

Adaptive Quadrature

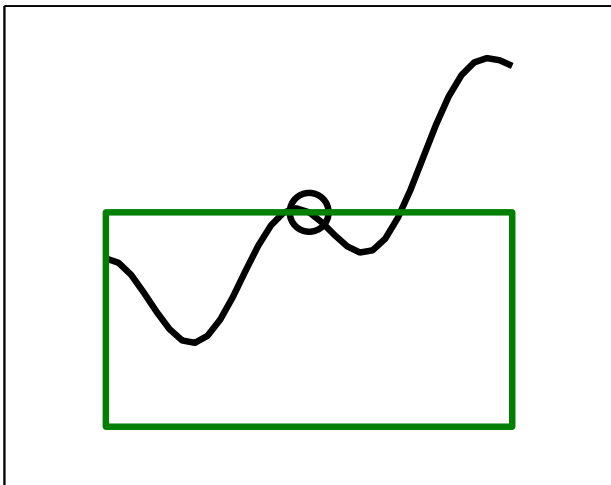
$$\int_a^b f(x)dx$$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

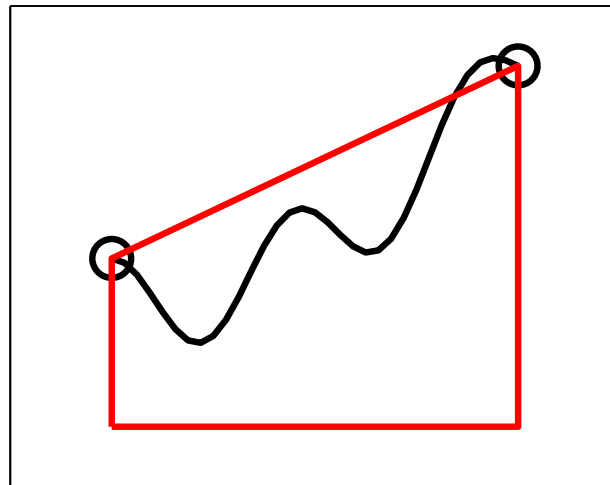
$$M = hf \left(\frac{a+b}{2} \right)$$

$$T = h \frac{f(a) + f(b)}{2}$$

Midpoint rule



Trapezoid rule



$$\int_0^1 x^2 dx = \frac{1}{3}$$

$$M = 1 \left(\frac{1}{2} \right)^2 = \frac{1}{4}$$

$$T = 1 \frac{0 + 1^2}{2} = \frac{1}{2}$$

$$S - T = -2(S - M)$$

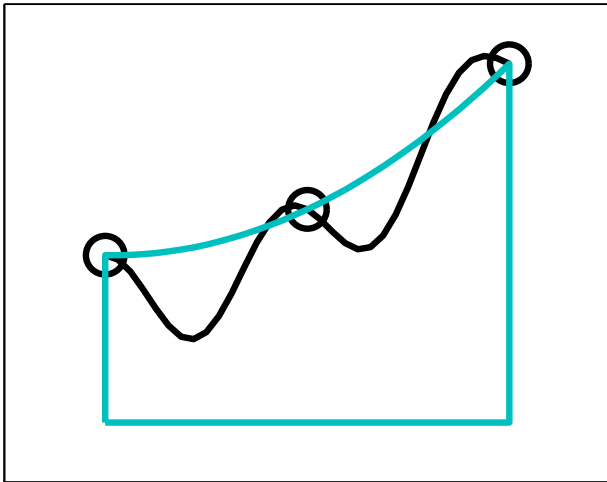
$$S = \frac{2}{3}M + \frac{1}{3}T$$

$$S = \frac{h}{6}(f(a) + 4f(c) + f(b))$$

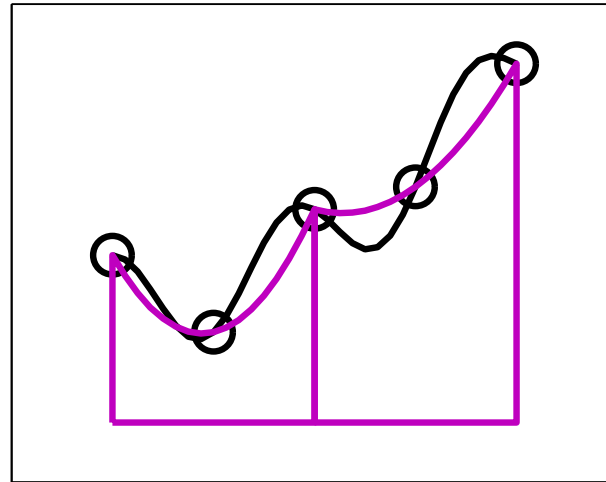
$$S = \frac{h}{6}(f(a) + 4f(c) + f(b))$$

$$S_2 = \frac{h}{12}(f(a) + 4f(d) + 2f(c) + 4f(e) + f(b))$$

Simpson's rule



Composite Simpson's rule



$$Q - S = 16(Q - S_2)$$

$$Q = S_2 + (S_2 - S)/15$$