Finite Fourier Transform

$$Y_{k+1} = \sum_{j=0}^{n-1} \omega^{jk} y_{j+1}$$

$$\omega = e^{-2\pi i/n}$$

$$Y = Fy$$

$$f_{k+1,j+1} = \omega^{jk}$$

$$F^H F = nI$$

$$F^{-1} = \frac{1}{n} F^H$$

$$y = \frac{1}{n} F^H Y$$

$$y_{j+1} = \frac{1}{n} \sum_{k=0}^{n-1} Y_{k+1} \bar{\omega}^{jk}$$

$$\bar{\omega} = e^{2\pi i/n}$$

Units

y	data	
Fs	sample rate	samples/time_unit
n = length(y)	number of samples	
k = 0:n-1	index	
t = k/Fs	time	time_units
dt = 1/Fs	time increment	time_units
Y = fft(y)	fourier transform	
abs(Y)	amplitude of fft	
P = abs(Y).^2	power	
f = k*(Fs/n)	frequency	cycles/time_unit
p = 1./f	period	time_units/cycle

Fast Finite Fourier Transform

$$Y_{k+1} = \sum_{j=0}^{n-1} \omega^{jk} y_{j+1}, \;\; k = 0, \ldots, n-1$$

$$\omega = \omega_n = e^{-2\pi i/n} = \cos\delta - i \sin\delta$$

$$\omega_{2n}^2 = \omega_n$$

$$\cos 2\delta = \cos^2\delta - \sin^2\delta$$
$$\sin 2\delta = 2\cos\delta \sin\delta$$

$$Y_{k+1} = \sum_{j=0}^{n-1} \omega^{jk} y_{j+1}, \;\; k = 0, \ldots, n-1$$

Assume that n is even and that $k \leq n/2 - 1$

$$\begin{aligned} Y_{k+1} &= \sum_{even\ \ j} \omega^{jk} y_{j+1} \; + \; \sum_{odd\ \ j} \omega^{jk} y_{j+1} \\ &= \sum_{j=0}^{n/2-1} \omega^{2jk} y_{2j+1} \; + \; \omega^{k} \sum_{j=0}^{n/2-1} \omega^{2jk} y_{2j+2} \end{aligned}$$

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omega = exp(-2*pi*i/n);
k = (0:n/2-1)';
w = omega .^ k;
u = fft(y(1:2:n-1));
v = w.*fft(y(2:2:n));

fft(y) = [u+v; u-v];
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