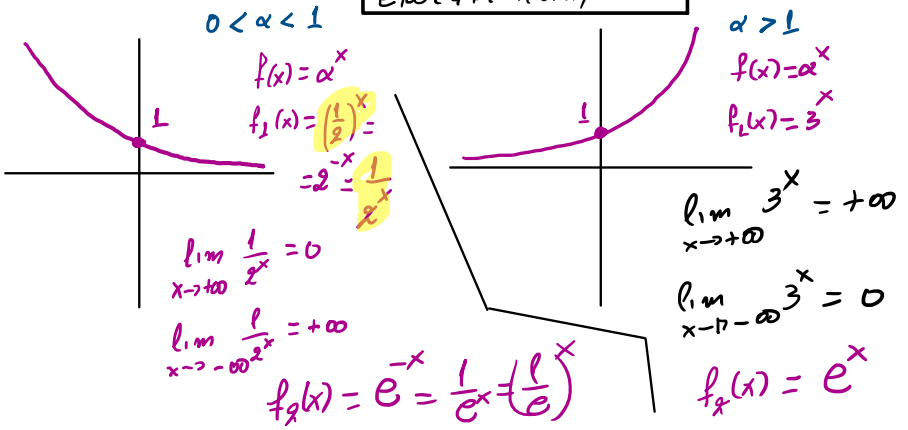
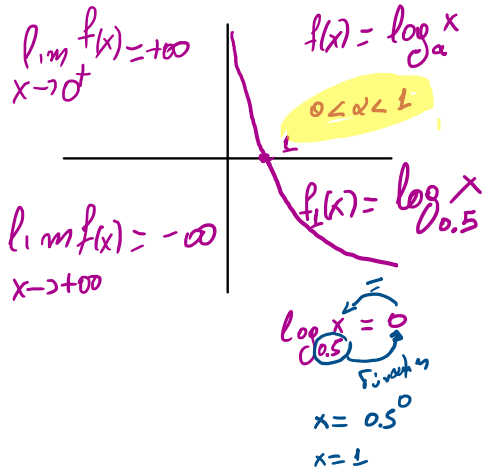
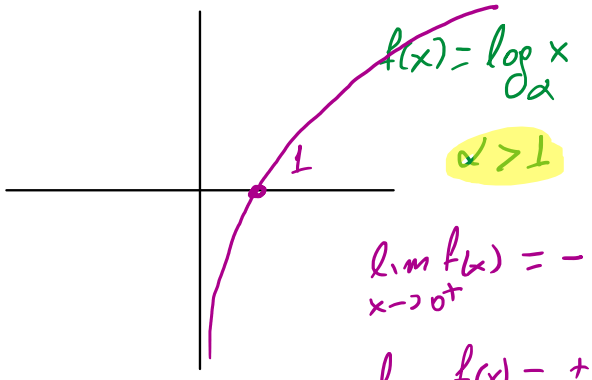
Εκθετική συνάρτηση

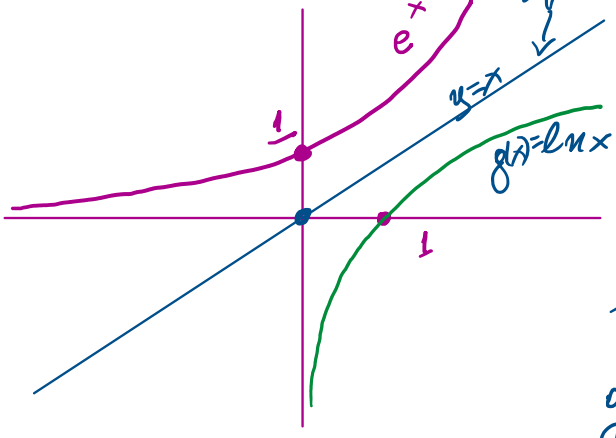


Λογαριθμική συνάρτηση

$f(x) = \log_\alpha \theta$







Αντίστροφες συναρτήσεις

$f(x) = e^x$

$g(x) = \ln x \quad (\log_e x)$

$e \simeq 2.71$

$f(g(x)) = e^{\ln x} = x$

$g(f(x)) = \ln e^x = x$

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ii. $f(x) = \dfrac{e^{(8+x)^2}}{x^3}$

$[(8+x)^2]' \quad 3 \quad e^{(8+x)^2} \quad (3)' \quad (8+x)^2 [(8+x)^2]' \quad 3 \quad e^{(8+x)^2} \quad 3x^2$

$$f'(x) = \frac{\left[e^{(8+x)^2}\right]' \cdot x^3 - e^{(8+x)^2} \cdot (x^3)'}{x^6} = \frac{e^{(8+x)^2}\left[(8+x)^2\right]' \cdot x^3 - e^{(8+x)^2} \cdot 3x^2}{x^6} =$$

$$= \frac{e^{(8+x)^2} \cdot 2(8+x) \cdot x^3 - e^{(8+x)^2} \cdot 3x^2}{x^6} =$$

$$= \frac{e^{(8+x)^2} \cdot x^2 \left[2(8+x) \cdot x - 3\right]}{x^6} = \frac{e^{(8+x)^2}\left(2x^2 + 16x - 3\right)}{x^4}$$

✓

i. $f(x) = ln(x^2 + 5)$ ***ii.*** $f(x) = ln\left[x + \sqrt{x^2 + 1}\right]$

iii. $f(x) = (x^2 - 1)\, ln(x^2 + 1)$

i) $f(x) = \ln(x^2+5)$

$$f'(x) = \left[\ln(x^2+5)\right]' = \frac{1}{x^2+5} \cdot (x^2+5)' = \frac{2x}{x^2+5}$$

ii) $f(x) = \ln\left[x + \sqrt{x^2+1}\right]$

$$\left(x + \sqrt{x^2+1}\right)' = 1 + \frac{1}{2\sqrt{x^2+1}} \cdot (x^2+1)' = 1 + \frac{x}{\sqrt{x^2+1}}$$

$$f'(x) = \frac{1}{x + \sqrt{x^2+1}} \cdot \left(x + \sqrt{x^2+1}\right)' =$$

$$= \frac{1}{x + \sqrt{x^2+1}} \cdot \left(1 + \frac{x}{\sqrt{x^2+1}}\right)$$

$$= \frac{1}{x + \sqrt{x^2+1}} \cdot \left(\frac{\sqrt{x^2+1} + x}{\sqrt{x^2+1}}\right) =$$

$$= \frac{1}{\sqrt{x^2+1}}$$

$$f(x) = (x^2 - 1)\ln(x^2 + 1)$$

$$f'(x) = 2x \cdot \ln(x^2+1) + (x^2-1)\frac{2x}{x^2+1}$$

$$= 2x\left[\ln(x^2+1) + \frac{x^2-1}{x^2+1}\right]$$

$(a+b)^2 = a^2 + 2ab + b^2$

$a^2 - b^2 = (a-b)(a+b)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - a^2 b + ab^2 + a^2 b - ab^2 + b^3$

$$= 2x\left[\ln(x^2+1) + \frac{x^2-1}{x^2+1}\right]$$

$$\alpha^3 - \alpha^2\beta + \alpha\beta^2 + \alpha^2\beta - \alpha\beta^2 + \beta^3$$

$$\alpha^3 - \beta^3 = (\alpha-\beta)\cdot(\alpha^2+\alpha\beta+\beta^2)$$

$$(\alpha+\beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$$

$$(\alpha-\beta)^3 = \alpha^3 - 3\alpha^2\beta + 3\alpha\beta^2 - \beta^3$$

$$(x^x) = x \cdot x^{x-1}$$

$$f(x)^{g(x)}$$

$$\alpha^{x^k}$$

$$2^x \quad x^{\ln x} \quad x^2$$

Παραγώγιση της συνάρτησης $h(x) = f(x)^{g(x)}$

Α' τρόπος:

$f(x) = x^{x^2+1}$

$\ln\theta^k = k\cdot\ln\theta$

$\theta = e^{\ln\theta}$

Βήμα 1ο:

$x^{x^2+1} = e^{\ln x^{x^2+1}} = e^{(x^2+1)\cdot\ln x}$

Βήμα 2ο:

$$\left[e^{(x^2+1)\ln x}\right]' = e^{(x^2+1)\ln x}\cdot\left[(x^2+1)\cdot\ln x\right]' =$$

$$= e^{(x^2+1)\cdot\ln x}\cdot\left[2x\cdot\ln x + (x^2+1)\cdot\frac{1}{x}\right]$$

Άρα, $\left[x^{x^2+1}\right]' = e^{(x^2+1)\ln x}\cdot\left[2x\ln x + \frac{x^2+1}{x}\right]$

$$\left[x^{x^2+1}\right]' = x^{x^2+1}\cdot\left[2x\ln x + \frac{x^2+1}{x}\right]$$

$$\left[x^{x^2+1}\right] = x^{x^2+1} \cdot \left[2x \ln x + \frac{1}{x}\right]$$