consumer's Surplus

- Approximating the net utility gain area under the reservation-price curve by the corresponding area under the ordinary demand curve gives the ***Consumer's Surplus measure of net utility gain***.

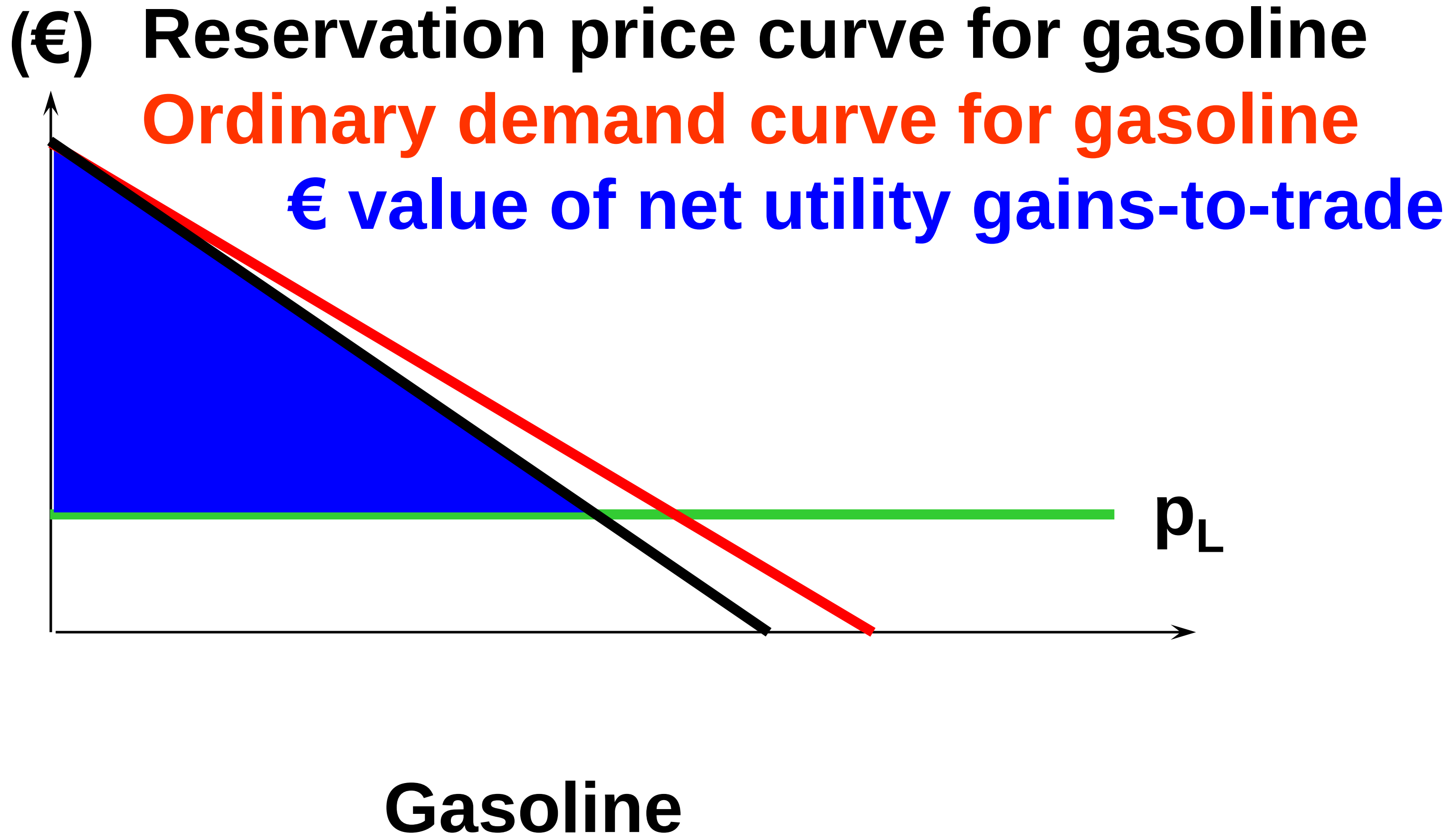
Consumer's Surplus



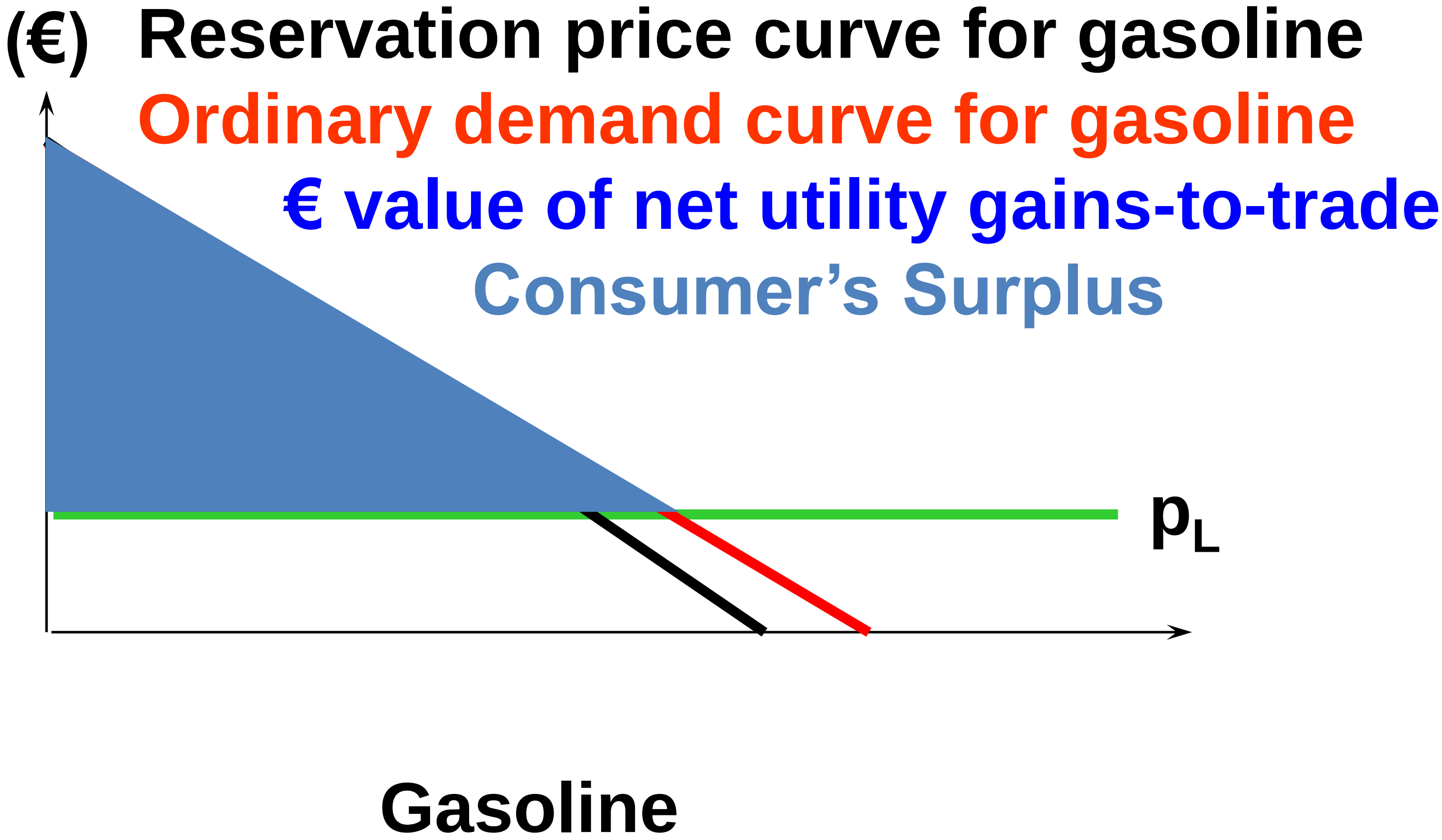
Consumer's Surplus



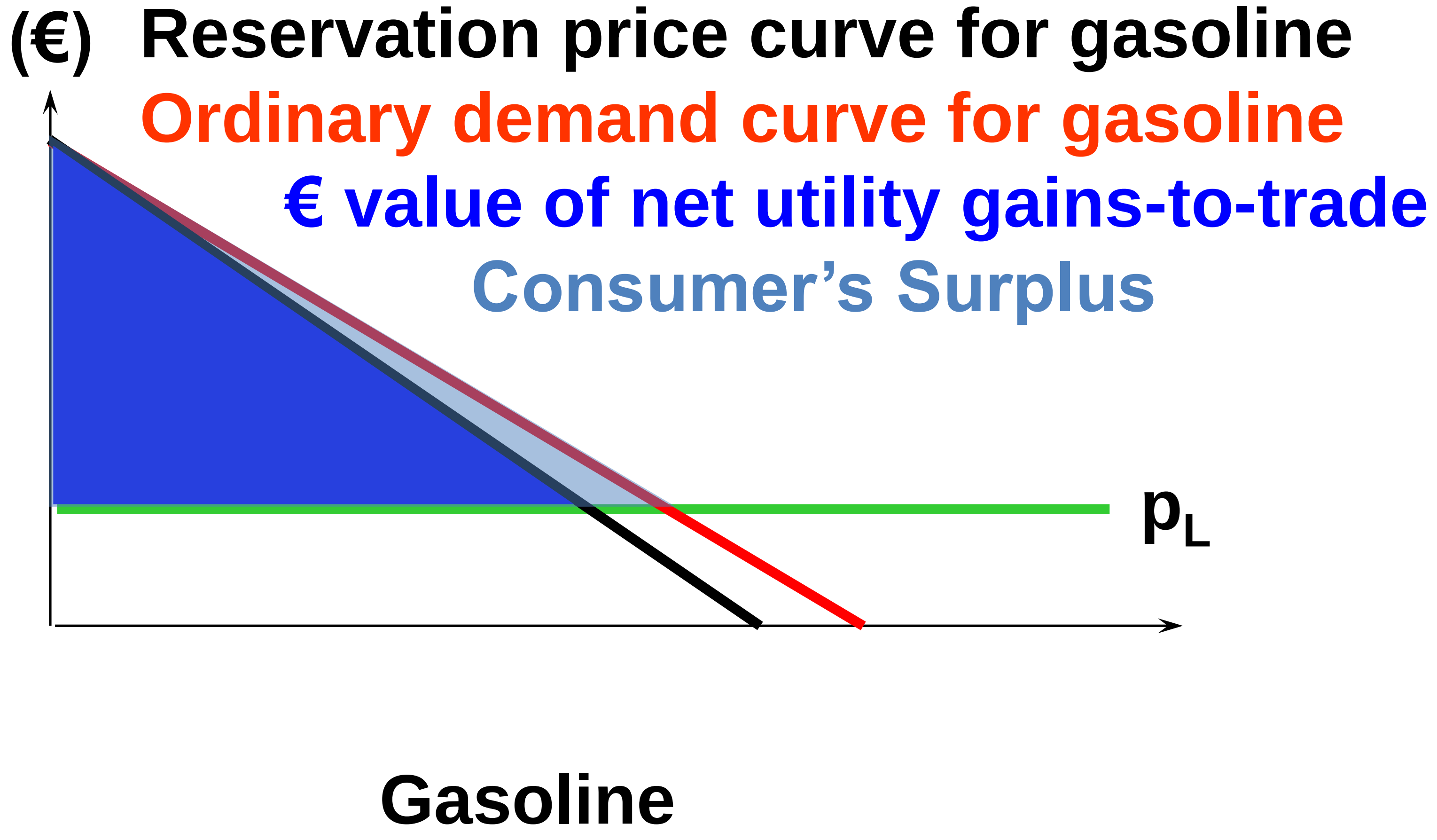
Consumer's Surplus



Consumer’s Surplus



Consumer's Surplus



Consumer’s Surplus

- The difference between the consumer’s reservation-price and ordinary demand curves is due to income effects.
- But, if the consumer’s utility function is quasilinear in income then there are no income effects and Consumer’s Surplus is an exact € measure of gains-to-trade.

Consumer's Surplus

The consumer's utility function is quasilinear in x_2.

$$U(x_1, x_2) = v(x_1) + x_2$$

Take $p_2 = 1$. Then the consumer's choice problem is to maximize

$$U(x_1, x_2) = v(x_1) + x_2$$

subject to

$$p_1 x_1 + x_2 = m.$$

Consumer's Surplus

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subject to

$$p_1 x_1 + x_2 = m.$$

Consumer's Surplus

That is, choose x_1 to maximize

$$v(x_1) + m - p_1 x_1.$$

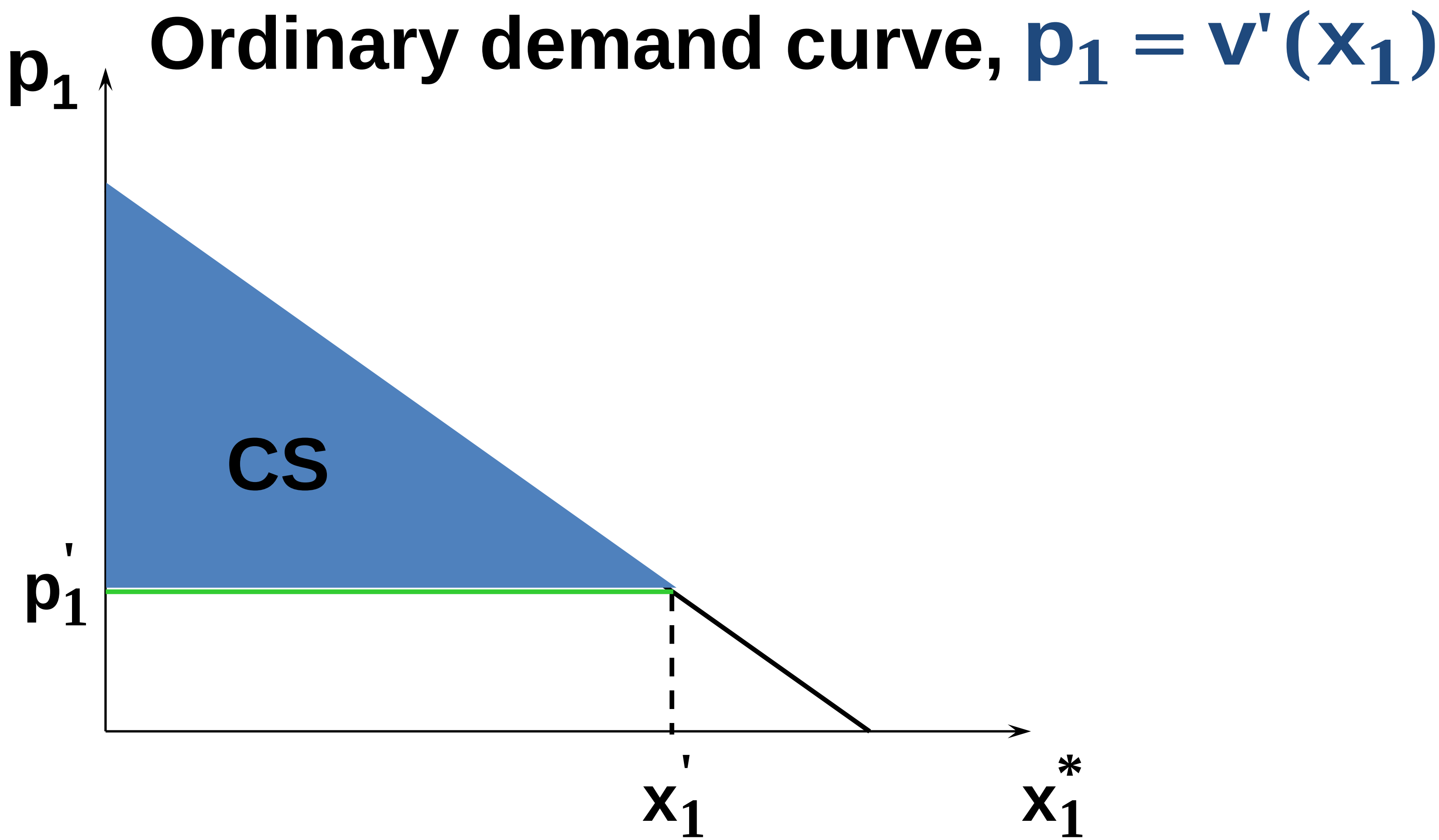
The first-order condition is

$$v'(x_1) - p_1 = 0$$

That is, $$p_1 = v'(x_1).$$

This is the equation of the consumer's ordinary demand for commodity 1.

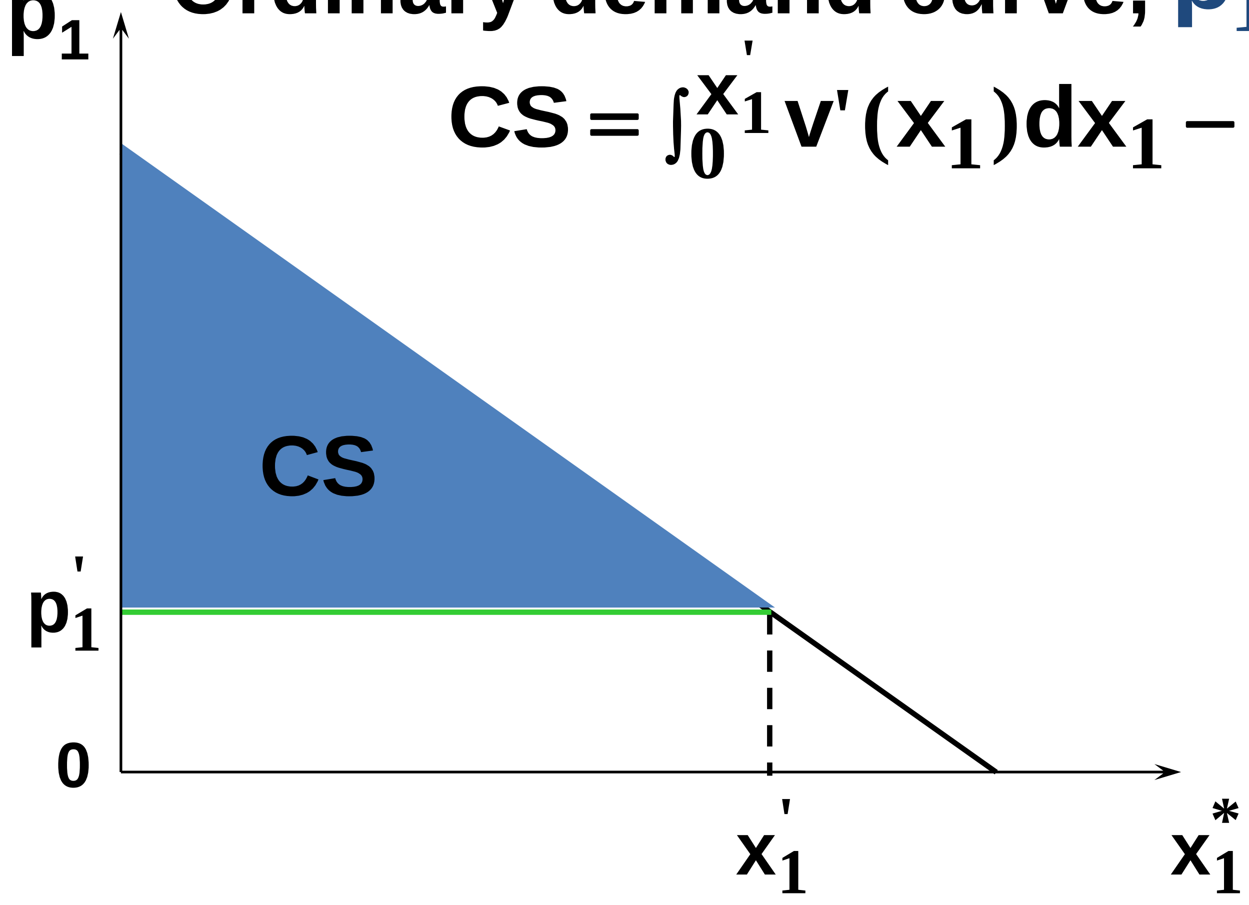
Consumer's Surplus



Consumer's Surplus

Ordinary demand curve, $p_1 = v'(x_1)$

$$CS = \int_0^{x_1'} v'(x_1)\,dx_1 - p_1' x_1'$$

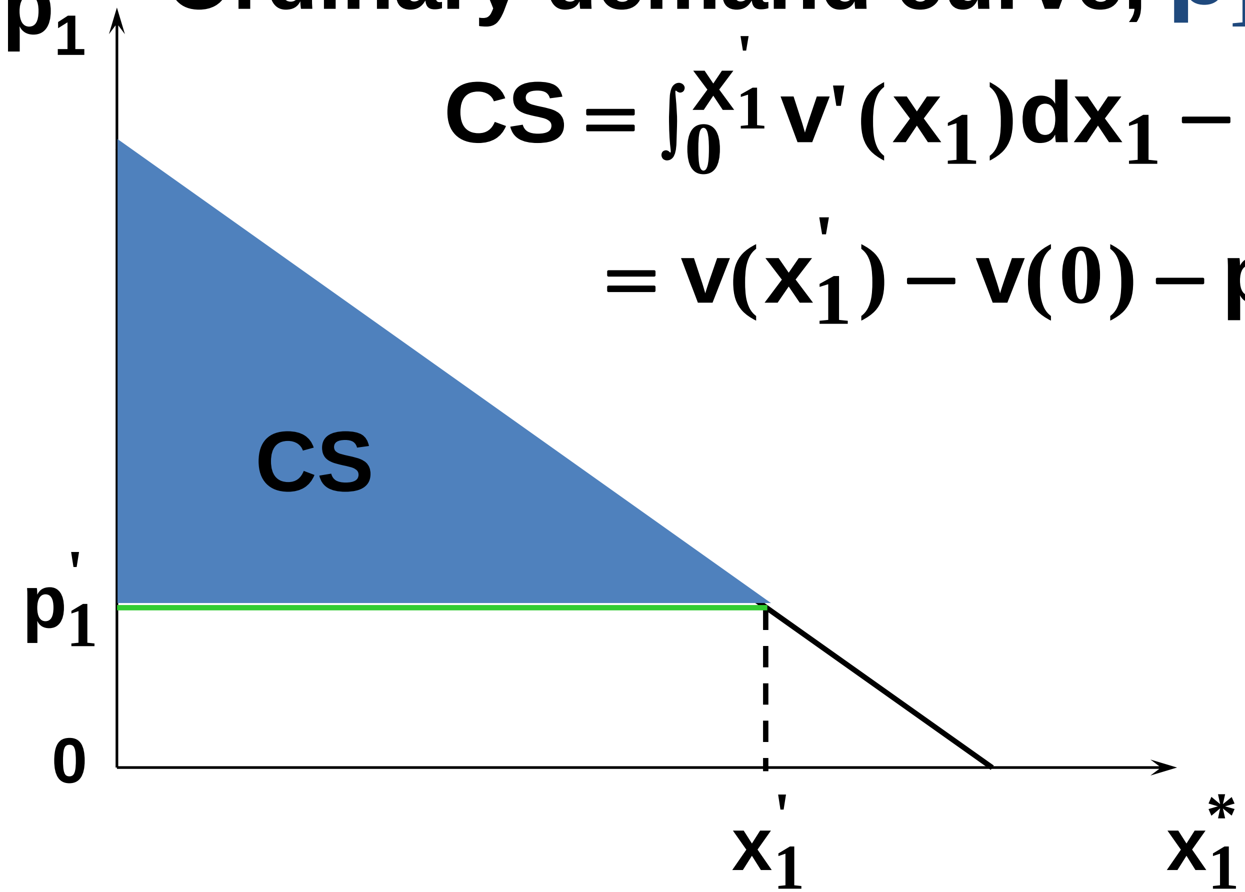


Consumer's Surplus

Ordinary demand curve, $\mathbf{p_1 = v'(x_1)}$

$$CS = \int_0^{x_1'} v'(x_1)\,dx_1 - p_1' x_1'$$

$$= v(x_1') - v(0) - p_1' x_1'$$



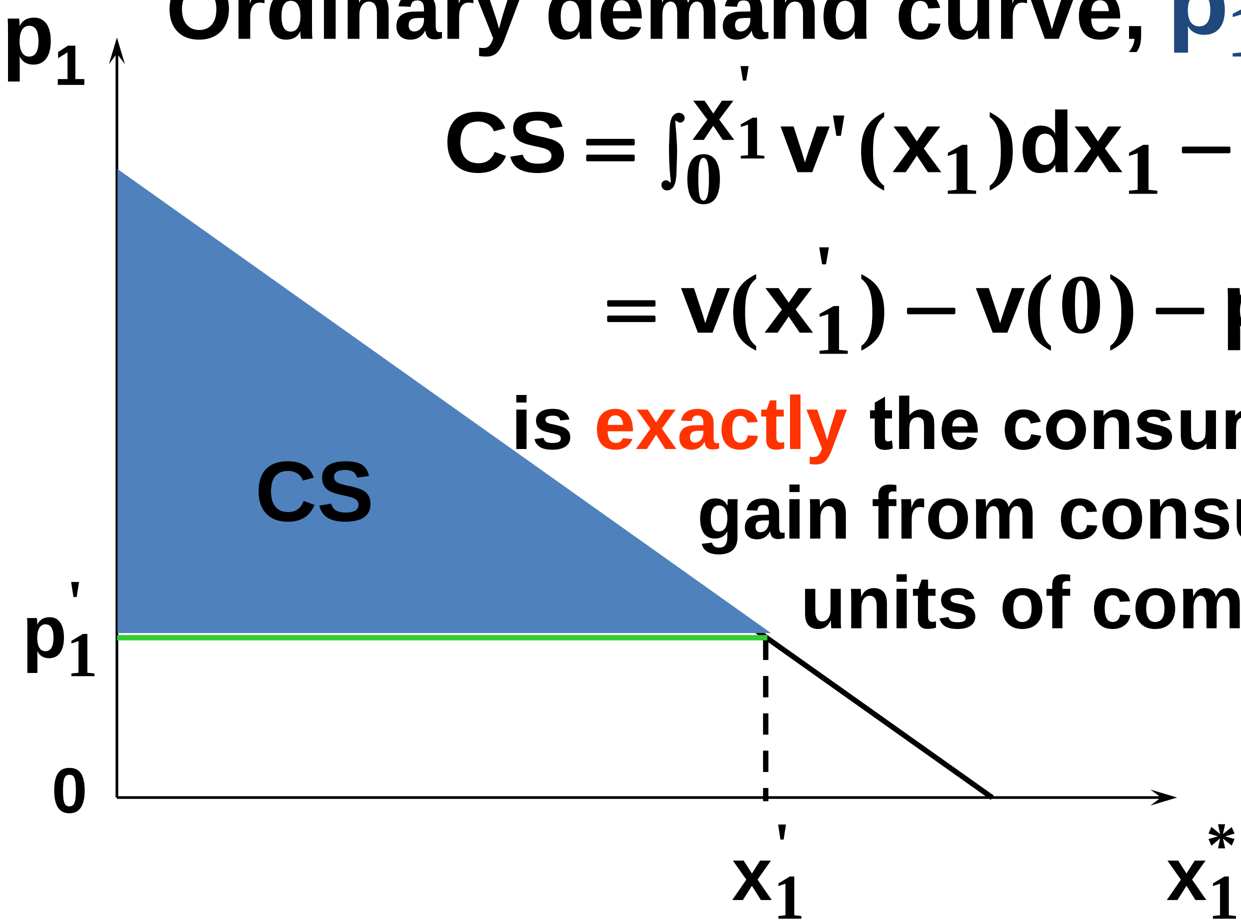
Consumer’s Surplus

Ordinary demand curve, $p_1 = v'(x_1)$

$$CS = \int_0^{x_1'} v'(x_1)dx_1 - p_1' x_1'$$

$$= v(x_1') - v(0) - p_1' x_1'$$

is exactly the consumer’s utility gain from consuming x_1’ units of commodity 1.



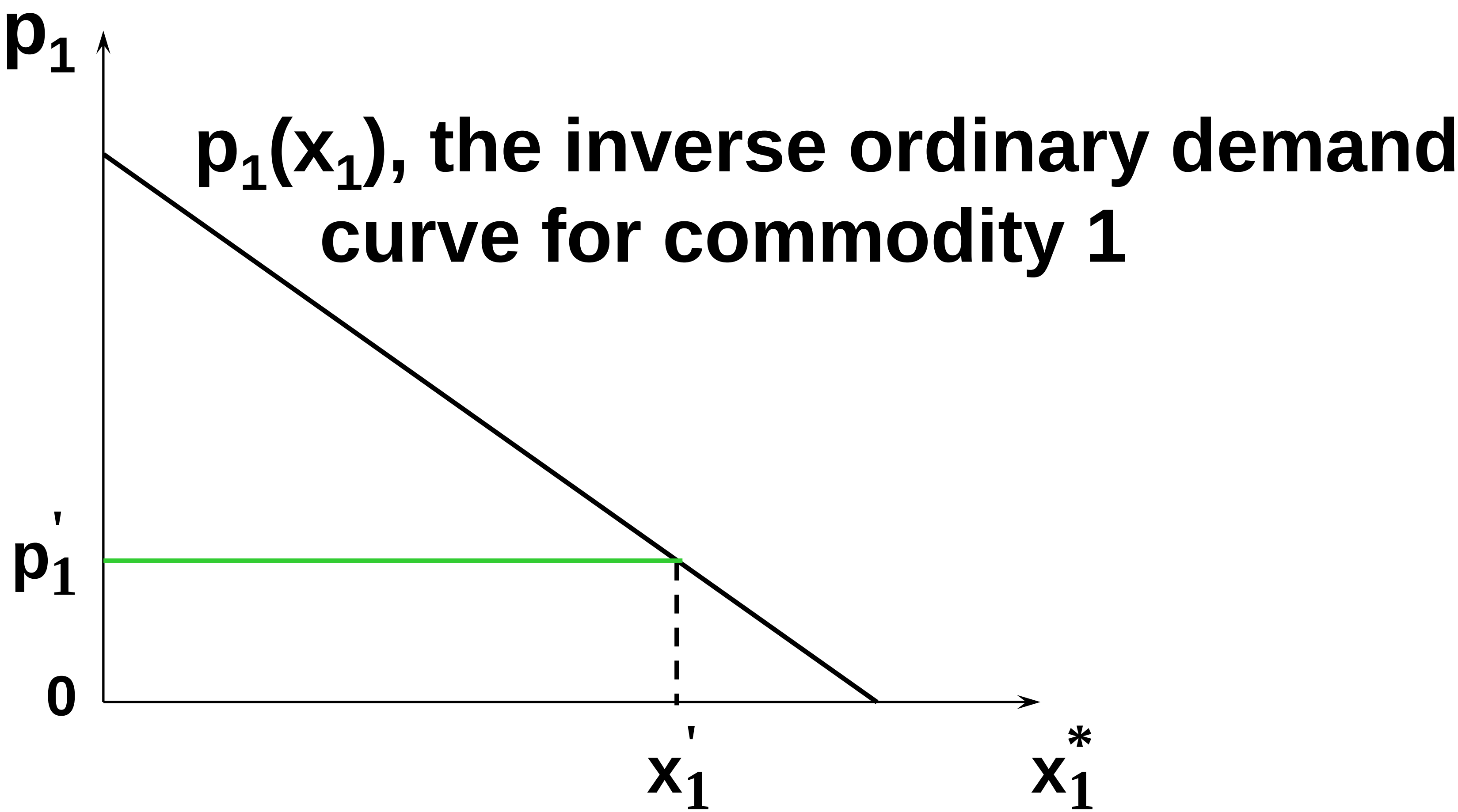
Consumer’s Surplus

- Consumer’s Surplus is an exact euro measure of utility gained from consuming commodity 1 when the consumer’s utility function is quasilinear in commodity 2.
- Otherwise Consumer’s Surplus is an approximation.

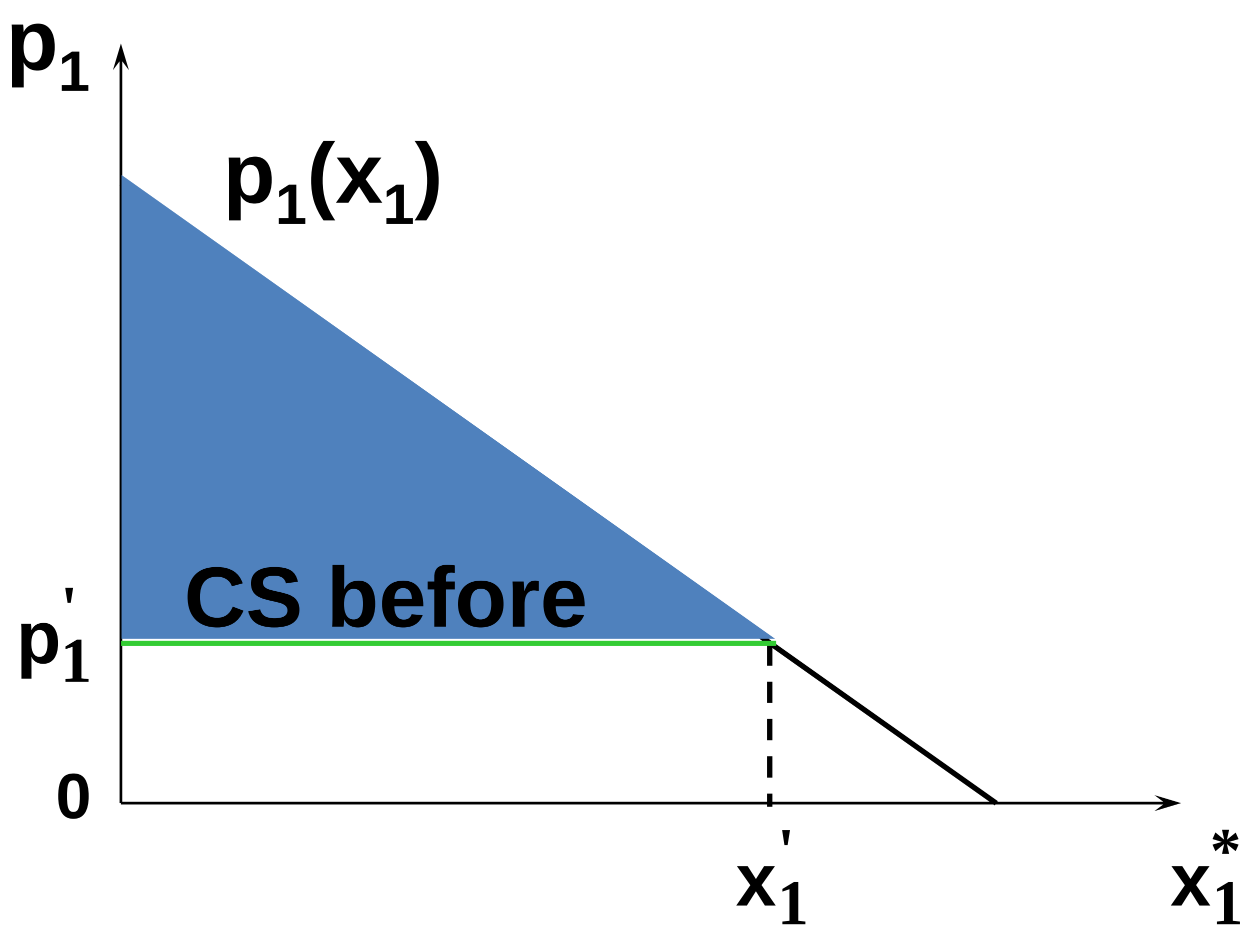
Consumer's Surplus

- The change to a consumer's total utility due to a change to p_1 is approximately the change in her Consumer's Surplus.

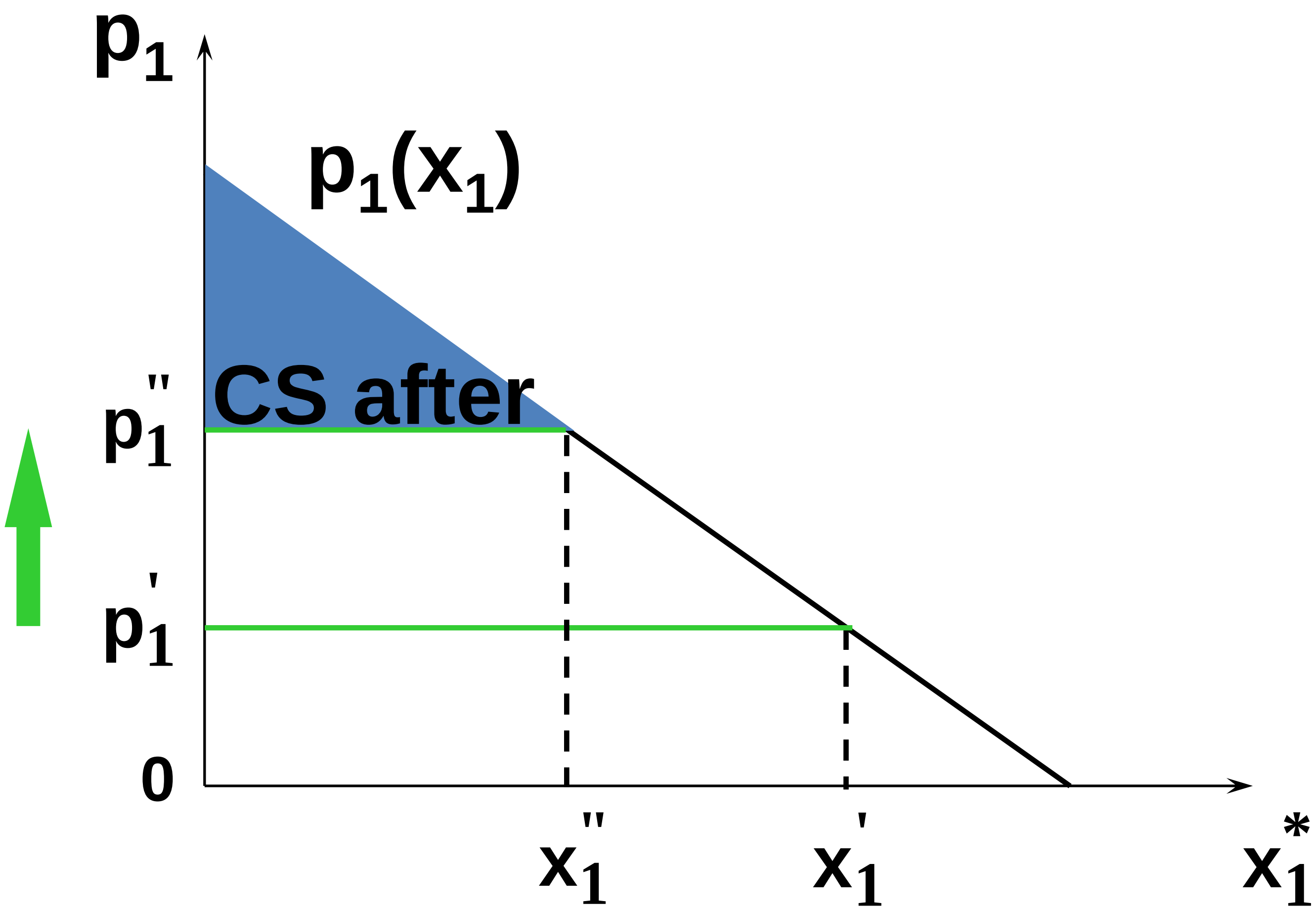
Consumer's Surplus



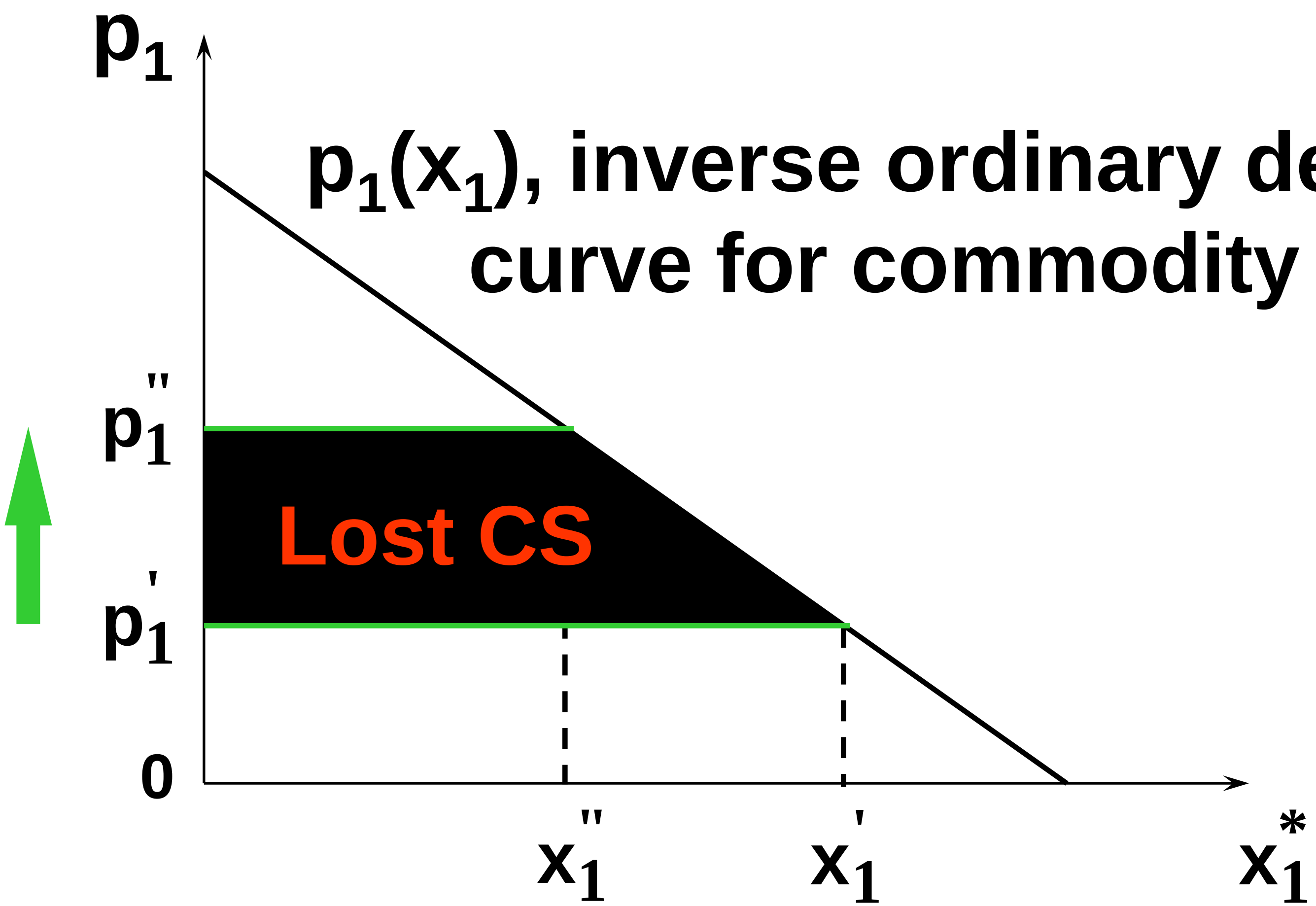
Consumer's Surplus



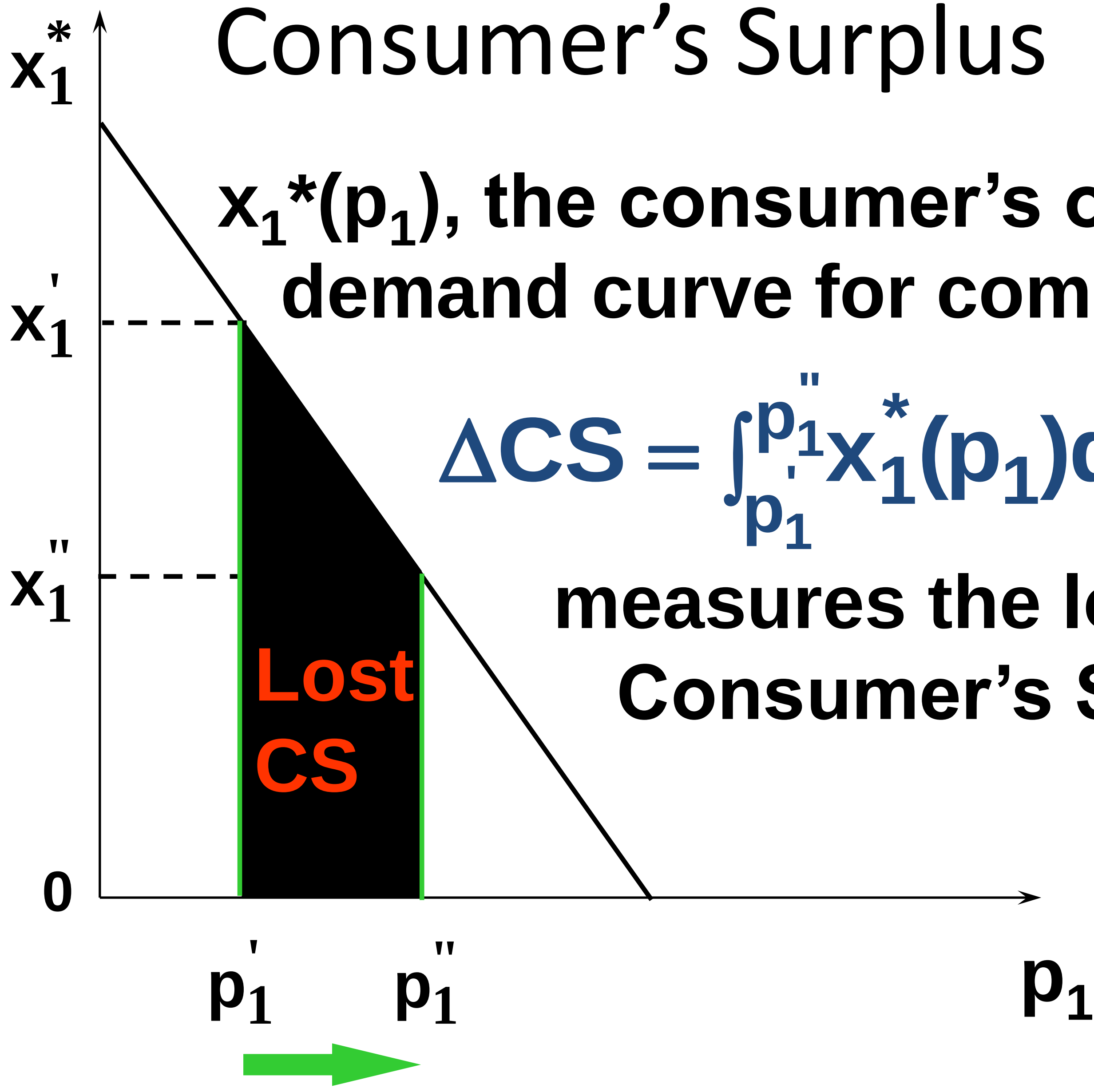
Consumer’s Surplus



Consumer's Surplus



Consumer's Surplus



$x_1^*(p_1)$, the consumer's ordinary demand curve for commodity 1.

$$\Delta CS = \int_{p_1'}^{p_1''} x_1^*(p_1)\,dp_1$$

measures the loss in Consumer's Surplus.

Compensating Variation and Equivalent Variation

- Two additional euro measures of the total utility change caused by a price change are ***Compensating Variation*** and ***Equivalent Variation***.

Compensating Variation

- p_1 rises.
- Q: What is the least extra income that, at the **new prices**, just restores the consumer's original utility level?

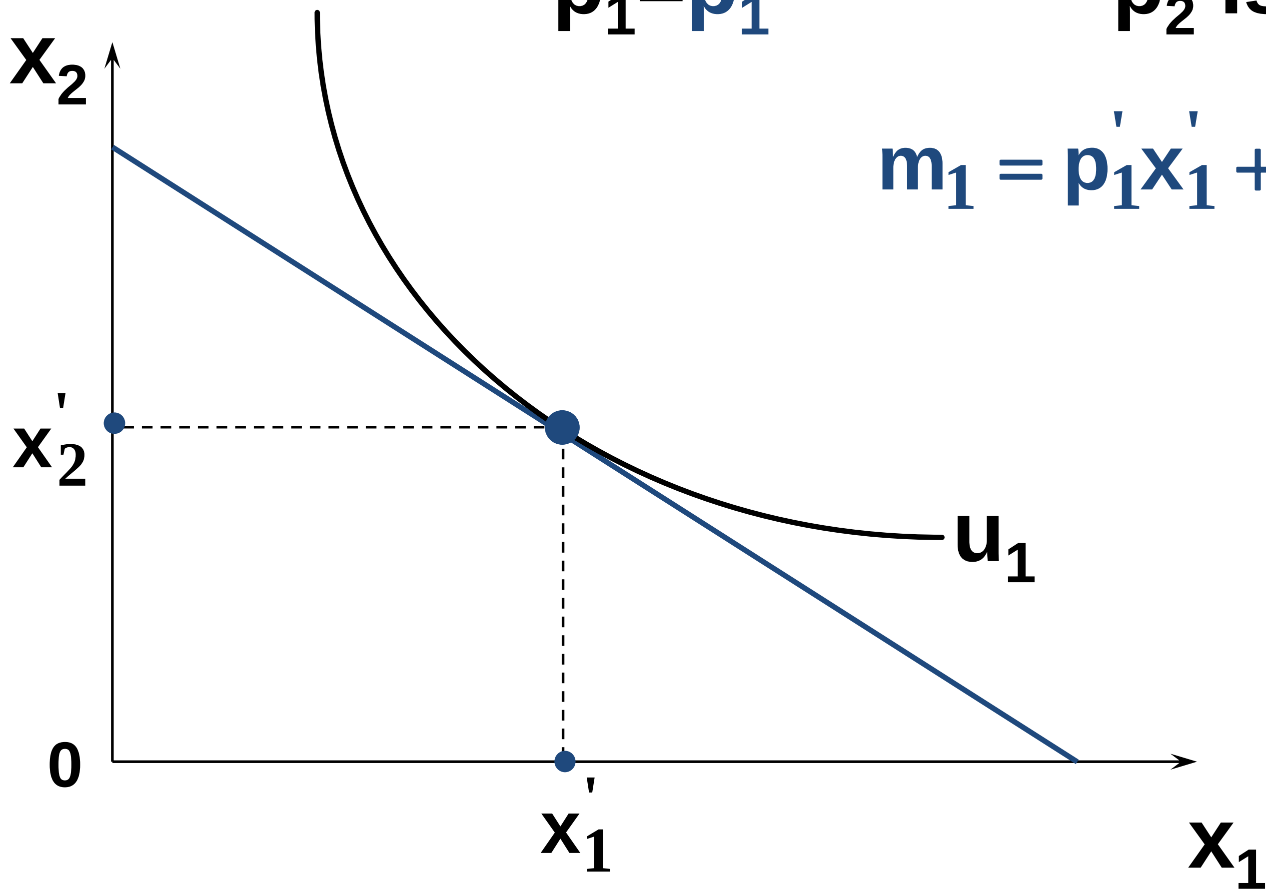
Compensating Variation

- p_1 rises.
- Q: What is the least extra income that, at the **new prices**, just restores the consumer's original utility level?
- A: The Compensating Variation.

Compensating Variation

$p_1 = p_1'$ p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$

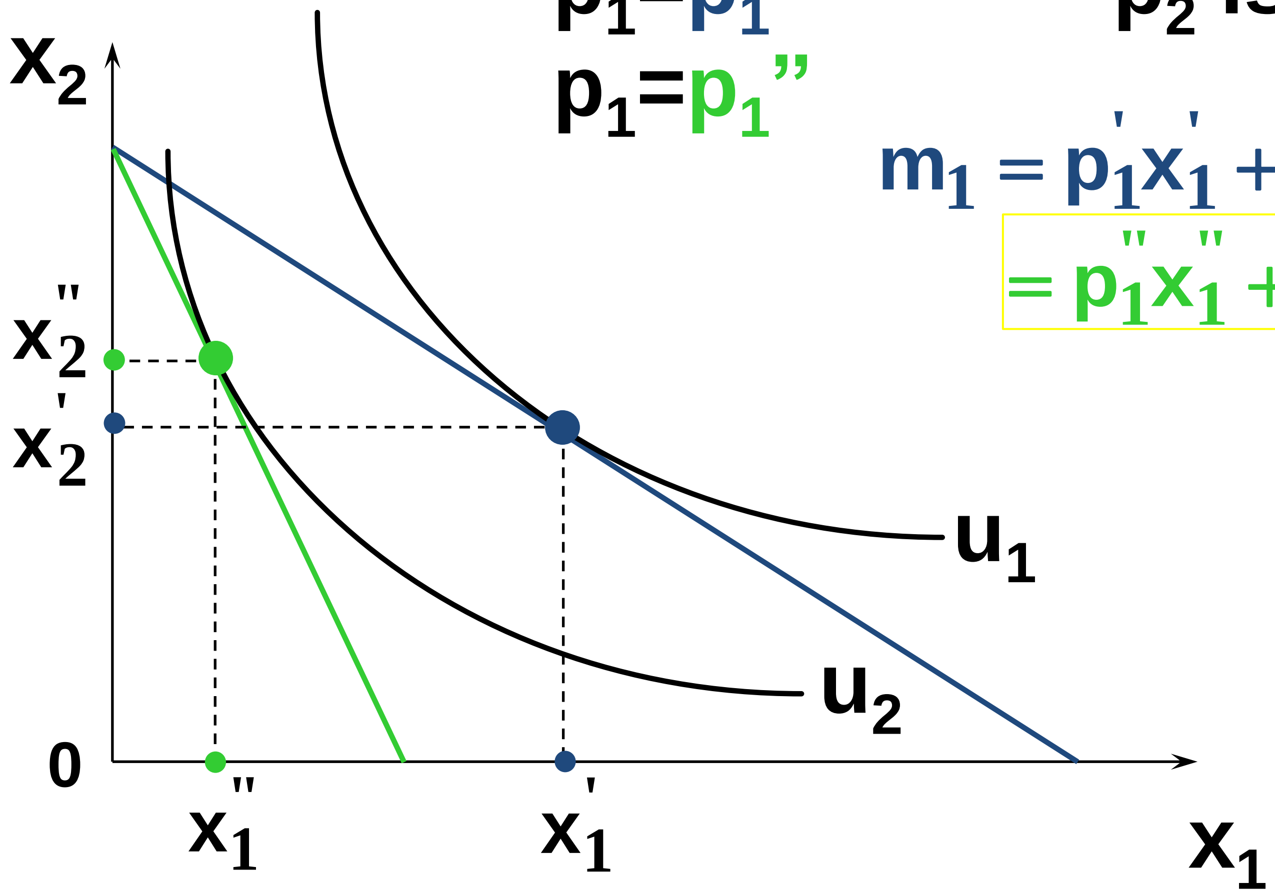


Compensating Variation

$p_1 = p_1'$
$p_1 = p_1''$

p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$
$$= p_1'' x_1'' + p_2 x_2''$$

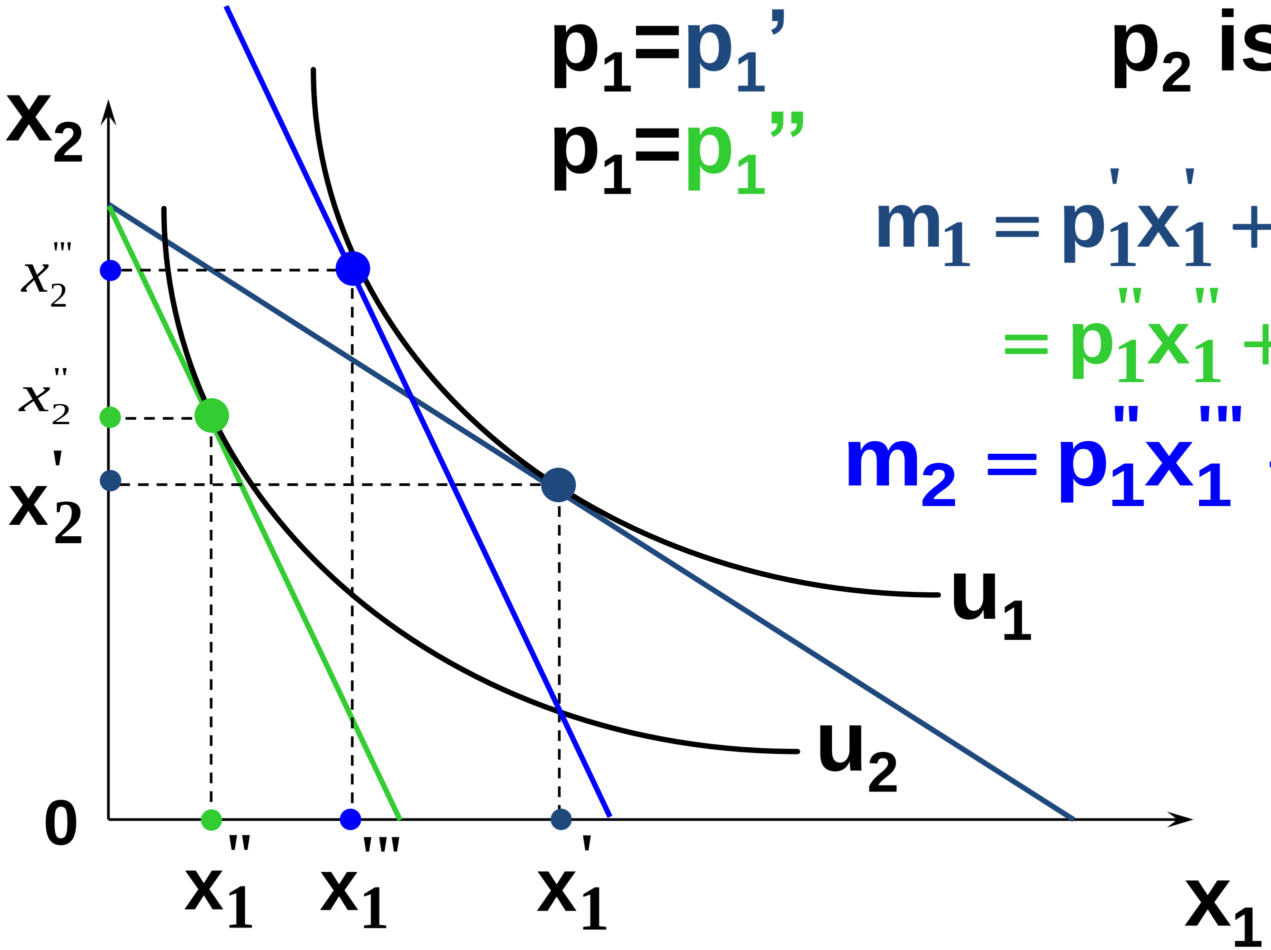


Compensating Variation

$p_1 = p_1'$
$p_1 = p_1''$

p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$
$$= p_1'' x_1'' + p_2 x_2''$$
$$m_2 = p_1'' x_1''' + p_2 x_2'''$$



Compensating Variation

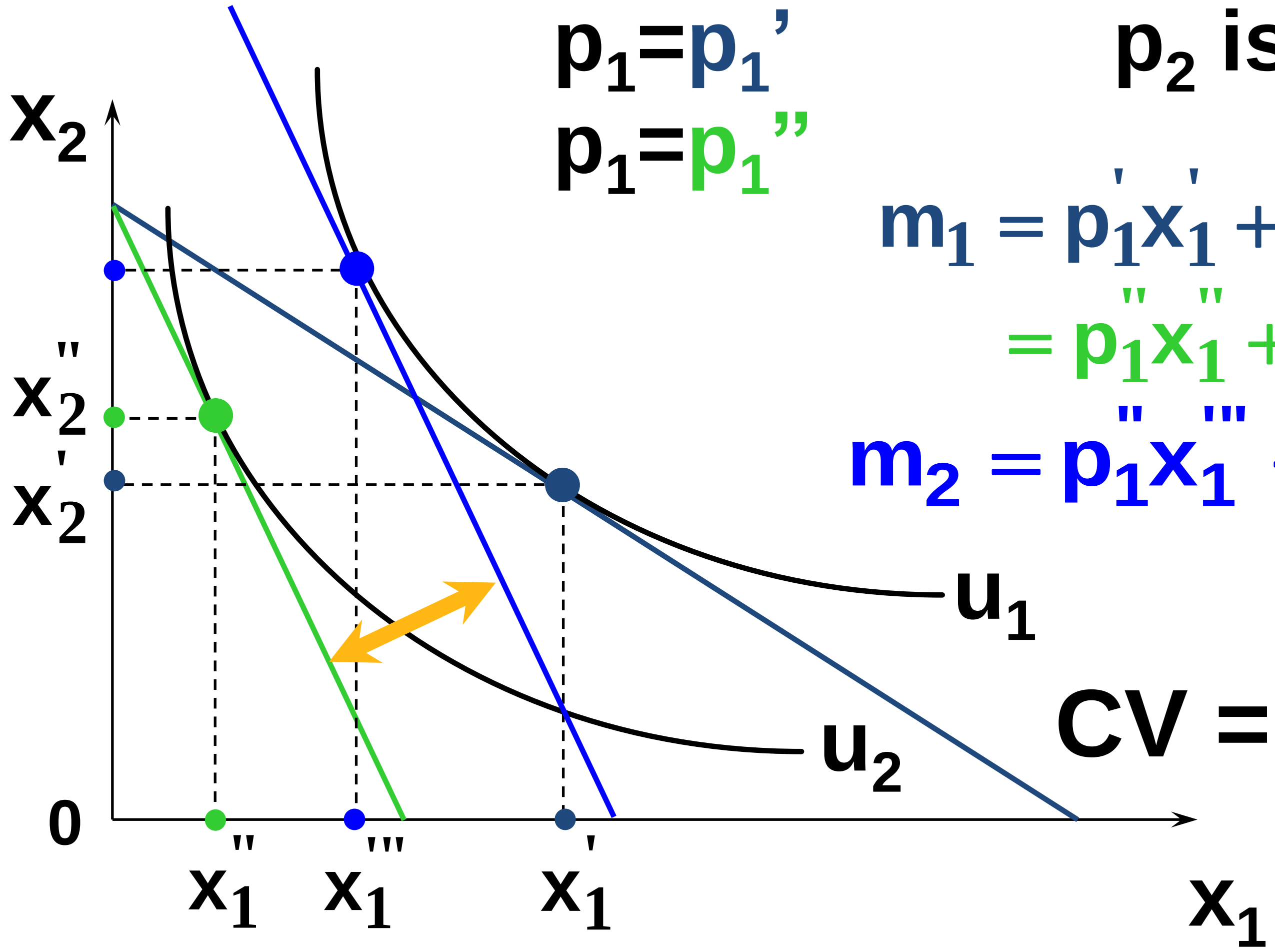
$p_1 = p_1'$

$p_1 = p_1''$

p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2' = p_1'' x_1'' + p_2 x_2''$$

$$m_2 = p_1'' x_1''' + p_2 x_2'''$$



CV = m_2 - m_1.

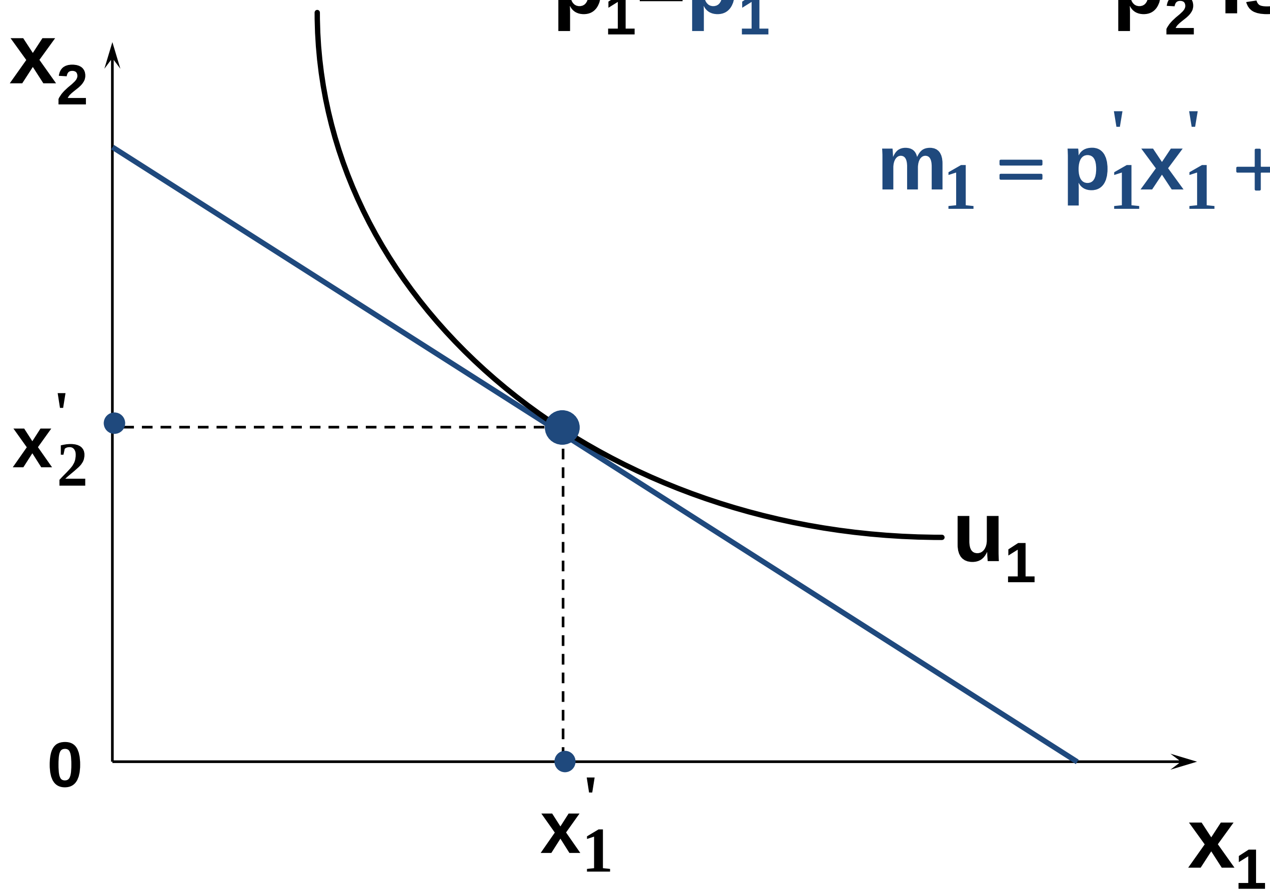
Equivalent Variation

- p_1 rises.
- Q: What is the least extra income that, at the **original prices**, just restores the consumer's original utility level?
- A: The Equivalent Variation.

Equivalent Variation

$p_1 = p_1'$ $\qquad$ p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$



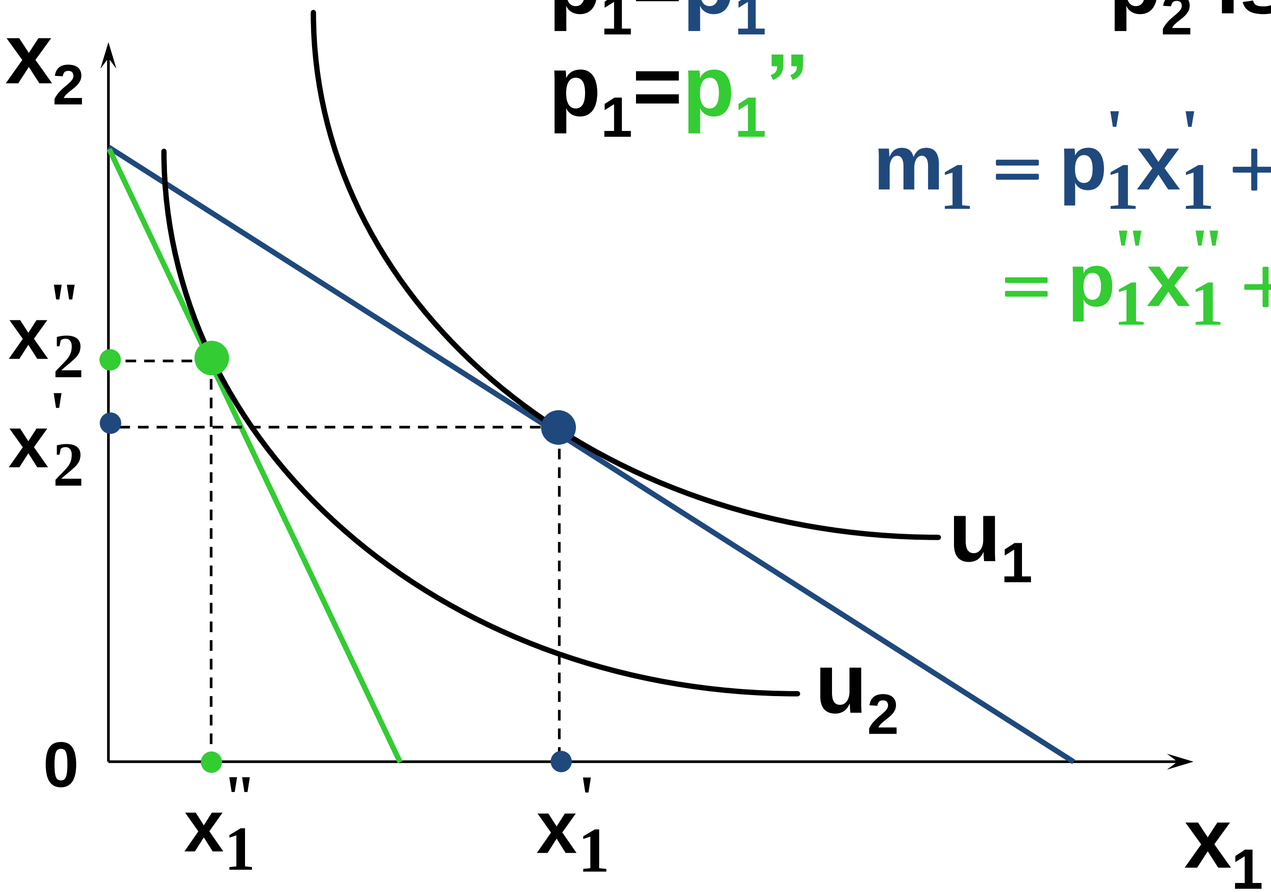
Equivalent Variation

$p_1 = p_1'$

$p_1 = p_1''$

p_2 is fixed.

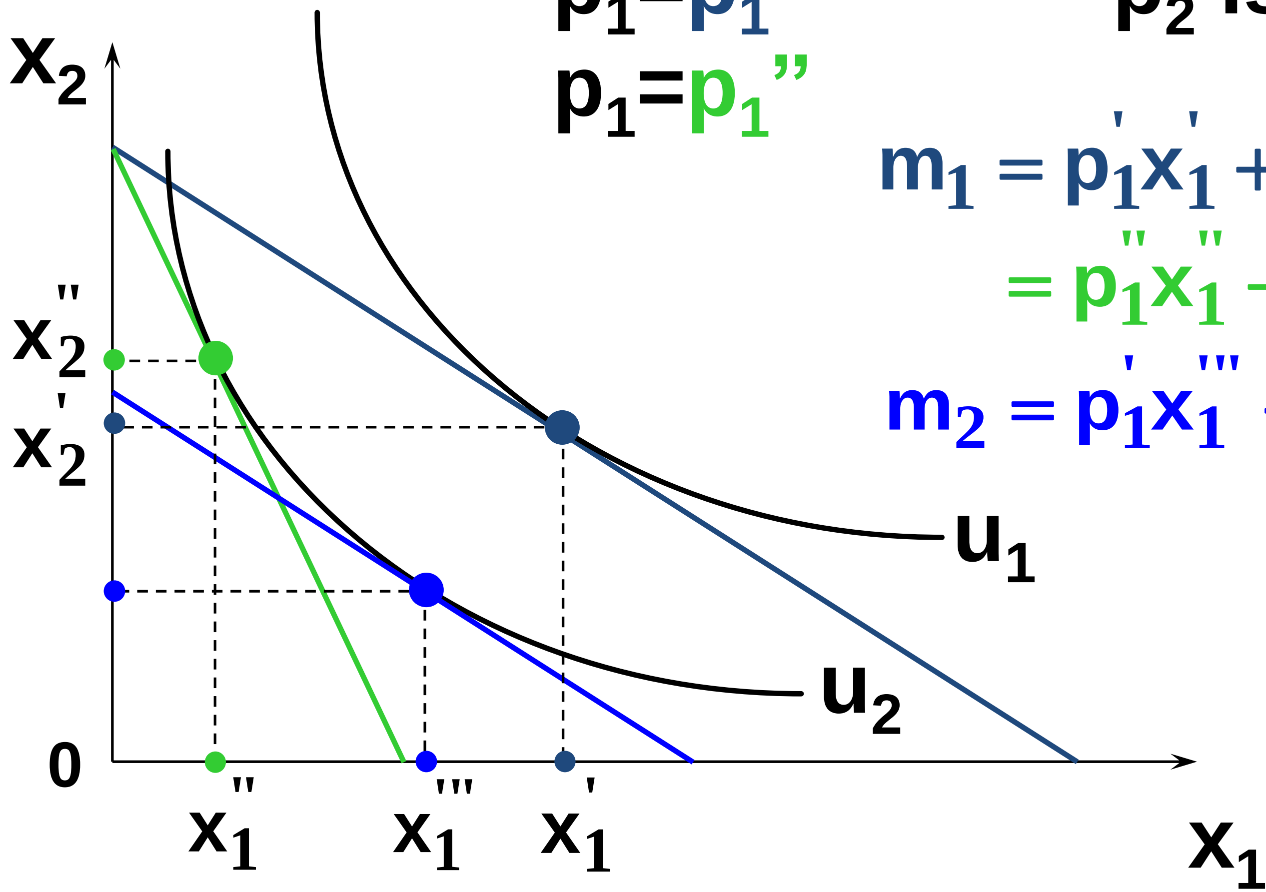
$$m_1 = p_1' x_1' + p_2 x_2'$$
$$= p_1'' x_1'' + p_2 x_2''$$



Equivalent Variation

$p_1 = p_1'$
$p_1 = p_1''$

p_2 is fixed.



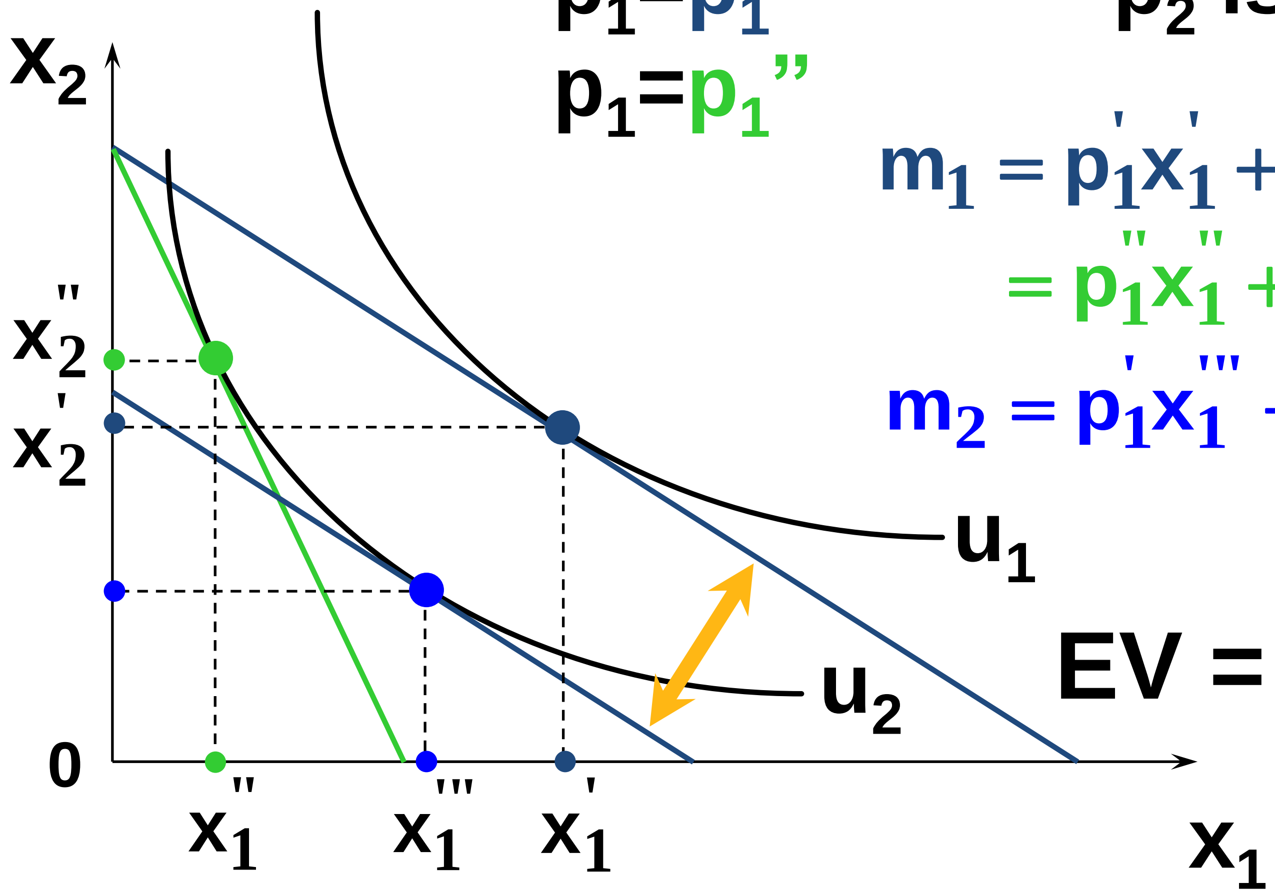
$$m_1 = p_1'x_1' + p_2x_2'$$
$$= p_1''x_1'' + p_2x_2''$$
$$m_2 = p_1'x_1''' + p_2x_2'''$$

Equivalent Variation

$p_1 = p_1'$
$p_1 = p_1''$

p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$
$$= p_1'' x_1'' + p_2 x_2''$$
$$m_2 = p_1' x_1''' + p_2 x_2'''$$



EV = m_1 - m_2.

Consumer's Surplus, Compensating Variation and Equivalent Variation

- Relationship 1: When the consumer's preferences are quasilinear, all three measures are the same.

Consumer’s Surplus, Compensating Variation and Equivalent Variation

- Consider first the change in Consumer’s Surplus when p_1 rises from p_1’ to p_1”.

Consumer's Surplus, Compensating Variation and Equivalent Variation

If $U(x_1, x_2) = v(x_1) + x_2$ **then**

$$CS(p_1') = v(x_1') - v(0) - p_1' x_1'$$

Consumer's Surplus, Compensating Variation and Equivalent Variation

If $U(x_1, x_2) = v(x_1) + x_2$ **then**

$$CS(p_1') = v(x_1') - v(0) - p_1' x_1'$$

and so the change in CS when p_1 rises from p_1' to p_1" is

$$\Delta CS = CS(p_1') - CS(p_1'')$$

Consumer's Surplus, Compensating Variation and Equivalent Variation

If $U(x_1, x_2) = v(x_1) + x_2$ **then**

$$CS(p_1') = v(x_1') - v(0) - p_1'x_1'$$

and so the change in CS when p_1 rises from p_1' to p_1" is

$$\Delta CS = CS(p_1') - CS(p_1'')$$

$$= v(x_1') - v(0) - p_1'x_1' - \left[v(x_1'') - v(0) - p_1''x_1''\right]$$

Consumer's Surplus, Compensating Variation and Equivalent Variation

If $U(x_1, x_2) = v(x_1) + x_2$ **then**

$$CS(p_1') = v(x_1') - v(0) - p_1'x_1'$$

and so the change in CS when p_1 rises from p_1' to p_1" is

$$\Delta CS = CS(p_1') - CS(p_1'')$$

$$= v(x_1') - v(0) - p_1'x_1' - \left[v(x_1'') - v(0) - p_1''x_1''\right]$$

$$= v(x_1') - v(x_1'') - (p_1'x_1' - p_1''x_1'').$$

Consumer's Surplus, Compensating Variation and Equivalent Variation

- Now consider the change in CV when p_1 rises from p_1' to p_1''.
- The consumer's utility for given p_1 is

$$v(x_1^*(p_1)) + m - p_1 x_1^*(p_1)$$

and CV is the extra income which, at the new prices, makes the consumer's utility the same as at the old prices. That is, ...

Consumer's Surplus, Compensating Variation and Equivalent Variation

$$v(x_1') + m - p_1'x_1'$$
$$= v(x_1'') + m + CV - p_1''x_1''.$$

Consumer's Surplus, Compensating Variation and Equivalent Variation

$$v(x_1') + m - p_1' x_1'$$
$$= v(x_1'') + m + CV - p_1'' x_1''.$$

So

$$CV = v(x_1') - v(x_1'') - (p_1' x_1' - p_1'' x_1'')$$
$$= \Delta CS.$$

Consumer's Surplus, Compensating Variation and Equivalent Variation

- Now consider the change in EV when p_1 rises from p_1' to p_1''.
- The consumer's utility for given p_1 is

$$v(x_1^*(p_1)) + m - p_1 x_1^*(p_1)$$

and EV is the extra income which, at the old prices, makes the consumer's utility the same as at the new prices. That is, ...

Consumer's Surplus, Compensating Variation and Equivalent Variation

$$\mathbf{v}(\mathbf{x}_1^{'}) + \mathbf{m} - \mathbf{p}_1^{'}\mathbf{x}_1^{'}$$
$$= \mathbf{v}(\mathbf{x}_1^{''}) + \mathbf{m} + \mathbf{EV} - \mathbf{p}_1^{''}\mathbf{x}_1^{''}.$$

Consumer's Surplus, Compensating Variation and Equivalent Variation

$$v(x_1^{'}) + m - p_1^{'}x_1^{'}$$

$$= v(x_1^{''}) + m + EV - p_1^{''}x_1^{''}.$$

That is,

$$EV = v(x_1^{'}) - v(x_1^{''}) - (p_1^{'}x_1^{'} - p_1^{''}x_1^{''})$$

$$= \Delta CS.$$

Consumer's Surplus, Compensating Variation and Equivalent Variation

So when the consumer has quasilinear utility,

$CV = EV = \Delta CS$.

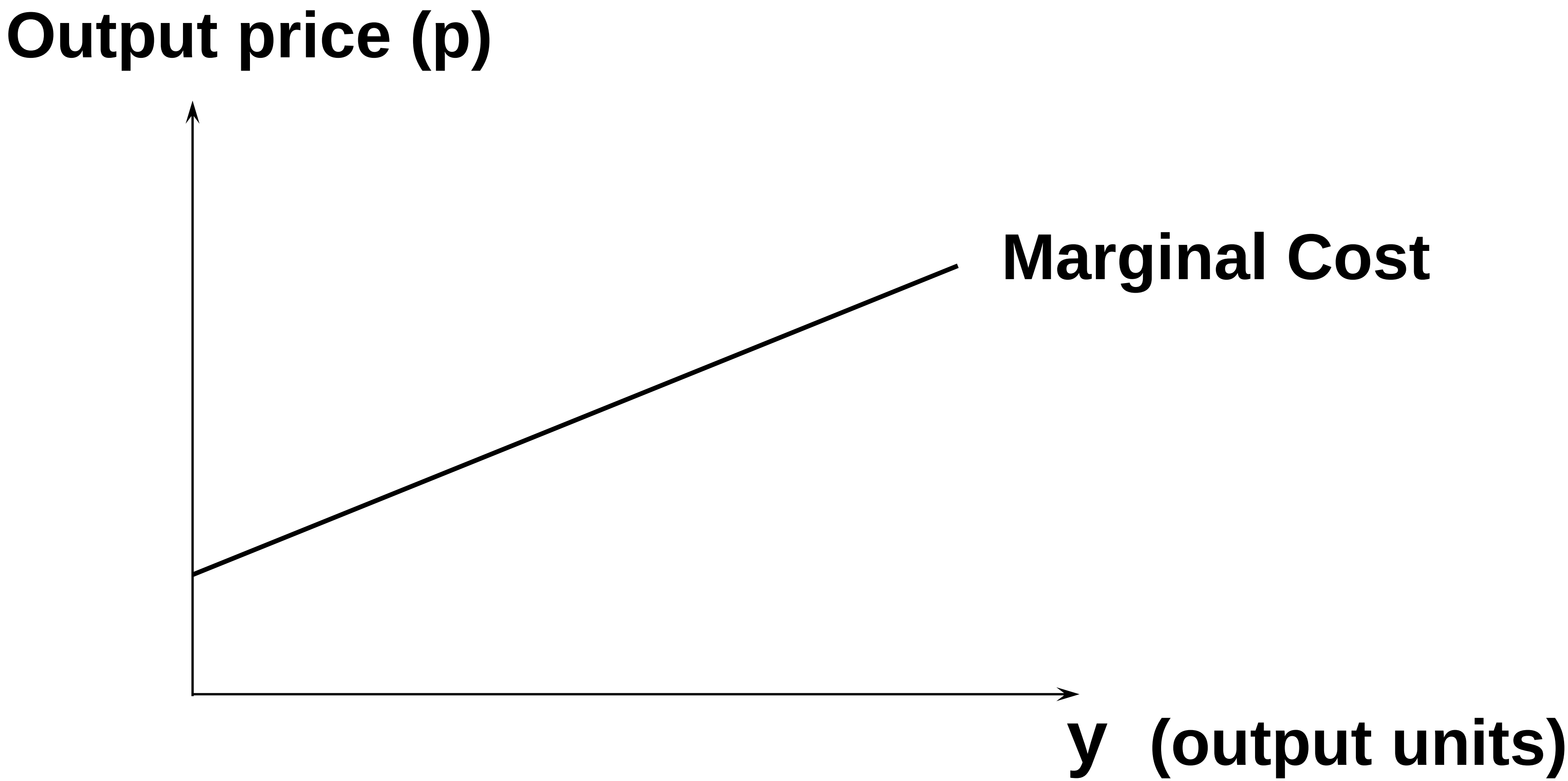
But, otherwise, we have:

Relationship 2: In size, $EV < \Delta CS < CV$.

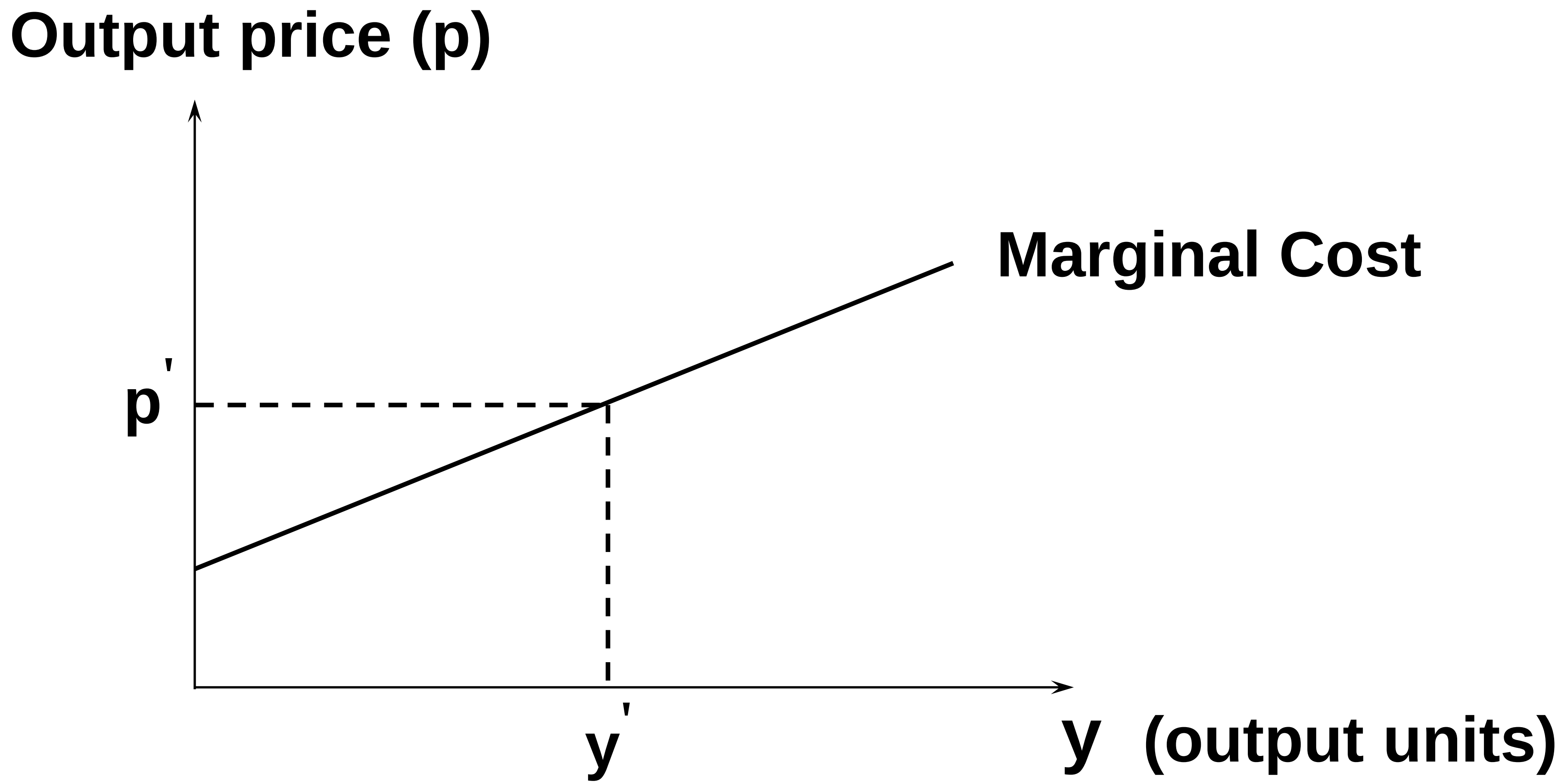
Producer's Surplus

- Changes in a firm's welfare can be measured in euros much as for a consumer.

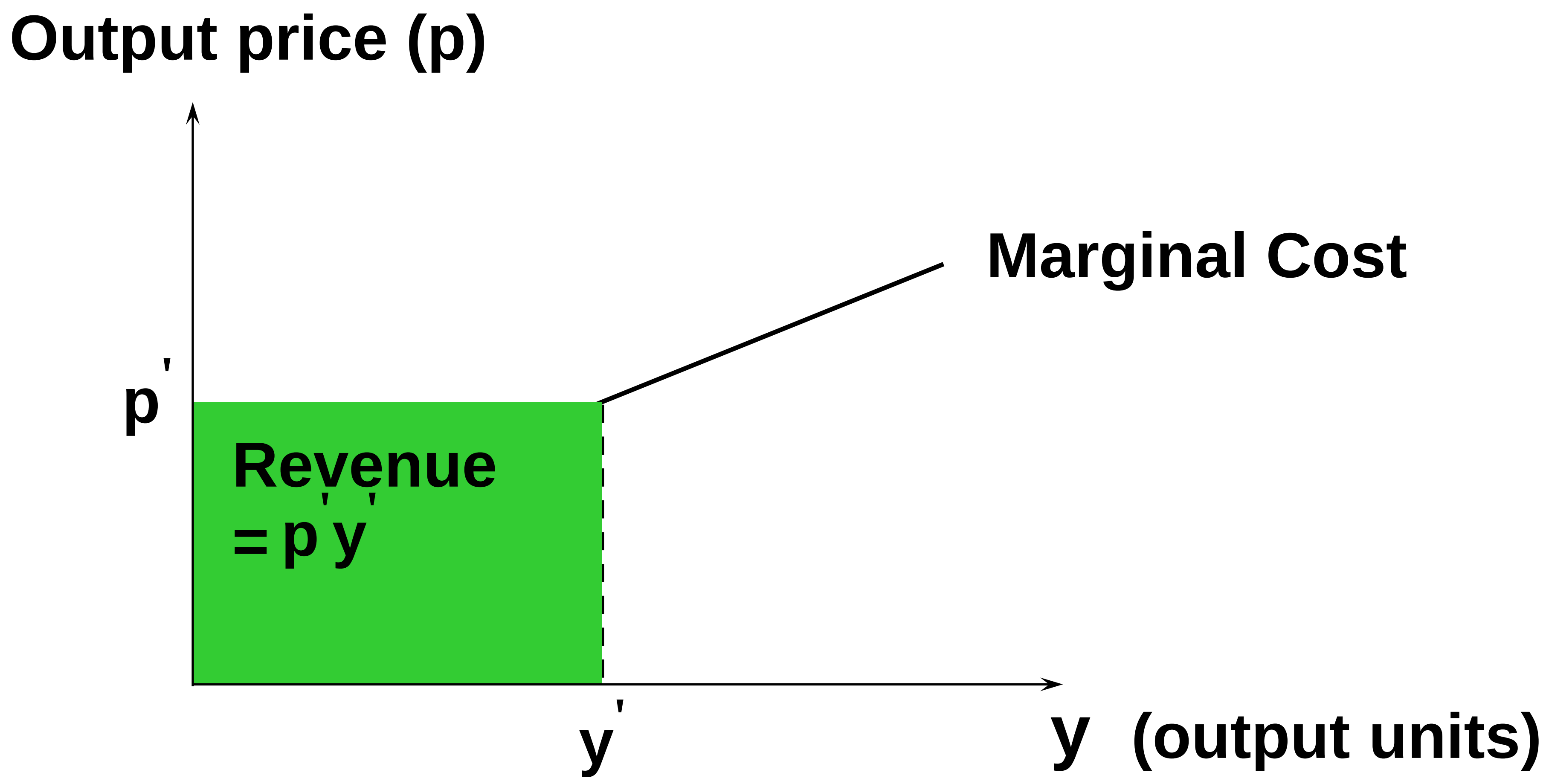
Producer's Surplus



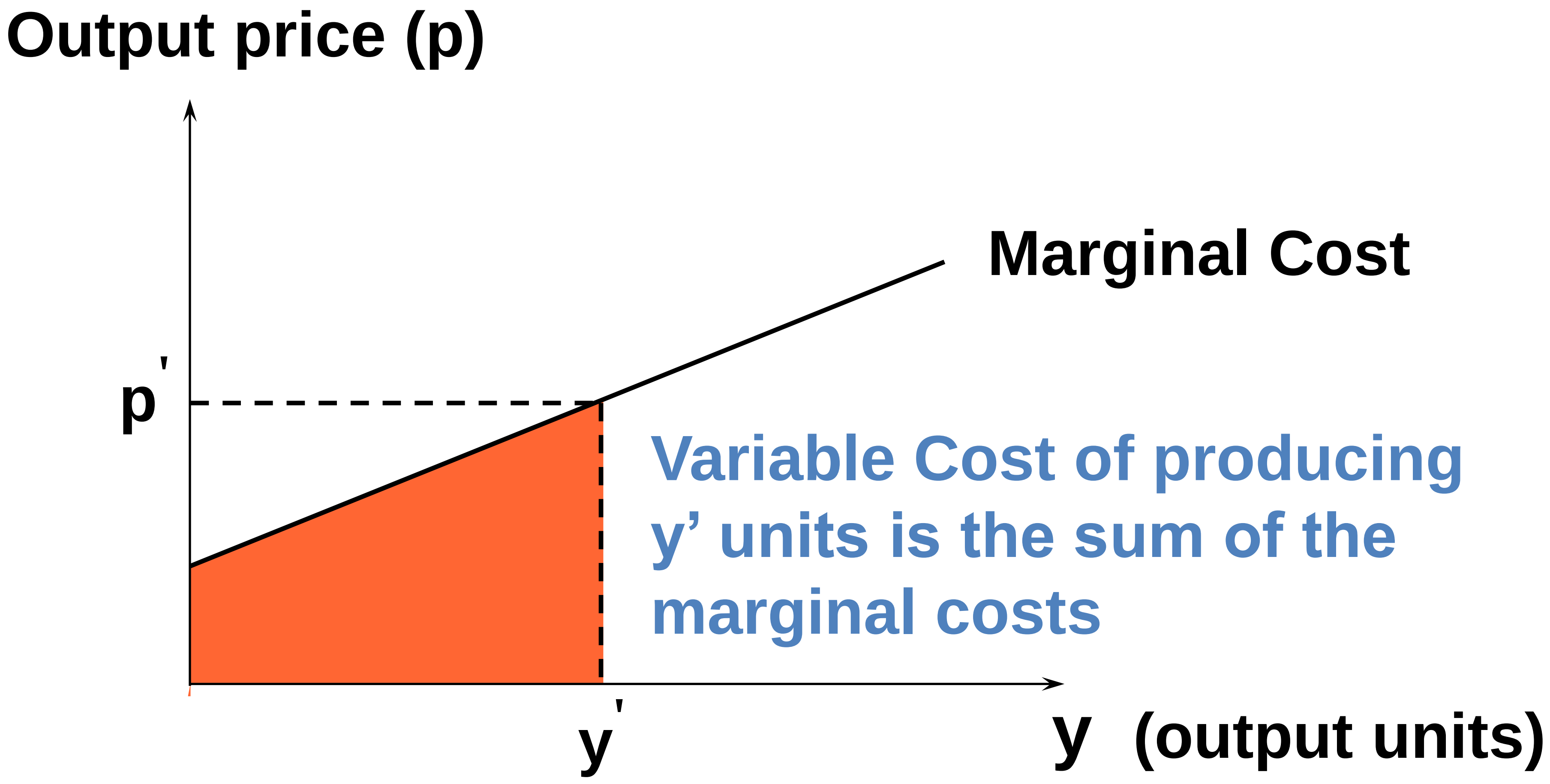
Producer's Surplus



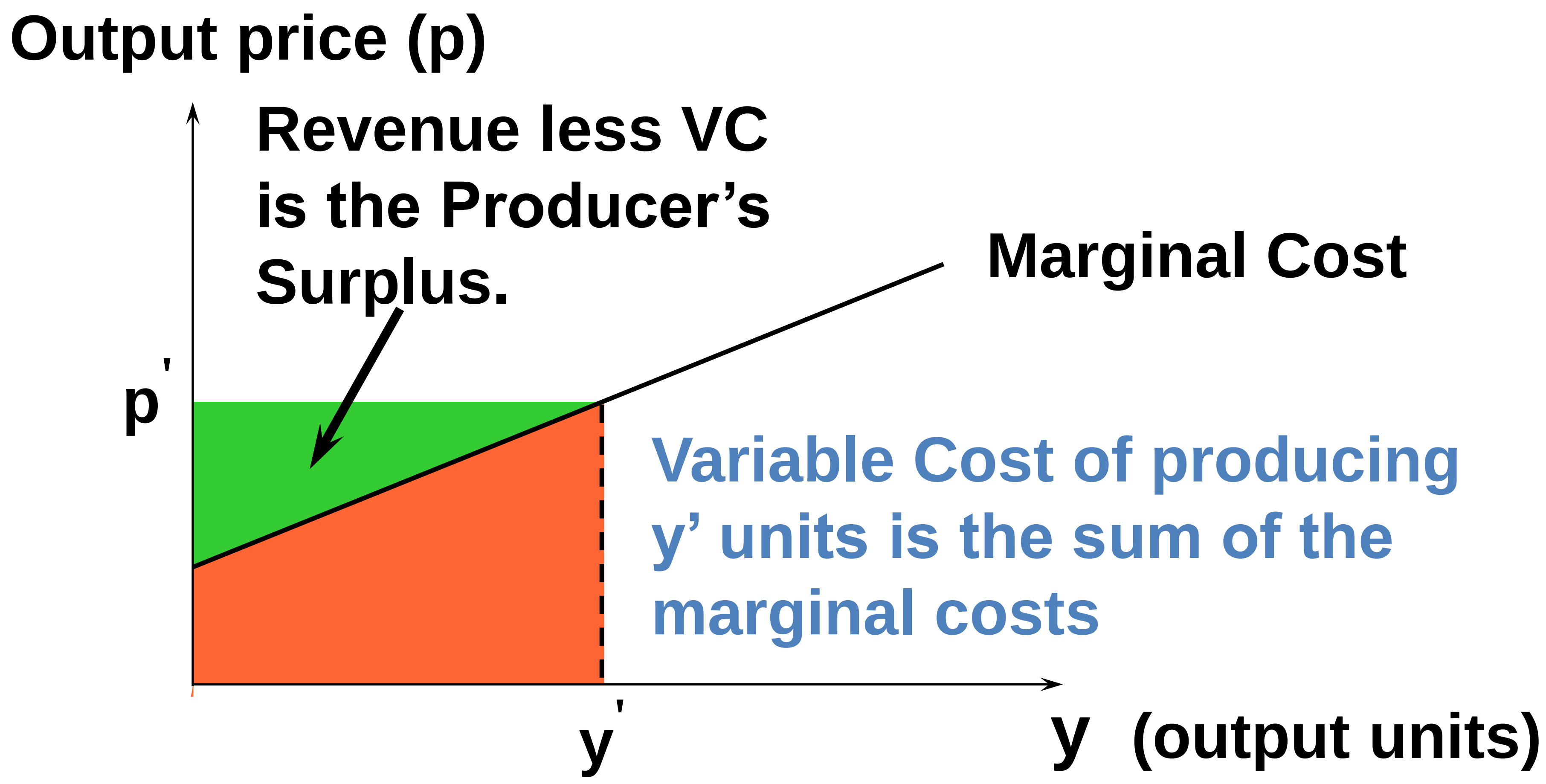
Producer's Surplus



Producer's Surplus



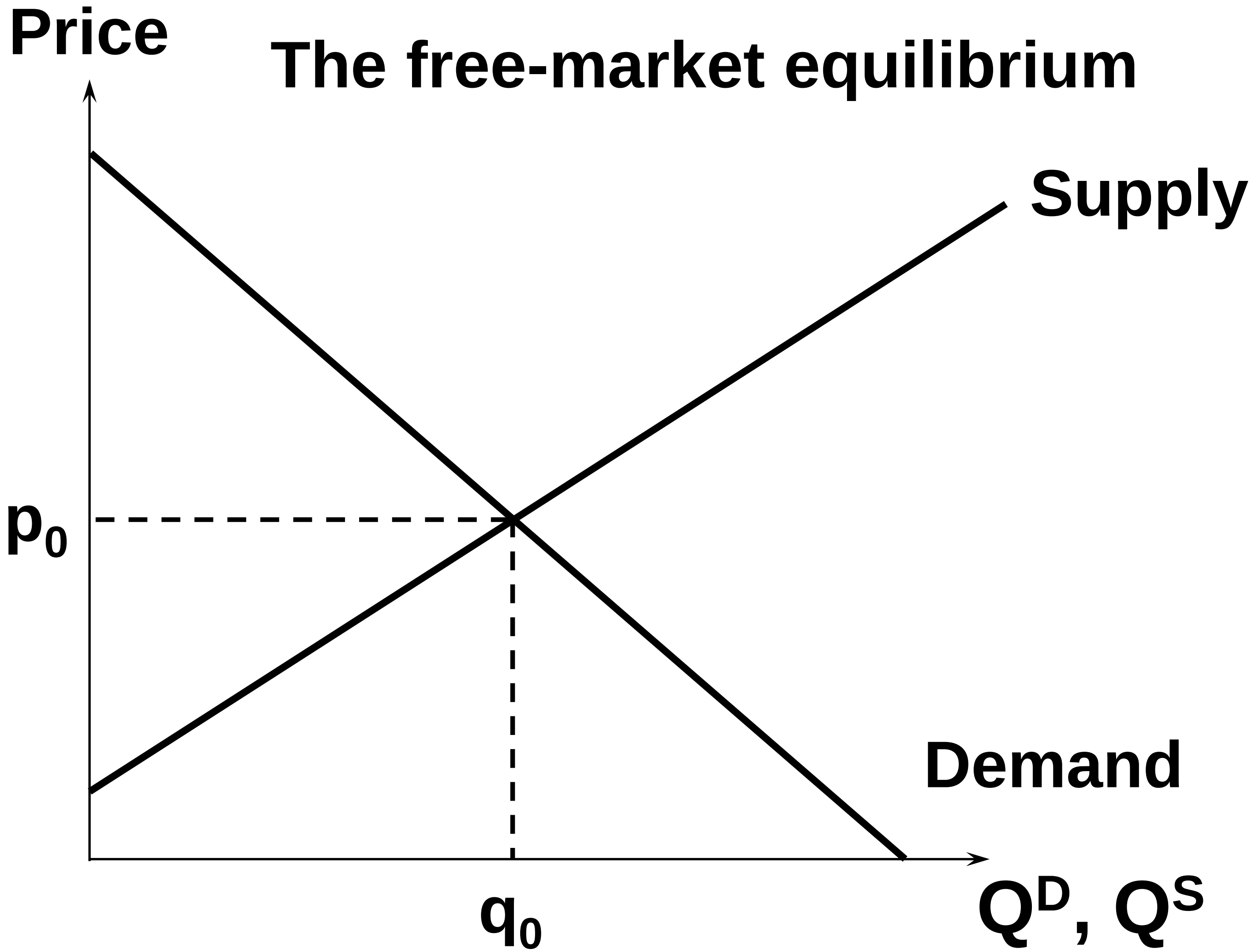
Producer's Surplus



Benefit-Cost Analysis

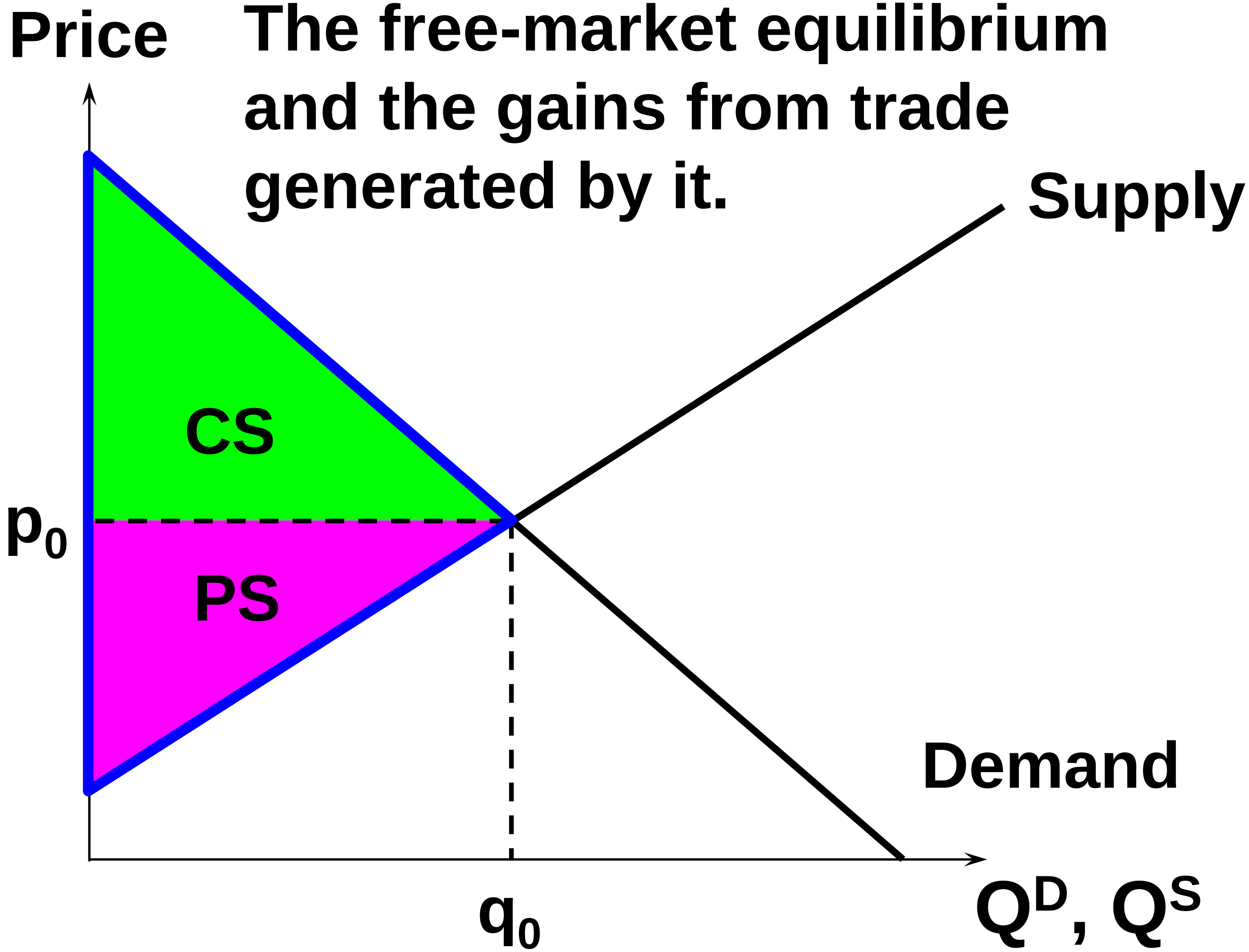
- Can we measure in money units the net gain, or loss, caused by a market intervention; *e.g.*, the imposition or the removal of a market regulation?
- Yes, by using measures such as the Consumer’s Surplus and the Producer’s Surplus.

Benefit-Cost Analysis

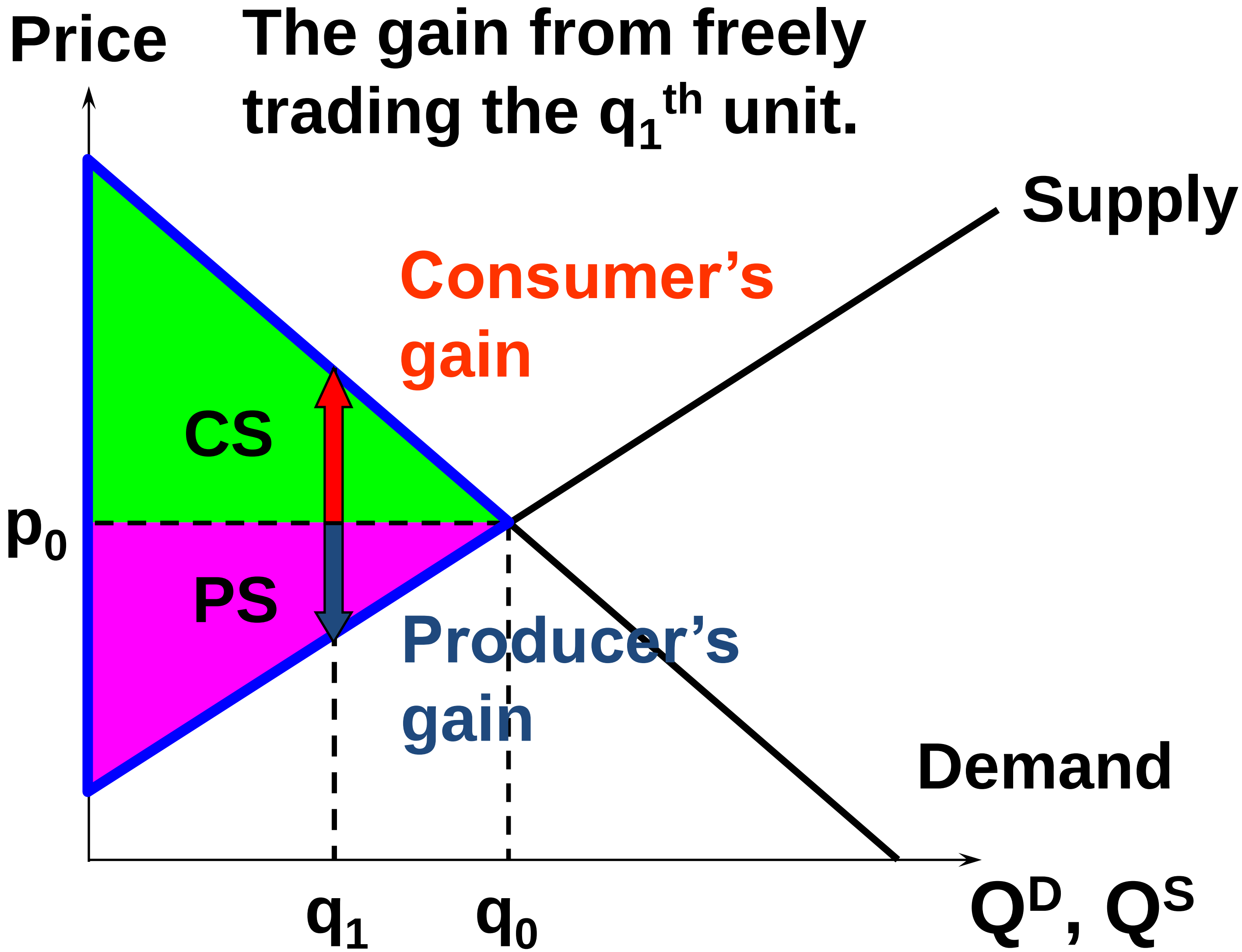


Benefit-Cost Analysis

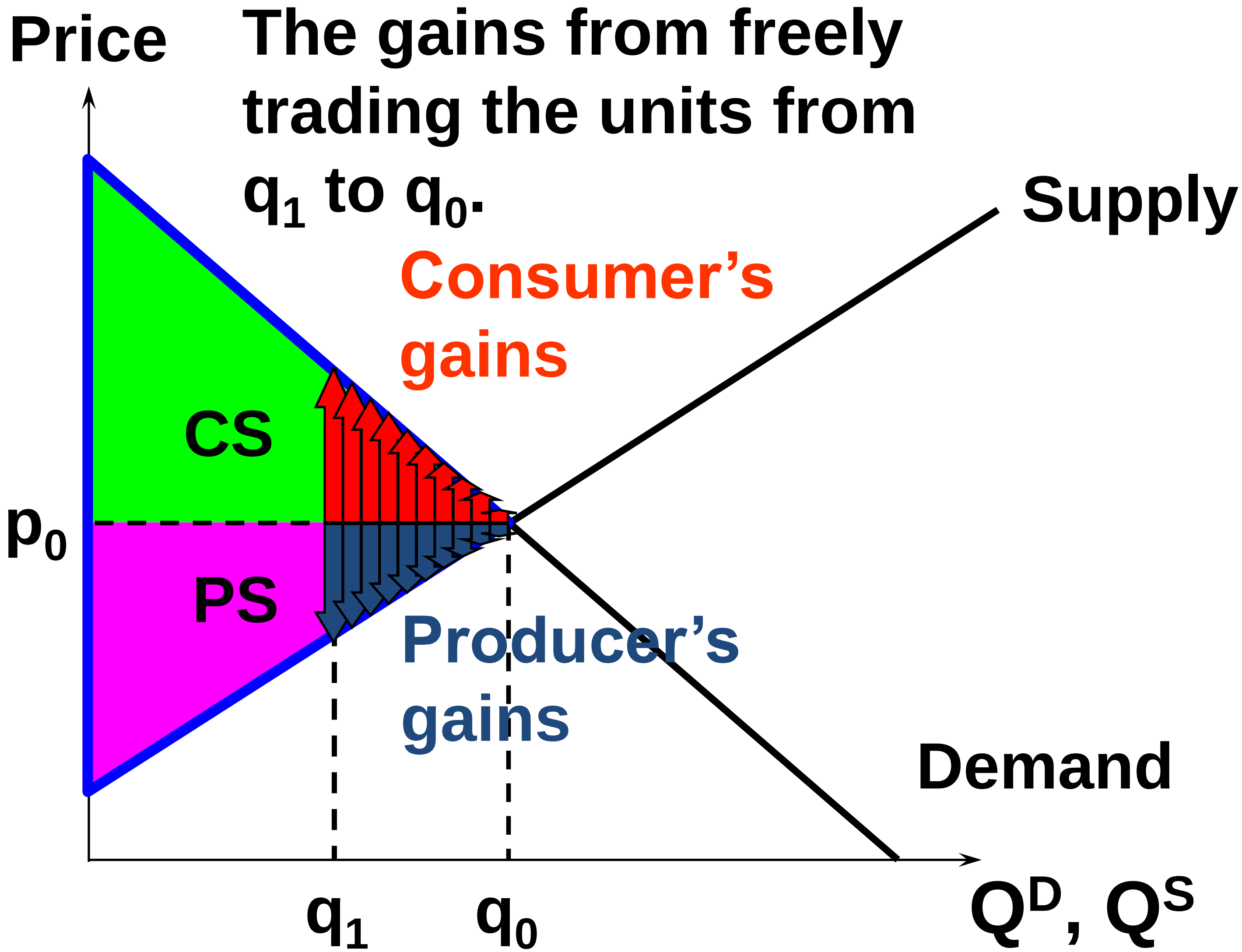
The free-market equilibrium and the gains from trade generated by it.



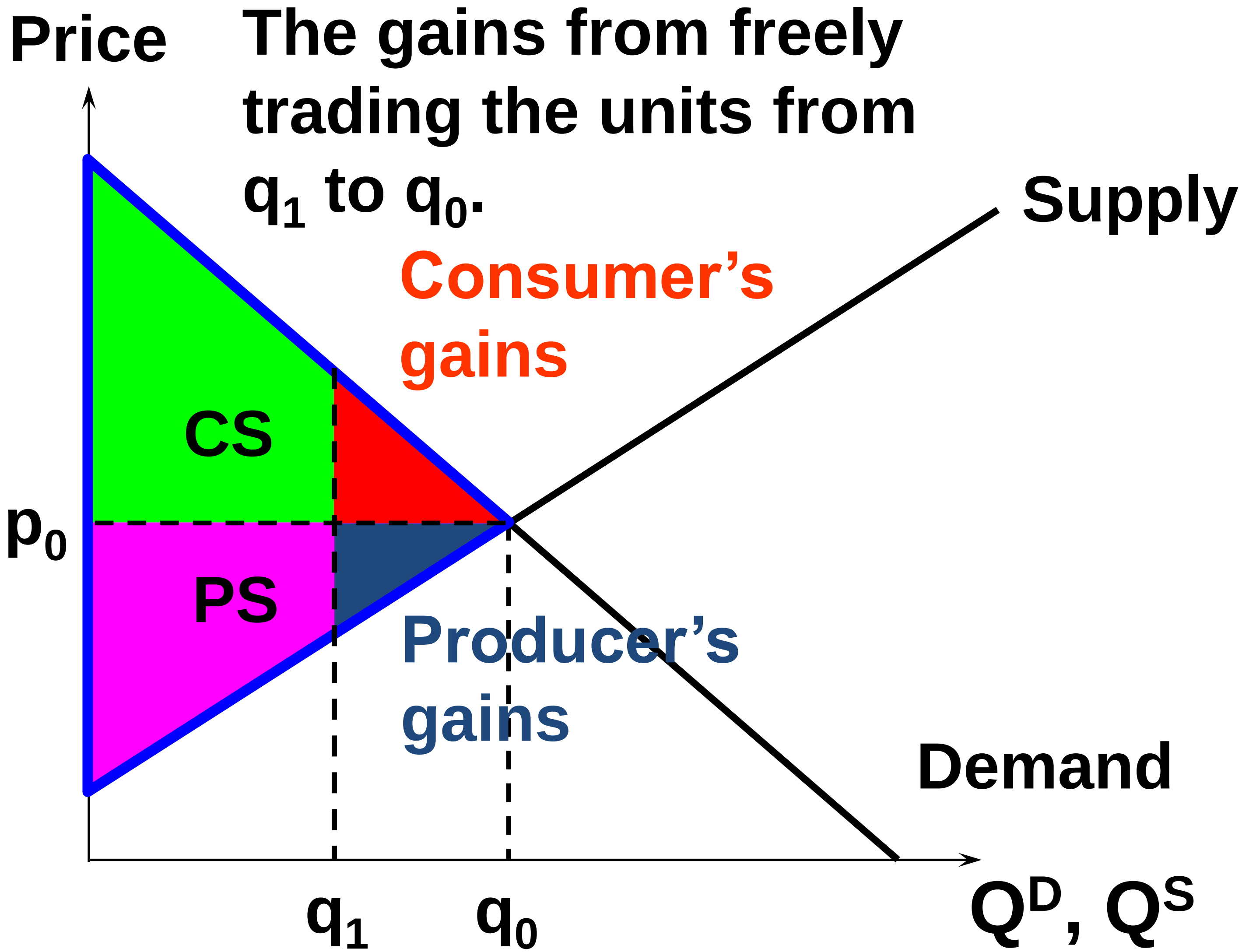
Benefit-Cost Analysis



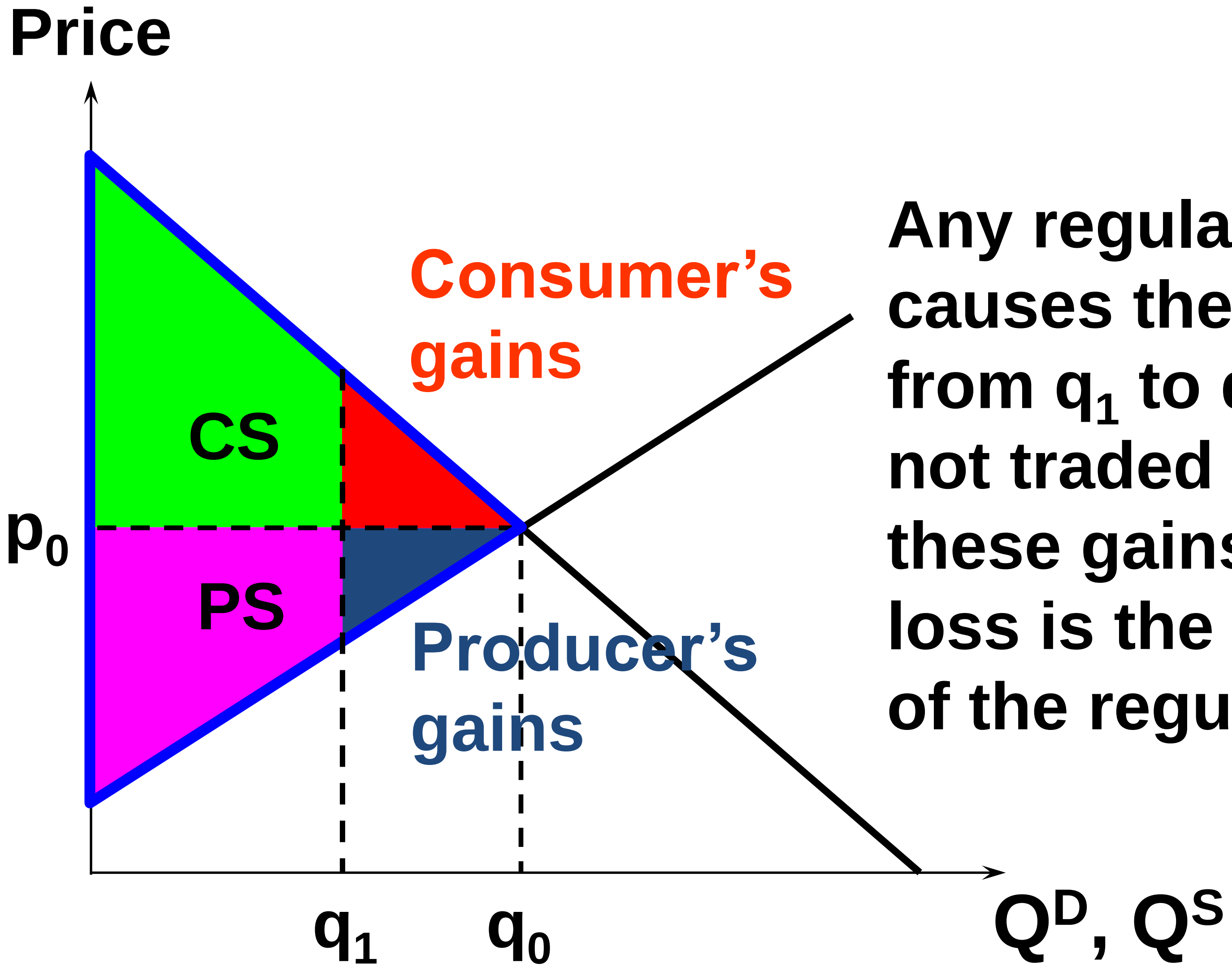
Benefit-Cost Analysis



Benefit-Cost Analysis



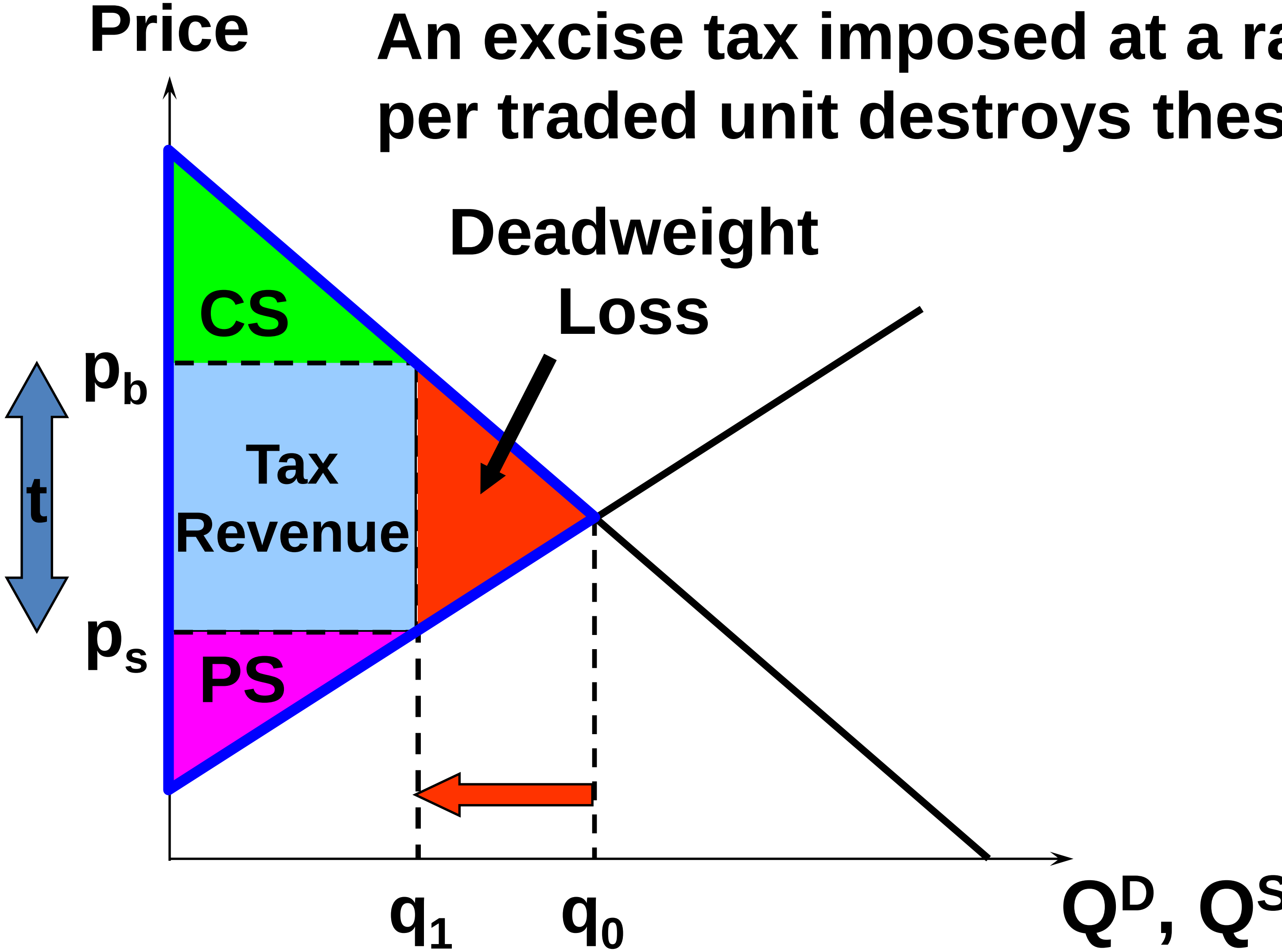
Benefit-Cost Analysis



Any regulation that causes the units from q_1 to q_0 to be not traded destroys these gains. This loss is the net cost of the regulation.

Benefit-Cost Analysis

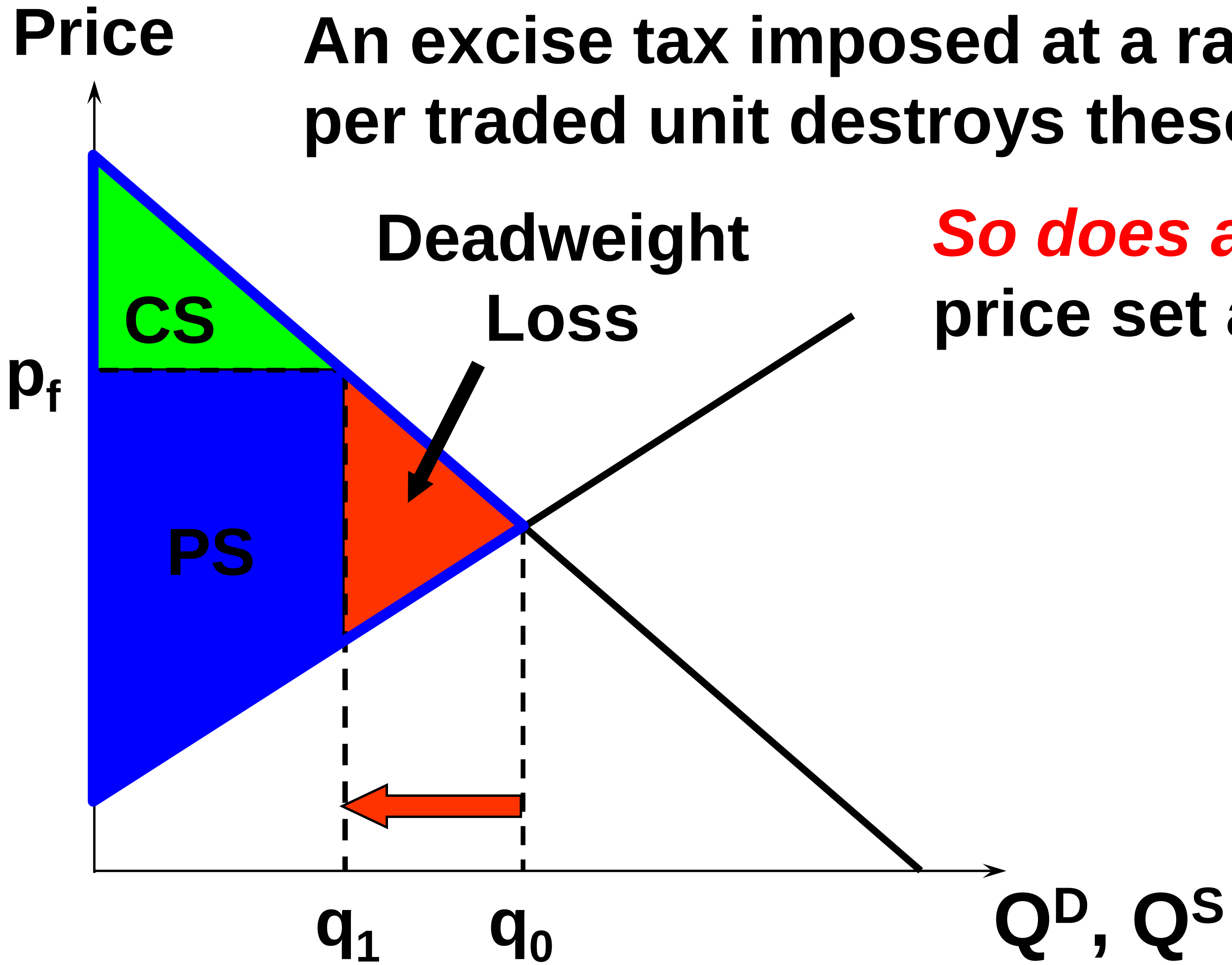
An excise tax imposed at a rate of €t per traded unit destroys these gains.



Benefit-Cost Analysis

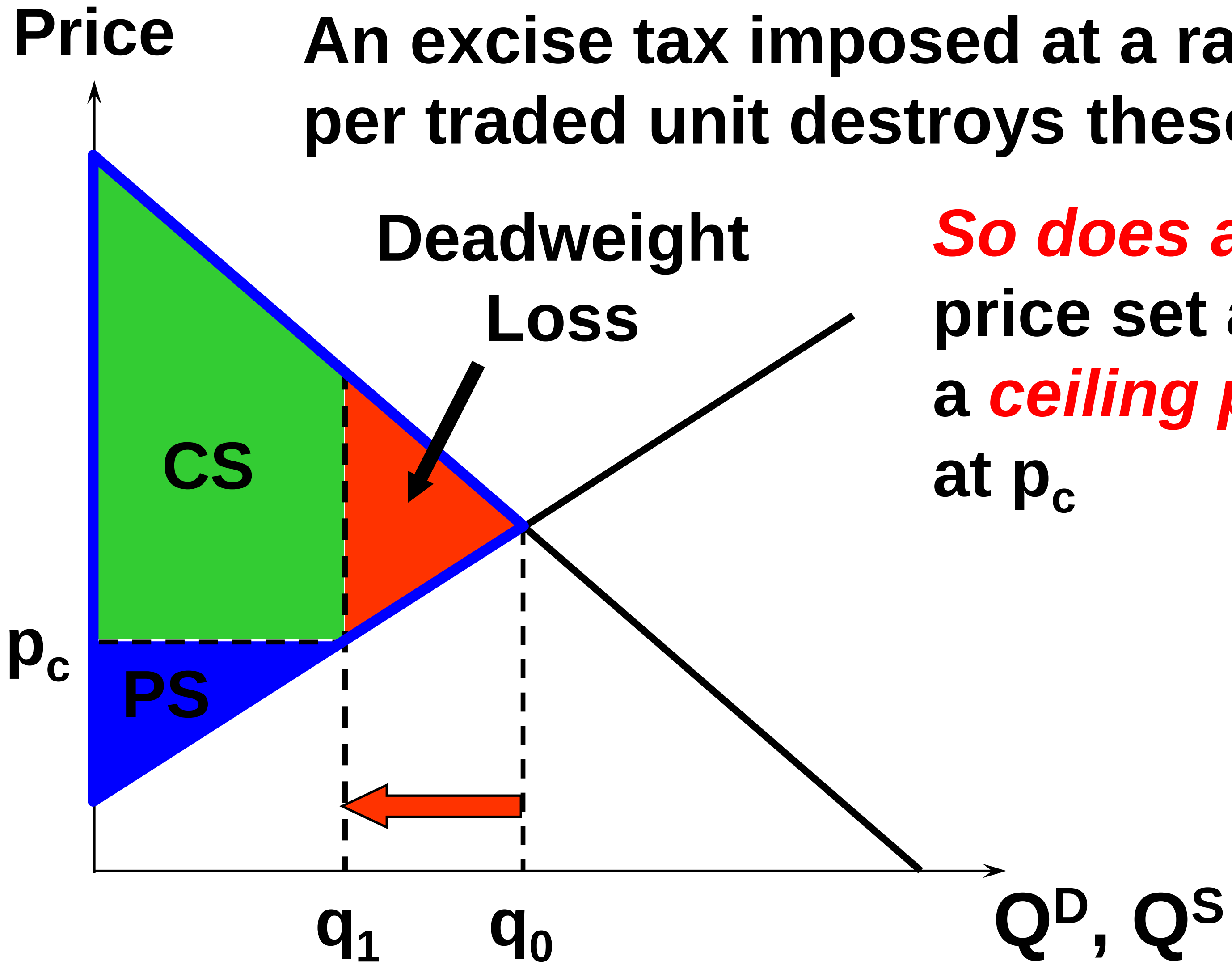
An excise tax imposed at a rate of €t per traded unit destroys these gains.

So does a floor **price set at p_f**



Benefit-Cost Analysis

An excise tax imposed at a rate of €t per traded unit destroys these gains.

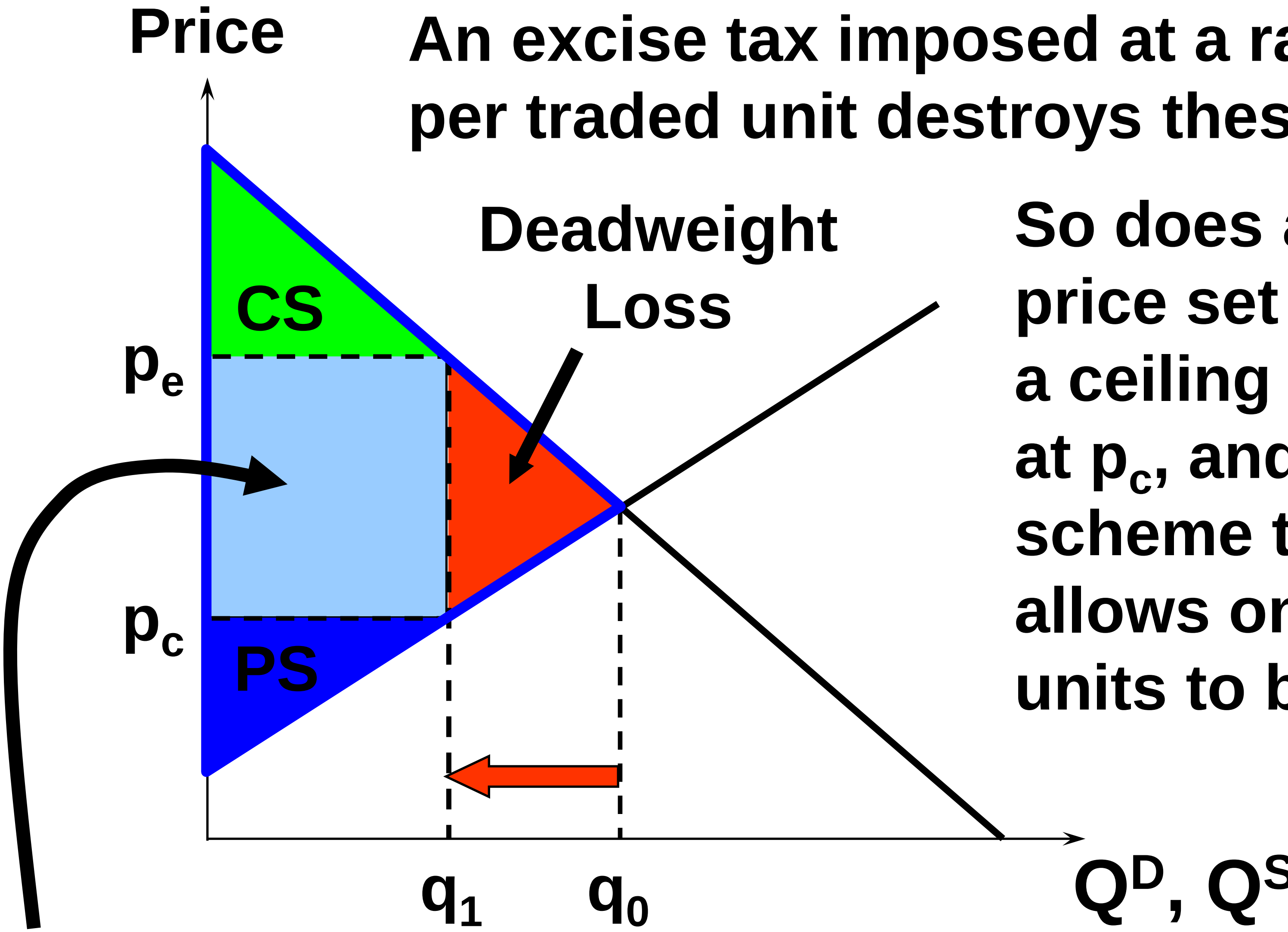


So does a floor **price set at p_f, a** ***ceiling price*** **set at p_c**

Benefit-Cost Analysis

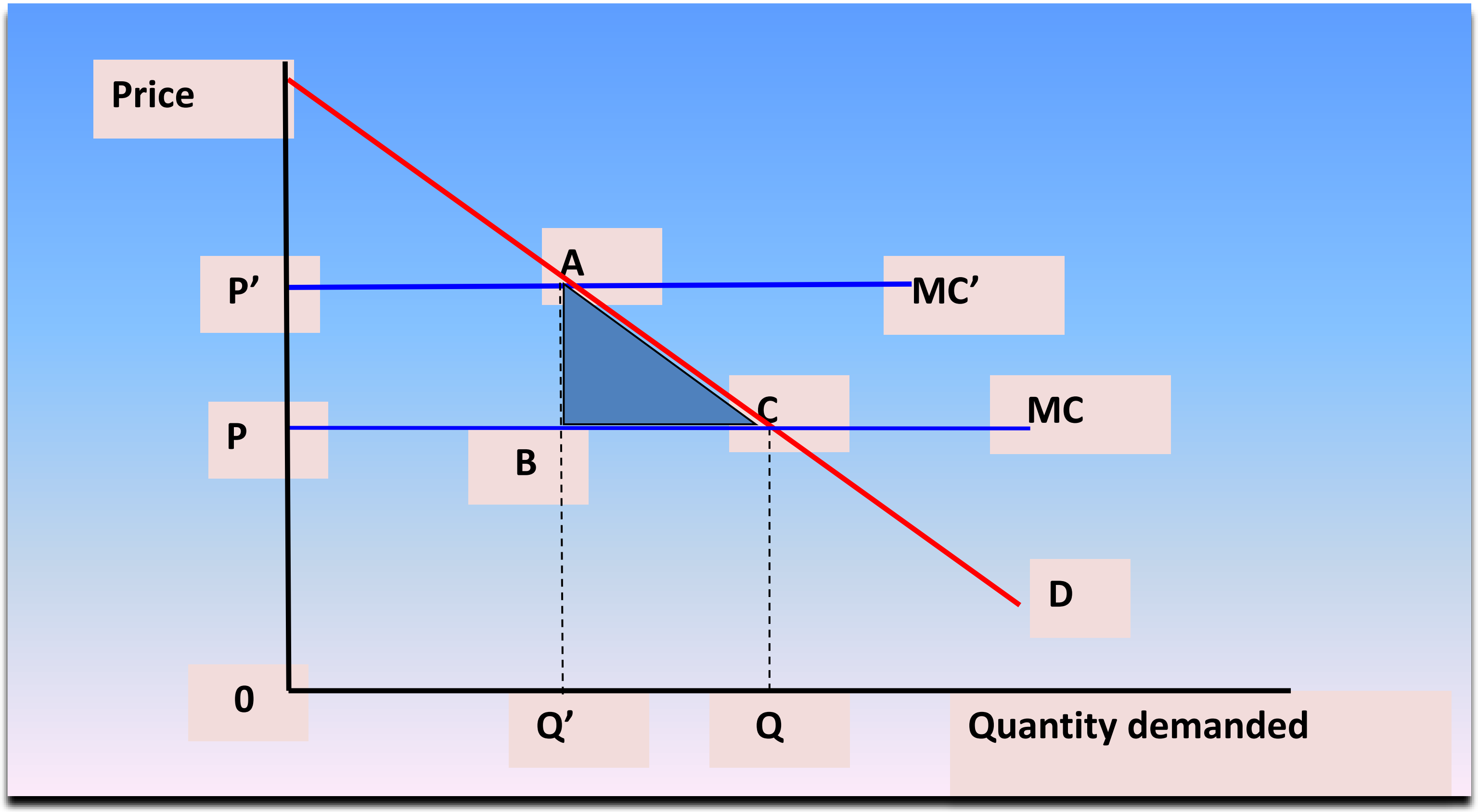
An excise tax imposed at a rate of €t per traded unit destroys these gains.

So does a floor price set at p_f, a ceiling price set at p_c, and a ration scheme that allows only q_1 units to be traded.



Revenue received by holders of ration coupons.

Measuring deadweight loss



Measuring deadweight loss

$$DWL = \frac{1}{2}(\Delta P)x(\Delta Q)$$

$$e = \frac{\Delta Q}{\Delta P}\frac{P}{Q}$$

$$EB = \frac{1}{2}(\Delta P)x(e\Delta P\frac{Q}{P})$$

$$EB = \frac{1}{2}e\frac{Q}{P}t^2$$