Curve Fitting

$$y(t) \approx \beta_1 \phi_1(t) + \ \ldots \ + \beta_n \phi_n(t)$$

$$x_{i,j} = \phi_j(t_i)$$

$$y \approx X\beta$$

```
beta = X\y
```

Seperable

$$y(t) \approx \beta_1 \phi_1(t, \alpha) + \ \ldots \ + \beta_n \phi_n(t, \alpha)$$

$$x_{i,j} = \phi_j(t_i, \alpha)$$

$$y(t) \approx \beta_1 t + \beta_2$$

$$\phi_j(t) = t^{n-j}, \;\; j = 1, \ldots, n$$
$$y(t) \approx \beta_1 t^{n-1} + \ldots + \beta_{n-1} t + \beta_n$$

$$\phi_j(t) = \frac{t^{n-j}}{\alpha_1 t^{n-1} + \ldots + \alpha_{n-1} t + \alpha_n}$$
$$y(t) \approx \frac{\beta_1 t^{n-1} + \ldots + \beta_{n-1} t + \beta_n}{\alpha_1 t^{n-1} + \ldots + \alpha_{n-1} t + \alpha_n}$$

$$\phi_j(t) = e^{-\lambda_j t}$$
$$y(t) \approx \beta_1 e^{-\lambda_1 t} + \ldots + \beta_n e^{-\lambda_n t}$$

$$y(t) \approx K e^{\lambda t}$$
$$\log y \approx \beta_1 t + \beta_2, \text{ with } \beta_1 = \lambda, \ \beta_2 = \log K$$

$$\phi_j(t) = e^{-\left(\frac{t-\mu_j}{\sigma_j}\right)^2}$$
$$y(t) \approx \beta_1 e^{-\left(\frac{t-\mu_1}{\sigma_1}\right)^2} + \ldots \beta_n e^{-\left(\frac{t-\mu_n}{\sigma_n}\right)^2}$$

$$r_i = y_i - \sum_1^n \beta_j \phi_j(t_i, \alpha), \;\; i = 1, \ldots, m$$

$$r = y - X(\alpha)\beta$$

$$\beta = X \backslash y$$

$$\|r\|^2 = \sum_1^m r_i^2$$

$$\|r\|_w^2 = \sum_1^m w_i r_i^2$$

```
X = diag(w)*X
y = diag(w)*y
```

$$\|r\|_1 = \sum_1^m |r_i|$$

$$\|r\|_\infty = \max_i |r_i|$$

censusgui

t	y
1900	75.995
1910	91.972
1900	105.711
1930	123.203
1940	131.669
1950	150.697
1960	179.323
1970	203.212
1980	226.505
1990	249.633
2000	281.422

$$y \approx \beta_1 t^3 + \beta_2 t^2 + \beta_3 t + \beta_4$$

$$s = (t - 1950)/50$$

$$y \approx \beta_1 s^3 + \beta_2 s^2 + \beta_3 s + \beta_4$$

Householder reflection

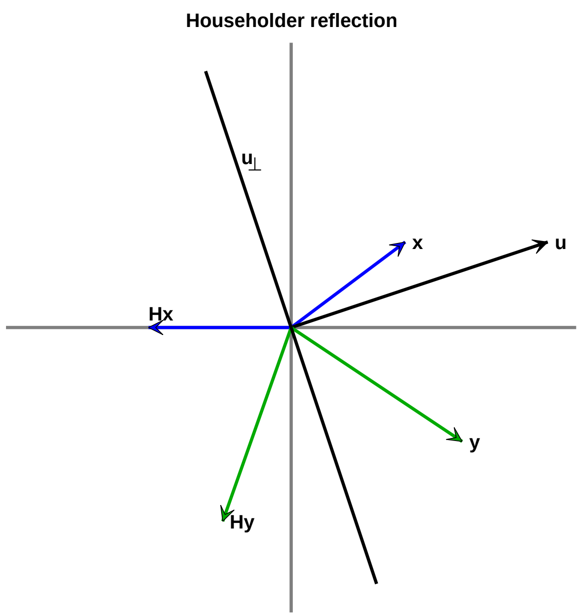
$$H = I - \rho u u^T$$

$$\rho = 2/\|u\|^2$$

$$H^T = H$$

$$H^T H = H^2 = I$$

$$\tau = \rho u^T x,$$
$$Hx = x - \tau u$$



$$\sigma = \pm\|x\|,$$
$$u = x + \sigma e_k,$$
$$\rho = 2/\|u\|^2 = 1/(\sigma u_k),$$
$$H = I - \rho u u^T$$

$$\text{sign } \sigma = \text{sign } x_k$$

Normal equations

$$X\beta \approx y$$

$$\min_\beta \|X\beta - y\|$$

$$X^T X\beta = X^T y$$

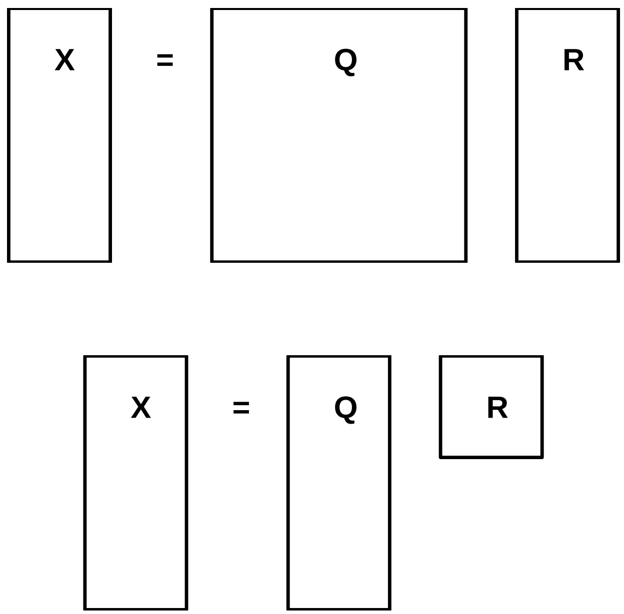
$$\beta = (X^T X)^{-1} X^T y$$

$$\kappa(X^T X) = \kappa(X)^2$$

$$X = \begin{pmatrix} 1 & 1 \\ \delta & 0 \\ 0 & \delta \end{pmatrix}$$

$$X^T X = \begin{pmatrix} 1+\delta^2 & 1 \\ 1 & 1+\delta^2 \end{pmatrix}$$

$$X = QR$$



$$H_n \cdots H_2 H_1 X = R$$

$$X\beta \approx y$$

$$R\beta \approx z$$

$$H_n \cdots H_2 H_1 y = z$$

$$Q = (H_n \cdots H_2 H_1)^T$$

$$y(s) \approx \beta_1 s^2 + \beta_2 s + \beta_3$$

```
     s         y
    0.0000  150.6970
    0.2000  179.3230
    0.4000  203.2120
    0.6000  226.5050
    0.8000  249.6330
    1.0000  281.4220
```

```
X = [s.*s s ones(size(s))]
  =
         0         0    1.0000
    0.0400    0.2000    1.0000
    0.1600    0.4000    1.0000
    0.3600    0.6000    1.0000
    0.6400    0.8000    1.0000
    1.0000    1.0000    1.0000
```

-1.2516	-1.4382	-1.7578
0	0.1540	0.9119
0	0.2161	0.6474
0	0.1863	0.2067
0	0.0646	-0.4102
0	-0.1491	-1.2035

-449.3721
160.1447
126.4988
53.9004
-57.2197
-198.0353

-1.2516	-1.4382	-1.7578
0	-0.3627	-1.3010
0	0	-0.2781
0	0	-0.5911
0	0	-0.6867
0	0	-0.5649

-449.3721
-242.3136
-41.8356
-91.2045
-107.4973
-81.8878

```
R =

  -1.2516   -1.4382   -1.7578
        0   -0.3627   -1.3010
        0         0    1.1034
        0         0         0
        0         0         0
        0         0         0

z =

-449.3721
-242.3136
 168.2334
  -1.3202
  -3.0801
   4.0048
```

```
beta = R(1:3,1:3)\z(1:3)

beta =
    5.7013
  121.1341
  152.4745

norm(z(4:6))
norm(X*beta - y)
```

$s = (2010 - 1950)/50 = 1.2$

$\beta_1 s^2 + \beta_2 s + \beta_3$

```
p2010 = polyval(beta,1.2)

p2010 =
  306.0453
```

Pseudoinverse

$$\|A\|_F = \left(\sum_i \sum_j a_{i,j}^2 \right)^{1/2}$$

$$Z = X^\dagger$$

```
Z = pinv(X)
```

If X is square and nonsingular,

$$X^{\dagger} = X^{-1}$$

If X is m-by-n with $m > n$ and full rank,

$$X^\dagger = (X^T X)^{-1} X^T$$

$$X^\dagger X = (X^T X)^{-1} X^T X = I$$

$$X X^\dagger = X (X^T X)^{-1} X^T \neq I$$

only has rank n.

Minimize

$$\|XZ - I\|_F$$

Also minimize

$$\|Z\|_F$$

Minimize both

$$|xz - 1| \text{ and } |z|$$

$$x^\dagger = \begin{cases} 1/x & : \quad x \neq 0 \\ 0 & : \quad x = 0 \end{cases}$$

rank deficiency

If X is rank deficient the least squares solution to the linear system

$$X\beta \approx y$$

is not unique.

null vector

$$X\eta = 0$$

$$X\beta \approx y$$

basic solution

```
beta = X\y
```

minimum norm solution

```
beta = pinv(X)*y
```

$$X = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \\ 13 & 14 & 15 \end{pmatrix}$$

$$y = \begin{pmatrix} 16 \\ 17 \\ 18 \\ 19 \\ 20 \end{pmatrix}$$

$$\eta = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

is a null vector.

```
beta = X\y

Warning: Rank deficient, rank = 2  tol =   2.4701e-014.

beta =
   -7.5000
         0
    7.8333

beta =
         0
  -15.0000
   15.3333

beta =
  -15.3333
   15.6667
         0
```

are also basic solutions.

```
beta = pinv(X)*y

beta =
   -7.5556
    0.1111
    7.7778

norm(pinv(X)*y) = 10.8440

norm(X\y) = 10.8449

X\y - pinv(X)*y =
    0.0556
   -0.1111
    0.0556
```

is a multiple of the null vector η.