Inderjit Singh, Lyn Squire
and John Strauss (editors)
Agricultural Household Models:
Extensions, Applications & Policy.
The John Hopkins University Press
for the World Bank, 1986, Baltimore

1

The Basic Model: Theory, Empirical Results, and Policy Conclusions

Inderjit Singh, Lyn Squire, and John Strauss

THE BASIC MODEL PRESENTED HERE is the analytical framework used in most of the early empirical efforts to investigate the behavior of agricultural households. A more general analytical framework is described in the appendix to part I. Many of the case studies presented in part II illustrate how the basic model can be expanded to treat a wider range of policy issues and how it can be modified to reflect more accurately the realities of agricultural production. For the present, however, attention is focused on fundamentals.

The Basic Model

For any production cycle, the household is assumed to maximize a utility function:

$$U = U(X_a, X_m, X_l) \qquad (1\text{-}1)$$

where the commodities are an agricultural staple (X_a), a market-purchased good (X_m), and leisure (X_l). Utility is maximized subject to a cash income constraint:

$$p_m X_m = p_a(Q - X_a) - w(L - F)$$

where p_m and p_a are the prices of the market-purchased commodity and the staple, respectively, Q is the household's production of the staple (so that $Q - X_a$ is its marketed surplus), w is the market wage, L is total labor input, and F is family labor input (so that $L - F$, if positive, is hired labor and, if negative, off-farm labor supply).

The household also faces a time constraint—it cannot allocate more time to leisure, on-farm production, or off-farm employment than the total time available to the household:

$$X_l + F = T$$

where T is the total stock of household time. It also faces a production constraint or production technology that depicts the relation between inputs and output:

$$Q = Q(L, A)$$

where A is the household's fixed quantity of land.

In this presentation, various complexities have been omitted. For example, other variable inputs—fertilizer, pesticide—have been omitted and the possibility that more than one crop is being produced has also been ignored. In addition, it has been assumed that family labor and hired labor are perfect substitutes and can be added directly. Production is also assumed to be riskless. Finally, and perhaps most importantly, it will be assumed that the three prices in the model—p_a, p_m, and w—are not affected by actions of the household. That is, the household is assumed to be a price-taker in the three markets and, as argued in the Introduction, this will result in a recursive model. At various points in this volume, each of these assumptions will be abandoned, but for much of the discussion in this chapter they will be retained.

The three constraints on household behavior can be collapsed into a single constraint. Substituting the production constraint into the cash income constraint for Q and substituting the time constraint into the cash income constraint for F yields a single constraint of the form

(1-2) $$p_m X_m + p_a X_a + w X_l = wT + \pi$$

where $\pi = p_a Q(L, A) - wL$ and is a measure of farm profits. In this equation, the left-hand side shows total household "expenditure" on three items—the market-purchased commodity, the household's "purchase" of its own output, and the household's "purchase" of its own time in the form of leisure. The right-hand side is a development of Becker's concept of full income in which the value of the stock of time (wT) owned by the household is explicitly recorded. The extension for agricultural households includes a measure of farm profits ($p_a Q - wL$) with all labor valued at the market wage, this being a consequence of the assumption of price-taking behavior in the labor market. Equations 1-1 and 1-2 are the core of all the studies of agricultural households reported in this volume.

In these equations, the household can choose the levels of consumption for the three commodities and the total labor input into agricultural production. We therefore need to explore the first-order conditions for maximizing each of these choice variables. Consider labor input first. The first-order condition is:

(1-3) $$p_a \partial Q / \partial L = w.$$

That is, the household will equate the marginal revenue product of labor to the market wage. An important attribute of this equation is that it contains only one endogenous variable, L. The other endogenous variables—X_m, X_a, X_l—do not appear and therefore do not influence the household's choice of L. Accordingly, equation 1-3 can be solved for L as a function of prices (p_a and w), the technological parameters of the production function, and the fixed area of land. This result parallels that described in the Introduction in that production decisions can be made independently of consumption and labor-supply (or leisure) decisions.

Let the solution for L be

(1-4) $$L^* = L^*(w, p_a, A).$$

This solution can then be substituted into the right-hand side of the constraint (equation 1-2) to obtain the value of full income when farm profits have been maximized through an appropriate choice of labor input. We could, therefore, rewrite equation 1-2 as

$$p_m X_m + p_a X_a + w X_l = Y^*$$

where Y^* is the value of full income associated with profit-maximizing behavior. Maximizing utility subject to this new version of the constraint yields the following first-order conditions:

(1-5)
$$\partial U / \partial X_m = \lambda p_m$$
$$\partial U / \partial X_a = \lambda p_a$$
$$\partial U / \partial X_l = \lambda w$$

and

$$p_m X_m + p_a X_a + w X_l = Y^*$$

which are the standard conditions from consumer-demand theory.

The solution to equation 1-5 yields standard demand curves of the form

(1-6) $$X_i = X_i(p_m, p_a, w, Y^*) \qquad i = m, a, l.$$

That is, demand depends on prices and income. In the case of the agricultural household, however, income is determined by the household's production activities. It follows that changes in factors influencing production will change Y^* and hence consumption behavior. Consumption behavior, therefore, is not independent of production behavior. This establishes the recursive property of the model described in the Introduction.

To complete this section, we derive the "profit effect" also mentioned in the Introduction. Assume that the price of the agricultural staple is increased. What is the effect on consumption of the staple? From equation 1-6,

$$\frac{dX_a}{dp_a} = \frac{\partial X_a}{\partial p_a} + \frac{\partial X_a}{\partial Y^*}\frac{\partial Y^*}{\partial p_a}. \tag{1-7}$$

The first term on the right-hand side is the standard result of consumer-demand theory and, for a normal good, is negative. The second term captures the profit effect. A change in the price of the staple increases farm profits and hence full income. From equation 1-7,

$$\frac{\partial Y^*}{\partial p_\alpha} dp_\alpha = \frac{\partial \pi}{\partial p_\alpha} dp_\alpha = Q\, dp_\alpha .$$

That is, the profit effect equals output times the change in price and is, therefore, unambiguously positive. As noted in the Introduction, the positive effect of an increase in profits—an effect that is totally ignored in traditional models of demand—will definitely dampen and may outweigh the negative effect of standard consumer-demand theory.

Estimation Issues

Given a recursive model, a set of output-supply and variable input-demand functions (equation 1-4) and a set of commodity-demand equations including leisure or labor supply (see equation 1-6) can be derived from the household's equilibrium. The output supplies and input demands are functions of input and output prices and of farm characteristics (including fixed inputs). They are derived from a profit function that obeys the usual constraints from the theory of the firm: homogeneity of degree one in prices, and convexity with respect to prices. The commodity demands are functions of commodity prices, full income, and possibly household characteristics (see below). When full income is held constant, these demands satisfy the usual constraints of demand theory: adding up to total expenditure; zero homogeneity with respect to prices and exogenous income; and symmetry and negative semidefiniteness of the Slutsky-substitution matrix. These results can be used as a guide when specifying the model for estimation.

If estimation is to be carried out by econometric means, errors have to be added to the model. The issues involved in specifying a sensible error structure are outside the scope of this chapter. For simplicity, suppose the errors are added to the demand and output-supply equations. If for a given household the errors on the input-demand and output-supply equations are uncorrelated with the errors on the commodity-demand equations, the entire system of equations is statistically block recursive. In this case, profits will be uncorrelated with the commodity-demand disturbances so that the latter equations may be consistently estimated as a system independent from the output-supply and input-demand equations. The practical advantage of estimating the demand and production sides of the model separately is that far fewer parameters need to be estimated for each. This can be important if the equations are nonlinear in parameters and have to be estimated using numerical algorithms, since expense is greatly reduced and tractability increased. Thus models with greater detail can be estimated.

Even though demand-side and production-side errors are uncorrelated, errors on different commodity-demand equations may still be correlated, as might errors of different output-supply and input-demand equations. This is intuitively plausible. Moreover, it is a necessary condition for the commodity-demand equations, since they must satisfy the adding-up constraint; that is, expenditures must add up to full income. If this constraint is to be met for every household, the errors, or a linear combination of them, must add up to zero for each household so that the result is nonzero correlations. This result is well known and is one reason for estimating either the commodity-demand equations or the output-supply and input-demand equations as a system: accounting for the error covariances will improve the statistical efficiency of the estimates. A second reason for estimating these equations as a system (or, more properly, two separate systems, one for the commodity demands and one for the output supplies and input demands) is to account for cross-equation parameter restrictions. These will occur because these equations are derived from a common optimizing problem. In particular, the adding up and the Slutsky symmetry constraints will impose certain cross-equation constraints on commodity-demand parameters, which, if used (and if they are correct), will again improve the statistical efficiency of the estimates. These advantages are well known and have given rise to an econometric literature on estimation of demand systems (see, for example, Brown and Deaton 1972; Barten 1977; Deaton and Muellbauer 1980).

One does not have to estimate a system of equations, since single, reduced-form equations can be consistently estimated as well. This will be advantageous when the underlying model is not recursive (see chapter 2).

The disadvantage of this approach is that it is usually not possible to solve for the reduced form analytically. Consequently, one cannot take full advantage of economic theory in imposing (or testing) parameter restrictions, although some of the restrictions may be readily apparent. Nevertheless, it is possible to specify what variables belong in the reduced form and thus to estimate a least squares approximation to it. In general, by not imposing parameter restrictions one sacrifices only statistical efficiency, and not consistency.

Even if the underlying model is recursive, estimating a single equation may be advantageous because it can economize data requirements. To estimate a complete set of commodity-demand, output-supply, and input-demand equations requires an enormous amount of data on consumption expenditures and prices for farm and nonfarm commodities; on household time allocation to on-farm and off-farm work and related wages; and on inputs and outputs of the production activities. To estimate a single equation, however, the analyst needs data on only one endogenous variable and the proper exogenous variables, but not on all the endogenous variables. (Other aspects of estimation—data requirements, specification of variables—are discussed in chapter 2.)

Empirical Results

The first empirical studies to give estimates of agricultural household models (Lau, Lin, and Yotopoulos 1978; Yotopoulos, Lau, and Lin 1976; Kuroda and Yotopoulos 1978, 1980; Adulavidhaya and others 1979; Adulavidhaya, Kuroda, Lau, and Yotopoulos 1984; and Barnum and Squire 1978, 1979a, b) are econometric studies that specify separable models and that estimate commodity demands and either output supply and input demands or a production function. They are highly aggregative on the demand side and use one agricultural commodity produced and consumed by the household (our X_a), one nonagricultural commodity that can only be purchased (our X_m), and leisure (our X_l). Kuroda and Yotopoulos decompose leisure into leisure of family members who work on the farm and leisure of these working off the farm. Those working off the farm are therefore different people with different labor quality than those working on the farm. To make the model separable they also implicitly assume hired labor is used on the farm. All the studies provide more detail on the production side and thus allow for several variable and fixed inputs.

Lau, Lin, and Yotopoulos (1978) look at Taiwanese household data averaged by farm size and by region for each of two years. Kuroda and Yotopoulos (1978, 1980) use cross-sectional household data from Japan, also grouped by farm size and by region. Adulavidhaya and others (1979, 1984) use cross-sectional household data from Thailand, but the cross sections differ for the production and consumption sides of the model. This approach considers that the two sets of households behave identically and is possible only because the model is recursive. Otherwise, data on the same set of households would be necessary. Barnum and Squire (1978, 1979a, b) use cross-sectional household data from the Muda River Valley in Malaysia. Both the Malaysian and Thai households practice monoculture (rice cultivation), so that aggregation on the production side is not a problem. It was possible to estimate price elasticities for Taiwan, Japan, and Thailand because prices vary by region (and over time in Taiwan). In Malaysia only, wages vary. By making sufficiently strong assumptions about preferences, however, price elasticities could be calculated.

These four studies use the systems approach to estimate commodity demands. Lau, Lin, and Yotopoulos (1978), Kuroda and Yotopoulos (1980), and Adulavidhaya and others (1984) use the Linear Logarithmic Expenditure System (LLES), whereas Barnum and Squire (1979a, b) use a Linear Expenditure System (LES). The LLES is derived from a translog indirect utility function that is homogeneous of degree minus one in prices. This implies that every expenditure elasticity with respect to full income is one—which is a restrictive assumption, particularly if one specifies many commodities. That fact that LLES is linear in parameters, however, makes estimation simpler. The LES is derived from an additive utility function, the Stone-Geary. It has fewer parameters to estimate than an LLES, but is nonlinear in parameters. Since the system is additive, Engel curves must be linear and no Hicks-complementarity between commodities is allowed for. As is true for the LLES, these conditions become less restrictive when commodities are highly aggregated. According to Deaton (1978), however, additivity should be rejected even then.

In all of these studies, household characteristics such as total size and its distribution are regarded as fixed, but they do affect commodity demands. The effects of demographic variables on demand can be modeled in different ways. Lau, Lin, and Yotopoulos (1978), for example, enter household characteristics as separate arguments into the utility function. This implies that they will be independent variables in the expenditure as well as indirect utility functions. Barnum and Squire use linear translation (see Pollak and Wales 1981) to enter household characteristics. This involves subtracting commodity-specific indices from each commodity in the utility function—that is, $U(X_0 - \gamma_0, \ldots, X_n - \gamma_n)$, where the X_i's are consumption of commodity i, and the γ's are the translation parameters that depend linearly on household characteristics. The associated indirect utility function looks like $V(p, Y - \Sigma_{i=1}^{n} p_i\gamma_i)$. In other words, everywhere that full income, Y, appears, one subtracts the sum of the values of these commodity indices (the p_i's being prices). Consequently, in this specification, the effect of household characteristics comes through full income. Other specifications of household characteristics are possible and perhaps are preferable. (For an excellent review, see Pollak and Wales 1981.

When demographic variables are used, an LLES share equation is given by

$$-\frac{p_j X_j}{Y} = \alpha_j + \sum_{k=1}^{n} \beta_{jk} \ln \frac{p_j}{Y} + \sum_{l=1}^{r} \sigma_{jl} \ln a_l$$

$$\sum_{j=1}^{n} \alpha_j = -1; \qquad \sum_{k=1}^{n} \beta_{jk} = 0, \forall_j; \qquad \sum_{j=1}^{n} \sigma_{jl} = 0, \forall_l$$

where p_j, X_j, and Y are defined as before, a_l is the lth household characteristic, and the α's, β's, and σ's are parameters to be estimated. An LES expenditure equation with linear translating is given by

$$p_j X_j = p_j(\theta_j + \gamma_j) + \beta_j[Y - \sum_{i=1}^{n} p_i(\theta_i + \gamma_i)], \qquad \sum_{j=1}^{n} \beta_j = 1.$$

Here the β's are the (constant) marginal budget shares, the θ's are parameters, and the γ's are the translation parameters that are a linear function of household characteristics, that is, $\gamma_i = \Sigma_{l=1}^{r} \sigma_{il} a_l$.

For the production side, Yotopoulos, Lau, and Lin (1976), Kuroda and Yotopoulos (1978), and Adulavidhaya and others (1979) estimate a profit function and associated input demand functions, which are derived from a Cobb-Douglas production function. Barnum and Squire (1978, 1979a) estimate a Cobb-Douglas production function directly since they do not have the necessary price data to estimate the dual functions.

Two other studies must be included here since they also estimate complete systems on both the demand and production sides. One, by Singh and Janakiram (see chapter 3), is based on Korean and Nigerian data, and the other, by Strauss (chapter 4), looks at data from Sierra Leone. Singh and Janakiram specify a linear expenditure system for the consumption side and use a linear program to model the production side; Strauss characterizes consumption behavior by a quadratic expenditure system and production behavior by a multiple output production function in which outputs are related by a constant elasticity of transformation and inputs by a Cobb-Douglas function.

As can be seen from table 1-1, the seven studies are nearly evenly split in their findings concerning the consumption of the agricultural commodity: four report a positive own-price elasticity and three a negative one. The magnitudes of both positive and negative elasticities are small. The positive response indicates that the profit effect has more than offset the traditional negative effect predicted by standard consumer-demand theory. For consumption of market-purchased goods, the most important result is the strongly positive cross-price elasticities. This result also attests to the strength of the profit effect in increasing total expenditure. The reported elasticities suggest that the level of farm incomes and the availability of nonfarm goods are important determinants of responsiveness. Sierra Leone, for example, has a much lower elasticity than the East Asian economies.

Elasticities of marketed surplus are strongly positive, whereas those for labor supply are negative. The positive elasticities of marketed surplus indicate that, even where the profit effect is strong enough to make consumption response positive, the total output response is always large enough to offset increased household consumption. The negative responses for labor supply suggest a strong profit effect and reflect the empirical fact that leisure is a normal good. (Other results are summarized in appendix tables 1A-1–1A-4.)

Table 1-1. *Selected Elasticities: Response to Changes in the Price of the Agricultural Commodity*

Economy	Agricultural commodity	Consumption of agricultural good	Consumption of market-purchased goods	Marketed surplus	Labor supply
Taiwan	Farm output	0.22	1.18	1.03	−1.54
Malaysia	Rice	0.38	1.94	0.66	−0.57
Korea, Rep. of	Rice	0.01	0.81	1.40	−0.13
Japan	Farm output	−0.35	0.61	2.97	−1.01
Thailand	Farm output	−0.37	0.51	8.10	−0.62
Sierra Leone	Rice	−0.66	0.14	0.71	−0.09
Northern Nigeria	Sorghum	0.19	0.57	0.20	−0.06

Do Agricultural Household Models Matter?

Agricultural household models integrate production and consumption decisions in rural farm households. As a result, they require a complex theoretical structure as well as a considerable amount of data for empirical estimation. The studies summarized here attest to the fact that, both theoretically and empirically, such models are difficult and costly to estimate. Is the effort justified? Can practitioners make do with far simpler techniques that have been traditionally used to model farm behavior—that is, with techniques that do not allow for even a recursive relation between the supply and demand sides? If our interest is empirical, we must ask whether agricultural household models, which account for the interdependence of production and consumption decisions, provide estimates of elasticities that are quite different from what could have been obtained otherwise. If we are interested in policy, we must ascertain whether the differences in these elasticity estimates have different policy implications from those that would have been arrived at by traditional methods. In this section we consider the empirical significance of agricultural household models.

As noted earlier, the distinctive feature of agricultural household models is that they include the profit effect (equation 1-7). When we compare elasticities with and without the profit effect (see table 1-2), the results clearly establish the empirical significance of agricultural household models. The estimates of the elasticity of demand with respect to

Table 1-2. *Selected Response Elasticities under Varying and Constant Profits*

Economy	Agricultural commodity		Nonagricultural commodity		Labor supply	
	A[a]	B[b]	A[a]	B[b]	A[a]	B[b]
With respect to agricultural price						
Taiwan	−0.72	0.22	0.13	1.18	0.21	−1.59
Malaysia	−0.04	0.38	−0.27	1.94	0.08	−0.57
Korea, Rep. of	−0.18	0.01	−0.19	0.81	0.03	−0.13
Japan	−0.87	−0.35	0.08	0.61	0.16	−1.00
Thailand	−0.82	−0.37	0.06	0.51	0.18	−0.62
Sierra Leone	−0.74	−0.66	−0.03	0.14	0.01	−0.09
Northern Nigeria	−0.05	0.19	−0.14	0.57	0.03	−0.06
With respect to wage rate						
Taiwan	0.14	−0.03	0.05	−0.12	−0.12	0.17
Malaysia	0.06	−0.08	0.29	−0.35	−0.07	0.11
Korea, Rep. of	0.16	0.01	0.77	0.05	0.00	0.11
Japan	0.29	0.15	0.39	0.25	0.15	0.45
Thailand	0.57	0.47	0.62	0.52	0.08	0.26
Sierra Leone	0.47	0.37	0.78	0.57	0.14	0.26
Northern Nigeria	0.06	0.02	0.04	0.01	0.01	0.01

a. Holding profits constant.
b. Allowing profits to vary.

own-price not only differ significantly in the cases of Japan, Thailand, and Sierra Leone, for example, but they also change sign in the case of Taiwan, Malaysia, Korea, and northern Nigeria. Thus, whereas traditional models of demand, as we would expect, predict a decline in own-consumption in response to an increase in agricultural commodity prices, for four cases, the agricultural household models predict an increase. This is because the profit effect—which is the result of the increase in income when crop prices are raised—offsets the negative price effects. Farm households end up increasing their own consumption as prices are raised. Whether or not the amounts they offer on the market will be reduced will depend on the elasticity of output, which we know remains positive in these cases (see table 1-1). The marketed surplus response, however, is dampened by the profit effect.

The differences in the elasticity of demand for nonagricultural goods with respect to the price of agricultural goods are also striking. The elasticities change sign in four cases, and in the other three cases the magnitudes are much larger when the profit effect is included. Whereas cross-price elasticities estimated using traditional demand models tend to be low or negative because of negative income effects, the estimates obtained with the agricultural household model are positive and large because of the positive profit effect. The elasticities of household labor supply with respect to the price of the agricultural good also differ greatly. In the traditional demand models, an increase in the price of the agricultural good reduces the consumption of both that good and leisure, and thus implies an increase in the family work effort (table 1-2). In contrast, agricultural household models predict a negative response of household labor supply to increased output prices because households are willing to take a part of their increased incomes in increased leisure, thereby reducing their work effort. Consequently, any increase in the demand for labor in agricultural production will have considerable spillover effect on the demand for hired labor.

Although fewer signs change when responses to agricultural wage rates are examined, the magnitudes change. In traditional demand models, an increase in the wage rate implies an increase in real household incomes, which induces a positive-demand response with respect to agricultural and nonagricultural goods and a negative or inelastic response where household labor supply is concerned. In agricultural household models these effects are partly offset because an increase in wages also affects the production side and reduces total farm incomes. As a result, demand responses for both agricultural and nonagricultural goods are either dampened or totally offset (as in Taiwan and Malaysia), and labor supply response becomes positive or more elastic.

Looking at the market (or off-farm) labor-supply responses of landed and landless households in rural India, Rosenzweig (1980) provides a different type of evidence that agricultural household models matter. After separately estimating reduced-form market-supply equations for landless and agricultural households, Rosenzweig compares coefficients between the two groups and finds that twenty-one out of twenty-two comparisons conform to the predictions of the agricultural household framework. For instance, the off-farm male labor response of landless households to increases in the market male wage is less than for agricultural households, as would be predicted because of the negative profit effect of raising male wages.

Furthermore, agricultural household models provide other elasticities that are not even defined for models that focus exclusively on consumption behavior. These are the elasticities of demand with respect to nonlabor input prices and stocks of fixed factors of production, including land and farm technology (see table 1-3). Although the absolute magnitudes are small in most cases, the important point is that they have no counterpart in models that do not integrate production and consumption. Thus, despite the fact that traditional demand models can predict demand responses to output prices, they tell us nothing about such responses to changes in the fixed factors of product or technology. Similarly, traditional supply models can predict supply responses to changes in output

Table 1-3. *Selected Response Elasticities with Respect to Variable Input Prices and Fixed Factors*

Economy	Agricultural commodity	Nonagricultural commodity	Marketed surplus	Labor supply
With respect to fertilizer price[a]				
Taiwan	−0.11	−0.11	−0.24	0.18
Malaysia	−0.03	−0.18	−0.15	0.05
Korea, Rep. of	−0.05	−0.23	−0.34	0.04
Japan	−0.03	−0.03	−0.09	0.07
Thailand	−0.03	−0.03	−0.41	0.05
With respect to land				
Taiwan	0.46	0.46	1.00	−0.77
Malaysia	0.26	1.37	1.15	−0.41
Korea, Rep. of	0.10	0.49	0.81	−0.08
Japan	0.19	0.19	0.96	−0.43
Thailand	0.11	0.11	1.48	−0.19
Sierra Leone	0.01	0.02	0.02	−0.01
Northern Nigeria	0.10	0.16	0.06	−0.08

a. Fertilizer is barely used in the Sierra Leone and northern Nigeria samples and therefore was not modeled.

and input prices or in fixed factors of production and technology, but they fail to tell us anything about the demand responses to these exogenous factors. Agricultural household models therefore provide a vital link between the demand and supply-side responses to exogenous policy changes. Although these links can be established informally between traditional supply-and-demand models, in agricultural household models they are handled directly within a consistent theory and framework of estimation.

When should a full agricultural household model be used? The answer is that, since the profit effect is its distinguishing feature, such a model is appropriate when the profit effect is likely to be important. Notice, however, that changes in some exogenous prices have a small effect on farm profits. The profit effect is much more important in Malaysia than in Sierra Leone (table 1-6), for example, partly because the effect of a price change on profits is much larger in Malaysia, where a 10 percent increase in output price results in a 16 percent increase in profits. In Sierra Leone, the same percentage increase in output price increases profits by only 2 percent.

Second, even if profits are affected by an exogenous price increase, they may be only a small part of full income (equation 1-2), and it is full income that appears in the demand equations. For our sample of economies, the share of profits in full income ranges from 0.5 in Malaysia to only 0.2 in Thailand. It follows that a given percentage increase in profits will have a much greater impact on total income in Malaysia than in Thailand.

Finally, the effect of full income on demand varies among commodities. It is much more important in the case of nonagricultural commodities than agricultural ones, for example, since the demand for agricultural commodities tends to be inelastic with respect to income. In Malaysia, the elasticity of demand for rice with respect to full income is only 0.52 compared with 2.74 for market-purchased goods. As a result, the profit effect is much more significant in the case of nonagricultural goods (table 1-6).

These remarks suggest that, if profits are relatively insensitive to producer prices and constitute a relatively small part of full income and if consumption of a particular item is relatively insensitive to full income, then an agricultural household model will not necessarily make our analysis more accurate. This proves to be the case, for example, with the elasticity of demand for agricultural goods with respect to changes in producer prices in Sierra Leone, although it is not true for low-income households in that study (see chapter 4). If these three conditions are reversed, however, a full agricultural household model is of critical importance, as the elasticity of demand for nonagricultural goods with respect to producer prices in Malaysia reveals.

Policy Results

Agricultural household models provide insight into three broad areas of interest to policymakers: the welfare or real incomes of agricultural households; the spillover effects of agricultural policies onto the rural, nonagricultural economy; and, at a more aggregate level, the interaction between agricultural policy and international trade or fiscal policy. The potential role of agricultural household models in this respect becomes evident when we look at these three dimensions in a "typical" agricultural policy such as taxing output (either through export taxes or marketing boards) in order to generate revenue for the central exchequer and simultaneously subsidizing a significant input (usually fertilizer) to restore, at least in part, producer incentives. The model could just as easily be applied to other policies, but this particular combination has been adopted by many developing countries and illustrates well the type of issue that can be analyzed within the framework of the agricultural household model.

Consider, first, the effect of pricing policy on the welfare or real full income of a representative agricultural household. For some price changes—for example, a change in the price of fertilizer—the resulting change in nominal full income is an accurate measure of the change in real income since the prices of all consumer goods have remained unchanged. In other cases, however, the commodity in question may be both a consumer good and a farm output or input. If the price of, say, an agricultural staple is increased, the household will benefit as a producer but lose as a consumer. As long as the household is a net producer of the commodity, its net benefit will be positive (see the appendix to part I). Nevertheless, to quantify the net gain to the household, one must allow for both the positive effect coming through farm profits and the negative effect coming through an increase in the price of an important consumer good.

Table 1-4 presents estimates of the elasticities of real full income with respect to changes in output price and fertilizer price for the six studies examined earlier. For marginal changes, the decrease in real income following an increase in the price of the agricultural output equals marketed surplus times the price increase, and the increase following a reduction in the price of an input equals the quantity of the input times the price reduction. Thus, if prices, marketed surplus, and full income are known, these elasticities can be calculated without reference to price and income elasticities. For nonmarginal changes, however, it would be necessary to

Table 1-4. *Effect on Real Income of Changes in Output and Fertilizer Prices*

Economy	*Response to output price*	*Response to fertilizer price*
Taiwan	0.90	−0.11
Malaysia	0.67	−0.07
Korea, Rep. of	0.40	−0.10
Japan	0.34	−0.03
Thailand	0.10	−0.03
Sierra Leone	0.09	—
Northern Nigeria	0.12	—

— Not applicable. Fertilizer is barely used in the Sierra Leone and northern Nigeria samples and therefore was not modeled.

use information on the underlying structure of preferences to calculate equivalent or compensating variation.

The percentage change in real income among the six countries under consideration is less than the percentage change in either the output price or the fertilizer price (table 1-4). In addition, it appears that the loss in real income arising from a given percentage reduction in the output price can be offset only if the price of fertilizer is reduced by a much larger percentage. In Malaysia, for example, a 10 percent reduction in output price would reduce real income by almost 7 percent, whereas a 10 percent reduction in the price of fertilizer would increase real income by only about 1 percent. This difference arises from the relative magnitudes of marketed surplus and fertilizer use. Thus, if policymakers are interested primarily in the welfare of agricultural households, intervention in output markets is likely to be much more important than intervention in the markets for variable, nonlabor inputs.

Policymakers are also concerned with the welfare of rural households that do not own or rent land for cultivation. Landless households either sell their labor to land-operating households or else engage in nonfarm activities (see, for example, Anderson and Leiserson 1980). Although governments have few policy instruments by which to improve the welfare of these households directly, price interventions and investment programs directed at land-operating households have spillover effects that may (or may not) be beneficial for these households. What can agricultural household models tell us about these effects?

An increase in the price of an important agricultural staple will obviously hurt households that are net consumers of that item. The direct effect of a price increase will therefore be unambiguously negative for landless households and nonfarm households. The policymaker thus faces a dilemma: if he wants to improve incentives and increase the incomes of agricultural households, he does so at the expense of other rural households. There are, however, offsetting indirect effects. If the price of the agricultural commodity is increased, for example, agricultural households increase their demand for total—hired and family—farm labor and reduce the supply of family labor; that is, they increase their leisure time (see table 1-5). As a result, the demand for hired labor can be expected to increase substantially to the benefit of landless households. In Malaysia, the reported elasticities of labor demand (1.61) and labor supply (0.57) imply an elasticity of demand for hired labor of 10.9. Although this figure in part reflects the initial small percentage of hired labor in total labor (19 percent), it nevertheless implies a substantial change in labor market conditions and would undoubtedly exert upward pressure on rural wage rates and would thereby offset, at least to some extent, the negative consequences, among landless households, of higher prices for agricultural commodities.

The policy implications of these findings are particularly significant because they also shed light on the extent to which the positive gains from technological improvements trickle down via the labor market to the rural landless. It is now widely accepted that the technological innovations associated with the green revolution (improved seeds, increased use of fertilizers and pesticides, increased irrigation and cropping intensity) have

Table 1-5. *Spillover Effects of Changes in Output and Fertilizer Price*

Economy	Labor demand	Labor supply	Consumption of nonagricultural goods
Output price			
Taiwan	2.25	−1.54	1.18
Malaysia	1.61	−0.57	1.94
Korea, Rep. of	0.57	−0.13	0.81
Japan	1.98	−1.01	0.61
Thailand	1.90	−0.62	0.51
Sierra Leone	0.14	−0.09	0.14
Northern Nigeria	0.12	−0.06	0.23
Fertilizer price[a]			
Taiwan	−0.23	0.18	−0.22
Malaysia	−0.12	0.05	−0.18
Korea, Rep. of	−0.12	0.04	−0.23
Japan	−0.13	0.07	−0.03
Thailand	−0.11	0.05	−0.03

a. Fertilizer is barely used in the Sierra Leone and northern Nigeria samples and therefore was not modeled.

had a great deal to do with increasing the demand for total labor, but the concern has been whether this increased demand would do much for hired labor, most of which comes from the smallest farms and the landless. The empirical findings show that it could. When an increase, either in the fixed factors of production or technologies, boosts farm incomes, the amount of family (household's own) labor effort tends to decline (see table 1-7). Therefore any increase in the demand for total labor means an even larger increase in the demand for hired labor. The labor supply-and-demand elasticities emerging from empirical applications of agricultural household models provide strong support for the view that trickle-down effects are both positive and significant.

A second indirect effect of increased output prices is a significant increase in the demand for nonagricultural goods (see table 1-5). The response elasticity is positive and greater than 1 in two economies (Taiwan and Malaysia) and is positive and greater than 0.5 in all economies except Sierra Leone, though for low-income households in Sierra Leone it is also high (0.9). Some of this demand will be for imports and urban-produced commodities. But a large part will be for rurally produced goods and services and will therefore increase demand for the output of nonfarm, rural households. Any increase in farm profits, whether caused by a price change or a technological improvement, can be expected to lead to a substantial increase in the demand for goods and services produced by nonagricultural households. Thus, spillover effects through output markets will, at least in part, offset the negative effects on nonfarm households of an increase in agricultural prices and will ensure that the benefits of technological improvements are dispensed throughout the rural community.

Table 1-5 also traces through the effects of a change in the price of fertilizer. The results suggest that changes in fertilizer prices can be made without generating large negative or positive spillover effects.

As mentioned earlier, governments often tax agricultural output in order to generate revenue and at the same time subsidize essential inputs such as fertilizer in order to restore production incentives. In this way, they hope to achieve self-sufficiency or earn foreign exchange. Can agricultural household models shed light on these and other policy options? Indeed, the information they provide with respect to the effect of pricing policy on marketed surplus and fertilizer demand can be used as inputs in calculations of self-sufficiency, balance of payment effects, and budgetary effects.

If a government's primary concern is self-sufficiency, it needs to know the marketed surplus available for procurement. When we look at the elasticity estimates for agricultural production, consumption, and marketed surplus (table 1-6), two points become clear. First, even where con-

Table 1-6. *Response of Output, Consumption, Marketed Surplus, and Input Demand to Price Changes*

Economy	*Agricultural output*	*Agricultural consumption*	*Marketed surplus*	*Fertilizer demand*
Output price				
Taiwan	1.25	0.22	1.03	2.25
Malaysia	0.61	0.38	0.66	1.61
Korea, Rep. of	1.56	0.01	1.40	1.29
Japan	0.98	−0.35	2.97	1.98
Thailand	0.90	−0.37	8.10	1.90
Sierra Leone	0.11	−0.66	0.71	—
Northern Nigeria	0.30	0.19	0.20	—
Fertilizer price[a]				
Taiwan	−0.23	−0.11	−0.23	−1.23
Malaysia	−0.13	−0.03	−0.15	−1.13
Korea, Rep. of	−0.30	−0.05	−0.34	−1.10
Japan	−0.13	−0.03	−0.09	−1.13
Thailand	−0.11	−0.03	−0.41	−1.11

— Not applicable.

a. Fertilizer is barely used in the Sierra Leone and northern Nigeria samples and therefore was not modeled.

sumption responds positively to an increase in the price of the agricultural commodity because of the profit effect, marketed surplus still responds positively. Where the consumption response is negative, the elasticities of marketed surplus are positive and large (see, for example, the case of Thailand). Governments can therefore use pricing policy in the output market to increase the marketed surplus even when it is unable to set consumer and producer prices independently. Second, efforts to offset disincentives in output markets through fertilizer subsidies will not be effective unless the fertilizer price is reduced by a much greater percentage than the output price.

Rough estimates of the effect of pricing policies on budget revenues and foreign exchange can also be derived from table 1-6. Assume, for example, that the output is exported and that the fertilizer is imported. According to table 1-10, an increase in output price will induce an increase in marketed surplus available for export, but only at the expense of increased use of fertilizer. The net foreign exchange effect, therefore, is given by the difference between the revenues from exporting the agricultural output and the costs of importing additional fertilizer. Similarly, if the output is taxed and fertilizer is subsidized, one can perform a similar calculation to arrive at a rough estimate of the net impact on the budget. In fact, the framework of the agricultural household model is highly flexible and can be adapted to fit many other circumstances and issues. One of the purposes of this volume is to present some of the extensions that have recently been tested (see chapters 3, 4, 5, 6, and 7).

Some Extensions

Most of the early work on agricultural household models ignored questions of choice among competing crops. These studies either examined monocultures or else treated farm output as an aggregate. Several important policy issues, however, are concerned with the *choice among alternative crops*. Many governments, for example, are concerned with the effect of export taxation on production when export crops compete with food crops destined for the domestic market. Similarly, if fertilizer intensity varies among crops, price-induced changes in the composition of output may have significant effects on the demand for fertilizer. The demand for hired labor can also be influenced by changes in crop composition.

The basic agricultural household model can be modified easily to accommodate multiple crops. Thus a production function for a single crop can be replaced with an implicit production function linking inputs and outputs:

$$G(Q_1, \ldots Q_n, V_1, \ldots V_m, A_1, \ldots A_k) = 0$$

where Q represents output, V is variable inputs, and A is fixed factors. Provided the household is a price-taker in the relevant markets, the introduction of multiple outputs does not affect the recursive property of the model. Two of the studies covered in this volume—those by Singh and Janakiram and by Strauss—allow specifically for multiple crops.

This is an important policy extension since pricing policies are often oriented around specific commodities, but other crops will likely be affected as well. These studies cover three countries: the Republic of Korea, Nigeria, and Sierra Leone, and in each case cover crops grown primarily for consumption, for sale, and crops both consumed and exchanged. The countries cover three dissimilar environments. In Korea, farm households are well integrated into product and factor markets. Crops are grown under irrigated conditions and in single stands. Considerable technological advance has occurred in rice production, with a consequent high-level use of purchased inputs. In addition, there are many sources for nonfarm incomes. Households in Sierra Leone are also fairly well integrated into product and labor markets, but the level of income is far lower than for Korean households. There has been little technological change in paddy rice (or other crop) production so there is little use for nonpurchased inputs. Irrigation is nonexistent and crops are grown in mixtures. Households in the state of Kaduna in northern Nigeria are far more isolated from factor and product markets. Production is mostly for subsistence, and intercropping is widespread. Northern Nigeria is also a semi-arid area, in contrast to Korea and Sierra Leone, so production risk is important (see chapter 9).

Table 1-7 reports own- and cross-price elasticities both for output supplies and marketed surpluses for households in the three countries. In all cases, the cross-price elasticities of output supply are very small, despite the crop disaggregation. For these studies, then, the need for comprehensive pricing policies is not evident. Korean households show a far greater own price output responsiveness than their northern Nigerian or Sierra Leone counterparts, which may be partly explained by the higher level of infrastructural development and the greater market integration in Korea.

The cross-price responses of marketed surpluses are small for the Sierra Leone case, but not negligible, and they are miniscule for the Korean and northern Nigerian samples. This reflects the very small cross-price elasticities both of output supply and commodity demands in the latter two studies. Again, the overwhelming impact of a commodity pricing policy is predicted by these studies to be on that commodity, without large spillover effects. It was necessary to disaggregate commodities in order to reach such a conclusion.

Table 1-7. *Cross-Price Elasticities of Supply and Marketed Surplus*

Economy	With respect to price of	Supply			Marketed surplus		
Korea, Rep. of		*Rice*	*Barley*	*Soybeans*	*Rice*	*Barley*	*Soybeans*
	Rice	1.56	−0.00[a]	0.00[a]	1.4	−0.00[a]	−0.00[a]
	Barley	−0.00[a]	0.50	−0.00[a]	−0.00[a]	0.50	−0.00[a]
	Soybeans	0.04	−0.10	−0.10	−0.03	−0.15	0.06
Nigeria		*Sorghum*	*Millet*	*Groundnuts*	*Sorghum*	*Millet*	*Groundnuts*
	Sorghum	0.30	−0.00[a]	−0.20	0.25	−0.00[a]	0.04
	Millet	−0.00[a]	0.25	−0.05	−0.00[a]	0.18	−0.07
	Groundnuts	−0.00[a]	−0.00[a]	0.18	−0.00[a]	−0.00[a]	0.09
Sierra Leone		*Rice*	*Rootcrops*	*Oil palm*	*Rice*	*Rootcrops*	*Oil palm*
	Rice	0.11	0.01	0.00[a]	0.71	0.06	−0.03
	Root crops and other cereals	0.02	0.10	0.00[a]	−0.08	0.46	−0.29
	Oil palm products	0.00[a]	0.00[a]	0.02	−0.05	−0.02	0.44

a. Insignificantly small.

Commodity disaggregation may also be important if calorie content varies among commodities and if governments are interested in the *nutritional status* of agricultural households. Strauss (chapter 4) shows how the basic model can be elaborated to investigate the effect of pricing policy on caloric intake. In his model, the utility function (see equation 1-1) becomes

$$U = U(X)$$

where X is a vector of consumer goods, including food items, nonfood items, and leisure. Caloric intake (K) can then be calculated from

$$K = \sum_i a_i X_i \qquad i = 1 \cdots m$$

where a_i is the calorie content of a unit of the i^{th} food and X_i, $i = 1 \cdots m$ are quantities of different food items.

With this extension, Strauss demonstrates that price changes exert a considerable effect on caloric intake and that the profit effect plays a significant role. One might expect that an increase in the price of an important food item would probably have a negative impact on caloric intake. According to table 1-8, however, in most cases, an increased price results in increased caloric intake because of an increase in profits. That is to say, even if the consumption of such a commodity declines, the extra profits can be used to purchase increased quantities of other foodstuffs, with the result that overall caloric intake will respond positively. Strauss is also able to demonstrate an important point regarding the distribution of calories among income groups in Sierra Leone: even if a price increase causes a reduction in the caloric intake of middle-income and high-income households (see the case of rice in table 1-8), the intake of low-income households is increased. This suggests that, if policymakers are concerned primarily with the nutritional status of low-income households, price increases for major food items may prove to be beneficial. Increases in the prices of food items toward, say, world prices may improve the nutri-

Table 1-8. *Response of Caloric Intake to Price Changes in Sierra Leone*

Food	Elasticity of caloric intake: Low income	Middle income	High income
Rice	0.19	−0.24	−0.20
Root crops and other cereals	0.43	0.13	0.11
Oils and fats	0.27	−0.03	−0.21
Fish and animal products	0.48	0.23	0.05
Miscellaneous foods	0.14	0.01	−0.01

tional status of low-income households and provide appropriate signals for resource allocation. The usual equity-growth tradeoff may be absent in this case.

Policymakers are interested in nutritional status presumably because it affects health and may also affect productivity at the individual level. Pitt and Rosenzweig (chapter 5) take the analysis one step further, therefore, and examine the interaction between *prices, health, and farm profits* in the context of an agricultural household model. To do so, they incorporate a health variable directly in the utility function (people prefer to be healthy) and in the production function (a healthy individual is more productive). To complete their model, they introduce a production function for health:

$$H = H(X_a, X_m, X_l, Z)$$

which says that health (H) depends on consumption (X_a and X_m) and hence on nutrition, leisure (or work effort, X_l), and a vector (Z) of other factors that affect health, some of which are chosen by the household (boiling water) and some of which are community-level services (well water).

When this model is applied to data from Indonesia, it is found that a 10 percent increase in the consumption of fish, fruit, and vegetables reduces the probability of illness by 9, 3, and 6 percent, respectively, whereas a 10 percent increase in the consumption of sugar increases the probability of illness by almost 12 percent. These results suggest that increases in consumption cannot automatically be assumed to contribute to health since the composition of consumption may also change in a manner detrimental to health.

In addition to estimating the health production function, Pitt and Rosenzweig also estimate a reduced-form equation that produces a direct link between prices and health. They show that a 10 percent reduction in the prices of vegetables and vegetable oil will decrease the probability of the household head being ill by 4 and 9 percent, respectively, whereas the same percentage reduction in the prices of grains and sugar will increase the probability of illness by 15 and 20 percent, respectively, albeit from a very low base. These results are calculated with profits held constant, however. In principle, when profits are allowed to vary, some of these results may be modified. In this particular application the coefficient on farm profits proved statistically insignificant. The results reported above are therefore reasonably accurate measures of the total effect of price changes on health.

Changes in health may also affect productivity and farm profits. Pitt and Rosenzweig demonstrate for their sample, however, that the effects of

ill-health on labor supply are not reflected in reduced farm profits when households have recourse to an active labor market. Thus, although family labor supply is significantly reduced by illness, total labor input, and hence farm profits, remain unaffected. In other words, in this Indonesian sample the benefits of improved health (or the costs of a deterioration in health) in agricultural households will be reflected in household income—if at all—only through labor-market supply.

Most of the policy issues mentioned thus far have been static in nature and have been couched in a single-period framework. Iqbal's work (chapter 6) represents a significant departure from previous studies in that it introduces another period to accommodate *borrowing, saving, and investment* decisions. Since governments and multinational agencies devote substantial quantities of funds to rural credit programs, this particular extension makes it possible to apply agricultural household models to a new set of policy issues of considerable importance in many countries.

In the first period of Iqbal's two-period model, the household may borrow and invest in farm improvements. In the second period, the loan must be repaid with interest and the household enjoys higher farm profits as a result of its investment in period one. Accordingly, the single full-income constraint is replaced by two full-income constraints, one for each period:

$$\Pi(K_1) + w_1 T_1 + B = C_1 + I$$

and

$$\Pi(K_1 + I) + w_2 T_2 = C_2 + B(1 + r[B])$$

where K_1 is capital in period one and I is investment, so that $K_1 + I$ is capital in period two. B is borrowing in period one and $B(1 + r[B])$ is repayment in period two. C is the value of consumption of goods and leisure. Iqbal draws a parallel between his treatment of household savings and borrowing and the treatment of own-consumption and marketed surplus or family labor supply and hired labor in the standard agricultural household model. He notes that the recursive property of the standard model carries over to his two-period extension, provided the household can borrow at a fixed rate of interest. In his application to Indian households, Iqbal argues that the interest rate is influenced by household borrowing decisions (r is a function of B in the second-period constraint) and therefore he adopts a nonrecursive specification.

Iqbal finds that borrowing is significantly reduced by increases in the interest rate, the elasticity being −1.2. His results support the view that interest rate policy can have a marked effect on the level of debt held by farmers. Iqbal also shows that farmers owning more than three hectares are highly sensitive to the interest rate, whereas the coefficient on borrowing by farmers owning less than three hectares is statistically insignificant. It follows that the elimination or reduction of subsidies to programs providing agricultural credit may serve the dual purpose of increasing efficiency in the capital market and simultaneously improving equity, since the reduction in borrowing by "large" farmers will exceed that by "small" ones.

As noted earlier, governments are also interested in the effects of agricultural pricing policy on more aggregate economic variables such as *budget deficits and foreign exchange earnings*. In Senegal, for example, agricultural products generate 70 percent of total export earnings, and deficits arising from the government's policy on agricultural pricing amount to more than 20 percent of government expenditure and 2 percent of GDP. Changes in agricultural prices can therefore be expected to have a considerable impact on these aggregates. Indeed, concern with the existing levels of foreign exchange earnings and a budget deficit may be the primary motivation for changing pricing policy in many countries. The government of Senegal has explored a number of ways, including pricing policy, to promote the production and consumption of millet in order to reduce imports of rice and hence improve the country's balance of payments.

The effect of pricing policy on foreign exchange and budget revenues discussed earlier in the chapter is further illuminated by Braverman and Hammer (chapter 8) through their addition of market-clearing conditions (for the major outputs and inputs) to the basic model. The changes in consumption, production, or labor supply at the household level following any change in an exogenous variable can then be aggregated and fed into the market-clearing equations. In some cases, the market is cleared through adjustments in international trade, and prices remain fixed at levels determined by the government, that is,

$$Q(\bar{p}_a) = X_a(\bar{p}_a) + E$$

where E represents net exports. In this event, a change in production or consumption has an immediate effect on foreign exchange earnings. Alternatively, the market may clear through adjustments in price, that is,

$$Q(p_a) = X_a(p_a).$$

Now a policy-induced change in production or consumption will bring about a change in price, which will generate second-round effects on production and consumption.

In their application to Senegal, Braverman and Hammer assume the first form of marketing clearing (quantity adjustment) for cotton, groundnuts, and rice and the second form (price adjustment) for maize and millet. The second-round effects flowing from induced changes in the prices of maize and millet are captured fully in their model. Table 1-9 provides a sample of their policy results. Compare, first, the effect of reducing the price of groundnuts or increasing the price of fertilizer on the deficit arising from the government's agricultural pricing policy. Both policies reduce the deficit. The reduction in the price of groundnuts, however, has a relatively small effect on net foreign exchange earnings (mainly because a reduction in rice imports offsets reduced exports), although it reduces the real incomes of farmers in the groundnut basin by almost 6 percent. Although an increase in the price of fertilizer causes a larger fall in net export earnings (in reflection of the fertilizer intensity of export crops), farm incomes are reduced by only 1 percent. This example illustrates the policy tradeoffs that can be explored within the framework of Braverman and Hammer's extension. It also confirms a point made earlier—that to be effective, changes in the prices of inputs such as fertilizer must be much larger than changes in the prices of the main outputs.

Another important point regarding the formulation of policy in Senegal is that the government has been eager to reduce imports of rice and hence save foreign exchange by increasing domestic production of rice and increasing consumption of domestic substitutes such as millet. How can this goal be achieved? One possibility is to increase the producers' price of rice. Such a measure does indeed reduce rice imports (by 7 percent), but net foreign exchange earnings fall (by 4.5 percent), because in order to increase rice production, farmers switch out of export crops (see table 1-9). The desired result, an increase in net foreign exchange earnings, fails to materialize because of substitution possibilities in production. In this case, failure to recognize substitution possibilities produces a

Table 1-9. *Effect of Agricultural Pricing Policy in Senegal*
(percentage change)

Policy	*Real income, groundnut basin*	*Export earnings*	*Government deficit*
15 percent decrease in producer price of groundnuts	−5.7	−1.9	−18.1
100 percent increase in price of fertilizer	−1.1	−5.2	−10.4
50 percent increase in producer price of rice	0.2	−4.5	−0.1
50 percent increase in consumer price of rice	−4.7	−0.2	−34.8

perverse result. In other situations, however, policy may be designed to take advantage of substitution possibilities. The government might increase the consumer price of rice in the hope that people would change their pattern of consumption in favor of millet. According to Braverman and Hammer's analysis, however, such a policy would have little impact on net export earnings, so that in this case a reliance on substitution possibilities would have been misplaced (see table 1-9).

These examples from the work of Braverman and Hammer illustrate the importance of placing agricultural household models in a multimarket framework, particularly where foreign exchange earnings and government revenues are of concern. Because the expansion of one crop is usually detrimental to another crop, changes in the quantities of internationally traded items and in the quantities of taxed or subsidized items will influence the overall impact of policy on foreign exchange and government revenue even if a change in a government-controlled price in one market leaves the prices in all other agricultural markets unchanged. More generally, changes in government-controlled prices will induce changes in other prices so that even measures of output response, labor supply response, consumer response, and changes in farm profits will have to allow for general equilibrium effects (see chapter 2). Thus the multimarket extension may well emerge as the most useful vehicle for generating relevant policy results from agricultural household models.

Appendix: Detailed Elasticities from Studies of Agricultural Household Models

The four tables in this appendix are on the following pages.

Table 1A-1. *Elasticities of Agricultural Commodity Consumption*

			Commodity price				*Fixed factors*				
Economy	*Commodity*	*Total expenditure*	*Own*	*Nonfarm*	*Wage*	*Fertilizer price*	*Workers*	*Dependents*	*Land*	*Capital*	*Scale technology factor*
Taiwan	Farm goods	n.a.	0.22	0.29	−0.03	−0.11	0.84	0.43	0.46	0.04	n.a.
Malaysia	Rice	n.a.	0.38	−0.15	−0.08	−0.03	0.44	0.23	0.26	n.a.	0.42
Japan	Farm goods	n.a.	−0.35	0.31	0.15[a]	−0.03	0.07[b]	0.14	0.19	0.07[c]	n.a.
Thailand	Farm goods	n.a.	−0.37	0.05	0.47	−0.03[d]	0.70	−0.16	0.11	0.10	n.a.
Korea, Rep. of	Rice	0.57	0.01	n.a.	0.01	−0.05	n.a.	n.a.	0.10[e]	n.a.	0.002[f]
Sierra Leone	Rice	0.52	−0.66	0.13	0.37	—	0.26[g]	0.13[h]	0.01	0.04	0.11
Northern Nigeria	Sorghum	1.80	0.19	n.a.	0.02	—	n.a.	n.a.	n.a.	n.a.	n.a.

n.a. Not available.
— Not applicable. Fertilizer was barely used by the Sierra Leone and northern Nigeria samples and therefore was not modeled.
a. Farm wages.
b. On-farm workers.
c. Machinery.
d. Price index of fertilizer, seed, and chemicals.
e. Average farm size.
f. With respect to increased tiller capacity.
g. Males 15 years and older.
h. Children 10 years and younger.

Table 1A-2. *Elasticities of Nonagricultural Commodity Consumption*

		Commodity prices				Fixed factors				
Economy	*Total expenditure*	*Own*	*Agricultural commodity*	*Wage*	*Fertilizer price*	*Workers*	*Dependents*	*Land*	*Capital*	*Scale technology factor*
Taiwan	n.a.	−0.58	1.18	−0.12	−0.11	0.84	0.0	0.46	0.04	n.a.
Malaysia	n.a.	−0.77	1.94[a]	−0.35	−0.18	−0.06	−0.05	1.37	n.a.	2.21
Japan	n.a.	−0.97	0.61	0.25[b]	−0.03	−0.12[c]	0.02	0.19	0.07[d]	n.a.
Thailand	n.a.	−0.89	0.51	0.52	−0.03[e]	0.69	−0.29	0.11	0.10	n.a.
Korea, Rep. of	2.76	−0.87	0.81[a]	0.05	−0.23	n.a.	n.a.	0.49[f]	n.a.	0.01[g]
Sierra Leone	1.18	−0.93	0.14[a]	0.57	n.a.	0.41[h]	0.09[i]	0.02	0.10	0.27
Northern Nigeria	3.30	n.a.	0.57[j]	0.01	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.

n.a. Not available.
a. Price of rice.
b. Farm wage.
c. On-farm workers.
d. Machinery.
e. Price index of fertilizer, seed, and chemicals.
f. Average farm size.
g. With respect to tiller capacity.
h. Males 15 and older.
i. Children 10 and younger.
j. With respect to sorghum price.

Table 1A-3. *Elasticities of Agricultural Commodity-Marketed Surplus*

		Commodity prices				Fixed factors				
Economy	*Commodity*	*Own*	*Nonfarm*	*Wage*	*Fertilizer price*	*Workers*	*Dependents*	*Land*	*Capital*	*Scale technology factor*
Taiwan	Farm goods	1.03	−0.05	−0.95	−0.24	−0.13	−0.07	1.00	0.08	n.a.
Malaysia	Rice	0.66	n.a.	−0.55	−0.15	0.09	−0.50	1.15	n.a.	1.85
Japan	Farm goods	2.97	−0.13	−0.77[a]	−0.09	−0.03[b]	−0.06	0.96	0.37[c]	n.a.
Thailand	Farm goods	8.10	−0.12	−3.62	−0.41[d]	−1.72	0.39	1.48	1.44	n.a.
Korea, Rep. of	Rice	1.40	n.a.	n.a.	−0.34	n.a.	n.a.	0.81	n.a.	n.a.
Sierra Leone	Rice	0.71	−0.12	−0.49	n.a.	−0.21[e]	−0.12[f]	0.02	0.11	0.32
Northern Nigeria	Sorghum	0.20	n.a.	n.a.	n.a.	n.a.	n.a.	0.06	n.a.	n.a.

n.a. Not available.
a. Farm wage.
b. On-farm workers.
c. Machinery.
d. Price index of fertilizer, seed, and chemicals.
e. Males 15 and older.
f. Children 10 and younger.

Table 1A-4. Elasticities of Labor Supply

Economy	Type of labor	Commodity prices: Agricultural commodity	Commodity prices: Nonfarm commodity	Farm wage	Off-farm wage	Fertilizer price	Fixed factors: Workers	Fixed factors: Dependents	Fixed factors: Land	Fixed factors: Capital	Scale technology factor
Taiwan	Total	−1.54	0.58	0.17	n.a.	0.18	1.27	0.20	−0.77	−0.06	n.a.
Malaysia	Total	−0.57[a]	0.24	0.11	n.a.	0.05	0.62	0.12	−0.41	n.a.	−0.65
Japan	Farm	−1.01	0.30	0.45	−1.97	0.07	−0.89[b]	0.34	−0.43	−0.17[c]	n.a.
Thailand	Total	−0.62	0.10	0.26	n.a.	0.05[d]	0.94	−0.28	−0.19	−0.19	n.a.
Korea, Rep. of	Total	−0.13[a]	n.a.	0.11	n.a.	0.04	n.a.	n.a.	−0.08[e]	n.a.	−0.002[f]
Sierra Leone	Total	−0.09[a]	−0.05	0.26	n.a.	n.a.	0.55[g]	0.13[h]	−0.01	−0.05	n.a.
Sierra Leone	Off-farm	−4.42[c]	−1.85	17.18	n.a.	n.a.	14.36[g]	3.78[h]	−0.94	−4.90	n.a.
Northern Nigeria	Total	−0.06	n.a.	0.10	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.

n.a. Not available.
a. Price of rice.
b. On-farm workers.
c. Machinery.
d. Price index of fertilizer, seed, and chemicals.
e. Average farm size.
f. With respect to tiller capacity.
g. Males 15 and older.
h. Children 10 and younger.

References

Adulavidhaya, Kamphol, Yoshimi Kuroda, Lawrence Lau, Pichit Lerttamrab, and Pan Yotopoulos. 1979. "A Microeconomic Analysis of the Agriculture of Thailand." *Food Research Institute Studies*, vol. 17, pp. 79–86.

Adulavidhaya, Kamphol, Yoshimi Kuroda, Lawrence Lau, and Pan Yotopoulos. 1984. "The Comparative Statics of the Behavior of Agricultural Households in Thailand." *Singapore Economic Review*, vol. 29, pp. 67–96.

Anderson, Dennis, and Mark Leiserson. "Rural Nonfarm Employment in Developing Countries." *Economic Development and Cultural Change*, vol. 28, pp. 227–48.

Barnum, Howard, and Lyn Squire. 1978. "Technology and Relative Economic Efficiency." *Oxford Economic Papers*, vol. 30, pp. 181–98.

———. 1979a. "An Econometric Application of the Theory of the Farm Household." *Journal of Development Economics*, vol. 6, pp. 79–102.

———. 1979b. *A Model of an Agricultural Household.* Washington, D.C.: World Bank.

Barten, Anton. 1977. "The Systems of Consumer Demand Functions Approach: A Review." *Econometrica*, vol. 45, pp. 23–52.

Brown, Alan, and Angus Deaton. 1972. "Surveys in Applied Economics: Models of Consumer Behavior." *Economic Journal*, vol. 82, pp. 1145–1236.

Deaton, Angus, and John Muellbauer. 1980. *Economics and Consumer Behavior.* Cambridge, Mass.: Cambridge University Press.

Kuroda, Yoshimi, and Pan Yotopoulos. 1978. "A Microeconomic Analysis of Production Behavior of the Farm Household in Japan: A Profit Function Approach." *The Economic Review* (Japan), vol. 29, pp. 116–29.

———. 1980. "A Study of Consumption Behavior of the Farm Household in Japan: An Application of the Linear Logarithmic Expenditure System." *The Economic Review* (Japan), vol. 31, pp. 1–15.

Lau, Lawrence, Wuu-Long Lin, and Pan Yotopoulos. 1978. "The Linear Logarithmic Expenditure System: An Application to Consumption Leisure Choice." *Econometrica*, vol. 46, pp. 843–68.

Pollak, Robert, and Terence Wales. 1981. "Demographic Variables in Demand Analysis." *Econometrica*, vol. 49, pp. 1533–51.

Yotopoulos, Pan, Lawrence Lau, and Wuu-Long Lin. 1976. "Microeconomic Output Supply and Factor Demand Functions in the Agriculture of the Province of Taiwan." *American Journal of Agricultural Economics*, vol. 58, pp. 333–40.

Heckman, James J., and Thomas MaCurdy. 1981. "New Methods for Estimating Labor Supply Functions: A Survey." In *Research in Labor Economics*, vol. 4. Edited by R. Ehrenberg. New York: JAI Press.

Jaynes, Gerald, 1982. "Production and Distribution in Agrarian Economies." *Oxford Economic Papers*, vol. 34, pp. 346–67.

Kirchner, James, I. J. Singh, and Lyn Squire. 1984. "Agricultural Pricing and Marketing Policies in Eastern Africa." World Bank. Processed.

Lau, Lawrence, Wuu-Long Lin, and Pan Yotopoulos. 1978. "The Linear Logarithmic Expenditure System: An Application to Consumption Leisure Choice." *Econometrica*, vol. 46, pp. 843–68.

Lau, Lawrence, Pan Yotopoulos, Erwin Chou, and Wuu-Long Lin. 1981. "The Microeconomics of Distribution: A Simulation of the Farm Economy." *Journal of Policy Modeling*, vol. 3, pp. 175–206.

Lee, Lung-Fei, and Mark Pitt. 1984. "Microeconometric Models of Consumer and Producer Demand with Limited Dependent Variables." Economic Development Center Bulletin no. 84-4. University of Minnesota, Minneapolis.

McFadden, Daniel. 1981. "Econometric Models of Probabilistic Choice." In *Structural Analysis of Discrete Data with Econometric Applications*. Edited by C. Manski and D. McFadden. Cambridge, Mass.: MIT Press.

Muth, Richard. 1966. "Household Production and Consumer Demand Functions." *Econometrica*, vol. 34, pp. 699–708.

Newbery, David, and Joseph Stiglitz. 1979. "Sharecropping, Risk Sharing, and the Importance of Imperfect Information." In *Risk, Uncertainty and Agricultural Development*. Edited by J. Roumasset, J. M. Boussard, and I. J. Singh. New York: Agricultural Development Council.

Pollak, Robert, and Michael Wachter. 1975. "The Relevance of the Household Production Function for the Allocation of Time." *Journal of Political Economy*, vol. 83, pp. 255–77.

Roumasset, James, Marc Boussard, and Inderjit Singh. 1979. *Risk, Uncertainty and Agricultural Development*. New York: Agricultural Development Council.

Stiglitz, Joseph. 1969. "Behavior toward Risk with Many Commodities." *Econometrica*, vol. 37, pp. 660–67.

Stiglitz, Joseph, and Andrew Weiss. 1981. "Credit Rationing in Markets with Imperfect Information." *American Economic Review*, vol. 71, pp. 393–410.

Strauss, John. 1982. "Determinants of Food Consumption in Rural Sierra Leone: Application of the Quadratic Expenditure System to the Consumption-Leisure Component of a Household-Firm Model." *Journal of Development Economics*, vol. 11, pp. 327–53.

Wales, Terence, and A. D. Woodland. 1983. "Estimation of Consumer Demand Systems with Binding Non-Negativity Constraints." *Journal of Econometrics*, vol. 21, pp. 263–285.

Wright, Brian. 1979. "Effects of Ideal Production Stabilization: A Welfare Analysis under Rational Behavior." *Journal of Political Economy*, vol. 87, pp. 1011–33.

Yotopoulos, Pan, and Lawrence Lau. 1974. "On Modeling the Agricultural Sector in Developing Economies." *Journal of Development Economics*, vol. 1, pp. 105–27.

Appendix

The Theory and Comparative Statics of Agricultural Household Models: A General Approach

John Strauss

THIS APPENDIX DEVELOPS the basic model of the agricultural household introduced in chapter 1. The recursive property and comparative statics are derived first. The concept of a shadow or virtual price is then explicitly defined, and it is shown how the response of the virtual price to exogenous variables can be obtained. It turns out that with a minimum of assumptions this response can be signed. Next, these results are used to examine the comparative statics of various farm-household models, when the household faces virtual rather than parametric prices. During this exercise, the difference in the comparative statics between recursive and nonrecursive models becomes clear. The next section presents the outline of a model in which the market for labor is absent. This follows the earliest modeling of an agricultural household, by Chayanov, and its later technical development by Japanese economists (for example, Nakajima). Models that incorporate Z-goods are subsequently discussed, along with the previously neglected topic of models with certain types of commodity heterogeneity, which lead to corner solutions. Finally, conditions under which agricultural household models are recursive are summarized.

A Basic Model: The Household as Price-Taker

All prices in the static model developed here are taken as exogenous. Assume the household maximizes its utility subject to its constraints. Three constraints are specified at first: a production function, a time, and a budget constraint. Since agricultural household models have not generally been used to address issues of intrafamily distribution (Pitt and Rosenzweig explore some of the conceptual problems involved), a household utility function is assumed to exist. Let

$$U(X_1, \ldots, X_L) \tag{IA-1}$$

be the utility function, which is well behaved: quasi-concave with positive partial derivatives. The arguments are household consumption of commodity i, with X_L denoting total leisure time. Clearly, the X_i's can be a vector of commodity consumption for different members of the family as well. For instance, we might want X_L to include male, female, or children's leisure time separately. We could also allow household characteristics such as number of members to enter the utility function separately. As long as these are viewed as fixed, this will not change the analysis.

Utility is maximized subject to a budget constraint:

$$Y = \sum_{i=1}^{L} p_i X_i \tag{IA-2}$$

where Y is the household's full income (see equation [IA-3]), and the p_i's are commodity prices (p_L being the wage rate). Full income of an agricultural household equals the value of its time endowment, plus the value of the household's production less the value of variable inputs required for production of outputs, plus any nonwage, nonhousehold production income such as remittances:

$$Y = p_L T + \sum_{j=1}^{M} q_j Q_j - \sum_{i=1}^{N} q_i V_i - p_L L + E \tag{IA-3}$$

where

- T = time endowment
- Q_j = output, for $j = 1, \ldots, M$
- V_i = nonlabor variable inputs, for $i = 1, \ldots, N$
- L = labor demand
- q_j = price of Q_j
- q_i = price of V_i
- E = exogenous income.

For the moment, it is assumed that L is total labor demanded by the household, both family and hired, which are assumed to be perfect substitutes, an assumption we relax later in the discussion on partly absent markets.

Outputs and inputs are related by an implicit production function

$$G(Q_1, \ldots, Q_M, V_1, \ldots, V_N, L, K_1, \ldots, K_0) = 0 \tag{IA-4}$$

where K_i's are fixed inputs. This is a general specification that allows for separate production functions for different outputs, or for joint production. G is assumed to satisfy the usual properties for production functions: it is quasi-convex, increasing in outputs and decreasing in inputs.

If the household maximizes utility (IA-1) subject to its full-income (IA-2 and IA-3) and production-function (IA-4) constraints and to prices (p, q) being fixed, then the household's choices can be modeled as recursive decisions, even though the decisions are simultaneous in time (Jorgenson and Lau 1969; Nakajima 1969). The household behaves as though it maximizes the revenue side of its full income, equation (IA-3), subject to its production-function constraint, and then maximizes utility subject to its full-income constraint, equation (IA-2). Since neither the value of endowed time nor exogenous income are household choice variables, maximizing full income is equivalent to maximizing the value of outputs less variable inputs (that is, profits).

To see that the model is separable between revenue and expenditure, the comparative statics are examined. Let the household consume three commodities: leisure, X_L; a good that is purchased on the market, X_m; and a good, X_a, produced by the household. (Obviously all these scalars could just as well be vectors.) The household uses labor, L, another variable input, V, and a fixed input K to produce both Q_a and another crop, Q_c. All Q_c is sold on the market (a commercial crop). The Lagrangian function can be written as

$$\begin{aligned} \pounds = U(X_L, X_m, X_a) + \lambda[p_L T + (q_c Q_c + p_a Q_a - p_L L - q_v V) \\ + E - p_L X_L - p_m X_m - p_a X_a] + \mu G(Q_c, Q_a, L, V, K). \end{aligned} \tag{IA-5}$$

If we assume interior solutions, the first-order conditions are:

$$\begin{aligned} \frac{\partial \pounds}{\partial X_L} &= U_L - \lambda p_L = 0 \\ \frac{\partial \pounds}{\partial X_m} &= U_m - \lambda p_m = 0 \\ \frac{\partial \pounds}{\partial X_a} &= U_a - \lambda p_a = 0 \\ \frac{\partial \pounds}{\partial \lambda} &= p_L(T - X_L - L) + q_c Q_c + p_a(Q_a - X_a) \\ &\quad - q_v V - p_m X_m + E = 0 \\ \frac{1}{\lambda}\frac{\partial \pounds}{\partial Q_c} &= q_c + \frac{\mu}{\lambda} G_c = 0 \end{aligned} \tag{IA-6}$$

$$\frac{1}{\lambda}\frac{\partial \pounds}{\partial Q_a} = p_a + \frac{\mu}{\lambda} G_a = 0$$

$$\frac{1}{\lambda}\frac{\partial \pounds}{\partial L} = -p_L + \frac{\mu}{\lambda} G_L = 0$$

$$\frac{1}{\lambda}\frac{\partial \pounds}{\partial V} = -q_v + \frac{\mu}{\lambda} G_v = 0$$

$$\frac{\partial \pounds}{\partial \mu} = G(Q_c, Q_a, L, V, K) = 0.$$

Totally differentiating (IA-6),

(IA-7)

$$\begin{bmatrix} U_{LL} & U_{Lm} & U_{La} & -p_L & 0 & 0 & 0 & 0 & 0 \\ U_{mL} & U_{mm} & U_{ma} & -P_m & 0 & 0 & 0 & 0 & 0 \\ U_{aL} & U_{am} & U_{aa} & -p_a & 0 & 0 & 0 & 0 & 0 \\ -p_L & -p_m & -p_a & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\mu}{\lambda}G_{cc} & \frac{\mu}{\lambda}G_{ca} & \frac{\mu}{\lambda}G_{cL} & \frac{\mu}{\lambda}G_{cv} & G_c \\ 0 & 0 & 0 & 0 & \frac{\mu}{\lambda}G_{ac} & \frac{\mu}{\lambda}G_{aa} & \frac{\mu}{\lambda}G_{aL} & \frac{\mu}{\lambda}G_{av} & G_a \\ 0 & 0 & 0 & 0 & \frac{\mu}{\lambda}G_{Lc} & \frac{\mu}{\lambda}G_{La} & \frac{\mu}{\lambda}G_{LL} & \frac{\mu}{\lambda}G_{Lv} & G_L \\ 0 & 0 & 0 & 0 & \frac{\mu}{\lambda}G_{vc} & \frac{\mu}{\lambda}G_{va} & \frac{\mu}{\lambda}G_{vL} & \frac{\mu}{\lambda}G_{vv} & G_v \\ 0 & 0 & 0 & 0 & G_c & G_a & G_L & G_v & 0 \end{bmatrix} \begin{bmatrix} dX_L \\ dX_m \\ dX_a \\ d\lambda \\ dQ_c \\ dQ_a \\ dL \\ dV \\ d\left(\frac{\mu}{\lambda}\right) \end{bmatrix} = \begin{bmatrix} \lambda dp_L \\ \lambda dp_m \\ \lambda dp_a \\ \psi \\ -dq_c \\ -dp_a \\ dp_L \\ dq_v \\ 0 \end{bmatrix}$$

where $\psi = -(T - X_L - L)dp_L + X_m dp_m - (Q_a - X_a)dp_a - dE - Q_c dq_c + V dq_v - \mu/\lambda\, G_k dk$. When differentiating the budget constraint we have substituted

$$-\frac{\mu}{\lambda}(G_c dQ_c + G_a dQ_a + G_L dL + G_v dV)$$

for

$$q_c dQ_c + p_a dQ_a - p_L dL - q_v dV.$$

This equals $\mu/\lambda\, G_k dK$ since $G(\cdot) = 0$. This system of equations is block diagonal, as can easily be seen from equation system (IA-7). The first set of equations, corresponding to the upper left block of the bordered Hessian matrix, gives the solution for commodity demands and the marginal utility of full income. The second (lower right) set of equations gives the solution for output supplies, variable input demands, and the associated multiplier. The assumptions concerning the utility and production functions ensures that second-order conditions are met. Hence, the two decision problems can indeed be solved recursively, despite their simultaneity in time.

Equation (IA-7) demonstrates the principal message of the farm-household literature, that farm technology, quantities of fixed inputs, and prices of variable inputs and of outputs do affect consumption decisions. Given recursiveness, however, the reverse is not true. Preferences, prices of consumption commodities, and income do not affect production decisions. Output supply responds positively to own-price at all times owing to the quasi-convexity assumption on the production function, $\partial Q_c/\partial q_c > 0$. The price of the cash crop, q_c, will be related to consumption of the purchased commodity, X_m, through changed income. From equation (IA-7) it can be seen that

$$\frac{\partial X_m}{\partial q_c} = Q_c \frac{\partial X_m}{\partial E}.$$

Likewise, changes in quantities of fixed inputs, K, will affect income, hence the consumption of X_m:

$$\frac{\partial X_m}{\partial K} = \frac{\mu}{\lambda} G_K \frac{\partial X_m}{\partial E}.$$

Assuming X_m is a normal commodity, increments to fixed inputs or to the cash crop price will induce higher consumption of X_m. For commodities that are also produced by the household, own-price effects are

(IA-8) $$\frac{\partial X_a}{\partial p_a} = \left.\frac{\partial X_a}{\partial p_a}\right|_U + (Q_a - X_a)\frac{\partial X_a}{\partial E}.$$

Thus, a change in the price of X_a has the usual negative substitution effect, and an income effect that is weighted by net sales (or marketed surplus) of X_a, not consumption of X_a. The income effect is positive for a net seller and negative for a net buyer. In consequence, for net sellers, consumption of X_a might respond positively to changes in its own price even though it is a normal good.

The income effect for a farm household has an extra term, $Q_a(\partial X_a/\partial E)$, as compared with the pure consuming household. This extra effect is introduced when the profits component of full income is raised; hence it can be referred to as a profit effect. To see this, note that from equation (IA-3) $dY = Tdp_L + d\pi + dE$, where π = profits, the value of outputs less the value of variable inputs. From equation (IA-3) and the first-order conditions,

$$d\pi = Q_c dq_c + Q_a dp_a - L\, dp_L - V\, dq_v + \frac{\mu}{\lambda} G_k dK.$$

Thus, the fourth element of the right-hand side of equation (IA-7) may be expressed as

$$\psi = -(T - X_L)dp_L + X_m dp_m + X_a dp_a - d\pi - dE.$$

It is then clear that the Marshallian demand for food can be written as

$$X_a(p_L, p_m, p_a, q_c, q_v, K, E) \quad \text{or as} \quad X_a(p_L, p_m, p_a, \pi, E)$$

with profits replacing nonlabor variable input prices and fixed inputs. The comparative statics are then

(IA-8a) $$\left.\frac{\partial X_a}{\partial p_a}\right|_{\pi} = \left.\frac{\partial X_a}{\partial p_a}\right|_{U} - X_a \frac{\partial X_a}{\partial Y}$$

which is identical to the pure consumer case, while

(IA-8b) $$\frac{\partial X_a}{\partial p_a} = \left.\frac{\partial X_a}{\partial p_a}\right|_{U} - X_a \frac{\partial X_a}{\partial Y} + \frac{\partial X_a}{\partial Y}\frac{\partial \pi}{\partial p_a}.$$

Since $\partial\pi/\partial p_a = Q_a$, from above, the extra effect does indeed come through changing farm profits. The comparative statics for leisure

(IA-9) $$\frac{\partial X_L}{\partial p_L} = \left.\frac{\partial X_L}{\partial p_L}\right|_{U} + (T - X_L - L)\frac{\partial X_L}{\partial Y}$$

are similar. The income effect is weighted by household labor supply minus labor demand (marketed surplus of labor), not by household labor supply. Assuming that leisure is a normal good makes a backward-bending supply curve less likely than if the household were solely a supplier of labor.

Deriving Virtual (Shadow) Prices

To explore the consequences of making prices endogenous to the household, it will be convenient to use duality results to express the equilibrium of the household. We can define the full-income function as the maximization of equation (IA-3) with respect to outputs and variable inputs subject to the production function, (IA-4), and can write

(IA-10)

$$Y = \Lambda(q_c, p_a, p_L, q_v, K, T, E) = p_L T + \pi(q_c, p_a, p_L, q_v, K) + E.$$

The full-income function is the sum of the value of endowed time, a restricted (or short-run) profits function, and exogenous income. The profits function has the usual properties—for example, it is convex in all prices. For the expenditure side of full income, we can define an expenditure function as the minimum expenditure (equation IA-2) required to meet a specified level of utility, $e(p_L, p_m, p_a, \bar{U})$. It obeys the usual properties; in particular it is concave in prices, and the partial derivatives with respect to price are the Hicksian (compensated) demand functions.

Now we are in a position to relax our assumption that prices are fixed market prices. The household's equilibrium is characterized by equality between the household's full-income function, Λ, and its expenditure function, e, where the expenditure function is evaluated at the utility level achieved at the household's optimum. This condition will hold whether or not households face given market prices. Now suppose that a household is constrained to equate consumption with production for some commodity(ies). One possible reason for this would be nonexistence of a market. Consequently, the household's equilibrium will be characterized by a set of additional conditions—equality of household demand and household supply for each commodity for which there is no market (see Dixit and Norman 1980, who use these conditions to characterize an economy under autarky). This second set of equilibrium conditions implicitly defines a set of virtual prices—or shadow prices (Neary and Roberts 1980; Deaton and Muellbauer 1980, chapter 4.3), which, if they existed, would induce the household to equate supply and demand for these commodities.

These virtual prices are not taken parametrically by the household as market prices are; rather, they are determined by the household's choices. From the household's equilibrium, it can be seen that they will be a function of market prices, time endowment, fixed inputs, and either exogenous income or utility. (They will also be a function of fixed household characteristics if these are introduced into the model.) Consequently, these prices depend on both the household's preferences and its production technology. Changes in market prices will now affect behavior directly, as before, and indirectly through changes in the virtual prices. Some mechanism of identifying the consequences of this additional effect is therefore needed to illuminate the significance of one's assumptions regarding price formation. That mechanism will be the comparative statics of the virtual price, which will now be developed.

Suppose, for the moment arbitrarily, that there exists no market for labor. The household equilibrium is characterized by

(IA-11) $$e(\bar{p}_L^*, p_m, p_a, \bar{U}) = \bar{p}_L^* T + \pi(q_c, p_a, \bar{p}_L^*, q_v, K) + E$$

$$e_L(\bar{p}_L^*, p_m, p_a, \bar{U}) = T + \pi_L(q_c, p_a, \bar{p}_L^*, q_v, K)$$

where $e_L = \partial e/\partial p_L^*$ and likewise $\pi_L = \partial \pi/\partial p_L^*$. The second equation gives the Hicksian leisure demand on the left-hand side and time endowment minus labor demand on the right. From this equation, $\bar{p}_L^*$, the compensated virtual price, can be solved for as

(IA-12) $$\bar{p}_L^* = \bar{p}_L^*(p_m, p_a, q_c, q_v, K, U).$$

Note that the utility level is being held constant, and not exogenous income. Alternatively, the Marshallian leisure demand

$$X_L(p_L^*, p_m, p_a, p_L^* T + \pi + E)$$

can be set equal to time minus labor demand, and a solution obtained:

(IA-13) $$p_L^* = p_L^*(p_m, p_a, q_c, q_v, K, E).$$

To relate the functions $\bar{p}_L^*$ and p_L^*, a somewhat different expenditure function is needed. Let

$$e'(p_L, p_m, p_a, q_c, q_v, K, T, \bar{U}) =$$

(IA-14) $$\min_{\substack{X_L, X_m, X_a \\ Q_c, Q_a, L, V}} p_L X_L + p_m X_m + p_a X_a - p_L T - q_c Q_c - p_a Q_a$$

$$+ p_L L + q_v V \quad \text{st } U(\cdot) = \bar{U} \text{ and } G(\cdot) = 0.$$

This represents the minimum exogenous income, E, necessary to achieve utility level $\bar{U}$, given the production function and prices. It is clear that e' meets all the conditions that a regular expenditure function does, and that

(IA-15) $$e'(p_L, p_m, p_a, q_c, q_v, K, T, \bar{U}) =$$

$$e(p_L, p_m, p_a, \bar{U}) - p_L T - \pi(q_c, p_a, p_L, q_v, K).$$

In equation (IA-13), if exogenous income E is evaluated at e' (hence full income, Y, at e) then Marshallian leisure demand equals the Hicksian demand and $p_L^* = \bar{p}_L^*$. Using this equality

(IA-16) $$\frac{\partial \bar{p}_L^*}{\partial Z} = \left.\frac{\partial p_L^*}{\partial Z}\right|_E + \frac{\partial p_L^*}{\partial E}\frac{\partial e'}{\partial Z} \qquad Z = p_m, p_a, q_c, q_v, K.$$

With utility constant, the response of the virtual price can be expressed in terms of second partial derivatives of the expenditure and profit functions. Using the implicit function rule and equation (IA-11),

(IA-17) $$\frac{\partial \bar{p}_L^*}{\partial Z} = -(e_{LZ} - \pi_{LZ})/(e_{LL} - \pi_{LL}) \qquad Z = p_m, p_a, q_c, q_v, K.$$

The denominator is unambiguously negative owing to the concavity of the expenditure function and the convexity of the profits function. The numerator can be either sign, but often the sign will be determinate if one is willing to assume that commodities are substitutes or complements in consumption or production. For instance, if $Z = p_m$, the price of the market-purchased commodity, X_m, the numerator is $-e_{Lm}$, which is negative if leisure and X_m are substitutes. If $Z = p_a$, the numerator is $\pi_{La} - e_{La}$. The first term is the response of output of X_a to wage, which should be negative. The second term is negative if leisure and X_a are substitutes. For an input price, q_v, the numerator is π_{Lv}, which can be positive or negative, depending on whether labor and input V are gross substitutes or complements.

Equation (IA-17) is a basic result that will be used repeatedly in the subsequent discussion to illuminate the effects of totally or partly absent markets. It allows one to sign the partial derivatives of the compensated virtual price, making this device useful in looking at the comparative statics. Moreover, it allows one to compare directly models that make differing assumptions about the nature of prices the household faces.

The sign of the response of the compensated virtual price, $\bar{p}_L^*$, to exogenous variables can be given an intuitive interpretation. If, for instance, the price of the cash crop rises, the demand schedule for labor should shift upward. Given that other market prices, fixed inputs, and utility are constant, the virtual wage has to rise in order to reequate compensated labor supply with demand. Such a rise will lower labor demand along the new schedule, while raising compensated, or Hicksian, labor supply.

The virtual prices are functions of both household preferences and production technology. Because these prices help to determine both consumption and production choices—they belong in both the expenditure and the full-income functions—the household commodity demands will depend on production technology, both through the virtual price and through full income. Output supplies and input demands will depend on preferences through the virtual price. If, however, the household faces only market prices or if it faces a virtual price for a commodity that is consumed but not produced (or vice versa), then production choices will not depend on household preferences, but consumption choices will depend on production technology through full income. The model is then recursive.

Models with Absent Markets: Labor

In the historical development of agricultural household models, partially autarkic behavior has been very important. One of the earliest models can be traced to the Russian economist A. V. Chayanov (1925) (see Millar 1970 for a reinterpretation). Chayanov was concerned with explaining the allocation of labor between work and leisure in Russian peasant households given his observation that virtually no hired labor was used in farm production activities. He recognized that such households were not simply maximizing profits as in the theory of the firm; rather, they had a subjective equilibrium in which they equated the marginal utility of household consumption with the marginal utility of leisure. His analysis was embellished by a group of Japanese economists, notably Tanaka (1951) and Nakajima (1957), during the 1950s and 1960s. Nakajima (1969), in particular, gave the model currency among English-speaking economists. He not only gave a mathematical formulation to Chayanov's model, but also proposed some additional models. Nakajima's (1969) model of a pure commercial family farm without a labor market assumed that households sold all of their output and purchased commodities from the market, and that they produced the output with family labor and a fixed amount of land. He also allowed for the possibility of a minimum subsistence consumption requirement as well as a target income. In a different version (his semisubsistence family farm), he allows the family to consume some of its output, and in another version introduces two outputs. Similar models of peasant households were advanced by Mellor (1963) and Sen (1966) and by economic anthropologists such as Fisk and Shand (1969). These models are thus special cases of the general form of the agricultural household model developed here.

These models in which the family supplied all of its labor were used primarily to explore the effects on labor supply (and hence output, since family labor was assumed to be the only variable input) of changes in different variables. The effect of output price was of particular interest because of the seemingly perverse possibility that output might respond negatively to output price. This might occur if the income effect, resulting in more leisure demand, was large enough. Nakajima showed that an exogenous increase in land input might also reduce output, because it too would have an income effect on leisure. Nakajima separated the response of labor supply to output price into substitution and income effects, showing that the income-compensated response of labor supply to output price was positive. Sen showed that output response to output price could be negative, and that there could be no output response to the withdrawal of family workers if the remaining family laborers worked sufficiently hard to offset the reduced number of hours worked as workers were withdrawn. This required that the virtual wage (or its ratio to output price, Sen's real cost of labor) be constant, as would be the case in

Sen's model if the marginal utilities of both income and leisure were roughly constant.

The possibility of a negative response of labor demand (and of output supply) to output price at the household level is dependent on the constrained equality of labor demand and labor supply. At the market level labor demand might respond negatively to output price if wage is bid up sufficiently (see Barnum and Squire 1980). If markets exist for all commodities, then the model is recursive and labor demand will respond positively to output price as long as it is not an inferior input. Nakajima noted this when discussing his model with a labor market and a cash crop. Both Jorgenson and Lau (1969) and Krishna (1964, 1969) proposed separable semisubsistence models in which labor is marketed and output is partly consumed at home. Jorgenson and Lau's study has formed the basis on which most of the empirical work to date has been conducted.

Consumption and Leisure Responses

The difference that absence of a labor market makes to the comparative statics of leisure and commodity demand can easily be seen by using the notion of a virtual wage. Write the Marshallian demand as

$$X_i[p_L^*, p_m, p_a, p_L^* T + \pi(q_c, p_a, p_L^*, q_v, K) + E], \quad i = L, M.$$

Differentiate this with respect to q_c to obtain

$$\text{(IA-18)} \qquad \frac{\partial X_i}{\partial q_c} = \frac{\partial X_i}{\partial p_L^*}\frac{\partial p_L^*}{\partial q_c} + Q_c \frac{\partial X_i}{\partial Y} \qquad i = L, M.$$

Cash output price has two effects on the demand for leisure or for the market purchased good: it has an income effect by changing profits (the second term), and it changes the virtual price for labor. Clearly, when the household is a price-taker in the labor market, the latter effect is zero.

Equation (IA-18) can be decomposed into substitution and income effects, which will help in signing the uncompensated changes in the demand for leisure and the market-purchased commodity. First, it can be shown that the uncompensated effect with respect to the virtual wage equals the compensated effect. To do this, it will be useful to equate Marshallian and Hicksian demands by evaluating full income, Y, and e and the virtual wage at $\bar{p}_L^*$ (that is, if both hold utility constant):

$$\text{(IA-19)} \quad X_i[\bar{p}_L^*, p_m, p_a, e(\bar{p}_L^*, p_m, p_a, U)] = X_i^c(\bar{p}_L^*, p_m, p_a, U) \qquad i = L, M.$$

Differentiating both sides of (IA-19) with respect to the cash crops price, q_c, and using $\partial e/\partial \bar{p}_L^* = X_L$ results in

$$\text{(IA-20)} \qquad \left.\frac{\partial X_i}{\partial \bar{p}_L^*}\right|_Y \frac{\partial \bar{p}_L^*}{\partial q_c} + X_L \frac{\partial X_i}{\partial Y}\frac{\partial \bar{p}_L^*}{\partial q_c} = \frac{\partial X_i^c}{\partial p_L^*}\frac{\partial \bar{p}_L^*}{\partial q_c} \qquad i = L, M.$$

Since

$$\text{(IA-21)} \qquad \frac{\partial X_i}{\partial p_L^*} = \left.\frac{\partial X_i}{\partial p_L^*}\right|_Y + (T - L)\frac{\partial X_i}{\partial Y}$$

and since labor supply equals labor demand, so that $X_L = T - L$, it can be shown by means of equation (IA-20) that $\partial X_i/\partial p_L^* = \partial X_i^c/\partial p_L^*$. Thus the income effect of a change in the virtual wage equals zero, which is intuitive since the net marketed surplus is zero when no labor market exists.

The term $\partial p_L^*/\partial q_c$ in equation (IA-18) can be made more transparent by noting from (IA-16) that

$$\frac{\partial p_L^*}{\partial q_c} = \frac{\partial \bar{p}_L^*}{\partial q_c} + Q_c \frac{\partial p_L^*}{\partial E} \qquad \left(\text{recall that } \frac{\partial e'}{\partial q_c} = -Q_c\right).$$

When this is substituted into (IA-18), one obtains

$$\text{(IA-22)} \qquad \frac{\partial X_i}{\partial q_c} = \frac{\partial X_i^c}{\partial p_L^*}\frac{\partial \bar{p}_L^*}{\partial q_c} + Q_c\left(\frac{\partial X_i}{\partial Y} + \frac{\partial X_i}{\partial p_L^*}\frac{\partial p_L^*}{\partial E}\right) \qquad i = L, M$$

$$\text{(IA-22a)} \qquad = \frac{\partial X_i^c}{\partial p_L^*}\frac{\partial \bar{p}_L^*}{\partial q_c} + Q_c \frac{\partial X_i}{\partial E} \qquad i = L, M.$$

Equations (IA-22) and (IA-22a) show the decomposed income and substitution effects. They also clarify the significance of one's view regarding the labor market. If the labor market does exist, then the household faces market prices so the substitution effect—the first term in (IA-22a)—is zero and the entire effect of the change in output price is captured by the income effect $[Q_c(\partial X_i/\partial Y)]$. This is positive, providing leisure or the purchased commodity are normal goods. When the labor market is absent, a substitution effect is caused by the change in the income-compensated virtual wage. Using equation (IA-17), we can rewrite this substitution effect as

$$\text{(IA-23)} \qquad \frac{\partial X_i^c}{\partial p_L^*}\frac{\partial \bar{p}_L^*}{\partial q_c} = e_{Li}\pi_{Lc}/(e_{LL} - \pi_{LL}) \qquad i = L, M.$$

If the compensated virtual wage rises—that is, if $\pi_{Lc} < 0$, in equation (IA-23)—then there is a substitution away from leisure or toward the purchased commodity (if it is a substitute for leisure). The income effect comes in two parts: first, a traditional looking income effect, and, second, a substitution-type effect due to an induced change in the uncompensated virtual wage, p_L^*. This two-part income effect is identical to equation (IA-24) of Neary and

Roberts (1980), once their equation (IA-19) has been substituted in. From equation (IA-22), we can see that when leisure is normal, $\partial p_L^*/\partial E > 0$, the income effect is smaller for leisure and larger for purchased goods (if we assume substitutability with leisure) when the labor market does not exist than when it does. An increase in exogenous income raises the uncompensated virtual wage, and this increase induces a substitution away from leisure or toward the purchased commodity. If we assume that the entire income effect is positive, the net effect of a rise in output price q_c on leisure is indeterminant, but it will be positive for the purchased commodity. This is the same result, of course, as is obtained by both Nakajima (1969) and Sen (1966).

Output Responses

If labor is the only variable input, then the sign of output response to output price must be the opposite to the leisure response. More generally, we can write output supply Q_c as

$$Q_c = \frac{\partial \pi}{\partial q_c}(q_c, p_a, p_L^*, q_v, K)$$

consequently,

$$\text{(IA-24)} \qquad \frac{\partial Q_c}{\partial q_c} = \pi_{cc} + \pi_{cL}\frac{\partial p_L^*}{\partial q_c}.$$

The first term is the output-supply response when the virtual wage is fixed, and is positive. The second term is negative, if we assume that output responds negatively to the virtual wage ($\pi_{cL} < 0$), so that the sign of the entire expression is indeterminant. It is possible to show that when household utility is held constant, the response is positive. (See Lopez 1980 for a somewhat different demonstration of this.) Substituting for $\partial p_L^*/\partial q_c$ from equation (IA-16),

$$\text{(IA-25)} \qquad \frac{\partial Q_c}{\partial q_c} = \left(\pi_{cc} + \pi_{cL}\frac{\partial \bar{p}_L^*}{\partial q_c}\right) + Q_c\pi_{cL}\frac{\partial p_L}{\partial q_c}.$$

The first two terms are the response of output supply when utility is held constant. The third term is an income effect, which is negative if π_{cL} is negative. The second term equals $\pi_{cL}^2/(e_{LL} - \pi_{LL})$, so it is negative. However, summing it with π_{cc} gives a nonnegative quantity because the function e' (equation [IA-15]) is concave in prices, so that

$$\frac{\partial^2 e'}{\partial q_c^2}\frac{\partial^2 e'}{\partial p_L^{*2}} - \left(\frac{\partial^2 e'}{\partial q_c \partial p_L^*}\right)^2 \geq 0.$$

Straightforward algebra shows that this expression is simply the first two terms in equation (IA-25) multiplied by $-\partial^2 e'/\partial p_L^{*2}$. The magnitude of π_{cL} and consequently the likelihood of a negative output response will be influenced by the number of variable inputs and the partial elasticity of substitution between labor and these other inputs. Presumably, the more inputs and the more substitutable they are, the less negative π_{cL} will be and the more likely will be a positive response to output price. Clearly, when the virtual wage is exogenous to the household, output response will be positive and greater than when the virtual wage is endogenous.

If the household consumes some of the output for which price is changing, Q_c, the comparative statics have an additional substitution effect, and the income effect is weighted by net output sold (marketed surplus) and not by total output:

$$\text{(IA-26)} \qquad \frac{\partial X_i^c}{\partial p_a} = \left.\frac{\partial X_i^c}{\partial p_a}\right|_{\bar{p}_L^*} + \frac{\partial X_i^c}{\partial p_L^*}\frac{\partial \bar{p}_L^*}{\partial p_a} \qquad i = L, M, A.$$

Again using equation (IA-17), $\partial \bar{p}_L^*/\partial p_a = (\pi_{aL} - e_{aL})/(e_{LL} - \pi_{LL})$, which is positive if Q_a and leisure are substitutes. Deriving the comparative statics as before, one finds

$$\text{(IA-27)} \qquad \frac{\partial X_i}{\partial p_a} = \left(\left.\frac{\partial X_i^c}{\partial p_a}\right|_{p_L^*} + \frac{\partial X_i^c}{\partial p_L^*}\frac{\partial \bar{p}_L^*}{\partial p_a}\right) + (Q_a - X_a)\left(\frac{\partial X_i^c}{\partial p_L^*}\frac{\partial p_L^*}{\partial E} + \frac{\partial X_i}{\partial Y}\right) \qquad i = L, M, A$$

$$\text{(IA-27a)} \qquad \frac{\partial X_i}{\partial p_a} = \frac{\partial X_i^c}{\partial p_a} + (Q_a - X_a)\frac{\partial X_i}{\partial E} \qquad i = L, M, A.$$

The substitution effect for leisure demand can be of either sign. It is not necessarily positive, even if X_a and leisure are substitutes holding the virtual wage constant. The income-compensated response of X_a can also be of either sign when the wage is virtual, since an increase in the price, p_a, will increase the compensated virtual wage leading to a substitution toward X_a. The substitution effect for X_a will be less negative than when the labor market exists, as Neary and Roberts (1980) found in the pure rationing case. The income effect has an extra term, which for X_a and X_m is positive if leisure is a substitute and is negative for leisure demand.

Marketed Surplus Responses

If we examine the response of marketed surplus of X_a, $Q_a - X_a$, to change in p_a, we obtain from (IA-25), (IA-16), and (IA-27)

$$\frac{\partial(Q_a - X_a)}{\partial p_a} = \left(\left.\frac{\partial Q_a}{\partial p_a}\right|_{\bar{p}_L^*} + \frac{\partial Q_a}{\partial p_L^*}\frac{\partial \bar{p}_L^*}{\partial p_a} - \left.\frac{\partial X_a^c}{\partial p_a}\right|_{\bar{p}_L^*} - \frac{\partial X_a^c}{\partial p_L^*}\frac{\partial \bar{p}_L^*}{\partial p_a}\right)$$

$$\text{(IA-28)} \qquad + (Q_a - X_a)\left(\frac{\partial Q_a}{\partial p_L^*}\frac{\partial p_L^*}{\partial E} - \frac{\partial X_a^c}{\partial p_L^*}\frac{\partial p_L^*}{\partial E} - \frac{\partial X_a}{\partial Y}\right).$$

The first four terms (in brackets) hold utility constant, and therefore comprise the substitution effect. It is straightforward to see that this effect equals

$$-\frac{\partial^2 e'}{\partial p_L^{*2}}\left[\frac{\partial^2 e'}{\partial p_a^2}\frac{\partial^2 e'}{\partial p_L^{*2}} - \left(\frac{\partial^2 e'}{\partial p_a \partial p_L^*}\right)^2\right]$$

and consequently is nonnegative (remember that e' is concave in prices). The last term equals

$$(Q_a - X_a)\left(\frac{\partial Q_a}{\partial E} - \frac{\partial X_a}{\partial E}\right)$$

and so is the income effect that should be negative if marketed surplus is positive and X_a is a normal good. Consequently, marketed surplus of X_a might respond positively or negatively to an increase in its own price. Comparing this result with that when the labor market exists, one can see that the extra substitution effects will be negative if X_a and leisure are substitutes, since the compensated virtual wage will then rise. The extra income effects should also be negative, so that a greater possibility exists of obtaining a negative own-price response of marketed surplus of X_a.

The comparative statics with respect to changes in p_m, q_v, K, and T are similar to equation (IA-22), except that the response of the compensated virtual wage is different, as is the term weighting the income effect. Specific formulae are left for the interested reader to derive.

Models with Absent Markets: Z-Goods

The market that one assumes not to exist clearly does not affect the foregoing argument. Hence, the existence of a labor market is a necessary but not a sufficient condition for an agricultural household model to be separable. *All markets must exist for separability*, although this is not a sufficient condition, as is discussed in the next section. Historically, economists thought that the labor market was the one least likely to exist for peasant farms. That view has been changing, however, since active rural labor markets have been found according to several studies (Rosenzweig 1978; Spencer and Byerlee 1977; Bardhan 1979; Squire 1981; Binswanger and Rosenzweig 1984), although they are not necessarily perfectly competitive ones. More recent studies have focused on the nonexistence of a market for so-called Z-goods. This was first formalized by Hymer and Resnick (1969), who refer to Z-goods as nonagricultural, nonleisure activities. In general the commodities Hymer and Resnick refer to, such as food processing and metalworking, are commodities for which small-scale rural industries have been found to exist (Anderson and Leiserson 1980; and Liedholm and Chuta 1976). Z-goods, however, refer equally as well to nontraded outputs of household production activities such as the number and quality of children, home maintenance, or food preparation. In this way, the household production models of Becker (1965) and Gronau (1973, 1977) can be incorporated into agricultural household models.

Hymer and Resnick (1969) were concerned with the increasing specialization of agricultural household activities, which they saw as occurring over time and resulting in an increasing marketed surplus from agricultural households. Rather than focus on the leisure-labor tradeoff, they concentrated on the Z-goods-food tradeoff. In terms of the general model specified here, households produce foods, Q_a, which they consume and sell the surplus in exchange for manufactured commodities, X_m. They produce Z-goods, our L, which they consume entirely at home, $L = X_L$. Labor supply does not enter their model, but implicitly it is assumed to be fixed in amount and to be equal to labor demand; thus it is not a choice variable. In terms of this model, labor is one of the fixed inputs, K, and it does not appear in the utility function. Alternatively, leisure can enter the utility as a fixed factor, similar to other fixed household characteristics such as household size and age distribution. In this case, the expenditure function will include leisure as a conditioning variable just as a short-run cost or profit function includes fixed inputs.

There are no other variable inputs, $V = 0$, nor does there exist a cash crop, $Q_c = 0$. These assumptions imply that the product transformation curve between foods and Z-goods has the usual downward-sloping, concave shape. Consequently, to find the sign of the effect of a change in the price of foods, p_a, on the output of foods, only the effect on demand (hence supply) of Z-goods needs to be considered, $\partial X_L/\partial p_a$, which is given by equation (IA-27). The substitution effect can be of either sign. If Z-goods and foods are substitutes, a rise in food prices will increase Z-goods consumption when the compensated virtual price of Z-goods is held constant. This will force up the virtual price, however, and lead to a substitution away from Z-goods consumption. The income effect is weighted by the marketed surplus of foods, which is presumed to be positive. Hymer and Resnick assume that Z-goods consumption is inferior and that the combined substitution effect is small, so that the net effect of

a rise in the price of foods will be a fall in the consumption (and production) of Z-goods, and hence a rise in food production. Of course, if foods are consumed by the household, the food consumption response to food price needs to be examined before what happens to marketed surplus of foods can be judged. As seen from equation (IA-28), marketed surplus of food can either rise or fall in response to an increase in food price, provided the household has a positive marketed surplus and Z-goods are normal (so $\partial p_L^*/\partial E > 0$). However, if Z-goods are inferior, then its virtual price falls when exogenous income rises, so that production of foods rises and compensated consumption of foods falls (provided foods and Z-goods are substitutes), making it more likely that the response of marketed surplus is positive.

The Hymer and Resnick assumption that leisure and labor demand are not choice variables can be relaxed. If it is assumed that no labor market exists, then two virtual prices exist, one for labor and one for household Z-goods. There are thus two equality constraints on supply and demand rather than one. Alternatively, the labor market may be assumed to exist.

As an alternative to the Hymer and Resnick interpretation, Z-goods might be interpreted as being synonymous with household production activities. The original work of Becker (1965), Lancaster (1966), and Muth (1966) emphasizes that the commodities that yield household utility are produced within the household by goods purchased in the market and by labor. In terms of this general model, X_c is a vector of commodities consumed and produced in the home. Market-purchased inputs are denoted by V ($X_m = 0$), and labor demand, L, is a vector of time allocated to the production of each commodity. Leisure usually is not considered, so total time is the sum of time spent in household production, plus market work. It is often assumed that Z-goods production is not joint and that it exhibits constant returns to scale. If no fixed inputs exist, the supply (and profit) functions will be ill-defined so that shadow (or implicit) prices cannot be defined in terms of equality between household supply and demands. Rather, they are defined implicitly by the partial derivatives of the cost functions with respect to output (Pollak and Wachter 1975). However if fixed inputs do exist, or the production functions are strictly convex, shadow (or virtual) prices can be implicitly defined from the equality of household demand and supply functions.

An elaboration of the household production framework by Gronau (1973) provides results almost identical to the model of Hymer and Resnick. Gronau's model amounts to relabeling food consumption as leisure and food production as labor demand. He, too, has a market-purchased and a home-produced (Z) commodity, with home production using labor and purchased inputs. As in the Hymer and Resnick model, a virtual price exists for the home-produced (Z) good. If no labor is supplied to the market, there will exist a virtual (shadow) wage as well, and the analysis is comparable to the Hymer and Resnick model when labor is a choice variable but no market for it exists. In a subsequent study, Gronau (1977) assumes that the market-purchased and the household-produced commodities are perfect substitutes in consumption and so may be added. As long as market purchases are positive and labor is sold on the market, this model is recursive. If labor is not sold on the market, a virtual (shadow) price for labor exists, and if market purchases of the home-produced commodity are zero, a virtual price for it exists. Huffman and Lange (1982) have a slightly different version of Gronau's model in which the household is explicitly an agricultural household. The household jointly produces a farm and a household commodity (X_c and X_a), selling the former and consuming the latter. Labor is sold on the market, but the only market purchases are for production inputs. A virtual price exists for the household commodity and the model is not separable. If, however, the farm and household commodities have separate production functions and fixed inputs could only be allocated to one enterprise, the model would be recursive between farm production decisions and the rest.

Partly Absent Markets: Commodity Heterogeneity

Even if all markets exist, households may face a virtual price that depends on both production technology and household preferences, so that again an agricultural household model would not be recursive. This can occur because markets are partly absent or because constraints are institutionally imposed (see Sicular, chapter 10, for an analysis of such constraints imposed on a production team in the People's Republic of China). In particular, a household may be able to sell a commodity but not buy it, or vice versa. If this commodity is both consumed and produced by the household, then the household's optimum may be at a corner at which consumption equals production. Such corner solutions are likely to occur especially when commodities are heterogeneous. For example, hired and family labor may be imperfect substitutes because of extra monitoring or search costs of hired labor. On-farm and off-farm labor may give different levels of disutility (see Lopez, chapter 11). Alternatively, a commodity consumed out of home production may have a different quality than the same commodity purchased on the market, and thus sales and purchase prices may differ.

Households can sell and consume family labor or home production, but they cannot purchase them. This suggests that, at the market price, supply might be less than demand, which is not possible. For such corner

solutions, the commodity in question has a virtual price that would equate supply and demand. The virtual price will be higher than the market price provided that the compensated marketed surplus responds positively to price.

If households have preferences between on-farm and off-farm labor, then even if hired and family labor are perfect substitutes in production there may exist excess supply of on-farm labor at the market wage, in which case the virtual wage will be lower.

It should be clear that the comparative statics for these equilibria are identical to those considered earlier for the cases in which no market exists. Also, if these corner solutions are not binding, then the model is separable, the market prices being the opportunity costs. This will complicate empirical work since, if such heterogeneity exists, a sample is likely to include both households at corners and households at interior solutions.

Recursive Conditions Summarized

This appendix has reviewed the comparative statics of some basic, static agricultural household models. A key modeling issue is under what circumstances a model is recursive. This is very important for applied empirical work since it makes the problem far more tractable (see chapter 1). It has been shown that a sufficient condition for recursiveness is that all markets exist for commodities that are both produced and consumed, with the household being a price-taker in each one, and that such commodities are homogeneous. As long as households can buy or sell as much as they want at given prices, production and consumption decisions can be treated as if they were sequential, production decisions being made first, even though they may be made simultaneously. Such strong conditions are not necessary, however. In particular, the homogeneity assumption can be dropped. In this case, however, the agricultural household model remains recursive only if the household does not choose to be at a corner for a commodity that it both produces and consumes (for example, consuming all of its output). If a corner solution is chosen, then a virtual price exists, which is a function of both preferences and technology, so that the household's decision is no longer separable. Note that even in the case of heterogeneity, it is still necessary to assume that all markets exist and that prices are given to households to achieve recursiveness. If even one market does not exist (for a commodity that is consumed and produced), then recursiveness from production to consumption decisions breaks down.

Historically, nonrecursive agricultural household models were thought to be relevant, primarily because labor markets were presumed not to exist. As more has been learned about rural labor markets in developing countries, this assumption has become increasingly questioned. This does not mean that empirically relevant models have to be recursive, but the reasons for nonrecursiveness need to be clearly spelled out (see chapter 2).

References

Anderson, Dennis, and Mark Leiserson. 1980. "Rural Nonfarm Employment in Developing Countries." *Economic Development and Cultural Change*, vol. 28, pp. 227–48.

Bardhan, Pranab. 1979. "Wages and Unemployment in a Poor Agrarian Economy: A Theoretical and Empirical Analysis." *Journal of Political Economy*, vol. 87, pp. 479–500.

Barnum, Howard, and Lyn Squire. 1980. "Predicting Agricultural Output Response." *Oxford Economic Papers*, vol. 32, pp. 284–95.

———. 1979. *A Model of an Agricultural Household*. Washington, D.C.: World Bank.

Becker, Gary. 1965. "A Theory of the Allocation of Time." *Economic Journal*, vol. 75, pp. 493–517.

Binswanger, Hans, and Mark Rosenzweig, eds. 1984. *Contractual Arrangements, Employment and Wages in Rural Labor Markets: A Critical Review*. New Haven, Conn.: Yale University Press.

Chayanov, A. V. 1925. "Peasant Farm Organization." Moscow: Cooperative Publishing House. Translated in *A. V. Chayanov: The Theory of Peasant Economy*, ed. D. Thorner, B. Kerblay, and R. E. F. Smith. Homewood, Ill.: Richard Irwin, 1966.

Deaton, Angus, and John Muellbauer. 1980. *Economics and Consumer Behavior*. Cambridge, Mass.: Cambridge University Press.

Dixit, Avinash, and Victor Norman. 1980. *Theory of International Trade: A Dual, General Equilibrium Approach*. Cambridge, Mass.: Cambridge University Press.

Fisk, E. K., and K. T. Shand. 1969. "The Early Stages of Development in a Primitive Economy: The Evolution from Subsistence to Trade and Specializations." In *Subsistence Agriculture and Economic Development*, ed. C. F. Wharton, Jr. Chicago: Aldine.

Gronau, Reuben. 1973. "The Intrafamily Allocation of Time: The Value of the Housewives' Time." *American Economic Review*, vol. 68, pp. 634–51.

———. 1977. "Leisure, Home Production and Work: The Theory of the Allocation of Time Revisited." *Journal of Political Economy*, vol. 85, pp. 1099–1124.

Huffman, Wallace, and Mark Lange. 1982. "Farm Household Production: Demand for Wife's Labor, Capital Services and the Capital-Labor Ratio." Yale University Economic Growth Center Discussion Paper no. 408. New Haven, Conn.

Hymer, Stephan, and Stephen Resnick. 1969. "A Model of an Agrarian Economy with Nonagricultural Activities." *American Economic Review*, vol. 59, pp. 493–506.

Jorgenson, Dale, and Lawrence Lau. 1969. "An Economic Theory of Agricultural Household Behavior." Paper read at 4th Far Eastern Meeting of the Econometric Society.

Krishna, Raj. 1964. "Theory of the Firm: Rappoteur's Report." *Indian Economic Journal*, vol. 11, pp. 514–25.

———. 1969. "Comment: Models of the Family Farm." In *Subsistence Agriculture and Economic Development*, ed. C. F. Wharton, Jr. Chicago: Aldine.

Lancaster, Kelvin. 1966. "A New Approach to Consumer Theory." *Journal of Political Economy*, vol. 74, pp. 132–57.

Liedholm, Carl, and Enyinya Chuta. 1976. *The Economics of Rural and Urban Small-Scale Industry in Sierra Leone*. African Rural Economy Working Paper no. 14. East Lansing: Michigan State University.

Lopez, Ramon. 1980. "Economic Behaviour of Self-Employed Farm Producers." Ph.D. dissertation, University of British Columbia, Vancouver.

Mellor, J. 1963. "The Use and Productivity of Farm Labor in Early Stages of Agricultural Development." *Journal of Farm Economics*, vol. 45, pp. 517–34.

Millar, J. 1970. "A Reformulation of A. V. Chayanov's Theory of the Peasant Economy." *Economic Development and Cultural Change*, vol. 18, pp. 219–29.

Muth, Richard. 1966. "Household Production and Consumer Demand Functions." *Econometrica*, vol. 34, pp. 699–708.

Nakajima, Chihiro. 1957. "Over-Occupied and the Theory of the Family Farm." *Osaka Daigaku Keizaigaku*, vol. 6.

———. 1969. "Subsistence and Commercial Family Farms: Some Theoretical Models of Subjecture Equilibrium." *Subsistence Agriculture and Economic Development*. Edited by C. F. Wharton, Jr. Chicago: Aldine.

Neary, J., and K. Roberts. 1980. "The Theory of Household Behavior under Rationing." *European Economic Review*, vol. 13, pp. 25–42.

Pollak, Robert, and Michael Wachter. 1975. "The Relevance of the Household Production Function for the Allocation of Time." *Journal of Political Economy*, vol. 83, pp. 255–77.

Rosenzweig, Mark. 1978. "Rural Wages, Labor Supply, and Land Reform." *American Economic Review*, vol. 68, pp. 847–61.

———. 1982. "Agricultural Development, Education and Innovation." In *The Theory and Experience of Economic Development*. Edited by M. Gersovitz and others. London: George Allen & Unwin.

Sen, Amartya K. 1966. "Peasants and Dualism with and without Surplus Labor." *Journal of Political Economy*, vol. 74, pp. 425–50.

Spencer, Dunstan, and Derek Byerlee. 1977. *Small Farms in West Africa: A Descriptive Analysis of Employment, Incomes and Productivity in Sierra Leone*. African Rural Economy Program Working Paper no. 19. East Lansing: Michigan State University.

Squire, Lyn. 1981. *Employment Policy in Developing Countries: A Survey of Issues and Evidence*. New York: Oxford University Press.

Tanaka, Osamu. 1951. "An Equilibrium Analysis of Peasant Economy." *Nogyo Keizai Kenkyu (Journal of Rural Economics)*, vol. 22.

World Bank. 1983. *World Development Report*. New York: Oxford University Press.