Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f(x) = x^2 - M$$

$$\begin{aligned} x_{n+1} &= x_n - \frac{x_n^2 - M}{2x_n} \\ &= \frac{1}{2}\left(x_n + \frac{M}{x_n}\right) \end{aligned}$$

$$e_{n+1} = -\frac{1}{2}\frac{f''(\xi)}{f'(x_n)}e_n^2$$

$$e_{n+1} = O(e_n^2)$$

A Perverse Example

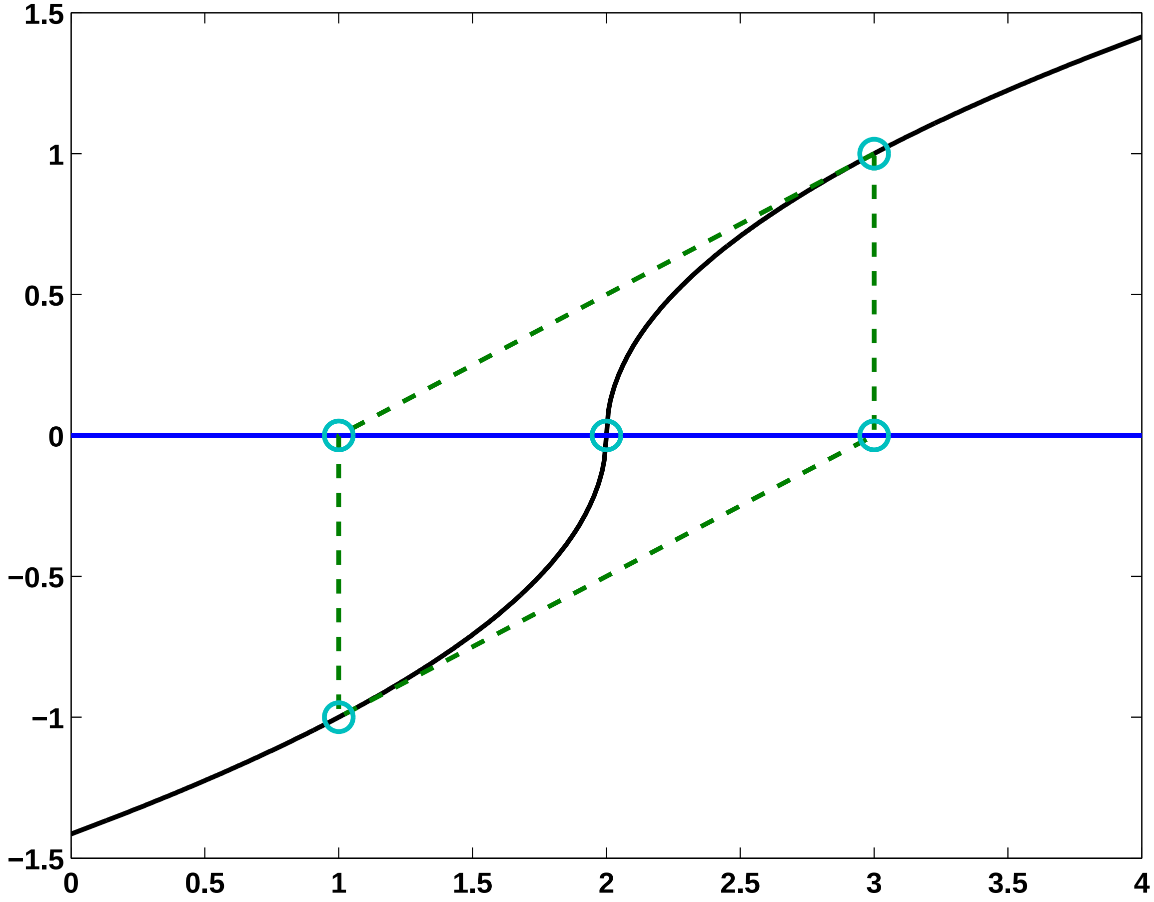
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} - a = -(x_n - a)$$

$$x - a - \frac{f(x)}{f'(x)} = -(x - a)$$

$$\frac{f'(x)}{f(x)} = \frac{1}{2(x - a)}$$

$$f(x) = \mathrm{sign}(x - a)\sqrt{|x - a|}$$


1.5
1
0.5
0
−0.5
−1
−1.5
0
0.5
1
1.5
2
2.5
3
3.5
4

Secant Method

$$
\begin{aligned}
s_n &= \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}} \\
x_{n+1} &= x_n - \frac{f(x_n)}{s_n}
\end{aligned}
$$

$$e_{n+1} = -\frac{1}{2}\frac{f''(\xi)f'(\xi_n)f'(\xi_{n-1})}{f'(\xi)^3}\; e_n e_{n-1}$$

$$e_{n+1} = O(e_n e_{n-1})$$

$$e_{n+1} = O(e_n^{\phi})$$

$$\phi = \frac{1+\sqrt{5}}{2}$$

Inverse Quadratic Interpolation

$$
\begin{aligned}
x_{n-2} &= P(y_{n-2}) \\
x_{n-1} &= P(y_{n-1}) \\
x_n &= P(y_n)
\end{aligned}
$$

$$x_{n+1} = P(0)$$

Zeroin

- Start with a and b so that $f(a)$ and $f(b)$ have opposite signs.

- Use a secant step to give c between a and b.

- Repeat the following steps until $|b-a| < \epsilon|b|$ or $f(b) = 0$.

- Arrange a, b, and c so that
  - $f(a)$ and $f(b)$ have opposite signs.
  - $|f(b)| \leq |f(a)|$
  - c is the previous value of b.

- If $c \neq a$, consider an IQI step.

- If $c = a$, consider a secant step.

- If the IQI or secant step is in the interval [a,b], take it.

- If the step is not in the interval, use bisection.

Arguments to function functions

- Expression string
- Inline function
- Function handle
- Symbolic expression