Computable General Equilibrium (CGE) Models:
A Short Course

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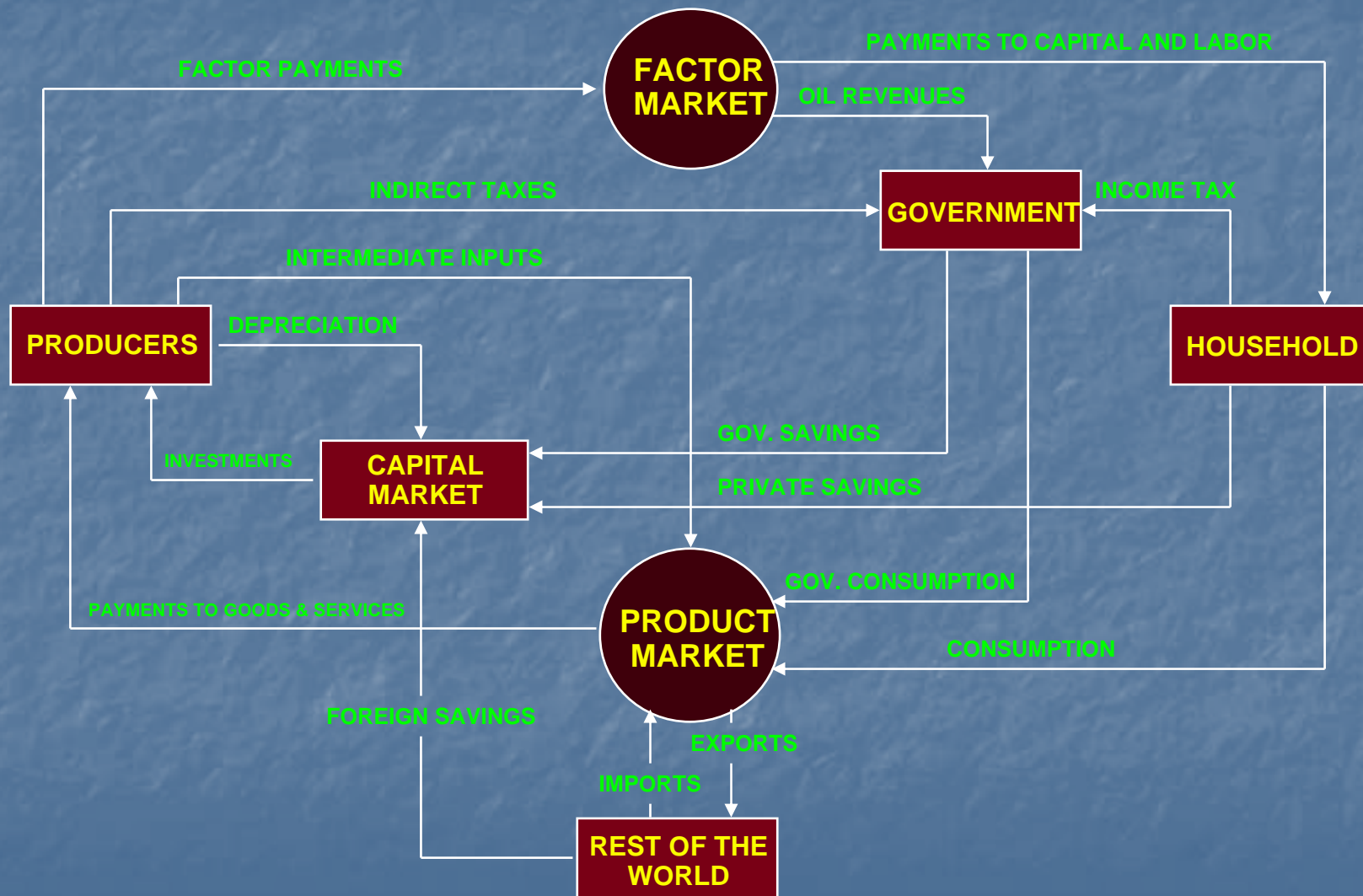# Session Two: SPECIFICATION

Session 2: **Specification**

- A taxonomy of models
- Components of a simple CGE
- Mathematical model statement

Review: CGE Keywords

- Multisectoral
- Nonlinear
- Economy-wide
- Autonomous decision making
- Walrasian competitive equilibrium

Structure of a CGE model

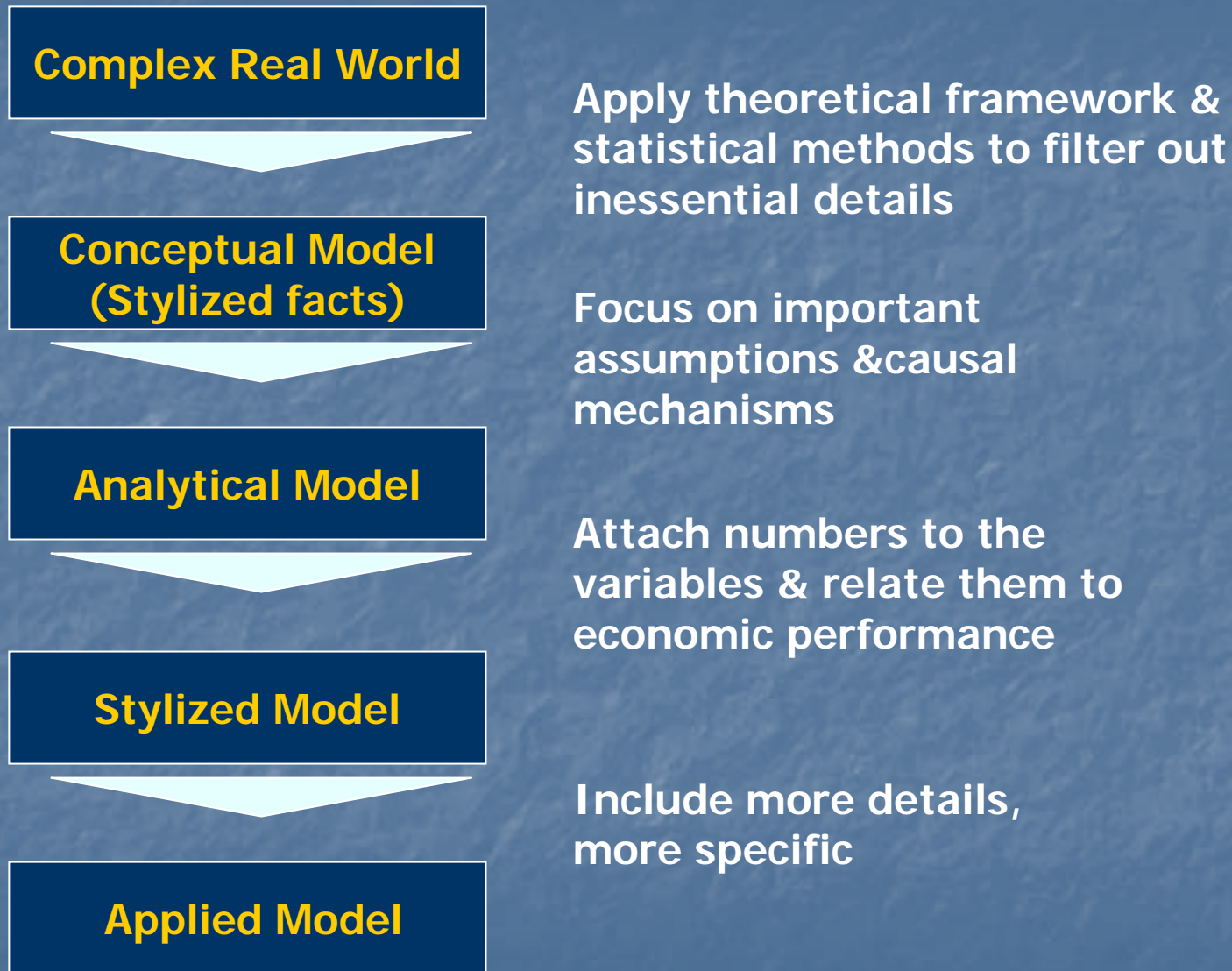# Circular Flow of Income

A taxonomy of models

Models and Policy Analysis

- Policy makers intend to change the way the economy operates, rather than just understand it

- Policy analysis: explain the links between instruments and targets

- Model-building: the process of abstraction and generalization required to provide structure to our empirical observation
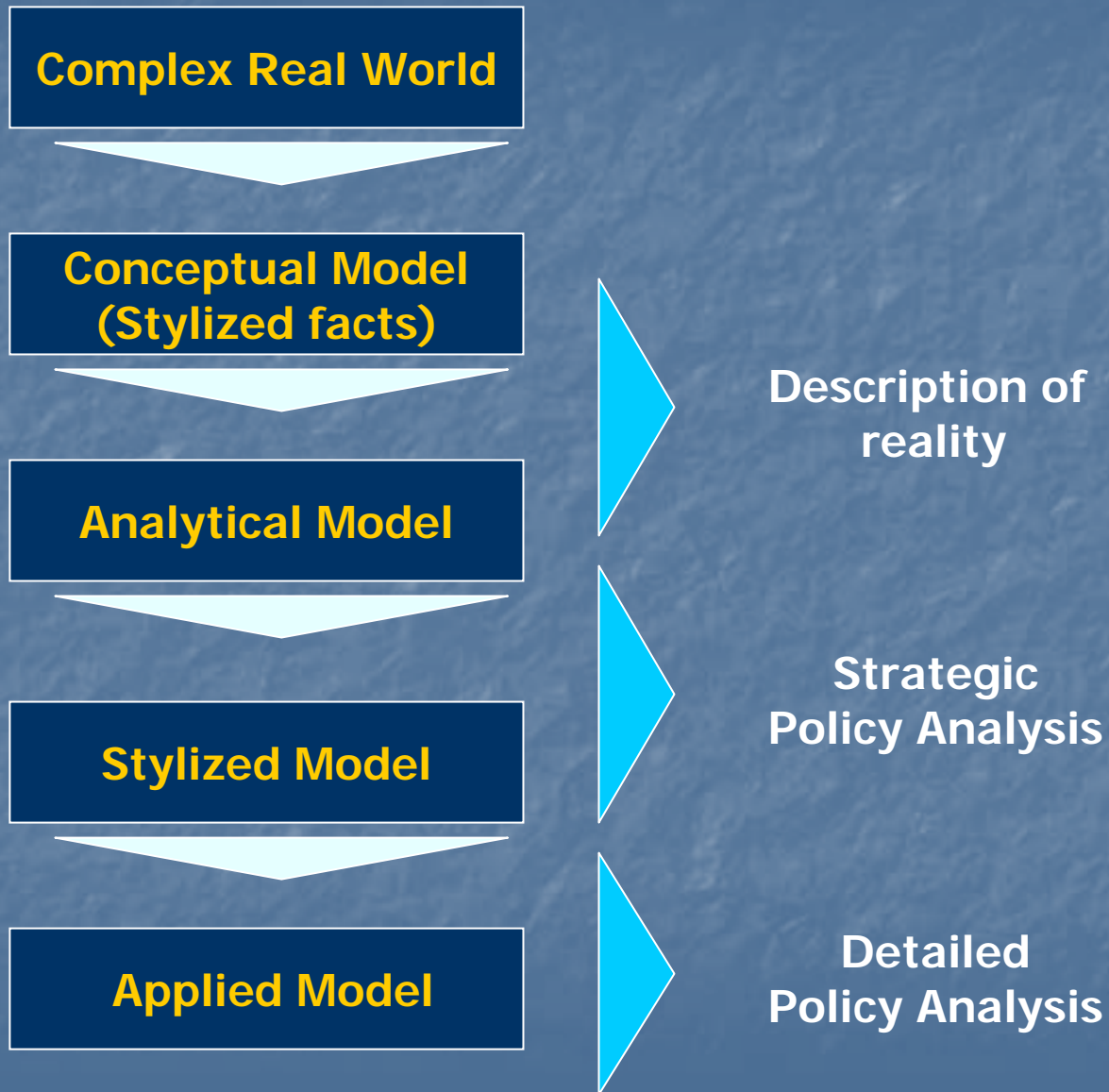
Economic Models

- Analytic models
- Stylized numerical models
- Applied models

A taxonomy of models

Complex Real World

Apply theoretical framework & statistical methods to filter out inessential details

Conceptual Model (Stylized facts)

Focus on important assumptions &causal mechanisms

Analytical Model

Attach numbers to the variables & relate them to economic performance

Stylized Model

Include more details, more specific

Applied Model

A taxonomy of models

Complex Real World

↓

Conceptual Model (Stylized facts)

↓

Analytical Model → Description of reality

↓

Stylized Model → Strategic Policy Analysis

↓

Applied Model → Detailed Policy Analysis

Session Two: Specification

Analytic models

- Cast economic relationships into a form susceptible to mathematical analysis
- Explore the implications of various sets of postulates
- Deliberately simplified to focus on important assumptions and causal mechanisms
- Empirical realism not an important criterion

Stylized numerical models

- Attach numbers to the variables and relate them to economic statistics

- Larger, more complex & more realistic

- Used to analyze problems too difficult to solve analytically

- Empirical, able to explore the size of various effects

- Give up simplicity to gain in applicability & generality

Applied models

- Include broader range of stylized facts
- More specific and narrow in application
- Include more institutional details
- Additional details may obscure the major causal mechanisms without adding any empirically significant effects

- CGE models fall into both stylized and applied categories

- In addition CGE models:
  - Have strong links with basic economic theory
  - Work by simulating the interaction of various actors as specified in neoclassical general-equilibrium theory
  - Derive "behavior" based on optimization as specified in micro-theory
  - Are fully "closed" in that the supply and demand sides of all markets are specified

Components of a simple CGE

- See a SAM structure

Mathematical model statement

- 123 Model

- Graphical Analysis

- Equations of an applied model

1-2-3 Model

- Capture mechanisms by which external shocks and domestic policies ripple through the economy

- Many problems (and solutions) are related to links between external sectors and domestic economy

- Based on: Devarajan-Go-Lewis-Ronbinson-Sinko (1997)

1-2-3 Model

- 1 country, 2 activities, 3 commodities
- 2 activities, producing D and E.
  - E not consumed domestically.
- Additional commodity, M, consumed domestically but not produced.

1-2-3 Model

- Very simplistic stylized model, but:
  - Mechanisms are transparent
  - Can be solved graphically, analytically, or with Excel
  - Behavior is similar to that of more complex models

1-2-3 Model

- Aggregate GDP (X) is fixed.
  - Full employment model.
- Trade balance set exogenously.
- World prices of M and E are fixed.
- Total absorption (Q) is endogenous.

Basic 1-2-3 CGE Model

Flows

1. $\bar{X} = G\left(E, D^S; \Omega\right)$

2. $Q^S = F\left(M, D^D; \sigma\right)$

3. $Q^D = \dfrac{Y}{P^q}$

4. $\dfrac{E}{D^S} = g_2\left(P^e, P^d\right)$

5. $\dfrac{M}{D^D} = f_2\left(P^m, P^d\right)$

6. $Y = P^x \square \bar{X} + R \square B$

Prices

7. $P^m = R \square pw^m$

8. $P^e = R \square pw^e$

9. $P^x = g_1\left(P^e, P^d\right)$

10. $P^q = f_1\left(P^m, P^d\right)$

11. $R \equiv 1$

Equilibrium Conditions

12. $D^D - D^S = 0$

13. $Q^D - Q^S = 0$

14. $pw^m \square M - pw^e \square E = B$

Basic 1-2-3 CGE Model

Identities

15. $P^x \square X \equiv P^e \square E + P^d \square D^S$

16. $P^q \square Q^S \equiv P^m \square M + P^d \square D^D$

17. $Y \equiv P^q \square Q^D$

Basic 1-2-3 CGE Model

Endogenous Variables

E: Export good

M: Import good

D^S: Supply of domestic good

D^D: Demand for domestic good

Q^S: Supply of composite good

Q^D: Demand for composite good

Y: Total income

P^e: Domestic price of export good

P^m: Domestic price of import good

P^d: Domestic price of domestic good

P^x: Price of aggregate output

P^q: Price of composite good

R: Exchange rate

Exogenous Variables

pw^e: world price of export good

pw^m: world price of import good

B: Balance of trade

σ: Import substitution elasticity
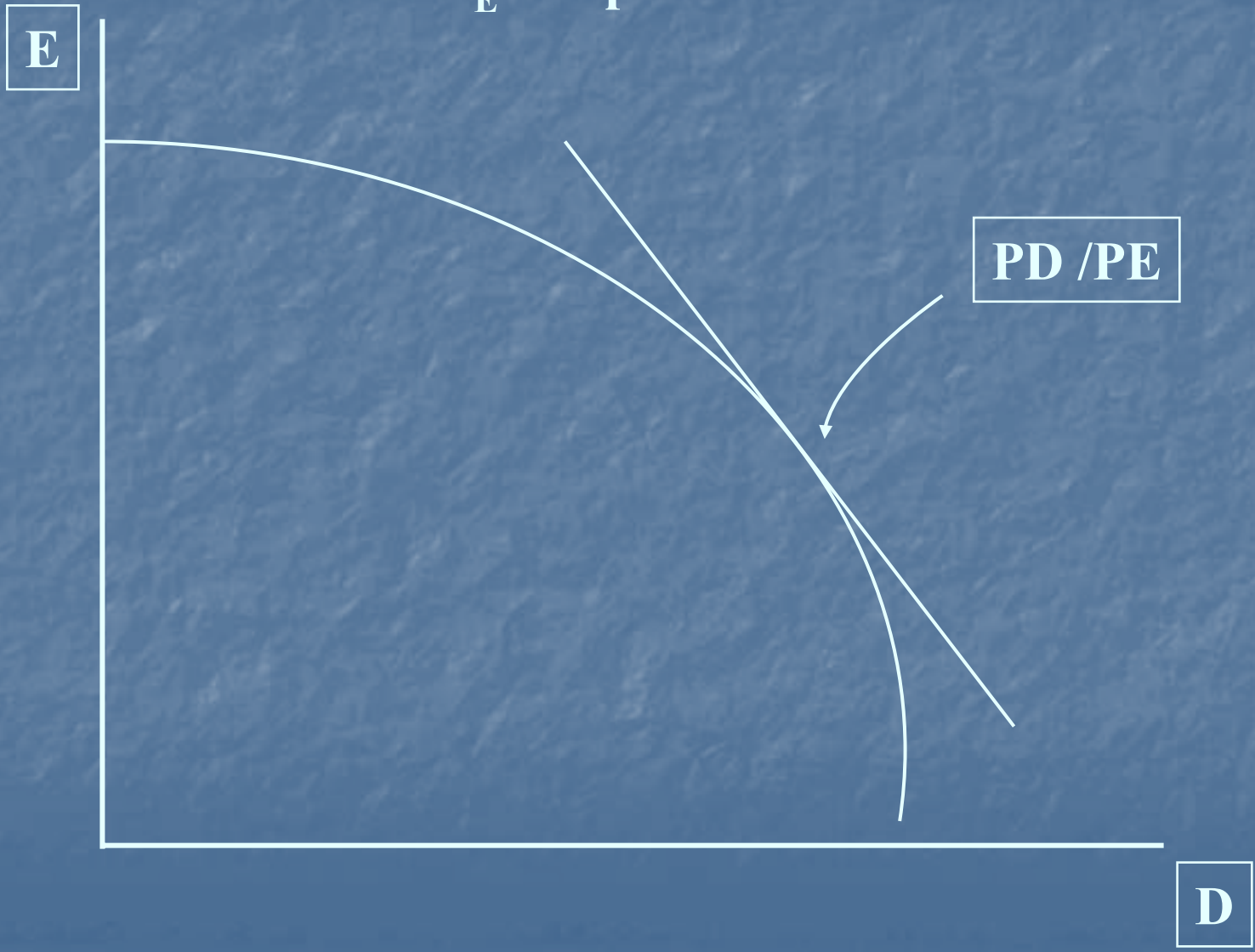
Ω: Export transformation elasticity

SAM for 1-2-3 Model

	Activities	Commod	Hshld	World
Activities		$P^d{\cdot}D^D$		$P^e{\cdot}E$
Commodities			$P^q{\cdot}Q^D$	
Households	$P^x{\cdot}\overline{X}$			$R{\cdot}B$
World		$P^m{\cdot}M$		
Total	$P^d{\cdot}D^S + P^e{\cdot}E$	$P^q{\cdot}Q^S$	Y	

Mathematical model statement

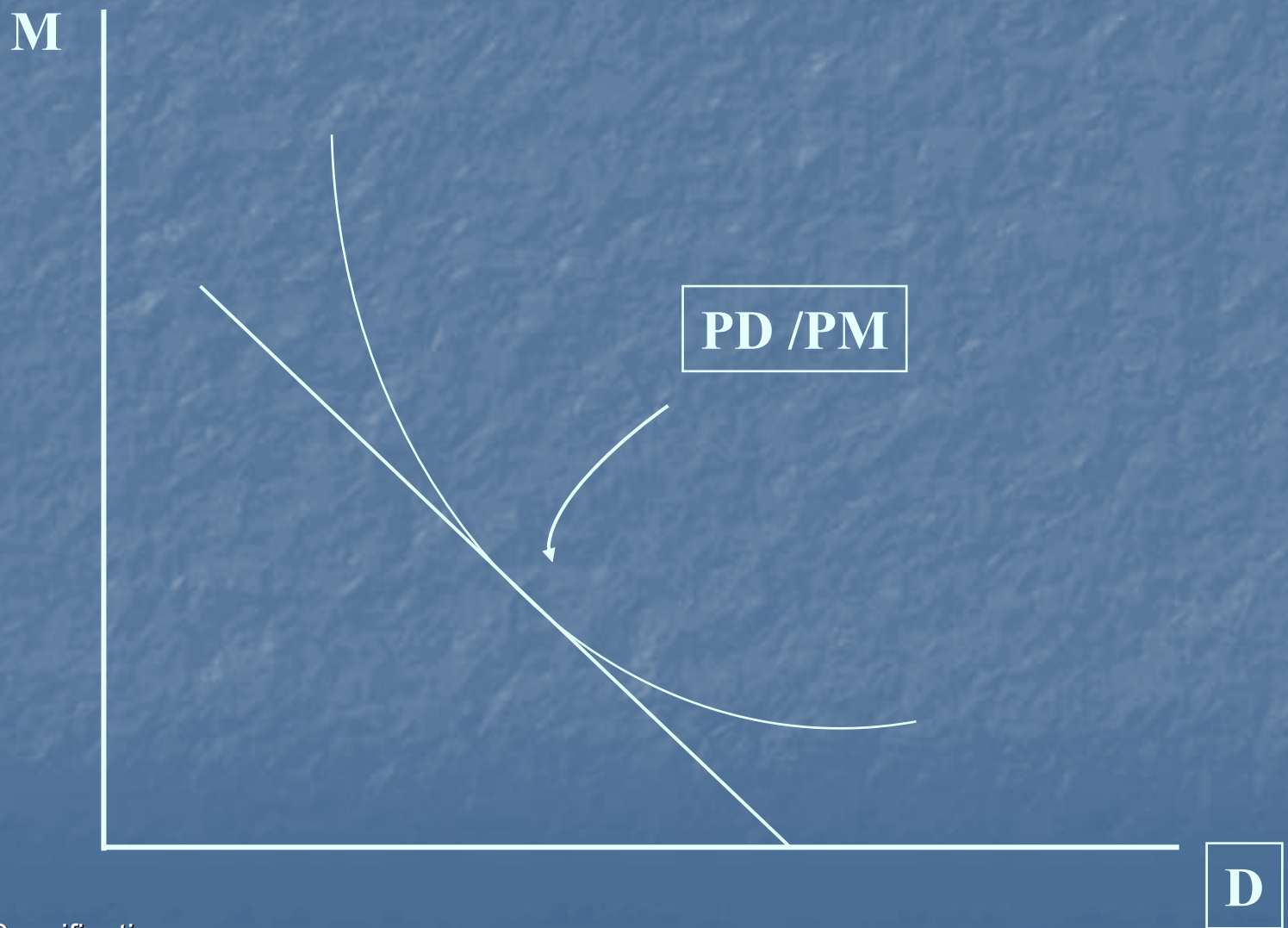$$E/D = k \, (P_E / P_D)^{\Omega}$$

$$P_E = R \cdot pwe$$

Mathematical model statement

$$M/D = k'(P_D / P_M)^\sigma$$

$$P_M = R \cdot pwm$$

M

PD /PM

D

1-2-3 Programming Model

Maximize $Q = F(M, D; \sigma)$

with respect to: M, E, D^D, D^S

subject to:

		Shadow Prices
1. $G(E, D^S; \Omega) \leq \overline{X}$	technology	$\lambda^x = P^x / P^q$
2. $pw^m \cdot M \leq pw^e \cdot E + \overline{B}$	balance of trade	$\lambda^b = R / P^q$
3. $D^D \leq D^S$	domestic market	$\lambda^d = P^d / P^q$

1-2-3
Model

P_D/P_M

M

$P_M M = P_E E$

D

E

P_D/P_E

D

Mathematical model statement

Quadrant labels: II, I (top), III, IV (bottom)

M

Balance of Trade

C*

C

Q = F(M,D)

B   B

D   E

X = G(E,D)

Domestic Market

P

P*

D

II

I

M

Balance of Trade

C

C*

Q = F(M,D)

D

E

P*

P

Domestic Market

X = G(E,D)

III

IV

D
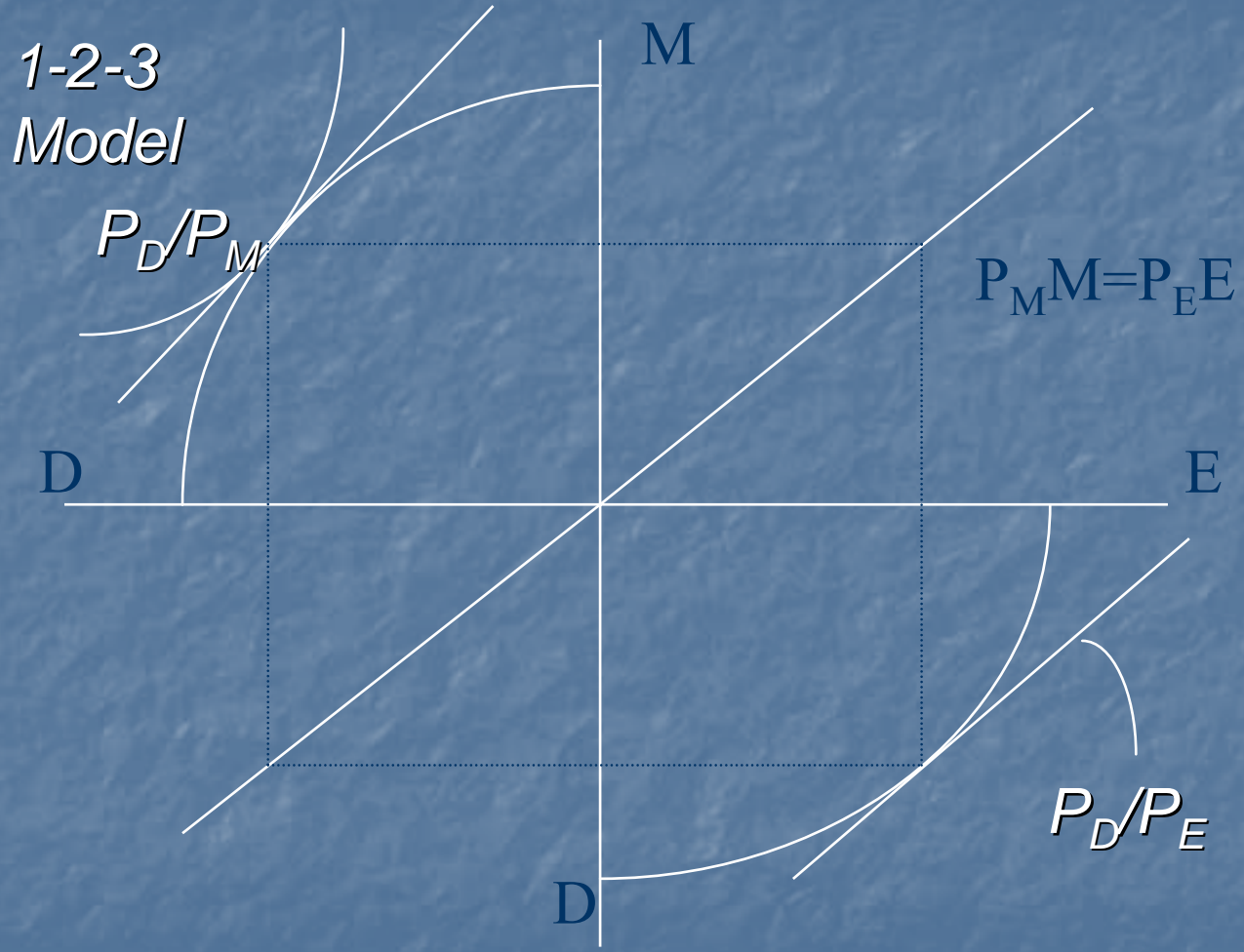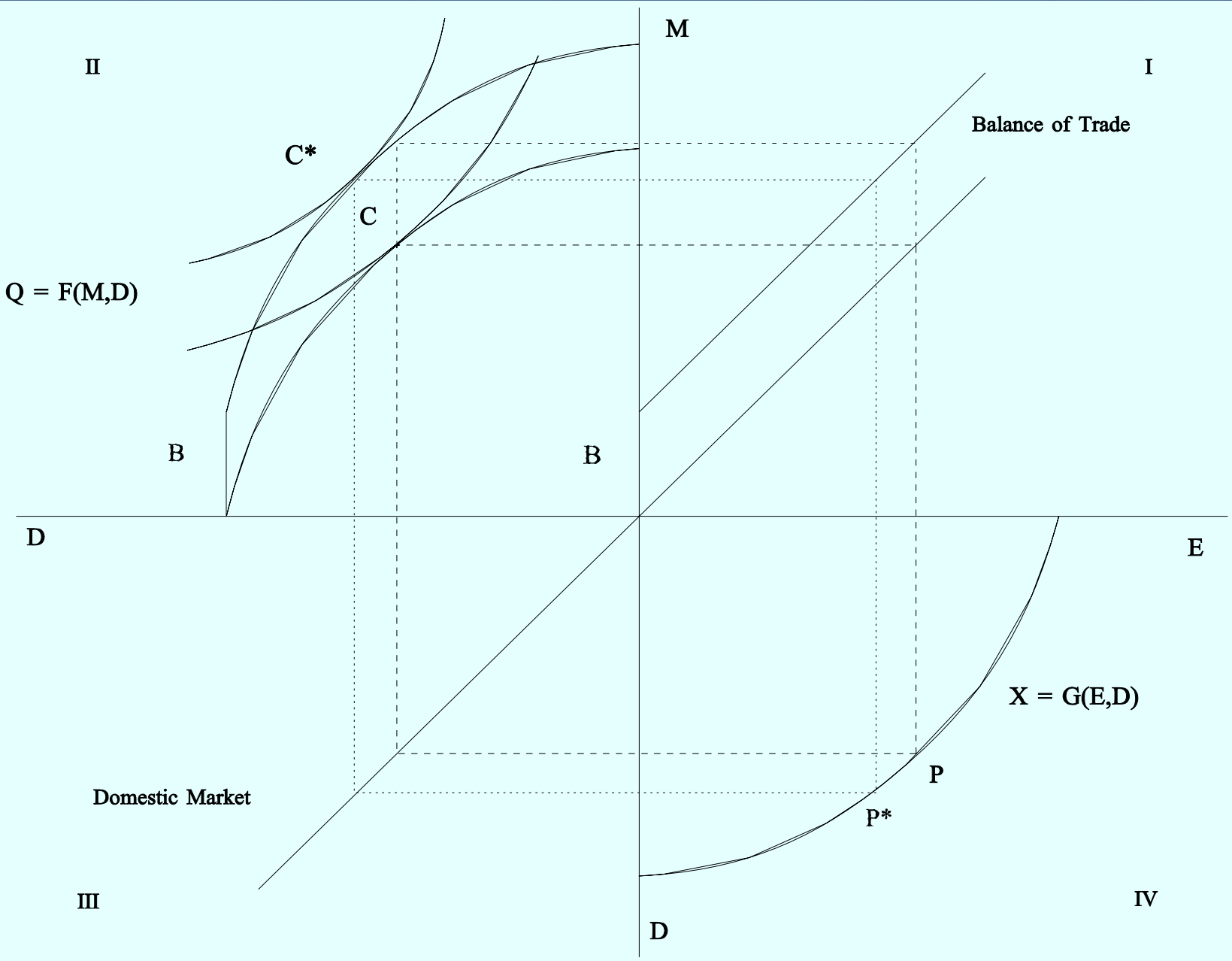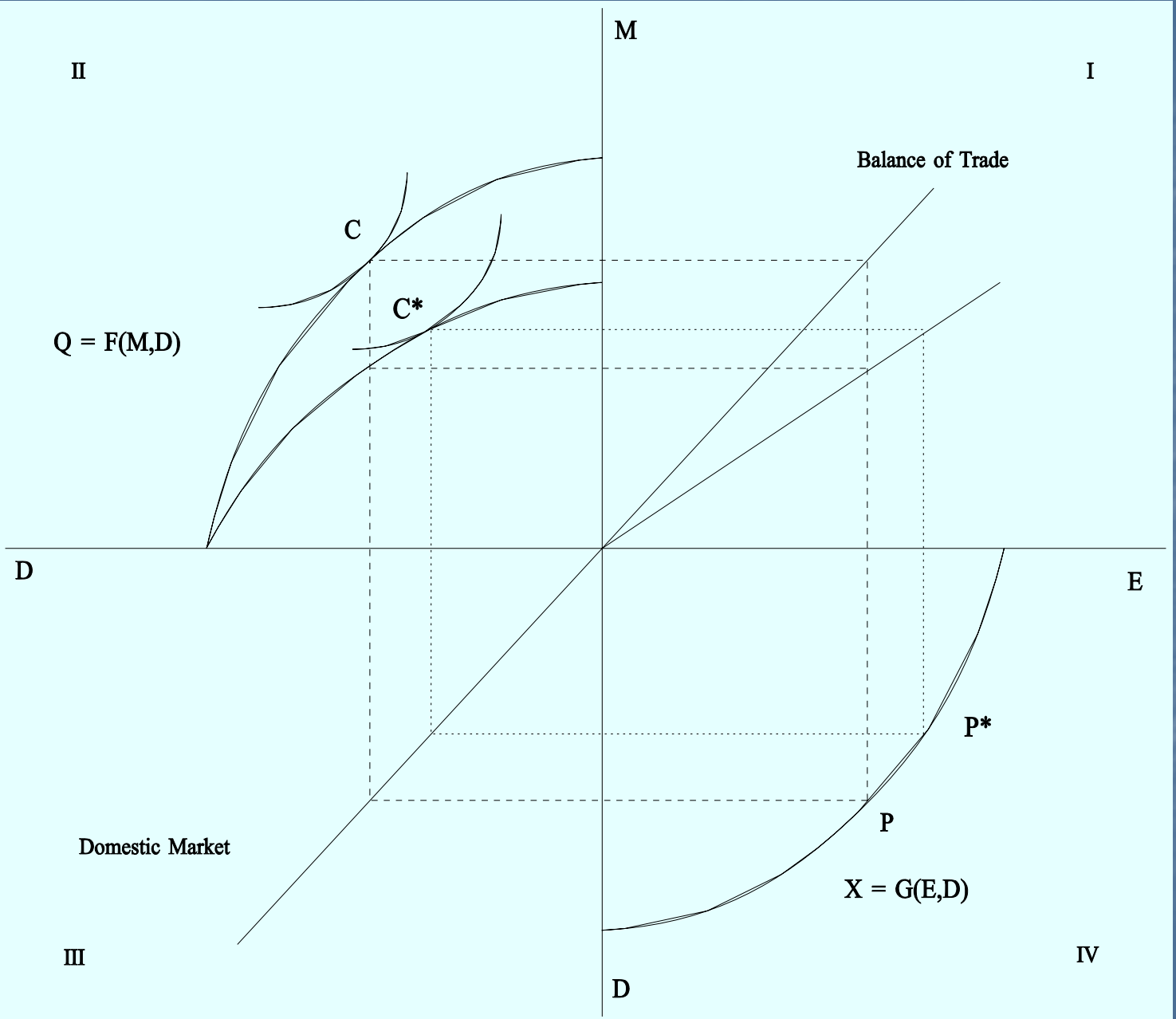
1-2-3 CGE Model with Consumption, Government, and Investment

Shantayanan Devarajan
Delfin S. Go
Jeffrey D. Lewis
Sherman Robinson
Pekka Sinko

1-2-3 CGE Model

Real Flows

1. $\bar{X} = G\left(E, D^S; \Omega\right)$

2. $Q^S = F\left(M, D^D; \sigma\right)$

3. $Q^D = C + Z + G$

4. $\dfrac{E}{D^S} = g_2\left(P^e, P^d\right)$

5. $\dfrac{M}{D^D} = f_2\left(P^m, P^d\right)$

1-2-3 CGE Model

$$6.\ T = t^m \square R \square pw^m \square M$$

$$+ t^q \square P^q Q^D$$

$$+ t^y \square Y$$

$$+ t^e \square P^e \square E$$

$$7.\ Y = P^x \square X + tr \square P^q + re \square R$$

$$8.\ S = \overline{s} \square Y + R \square \overline{B} + S^g$$

$$9.\ C \square P^t = \left(1 - \overline{s} - t^y\right) \square Y$$

1-2-3 CGE Model

Prices

10. $P^m = \left(1 + t^m\right)\square R\square pw^m$

11. $P^e\square\left(1 + t^e\right) = R\square pw^e$

12. $P^t = \left(1 + t^q\right)\square P^q$

13. $P^x = g_1\left(P^e, P^d\right)$

14. $P^q = f_1\left(P^m, P^d\right)$

15. $R \equiv 1$

1-2-3 CGE Model

Equilibrium Conditions

16. $D^D - D^S = 0$

17. $Q^D - Q^S = 0$

18. $pw^m \square M - pw^e \square E - ft - re = B$

19. $P^t \square Z - S = 0$

20. $T - P^q \square \overline{G} - tr \square P^q + ft \square R - S^g = 0$

1-2-3 CGE Model

Identities

21. $P^x \Box X \equiv P^e \Box E + P^d \Box D^S$

22. $P^q \Box Q^S \equiv P^m \Box M + P^d \Box D^D$

Endogenous Variables	Exogenous Variables
E: Export good	pw^e: world price of export good
M: Import good	pw^m: world price of import good
D^S: Supply of domestic good	t^m: Tariff rate
D^D: Demand for domestic good	t^x: Export tax rate
Q^S: Supply of composite good	t^q: Sales tax rate
Q^D: Demand for composite good	t^y: Direct tax rate
P^e: Domestic price of export good	tr: Government transfers (real)
P^m: Domestic price of import good	ft: Foreign transfers to government
P^d: Domestic price of domestic good	re: Foreign remittances to private sector
P^x: Price of aggregate output	s: Average savings rate
P^q: Price of composite good	X: Aggregate output (GDP)
P^t: Sale price of composite good	G: Real government demand
R: Exchange rate	B: Balance of trade
T: Tax revenue	σ: Import substitution elasticity
S^g: Government savings	Ω: Export transformation elasticity
Y: Total income	
C: Aggregate consumption	
S: Aggregate savings	
Z: Aggregate real investment	

IRAN CGE