The Interpolating Polynomial

$$P(x_k) = y_k, \;\; k = 1, \ldots, n$$

$$P(x) = \sum_k \left(\prod_{j \neq k} \frac{x - x_j}{x_k - x_j} \right) y_k$$

$$
\begin{aligned}
P(x) \;=\; & \frac{(x-1)(x-2)(x-3)}{(-6)}(-5) \;+ \\
& \frac{x(x-2)(x-3)}{(2)}(-6) \;+ \\
& \frac{x(x-1)(x-3)}{(-2)}(-1) \;+ \\
& \frac{x(x-1)(x-2)}{(6)}(16)
\end{aligned}
$$

$$P(x) = x^3 - 2x - 5$$

$$P(x) = c_1 x^{n-1} + c_2 x^{n-2} + \cdots + c_{n-1} x + c_n$$

$$\begin{pmatrix} x_1^{n-1} & x_1^{n-2} & \cdots & x_1 & 1 \\ x_2^{n-1} & x_2^{n-2} & \cdots & x_2 & 1 \\ \cdots & \cdots & \cdots & \cdots & 1 \\ x_n^{n-1} & x_n^{n-2} & \cdots & x_n & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

$$v_{k,j} = x_k^{n-j}$$

Piecewise linear interpolation

$$L(x) = y_k + s\delta_k$$

$$x_k \le x < x_{k+1}$$

$$s = x - x_k$$

$$\delta_k = \frac{y_{k+1} - y_k}{x_{k+1} - x_k}$$

$$L(x) = y_k + (x - x_k)\frac{y_{k+1} - y_k}{x_{k+1} - x_k}$$

Piecewise cubic interpolation

$$h_k = x_{k+1} - x_k$$

$$\delta_k = \frac{y_{k+1} - y_k}{h_k}$$

$$d_k = P'(x_k)$$

$$P(x) = \frac{3hs^2 - 2s^3}{h^3} y_{k+1} \quad + \quad \frac{h^3 - 3hs^2 + 2s^3}{h^3} y_k + \\ \frac{s^2(s-h)}{h^2} d_{k+1} \quad + \quad \frac{s(s-h)^2}{h^2} d_k$$

$$P(x_k) = y_k, \;\; P(x_{k+1}) = y_{k+1}$$
$$P'(x_k) = d_k, \;\; P'(x_{k+1}) = d_{k+1}$$

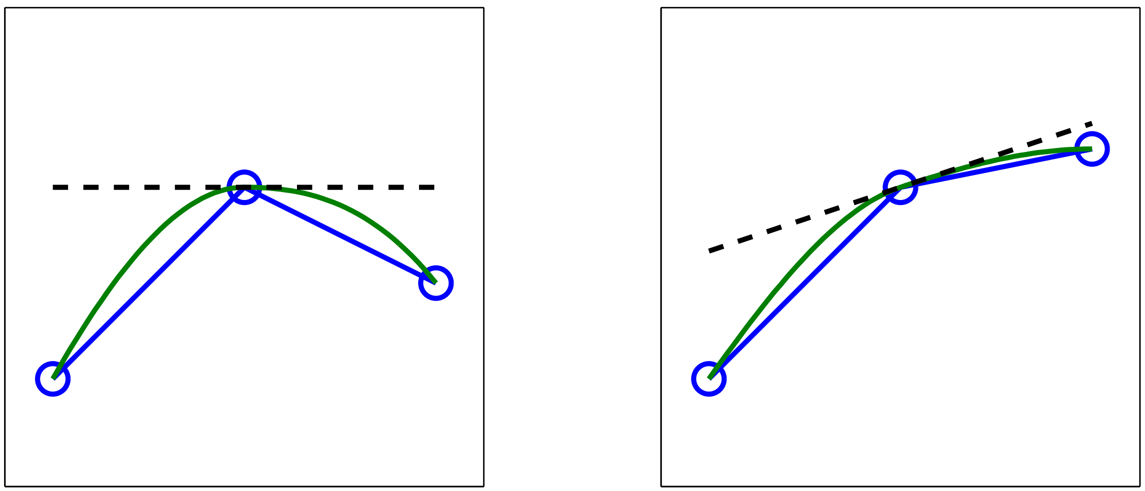
pchip

If δ_k and δ_{k-1} have opposite signs

$$d_k = 0$$

else

$$\frac{1}{d_k} = \frac{1}{2}\left(\frac{1}{\delta_{k-1}} + \frac{1}{\delta_k}\right)$$



$$\frac{w_1 + w_2}{d_k} = \frac{w_1}{\delta_{k-1}} + \frac{w_2}{\delta_k}$$

$$w_1 = 2h_k + h_{k-1}$$
$$w_2 = h_k + 2h_{k-1}$$

Local power form

$$P(x) = y_k + s d_k + s^2 c_k + s^3 b_k$$

$$\begin{aligned} c_k &= \frac{3\delta_k - 2d_k - d_{k+1}}{h} \\ b_k &= \frac{d_k - 2\delta_k + d_{k+1}}{h^2} \end{aligned}$$

spline

$$P''(x) = \frac{(6h - 12s)\delta_k + (6s - 2h)d_{k+1} + (6s - 4h)d_k}{h^2}$$

$$P''(x_k+) = \frac{6\delta_k - 2d_{k+1} - 4d_k}{h_k}$$

$$P''(x_{k+1}-) = \frac{-6\delta_k + 4d_{k+1} + 2d_k}{h_k}$$

$$P''(x_k-) = \frac{-6\delta_{k-1} + 4d_k + 2d_{k-1}}{h_{k-1}}$$

$$h_k d_{k-1} + 2(h_{k-1} + h_k) d_k + h_{k-1} d_{k+1} = 3 h_k \delta_{k-1} + 3 h_{k-1} \delta_k$$

$$d_{k-1} + 4 d_k + d_{k+1} = 3 \delta_{k-1} + 3 \delta_k$$

$$d_1 + 2 d_2 = \frac{5}{2} \delta_1 + \frac{1}{2} \delta_2$$

$$2 d_{n-1} + d_n = \frac{1}{2} \delta_{n-2} + \frac{5}{2} \delta_{n-1}$$

$$A = \begin{pmatrix} h_2 & h_2 + h_1 & & & \\ h_2 & 2(h_2 + h_1) & h_1 & & \\ & \ddots & \ddots & \ddots & \\ & & h_{n-1} & 2(h_{n-1} + h_{n-2}) & h_{n-2} \\ & & & h_{n-1} + h_{n-2} & h_{n-2} \end{pmatrix}$$

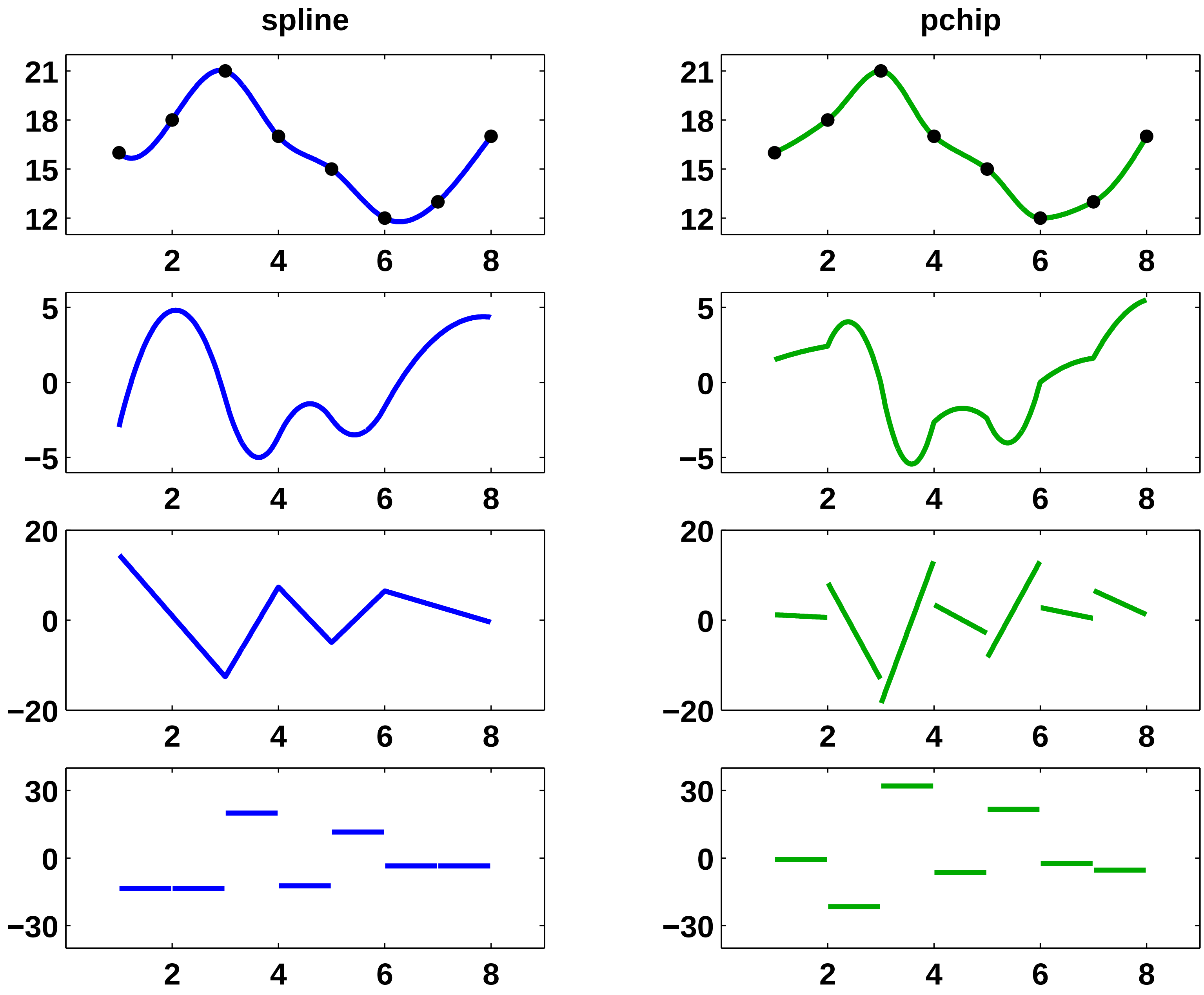
$$r = 3 \begin{pmatrix} r_1 \\ h_2\delta_1 + h_1\delta_2 \\ \vdots \\ h_{n-1}\delta_{n-2} + h_{n-2}\delta_{n-1} \\ r_n \end{pmatrix}$$

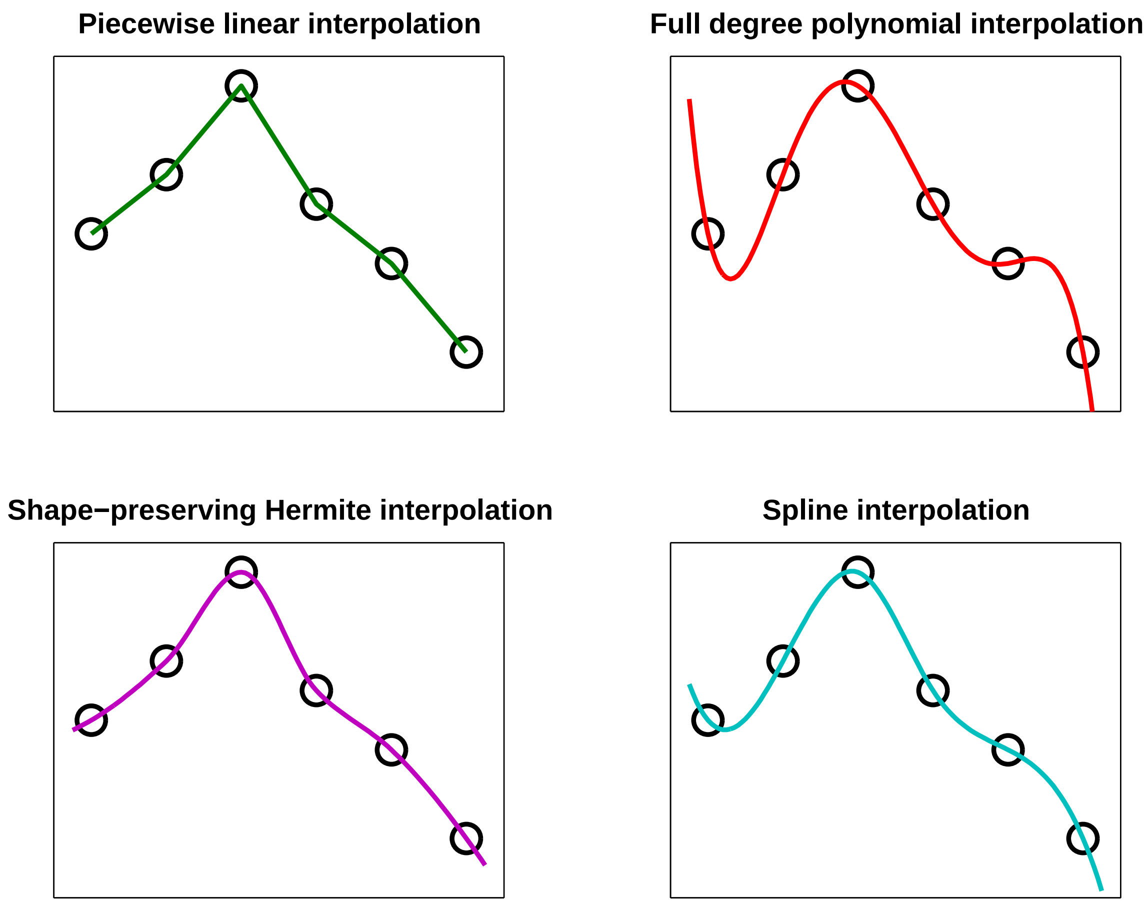
$$d = A \backslash r$$

$$A = \begin{pmatrix} 1 & 2 & & & & \\ 1 & 4 & 1 & & & \\ & 1 & 4 & 1 & & \\ & & \ddots & \ddots & \ddots & \\ & & & 1 & 4 & 1 \\ & & & & 2 & 1 \end{pmatrix}$$

$$r = 3 \begin{pmatrix} \frac{5}{6}\delta_1 + \frac{1}{6}\delta_2 \\ \delta_1 + \delta_2 \\ \delta_2 + \delta_3 \\ \vdots \\ \delta_{n-2} + \delta_{n-1} \\ \frac{1}{6}\delta_{n-2} + \frac{5}{6}\delta_{n-1} \end{pmatrix}$$

$$d = A \backslash r$$


spline
pchip
21
18
15
12
5
0
−5
20
0
−20
30
0
−30
2
4
6
8


Piecewise linear interpolation
Full degree polynomial interpolation
Shape−preserving Hermite interpolation
Spline interpolation