A Gentle Introduction to Formal Semantics

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Preface

[to be written]

[A first version of these lecture notes was written in German by the first author and can be downloaded from www2.sfs.uni-tuebingen.de/sternefeld/Downloads/Ede_Semantik1_WS00-01.pdf. The present text has been modified, shortened, extended, and translated into English by the second author. For ease of comparison we sometimes added German translations in brackets. Style and exposition could further be improved, still awaiting the help of a native speaker of English. The pointing finger ☞ that occasionally accompanies proper names or technical terms is a request to look up the highlighted keyword in wikipedia (preferably the German version, which is much better than the English one), which will provide for invaluable background information that should not be ignored by any serious student of semantics.]

1 Literal Meaning

The subject of **semantics** is the systematic study of the **meaning** of linguistic expressions like morphemes, words, phrases, sentences or even texts. Before we can start with the investigation of their meaning, we will have to narrow down the object of our subject of interest. The reason for this is that not everything that can be said about our understanding of linguistic expressions is relevant for a theory of *meaning*. Rather, we will only be interested in that part of "meaning" of a linguistic item that is associated with it by virtue of certain **linguistic conventions** of a specific type—this is what we will be calling the **literal meaning** of an expressions.

1.1 Hidden Sense

Humans understand utterances automatically, immediately, effortlessly, and without explicitly thinking about meaning or about what they are doing when understanding language. Rarely are we forced to consciously reflect on meaning in a systematic way; sometimes such a situation arises when being concerned with the "interpretation" of literary texts, e.g. **poems** or **lyrics**. Here is a case in point:

(1)

Schwerer Päonienduft	The heavy scent of peonies
Von fern	From far away
Le Ta	Le Ta
Gatte und Kind	Spouse and child
Verlassen	Lonesome
Wenn der Schwan ruft	When the swan calls
Tusche von Meisterhand	A print by a master
Im Schnee	In the snow
Mädchen	Girl
Deiner Geburt	Of your birth
Erinnern	Remembering
Schriftzeichen im Sand	Writing in the sand

And indeed, an obvious question concerning these lines raise is: What do they mean? We clearly have to *interpret* the above lines in order to make sense out of them (and we implicitly assume that *some* sense *can* be made out of all this.)

The term "interpretation", understood in this way, means that we unearth some **hidden meaning** that is not at all obvious to anyone confronted with (1). Given that *Le Ta* has no obvious meaning (its not an expression we could find in any dictionary), how are we to interpret it? Is it a proper name? Perhaps, after all, this seems plausible. But what about the other terms that do have a plain meaning? What about the connections between these words? How does all this make sense, and, if it does, what is the hidden meaning of these words that seem to contribute to an additional sense yet to be discovered? This is the kind of question literary criticism is concerned with.

The above poem is is taken from Klaus Döhme: *Leda & Variationen* (Trier 1978). There we also find the following contribution to an "interpretation" of the poem:

(2) The redundancy-purged denseness of the Middle Chinese bi-stanza Ritornello (I Shing Min) with the classical AXXXA rhyme scheme endows this archetypal mythotope preeminently with its lyricalalness par excellence[1]

Whether this comment improves our understanding of the poem is doubtful: this commentary is at least as difficult to understand as the poem itself. At any rate, there is no need to bother about these questions too much here: both the poem and its interpretation are spoofs! The author's intention is precisely to create some feeling of hidden meaning, although he only mimics a certain style that pretends to be deep and meaningful (cf. Döhmer (1978) for details).

One of the most famous real-life examples of a poem crying out for additional sense creation is the following one by William Carlos Williams (1923):

(3) so much depends
upon

a red wheel
barrow

glazed with rain
water

beside the white
chickens

You'll find thousands of internet pages in search of a hidden meaning (☞ The Red Wheelbarrow).

Fortunately, in semantics we are not interested in hidden meaning, but only in the ostensible, primary meaning which is what the poem **literally** says. But even this is not easy to find out in the case at hand. One problem is to identify the sentences or phrases in (2)—what shall we make out of incomplete sentences and incomplete phrases? Can we be sure that (4) is a complete sentence?

(4)

Deiner Geburt
Erinnern

[1] "In der redundanzfeindlichen Dichte des mittelchinesischen Doppelstrophen-Ritonells (I Shing Min) mit dem klassischen Reimschema AXXXXA gewinnt jenes archetypische Mythotop katexochen seine Lyrizität par exzellence." Thanks to Janina Rado for the translation of this into English.

And if so, can this be interpreted as an old-fashioned way of saying the same as (5)?

(5) An deine Geburt erinnern Schriftzeichen im Sand
of your birth remind characters in-the sand

If this interpretation is correct, everyone can understand its literal meaning, namely that there are signs in the sand that are reminiscent of your birth. And that's all about it. Nonetheless, many details may still remain unclear: What does *your/deine refer* to (the reader of the poem)? How does this sentence relate to the meaning of the entire poem (and to the intentions of the author)? Are the signs scratched into the sand or are they mere shadows? All this does not belong to the literal meaning of the sentence.

The general point to be illustrated here is that lyrics or poems seem to bear some surplus meaning not contained in the literal meaning of the words. This extra sense is the topic of literary studies, which is in search of meaning behind the scene—which might be interesting enough. But fortunately it's not what we are doing in semantics. Semanticists are primarily concerned with aspects of the **literal meaning** of words, phrases, and sentences: There are some signs or characters, there is some sand, there is an addressee referred to by "your" etc. Although literal meaning can be quite unhelpful in the context of poetry, this does not bother us in semantics. In semantics, we aim low and are content with dealing with the obvious only.

Now, compared with the discovery of *hidden* meaning, the description of *literal* meaning seems to be a thoroughly boring enterprise that does not deserve any scientific occupation. Given that *anyone can understand literal meaning in an effortless way*, why should scientists care for (literal) meaning? Is it worth the effort to study something that is grasped by anyone without the least difficulty?

The answer is that, although understanding utterances proceeds automatically and effortlessly, we still have no **explanation** for why and how this is possible at all. To mention an analogy from human perception: When hearing a noise we can often identify the direction of its source. However, how this can be achieved by the human organism is quite far from trivial and has not been understood until recently (for more information, cf. ☞ "Räumliches Hören", Ackern, Lindenberg). This kind of ignorance also holds for almost any aspect of human cognition: we have no idea how exactly the mind or brain works, and it's only recently that aspects of the working of human perception have been explained by reference to certain neuro-physiological mechanisms.

Consequently, there is something to be *explained* if we want to understand why and how humans can succeed in understanding phrases and sentences—in particular, sentences *they might have never heard before.* This is one of the central topics in linguistic theorizing. In fact, some of you might recall from your elementary introduction to linguistics that in syntax, one of the basic issues was "recursiveness", namely the fact that there is no upper limit to the length of grammatical sentences, although any dictionary of a particular language only contains finitely many words. Put another way:

(6) *Foundational research question in syntax:*
How comes that we can, at least in principle, decide for arbitrarily long sentences (which we might have never heard before), whether or not they are syntactically well-formed?

Now, in semantics, we may ask the parallel question:

(7) *Foundational research question in semantics:*
Given our restriction on literal meaning, how come that we can understand arbitrarily long sentences we have never encountered before, and, in particular, how come that we can tell whether or not they make sense (are semantically well-formed)?

This introduction tries to give an answer to this question.

1.2 Irony and Implicature

Before we can embark on such an endeavor, let us explain more precisely our understanding of *literal meaning*. Suppose Fritz is leaving the Mensa and meets his friend Uwe who is asking about the quality of the meal. Then Fritz says:

(8) Das Steak war wie immer zart und saftig
the steak was as always tender and juicy

Now, according to the literal meaning, the quality of the food should have been excellent. But this is not the intended message: rather, Fritz wants to convey that the steak was as it always is, namely *neither* tender *nor* juicy. And Uwe, his friend, easily understands the message conveyed by (8). How does that happen?

As a prerequisite for such an understanding it is absolutely necessary for Uwe to first understand the literal meaning. Knowing his friend and the usual quality of the food in the Mensa very well and having no evidence for a sudden lapse of taste on Fritz's part, he also knows that the literal meaning cannot possibly be the intended meaning. Besides, Uwe might detect a waggish expression on Fritz's face. He therefore legitimately concludes that the utterance is meant in an **ironic** manner (☞Irony). And this implies that the conveyed meaning is exactly the **opposite** of the literal meaning. In order for this to work properly it is necessary for the literal meaning to come first: only on the basis of an understanding of the literal meaning is it possible to understand the utterance as saying the opposite of the literal meaning.

NB: In classical rhetoric, irony is always defined as expressing the opposite of the literal meaning. In ordinary language, however, the term *irony* is used in a much broader sense. Suppose Fritz continues his description of the menu by saying:

(9) Auch der Nachtisch war nicht giftig
Also the dessert was not poisonous

Although this utterance can be called ironic, the term *irony* in its traditional narrow sense is not adequate in this case, because Fritz does not want to say the exact opposite of (9), namely, that the dessert was poisonous. Nor does he want to convey the literal meaning, namely that the quality of the dessert is such that one is not in danger of being poisoned. Rather, what he wants to say is something like:

(10) The quality of the dessert cannot be categorized as much better than not poisonous.

Which implies that it is very bad.

In linguistics, this is called an instance of an ☞**implicature**. An implicature is something that goes beyond the literal meaning, but cannot contradict the literal meaning. The above implicature is of a special type; it is called *scalar* (skalare Implikatur) because the conveyed meaning involves a scale of grades; in this case a scale that characterizes the edibility of food, ranging from deathly to three Michelin stars. “not poisonous” seems to range somewhere in the lowest range of the scale.

What we see from these examples is that the literal meaning often does not suffice to really understand an utterance; it must be augmented in some way or other. How this is done is explained in **pragmatics**, which is concerned with systematic aspects of the use of linguistic forms. Within semantics we stick to the literal meaning, which is, as we have seen, a prerequisite for a full and correct understanding of an utterance.

1.3 The Way You Say It

We have seen above that the intended effects of an utterance may go far beyond its literal meaning:

- Numerous texts (in particular literary ones) exhibit a hidden meaning that reveals itself only to an educated person
- Rhetorical effects like irony, exaggeration or scalar implicatures can reverse, augment or modify the literal meaning
- A certain choice of words or a stylistic register can express the speaker’s attitude, over and above the literal content of the word

As an example for the last point, imagine that the manager of the *Studentenwerk* is interviewed by a journalist from a student’s journal, the Campus Courier. The junior editor was supposed to ask something like:

(11) Planen Sie tatsächlich eine Anhebung der Essenspreise?
Plan you really a raise of meal prices
Are you really planning to raise the meal prices?

Now, what the journalist actually utters is this:

(12) Willst Du allen Ernstes für den Fraß noch mehr Zaster verlangen?
(*Fraß* = coll. *food*; *Zaster* = coll. *money*)
Are you serious about demanding even more dough for your grub?

There are of course several features of (12) that render this utterance inappropriate (can you descibe them?). But thinking about the literal meaning of (12) will reveal that by and large its relevant content is in fact the same as that of (11).

That is, both questions "mean" more or less the same. But in what sense of more or less? This again is a topic that is dealt with in pragmatics. It's not what you say, it's the way you say it that is relevant for pragmatics. From the viewpoint of linguistic semantics we may say that the literal meaning of both sentences is almost identical, and that small differences can be neglected. Nonetheless the expressions used in (11) and (12) have different **connotations**. Although we may refer to the same kind of thing with two different expression (e.g. the same person referred to with the personal pronoun *du* (engl. *you*) and its polite alternate *Sie* (engl. *you* again)), the connotations of these expressions may differ (cf. ☞ *connotation (usage)*; ☞ Konnotation im Sinne von Nebenbedeutung).

HOMEWORK: Another case of non-literal meaning is exemplified by so-called metaphors. Browse the internet for definitions of the term "'metaphor". What is the difference between metaphoric and ironic use of expressions? Consider the meaning of the adjectives in (13). Is the use of the adjectives metaphoric, ironic, or idiomatic? Should the extra meaning of these expressions be listed in a good dictionary of German? (And are they indeed in yours?)

(13) schreiende Farben (jazzy colours), purzelnde Preise (falling prices), schlagende Argumente (telling arguments)

1.4 Difficult Sentences

In general, and as claimed above, the understanding of the literal meaning proceeds automatically, unconsciously, and effortlessly, similar to other acoustic, visual or sensual perception—but unlike the understanding of hidden sense. However, although in practice this seems to be true, we might come across sentences whose meaning is still difficult to decipher, even when considering only the literal meaning of the words they contain. Consider eg. run-on sentences like:

(14) The woman, whose sister, whose son, whose girl friend studies in France, emi-

grated to Australia, resides in Italy, lives next door

An equivalent German translation is (15):

(15) Die Frau, deren Schwester, deren Sohn, dessen Freundin in Frankreich studiert, nach Australien ausgewandert ist, in Italien lebt, wohnt nebenan.

In both languages these sentences are extremely hard to process. After a while, having parsed their syntactic structure, one may find out that (15) means the same as:

(16) Die in Italien lebende Schwester der Frau nebenan hat einen Sohn, dessen Freundin in Frankreich studiert und der selbst nach Australien ausgewandert ist.

This sentence is much easier to understand and is not much longer than the original one, so the problem is not length! Rather, it is the kind of construction that makes the sentence incomprehensible (without a tedious linguistic analysis).[2]

Upon further reflection, however, one might argue that the problem with understanding (14)/(15) should not be located within semantics, but rather in syntax: after all, the complexity of the difficult sentences already arises with the *syntactic* parsing of (14)/(15), leading to a kind of recursive self-embedding structure that is difficult to parse syntactically and that is avoided in (16). On the other hand, since syntactic parsing also normally proceeds unconsciously and quickly, the memory overload that may cause the problem in (14)/(15) might not only involve syntax, but probably semantics as well. The two go hand in hand, and a priori it is not clear whether the difficulty should be located in syntax or in semantics. However, we can make a point in favor of additional semantic complexity by considering the following self-embedding structures:

(17) a. The woman, the man, the host knew, brought, left early
b. The woman, someone I knew brought, left early

(18) a. Die Frau, die der Mann, den der Gastgeber kannte, mitbrachte, ging früh
b. Die Frau, die jemand, den ich kannte, mitbrachte, ging früh

In (17-b), we replaced *the man* with *someone*, and *the host* with *I*. Intuitively, (17-b) is much easier to understand than (17-a), although the replacement of these elements does not change the relevant syntactic structure. The difference must then somehow be related to semantics. This implies that semantics does play a role in calculating the complexity of (17) (and (18)), although of course syntax and prosody may still be involved as additional factors that influence the comprehensability of the construction.

[2] For the English equivalent in (14) it seems to be very difficult to find a paraphrase that is both easy to understand but not considerably longer than the original sentence. Try by yourself! It's for this reason that we preferred a German example.

Apart from constructional complexities as exemplified above, there might be other reasons that make it difficult to grasp the literal meaning of a sentence. The American classical scholar (Altphilologe) Moses Hadas once started a book review with the following sentence:

(19) This book fills a much-needed gap

That a book fills a gap is normally understood as something positive and it is this positive expectation that drives our interpretation of the sentence. Moreover, the expression *much needed* is normally understood as something positive as well, except that—in this particular case—*much needed* is not attributed to the book but to a gap, that is, to the non-existence of the book. So the literal meaning of the sentence is that *we do not need the book* (but the gap). In fact, the review is totally devastating. (Other memorable and facetious quotes of Moses Hadas include: "I have read your book and much like it." and "Thank you for sending me a copy of your book. I'll waste no time reading it.")

An even more complex case of semantic processing difficulty is exemplified by:

(20) No head injury is too trivial to ignore
Keine Kopfverletzung ist zu trivial um ignoriert zu werden

The example works in both languages, the only difference in structure being that the infinitive at the end must be expressed by using a passive voice in German.

At first hearing this sentence seems to say that we shouldn't trifle with brain injuries. But in reality, analysing the literal meaning, we may discover that the proposition made is very cynical, namely that any brain injury should be ignored! In order to see this, compare (20) with:

(21) No beverage is too cold to drink
Kein Getränk ist zu kalt, um getrunken zu werden

Now, a beverage that is too cold to drink is one that should not be drunk, and accordingly, (21) says that

(22) Any beverage—as cold as it may be—can be drunk

But now, by analogy, (20) means the same as:

(23) Any head injury—as harmless as it may be—can be ignored

The message that should be conveyed by the original sentence seems to be that even harmless injuries have to be taken particularly seriously and should not be ignored. But thinking about it and taking into account the analogy between (22) and (23), you will find out that this is just the opposite of the literal meaning!

In a blog post on this example (☞http://semantics-online.org/2004/01/no-head-injury-is-too-trivial-to-ignore), Mark Liberman adds on to this a naturally occurring example he found (uttered by a certain Mr. Duffy):

(24) I challenge anyone to refute that the company is not the most efficient producer in North America

Mark Liberman asks:

> Is this a case where the force of the sentence is logically the same with or without the extra not? Or did Mr. Duffy just get confused?
>
> I would certainly lean towards the latter explanation. But it's quite well-known that it is hard not to be confused. The coolest case I know is [(20)]. I believe it was brought into the literature by Wason and Reich:
> Wason, P. C., and Reich, S. S., 'A Verbal Illusion,' Quarterly Journal of Experimental Psychology 31 (1979): 591-97.
> It was supposedly found on the wall of a London hospital. Actually, a Google search suggests that the ultimate source of the quote is Hippocrates (460–377 BC). By the way, a number of the Google hits seem to come from sites run by injury lawyers. Also, by the way, the full quote appears to be "No head injury is too severe to despair of, nor too trivial to ignore", which is even more mind-boggling, at least for my poor little brain.

So what we have learned in this section is that our normal, unconscious understanding of such sentences might go wrong in various ways, the result being that we are mistaken about the (literal) meaning of a sentence.

As a methodological side effect, these considerations also have established that the literal meaning can be detected and analysed in a systematic way without recourse to mere intuition (which, as we have seen, can be misleading); there is something systematic in the way meaning is built up that needs to be analysed and be explained. This is what semanticists do. They try to build up meaning in a systematic fashion so that the exact content reveals itself in a way that is predicted by a semantic theory, not by intuition alone.[3]

[3]Perhaps the beginner should not be too optimistic in expecting to be able to formally analyse these difficult sentences (by the end of the course). The semantics of the above quoted examples is extremely difficult to analyse and even remains so for the trained semanticist. It is only by specializing in certain kinds of constructions that one may be able to come to grips with them, perhaps at the end of your academic studies in semantics.

2 Lexical Semantics

Sentences, as long and complicated as they may be, always consist of (structured sequences of) single words. Therefore it seems natural to start off an investigation of literal meaning with the study of **word meaning** (as opposed to the meaning of phrases or sentences). Using linguistic terminology, the entirety of words of a specific language is called a **lexicon**, therefore the investigation of word meaning is often called ☞**lexical semantics**. That's the topic of this section.

Let us start with a simple, but potentially confusing question:

2.1 What's in a Word?

This question is far from trivial, and there is no general answer to it. This can be illustrated by the fact that speakers of German and English tend to have different intuitions about whether a string of two words X+Y like *linguistics department* is to be analysed as one word or as two. Speakers of German normally seem to have firmer intuitions about such compounds, because compounds are written without a blank between X and Y, qualifying them as a single word. This is normally not the case in English, which seems a potential reason for why speakers of English have less firm intuitions about words. However, recent discussion of German orthography has revealed that there may very well be borderline cases even in German (eg. German speaking readers may ask themselves: are *fallen+lassen, Rad+fahren* or *liegen+bleiben* one word or two?).

Another issue related to the question of wordhood is this: Is a single sequence of phonemes the realization of one or two words? Consider the following examples:

(1) a. German: /razen/, written as Rasen; English: *meadow*
b. German: /razen/; written as rasen; English: *rage*

Now, if a word consists only of a sequence of phonemes, (a) and (b) illustrate the same word. But this of course is absurd! Clearly, (a) and (b) contain different words. Although the pronunciation is identical, we (fortunately) still have two different spellings, and this constitutes clear evidence for two different words (which, as it happens, also belong to two different syntactic categories: *Rasen* is a noun, and *rasen* a verb). The same difference in syntactic category can be observed in the English examples in (2):

(2) a. *light* = *not heavy* vs. *illumination*
b. *rose* = a flower vs. past tense of *rise*
c. *left* = opposite of *right* vs. past tense of *leave*

But now consider the following German examples:

(3) a. German: *Bank*$_1$ (plural = *Banken*) = *bank(ing house)*,

German: *Bank*$_2$ (plural = *Bänke*) = *bench*

b. German: *Schloss*$_1$ = *castle*

German: *Schloss*$_2$ = *lock*

The words in (3-a) and the words in (3-b) do not differ in syntactic category, but still in meaning. For each of them you will find two lexical entries in your dictionary, and therefore we feel entitled to conclude that these are two different words. For (3-a) this becomes apparent by looking at the different plural forms of *Bank*. As illustrated in (3-b), however, it may happen that we do not find any grammatical difference between two words at all, except meaning. Thus, the two words *Schloss*$_1$ and *Schloss*$_2$ have the same syntactic category, the same gender (neutral), and the same inflection, though different meanings. In this case one often says that *the* word *Schloss* has two meanings. Saying this implies there is only one word, whereas above we insisted that *Schloss* represents not one word, but two. This is of course a pure matter of terminology. If we understand the term "word" as including meaning, then we have two words; if by "word" we understand only its form (with the exclusion of meaning), there is only one word. Unfortunately, the difference is mostly neglected in everyday talk.

In these lectures we prefer to include meaning, so that diffence in meaning suffices for there to be two words with one spelling. Different words with the same spelling are called **homographs**; different words with the same pronunciation are called ☞**homophones**. Note that homophones may differ in spelling:

(4) Homophones, also called homonyms:

a. *four* vs. *for*

b. *break* vs. *brake*,

c. … see the list in Wikipedia

And homographs may differ in pronunciation, cf.

(5) Homographs, also called heteronyms or heterophones: [4] *desert* (to abandon; with stress on the second syllable) vs. *desert* (arid region; with stress on first syllable)

2.2 Ambiguity and Polysemy

The words discussed in the last section have one thing in common: they differ in meaning and thereby illustrate what is often called **ambiguity**. Quoting from Wikipedia, ☞*Ambiguity (Linguistic forms)*:

[4] Quoted from Wikipedia: "There is considerable confusion and contradiction in published sources about the distinction between homonyms, homographs, homophones and heteronyms." See ☞Homonym for details and discussion.

As a rich source for homonyms in German we recommend: *Bilden Sie mal einen Satz mit … 555 Ergebnisse eines Dichterwettstreits*. Ed. by Robert Gernhardt and Klaus Cäsar Zehrer. Fischer Verlag 2007.

> Lexical ambiguity arises when context is insufficient to determine the sense of a single word that has more than one meaning. For example, the word “bank” has several distinct definitions, including “financial institution” and “edge of a river,” but if someone says “I deposited 100 dollar in the bank,” most people would not think you used a shovel to dig in the mud. The word “run” has 130 ambiguous definitions in some lexicons. “Biweekly” can mean “fortnightly” (once every two weeks - 26 times a year), OR “twice a week” (104 times a year).

Note that in this definition **a single (sic!) word** is assumed to have more than one meaning. Above, however, we argued that there are **two** words $bank_1$ and $bank_2$ which happen to have the same pronunciation. As noted above, this is a matter of terminology only; but it seems to me that our terminology is more precise. In linguistic texts, we use indices, eg. $bank_1$ and $bank_2$ as a sign to indicate ambiguity, but in normal speech the use of indeces is out of the question. Therefore, in simple texts, the less precise notion seems to be preferred.[5]

Apart from this, there is yet another peculiatity in the quote above that might bother us: the assumption that ambiguity has to do with the *context* of an utterance seems to be misguided. Lexical ambiguity does not only arise when the context of use is insufficient to decide between different meanings: one can easily imagine that there is never any kind of misunderstanding in the use of $bank_1$ and $bank_2$, so that in every single utterance of one of them it is clear (and unambiguously determined by the context!) which meaning is intended. Even then we would still say that the sequence written as *bank* is lexically ambiguous. The problem with the above quote is that it cannot serve as a *definition* of the term “lexical ambiguity”; rather it may serve as a kind of illustration: Of course, there might be contextually and referentially ambiguous cases like (6):

(6) Give me the glasses!

Imagine a particular situation with two wine glasses on a table and a pair of spectacles. Then, it might still be unclear whether *glasses* is the plural of (wine) *glass*, or whether we mean (eye)glasses, i.e. spectacles. If the ambiguity has not been resolved, I would not know what to bring; but fortunately the circumstances allow for the **disambiguation** of an ambiguity.

The above example points to another difficulty. Translating the sentence (6) into German, we would have to decide between two terms: *Gläser* and *Brille*, the latter being the term for eyeglasses. Therefore one might be entitled to conclude that there is an ambiguity. However, without this criterion, we would be less sure. Indeed, there are

[5]Interestingly, still further (and totally different) criteria have been used to define the notion word; see Di Sciullo and Williams (1987) for a thorough discussion. These different criteria also seem to play a role in the ongoing discussion of the “New German Orthography”; see Jacobs (2005).

archaic dialects of German that would permit for the same sort of use, so that *Gläser* could also mean *Augengläser*. Can we still say that there is a real ambiguity envolved here? After all, as spectacles are also made out of glass, one might say that the term *Gläser* is not ambiguous, rather it is underspecified with respect to the *kind* of glasses that is intended.

So in many cases it holds that the different meanings are somehow related to each other, or are very similar, so that there seems to be some vagueness involved. Therefore linguists have strived to develop criteria that ideally should decide whether two terms are ambiguous. We will only discuss one of them here.

(7) At the end of the day we had to deplore that John destroyed glasses and Bill too destroyed glasses

This sentence seems to be okay even in the case where *glasses* may have the two different interpretations:

(8) At the end of the day we had to deplore that John destroyed glasses$_1$ and Bill too distroyed glasses$_2$

But now, we may ask whether we can conclude from (8) that

(9) Bill and John destroyed glasses

Can (9) be used to express the same as (8)? This seems hardly possible, and the reason for this unability seems to be that *glasses* is indeed ambiguous![6]

Another potential case of ambiguity is illustrated in (10):

(10) a. Er geht noch zur Schule (= the institution)
He still goes to school
b. Die Schule streikt heute (= all pupils, teachers etc.)
School is on strike today
c. Unsere Schule steht unter Denkmalschutz! (= the building)
Our school is classified as a historical monument
d. Schulen sollten von außen als solche erkennbar sein (= the building, but because of ”als solche” at the same time also the institution)
Schools should be identifiable as such from the outside

These differences in meaning seem to be systematic, and it may well be that they are not listed in your dictionary; if being distinguished in a dictionary is a valid critereon, the differences we observe in (10) do not give rise to different words. Nonetheless, thinking

[6]Unfortunately, most ambiguity tests are unreliable; cf. Sadock and Zwicky (1975) for further discussion.

of what the term expresses and of what kinds of things we refer to with the expression in these sentences, it is quite obvious that some distinction in meaning is undeniable. Such systematic differences, arising as variants of one core meaning (= the institution), have a special name: the phenomenon is called ☞**polysemy**. The difference between ambiguity and polysemy is this: ambiguities can be arbitrary, as with $bank_1$ and $bank_2$, whereas polysemy is something systematic that can be observed with a whole range of expression (*Schule/school, Krankenhaus/hospital, Kirche/church* etc.).

Polysemy is often contrasted with homophony. Both require identical pronunciation, but whereas in homophonous pairs the different meanings are not related to one another, polysemous pairs require a close semantic relationship between the meanings of the words, ideally of the sort exemplified in (10). Here are some more examples where the semantic relation between the two meanings is of the more opaque sort:

(11) a. bright: *shining* or *intelligent*
b. to glare: *to shine intensely* or *to stare angrily*
c. a deposit: *minerals in the earth* or *money in the bank* or *a pledge* or …

For the linguistic layman this kind of relationship between words seems to be the most interesting aspect of semantics, giving rise to endless debates and historical speculations about the nature of the similarity.[7] Since we are not concerned with diachronic linguistics and etymology, we will refrain from any discussion of polysemy.

2.3 Sense Relations

It is often implied in the literature that semantic theories should account for certain kinds of intuitive judgments of native speakers of a particular language. These judgments and the corresponding intuitions can be of several sorts. One is **semantic** (as opposed to *syntactic*) **well-formedness**, the other is the language user's ability to assess certain systematic aspects of the meanings of words to which we will return. Starting with the first, the reader will agree that the following sentences are strange:

(12) a. Der Koch singt ein Gewürz
the cook is singing a spice
b. Die Gabel bezweifelt das
the fork doubts it

The meaning of (12-a) is unclear because one can sing only songs or texts. Somehow the verb and the object do not fit together. In (12-b) there is a mismatch between

[7]As it turns out, many ambiguities evolved from polysemies; eg. the German example *Schloss* (lock/castle) has started off with a single basic meaning corresponding to *lock*; the castle reading then evolved from a building that locks the way out of a valley.

the verb and the subject. The reason for the awkwardness of these (syntactically well-formed) sentences is that they violate certain semantic well-formedness conditions that accompany the verbs. These conditions are called **selectional constraints** (Selektionsbeschränkungen) of the verb: *to doubt/bezweifeln* requires the subject to be human, and the object of *to sing/singen* has to be something like a song. Native speakers who have learned the meaning of the verbs are clearly able to activate intuitions of this sort, ie. intuitions about selectional constraints. All selectional constraints are part of the meaning of particular lexical items.

Another semantic skill of speakers (and hearers) is their ability to make claims about the meaning of two words in comparison. This is exemplified in:

(13) a. Groundhog means the same as woodchuck
b. Professor and bachelor differ in meaning
c. Precipitation is a more general term than drizzle
d. Dog and cat are incompatible with-each-other

These sentences are statements about so-called *sense relations.* (13-a) states that two words have the same meaning, they are *synonymous.* The sense relation expressed is **synonymy** (☞Synonymie). Synonymy between simple lexical items seems to be very rare in natural language. It is sometimes said that "true synonyms", i.e. those whose connotations do not differ too much, are extremely rare (the phenomenon has been dubbed "Synonymenflucht"). This has been explained by an economy principle to the effect that language does not contain redundant material in the lexicon. Given the host of synonym pairs in (certain) closed categories (*obschon, obzwar, obgleich*), this claim needs some qualification; but it does seem that fully synonymous content words are a rare species. Also, we observe that many new terms have been coined that were initially intended to replace old ones with the same meaning: Compare *Fahrkarte* vs. *Fahrausweis*; *Schaffner* vs. *Fahrdienstleiter*; *Mülleimer* vs. *Wertstoffbehälter*; *Toilette* vs. *WC-Center*, *Hotel* vs. *Beherbergungsbetrieb*, etc. This has been called semantic environmental pollution (*semantische Umweltverschmutzung*). Note that most of these terms are compounds. It therefore remains true that there are hardly any two synonymous *simplex* (uncompounded) content words.

The next sentence (13-b) states a *non-identity of meaning*; this can also be called a sense relation, albeit normally a very uninformative one. (12) is more interesting. It says that one notion includes the other, or in other words, it logically implies the other. The more general including term is called a ☞**hyperonym** (Oberbegriff), the more special included term is called a ☞**hyponym** (Unterbegriff). If a notion A is a hyperonym of B, then B is a hyponym of A. The relation of inclusion is called **hyponomy**. The reverse relation of being included is called☞**hyperonomy** (Hyperonomie).

Finally considering (13-d), assume that the utterance is not meant to report something about the behavior of cats and dogs; rather, one wants to say that the notions exclude each other. That is, if something is a cat it cannot be a dog, and vice versa. Similarly, what is white cannot be red. The relevant sense relation is **incompatibility**. The attentive reader should note that this relation is stronger than simple difference in meaning. For example, *bachelor* and *professor* are not incompatible, but do differ in meaning, as was just noted.

Apart from these typical relations there are a number of other relations between words, like the one illustrated in (14):

(14) a. John **kills** Bill
b. Bill **dies**

Here one would say that *kill* means something like (or is almost synonymous to) *cause to die*. Thus, dying is a sort of causal consequence of killing; hence the semantic relation is *causation* (Verursachung, Kausativierung).

2.4 Semantic Networks

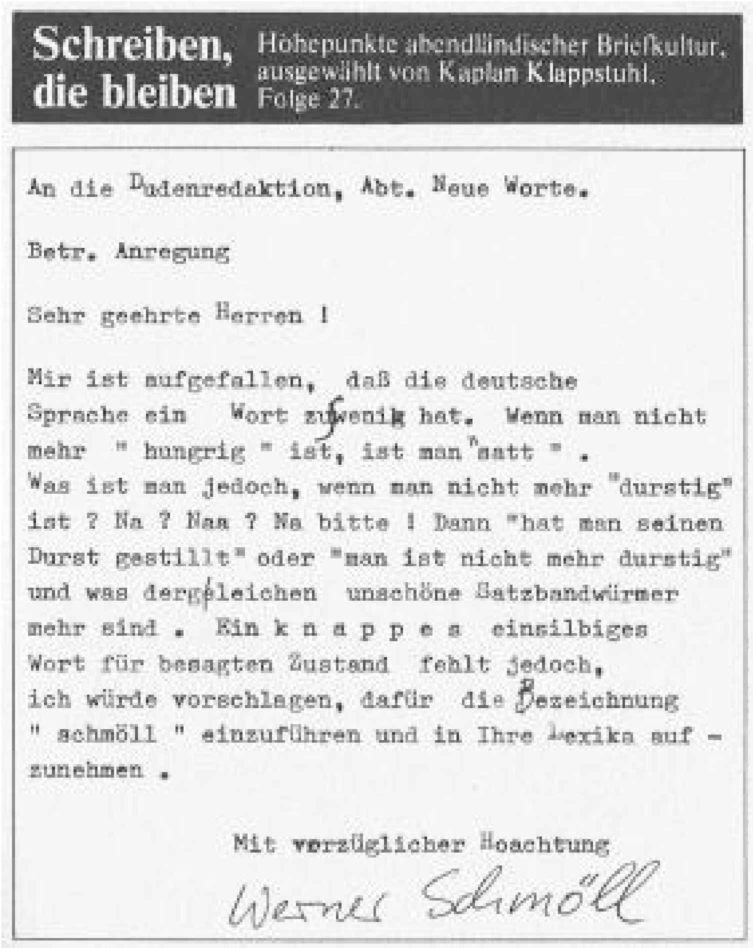
As a result of establishing more and more sense relations, linguists have proposed that all sense relations that hold between words should be organized in a kind of ☞**semantic network** (☞semantisches Netz). Networks consist of nodes labelled with lexical items and connected by semantic relations. These relations may contain all sorts of relevant information about the meaning of a lexical item; the most primitive networks represent sense relations only. Here is an example from an electronic data base called GermanNet:

(15)
wandeln (*transform*)
verändern (*change*)
↑
schließen öffnen sich öffnen (*shut*, *open* (transitive), *open* (intransitive))
↕ ←→ ↕ ← ↕
zumachen aufmachen aufgehen (*shut, make open, come loose*)
↓
aufstoßen *push open*
aufbrechen *break open*
aufsperren *unbar*

It is the task of lexical semantics to describe the network of a given language.

HOMEWORK: Discuss which sense relations (including causation) are represented by the arrows in (15).

One of the potentially interesting things about networks is that they may have gaps. This is illustrated in the following letter taken from Robert Gernhard, Welt im Spiegel 1975:


Schreiben, die bleiben Höhepunkte abendländischer Briefkultur, ausgewählt von Kaplan Klappstuhl. Folge 27.

An die Dudenredaktion, Abt. Neue Worte.

Betr. Anregung

Sehr geehrte Herren !

Mir ist aufgefallen, daß die deutsche Sprache ein Wort zuwenig hat. Wenn man nicht mehr " hungrig " ist, ist man "satt " . Was ist man jedoch, wenn man nicht mehr "durstig" ist ? Na ? Naa ? Na bitte ! Dann "hat man seinen Durst gestillt" oder "man ist nicht mehr durstig" und was dergleichen unschöne Satzbandwürmer mehr sind . Ein k n a p p e s einsilbiges Wort für besagten Zustand fehlt jedoch, ich würde vorschlagen, dafür die Bezeichnung " schmöll " einzuführen und in Ihre Lexika auf - zunehmen .

Mit vorzüglicher Hochachtung

Werner Schmöll

(Translation [to be done]: I noticed that the German language lacks a word. When you aren't hungry any more, you are full (satiated); but when you are not thirsty anymore, you are ... I'de like to ask you to introduce the term "schmöll" into your dictionaries.

Yours faithfully

Werner Schmöll)[8]

The remarkable thing is not the fact that we do not have words for particular (kinds of) things. This is quite normal. E.g. we do not have a word for blond girls that were born on April 1st. Although we could of course invent a notion like first-april-girl, this is not a single word but an ad hoc compound. The problem is rather that *thirsty* lacks an **antonym**, a word that is incompatible and expresses the opposite. Thus, the opposite of black is white, the opposite of slow is fast, etc. This relation is another sense relation; cf. ☞Antonym, ☞Opposite (semantics))

[8] The question is whether we really need such a notion. After all, you can booze till you drop. On the other hand we do have the notion "abgefüllt" (filled) in German, though it has a broader meaning than schmöll. (Bottles can be filled, but they cannot be schmölled.) The German speaking readers should consult wikipedia's ☞Sitt.

Any parsimonious description of a semantic network will take advantage of the fact that we can reduce some sense relations to others. A particularly useful method for doing so is to describe sense relations between words by analyzing synonymous paraphrases. For example, what is the sense relation between *brother* and *sister*? One way of approaching the problem is by using the synonymous expressions *male sibling* and *female sibling*. Since *male* and *female* are incompatible (in fact even antonyms), we can automatically infer that *brother* and *sister* are also incompatible.

This implies a definition of sister as female sibling, and we might now go on and describe (or define) *siblings* as people having the same parents. Continuing in this fashion one might try to find more and more primitive basic notions (and relations) that can be used to express (or define) large parts of the dictionary, which in turn helps us to find semantic relations between individual lexical items. For example, the part-hole relation is an important one holding between all sorts of things; this further sense relation is called **meronymy**. For example, *toe* is a meronym of *foot*, since a toe is part of a foot.

Going on this way we will eventually arrive at lexical items that cannot be decomposed any further. The atomic primitives we arrive at at the end of such a procedure have been called **semantic markers** or **semantic primitives**. According to what was said above, one may expect *mail, sibling, part of,* or *toe* to be such primitives. In general, though, it is not clear where this strategy will lead us. E.g. will *clean* be defined in terms of *dirt* of vice versa? At the end of the day, all sorts of relations between items in such a web of semantic markers can be said to express sense relations.

Sense relations do not only hold between single words but also between complex expressions:

(16) mare
female horse
Pferd weiblichen Geschlechts

(17) black mail showhorse
black stallion
mammal

It is clear that any complete semantic theory must give an account of these relations. However, this cannot be achieved by simply describing the meaning of words alone. What we need in addition is a way to describe **combinations** of meaning that make up a phrase and ultimately a sentence.

As it turned out in the history of the discipline, it is not possible to develop a method of doing so by simply manipulating semantic primitives like markers. Rather, one might instead persue the opposite strategy, starting with a comparison between the meaning of entire sentences and then finding out more about the meanings of their parts and how they combine. This way of approaching the problem of word meaning turned out much more successful. The method is called **compositional semantics**, and it is this kind of semantics we will be concerned with in the remainder of this text.[9]

3 Structural Ambiguity

3.1 Some Elementary Examples

We have seen that the same sequence of sounds or letters can express different meanings: words can be ambiguous. But ambiguity is not only found with words—as a phenomenon to be recorded in a dictionary. We also find ambiguity with sequences of identical words; a sequence of words may express two different meanings without containing any ambiguous words. As it turns out, such **structural** ambiguities are particularly revealing when it comes to analysing the meaning of complex expressions. Let us look at an example:[10]

(1) John told the girl that Bill liked the story

We will say that this sentence can have two different **readings**, meaning that it can be understood in two different ways. In one reading, a girl is being told (by John) that Bill liked the story. In this reading, the direct object of the verb *tell* is the *that*-clause. But there also is another reading: In this reading, a certain girl that is liked by Bill is told a story (by John). In the second reading, the direct object of *tell* is the noun phrase *the story* and the *that*-clause is a relative clause that modifies the expession *the girl*.

The different readings arise from different syntactic relations between the parts (or ”constituents”) of the sentence among each other. We will represent these relations using boxes that indicate which parts of speech belong together in the relevant syntactic analysis of the sentence. For example, the first reading of (1) can be associated with the boxes in (2):[11]

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[9]Proponents of this kind of theory have criticised markerese semantics for not being a semantics at all, because it does not deal with the relations between symbols and the world of non-symbols—that is, with purportedly “genuinely semantic” relations (cf. the criticism in Lewis (1972))—a matter to which we return.

[10]All example sentences in this section are taken from Frazier and Clifton (1996).

[11]A word on the representation of constituent structure by boxes is in order. Here and elsewhere we will only represent that aspects of construction that are relevant to our immediate concerns. This is why we don’t have a box around *the girl* in (1). This way we can also be neutral as to syntactic details like a “flat” (ternary) structure as in (i-a) or the more articulated structure in (i-b):

(i) a. $\boxed{\text{told } \boxed{\text{the girl}} \quad \boxed{\text{a story}}}$

(2) John $\boxed{\text{told the girl } \boxed{\text{that Bill liked the story}}}$

(2) contrasts with the structure in (3) which represents the second reading:

(3) John $\boxed{\text{told } \boxed{\text{the girl } \boxed{\text{that Bill liked}}} \text{ the story}}$

The main point we want to make is that the syntactic structure, i.e. the particular way in which words are combined to form a sentence, influences the meaning of the sentence. Of course, syntactic structure will also influence other properties of an utterance, e.g. intonation. Reading (2) aloud, you will observe that the reading associated with (2) is likey to come along with a pause after the word *girl*, whereas the structure in (3) would require a pause after *liked*. This way, intonation may help to disambiguate syntactic structure and meaning.

Here is another famous example:

(4) I saw the man with the binoculars

This sentence allows the two readings in (5):

(5) a. John used binoculars to observe a man
b. John observed a man who had binoculars with him

Again, the ambiguity goes hand in hand with different syntactic structures:

(6) I $\boxed{\text{saw } \boxed{\text{the man}} \text{ with the binoculars}}$

In this structure, the prepositional phrase *with the binoculars* pertains to the event of seeing; this corresponds to the paraphrase in (5-a). In the alternative reading, the prepositional phrase modifies *the man*, which can be represented by putting *the man* and *with the binoculars* into the same box.

(7) I $\boxed{\text{saw } \boxed{\boxed{\text{the man}} \text{ with the binoculars}}}$

---

b. $\boxed{\boxed{\text{told } \boxed{\text{the girl}}} \ \boxed{\text{a story}}}$

Unfortunately, this time intonation is of little help for disambiguation, but nonetheless it is true that some sort of disambiguation is performed almost automatically whenever a reader or listener encounters ambiguous sentences.

When hearing a sentence, we will most often think it is unambiguous. There are many reasons for that, one being that a reading is pragmatically more salient than the other. Consider the following sentence:

(8) The tourists didn't know the museum of the city they visited last year

A natural interpretation, surely the more prominent one, is induced by the following structure:

(9) The tourists don't know $\boxed{\text{the museum of } \boxed{\text{the city that they visited last year}}}$

The sentence does not claim that the tourists visited the museum last year; in fact, such an interpretation would be quite implausible unless we assume some sort of collective amnesia. But now consider *The tourists recognized the museum of the city that they visited last year*. This sentence more easily allows for a different boxing, the one shown in (10),

(10) The tourists recognize $\boxed{\boxed{\text{the museum of the city}} \text{ that they visited last year}}$

and another reading of the boxed constituent, namely one that implies that the tourists did visit the museum last year. The differences of interpretation occur because we naturally intend to interpret the boxed constituent in a way as plausible, relevant, and true in a given situation or context of utterance so that an alternative reading making the utterance implausibe, irrelevant or false is not taken into consideration.

Another reason for preferring one interpretation over another is that some syntactic structures are inherently more complex to process than others. For example,

(11) The tourists admired the museum of the city that they visited last year

seems relatively neutral with respect to possible interpretations, yet there seems to be a bias towards assuming a structure like the one in (9) with the relative clause attached to the rightmost noun, a structure which is claimed to be inherently easier to process than the one in (10) where the relative clause attaches to the more remote noun *museum*. This hypothesis lead to a vast amount of psycholinguistic investigation; cf. Frazier and Clifton (1996) for further reading.

The discussion so far seems to suggest that we become aware of ambiguities by *syntactic* analysis. This might be true in some cases, but it is not the rule. Although syntax and semantics seems to go in tandem, our intuitions are primarily about meaning, and these intuitions guide our way to syntax. In order to detect ambiguities we do not primarily look at syntactic structure but instead try to establish different readings by using **paraphrases**. Such paraphrases have already been used in our discussion of (4) (*John observed a man with binoculars*) when explaining the two meanings in (5).

Looking at the linguistic paraphrases in (5), we first ask ourselves whether any situation described by (5-a) can also be reported by using (4) (restricting ourselves to the literal meaning of the sentences). Our intuition about the meaning of the sentences should tell us that the answer is YES, and the same must hold for (5-b). We now "see" that (4) can be interpreted either as (5-a) or as (5-b).

It is not always so obvious, however, that paraphrases really have different meanings. In order to test this, we rely on a principle that has also been called "**the most certain principle**" in semantics (cf. B"auerle and Cresswell (1989)):

(12) If a sentence A is true and another sentence B is false in the same situation, then A and B differ in meaning.

(12) is an axiom in our theory of meaning; we'll come back to this connection between meaning and truth and falsity on many other occasions.

Applying this principle to the case at hand, it is easy to imagine a situation with only John having binoculars, so one of the paraphrases is true and the other is false; likewise, when only the man has binoculars, the previously true sentence now turns false and the formerly false sentence becomes true. In such a case the method of paraphrases can be used as a water-proof method for identifying ambiguties. We will rely on this method in other cases as well.

Summarizing so far, one might be tempted to say:

(13) If a sentence may both be true and false in the same situation, it is ambiguous.

This is slightly simplified: The one apparent problem with it parallels the terminological difficulty we already discussed in the context of ambiguous words in the last chapter: is there only *one* sentence that is ambiguous, or do we have to assume *two* different sentences with different structures? Again this is a matter of terminology: if we abstract away from structure, we only have one sentence, if not, we have two. Above we decided that $bank_1$ and $bank_2$ are two different words. By analogy, an ambiguous "sentence" should rather be two sentences. But unfortunately, there is no commonly accepted term for what it is that is ambiguous (a linear string of identical words having the category sentence). So we reluctantly accept the common usage, leaving (13) as it is.

By definition, then, an ambiguitiy is purely structural if and only if the ambiguous sentence contains identical sequences of words, with no word-form being itself ambiguous. By contrast, ambiguities like

(14) They can fish

a. They put fish in cans
b. They are able to fish

are lexical (and structural), because they involve different syntactic categories.

Relying thus on our previous discussion of ambiguity of word-forms we should critically assess our former examples from the perspective of clear unamgiguity at the level of morphems. E.g., going back to (4), one may well ask whether the preposition *with* has the same meaning in both construals: Does it mean something like *belonging to* in one reading, and *using as an instrument* in the other? Or is there a very abstract common meaning that covers both occurrances of *with*?

Another potentially problematic example in this respect is (15):

(15) He put the block in the box on the table

This can be used to describe two different scenarios: (a) He put something on the table, namely the block in the box. (b) He put something in the box standing on the table, namely the block. This looks like a purely structural ambiguity:

(16) a. He $\boxed{\text{put } \boxed{\text{the block in the box}} \text{ on the table}}$

b. He $\boxed{\text{put the block } \boxed{\text{in the box on the table}}}$

However, when translating these sentences into German, we observe that in (16-a) the preposition *in* comes with dative case marking, whereas *in* in (16-b) *the box* has accusative case marking, and conversely for the German translation of *on*: *auf* in (17-a) assign accusative, whereas *auf* in (17-b) assigns dative case marking.

(17) a. Er tat den Block in der Box auf den Tisch (= (16-a))
b. Er tat den Block in die Box auf dem Tisch (= (16-b))

This correlates with a difference in meaning: Accusative is used to express an additional directional "meaning" of *in* and *auf*, whereas dative case expresses a purely local meaning. The question then arises whether there are four prepositions in_1, in_2 (= *into*), on_1, and on_2 (= *onto*) or only two. In the first case, the ambiguity would not be **purely structural**.

We believe that most semanticists today would agree that there should be only one core meaning for each preposition, but of course it remains to explain how this meaning interacts with the semantics of the construction it interacts with. Likewise, most syntacticians would agree that the word *that* in (1), repeated in (18), is not lexically ambiguous:

it's simply a subordinating functional element in clause initial position.

(18) John told the girl that Bill liked the story

a. John $\boxed{\text{told the girl } \boxed{\text{that Bill liked the story}}}$

b. John $\boxed{\text{told } \boxed{\text{the girl } \boxed{\text{that Bill liked}}} \text{ the story}}$

In particular, it is **not** assumed that *that* in (18-b) is a kind of relative pronoun, in contrast to *that* in (18-a), which is a so-called complementizer. If this were the case, the ambiguity would not be purely structural. Observe that all ambiguities considered so far are genuinely structural.

A final remark about the role of syntax and the notion "structural" might be in order. Consider the following sentences:

(19) a. John ate the broccoli raw
b. John ate the broccoli naked

The most plausable reading of (19-a) attributes *raw* to the broccoli, whereas the predicate *naked* in (19-b) applies to John. Does this imply that the sentences in (a) and (b) have different structures? This seems to depend on the underlying syntactic theory. Following our semantic intuitions we might propose the following structures:

(20) a. John $\boxed{\text{ate } \boxed{\text{the broccoli wet}}}$

b. $\boxed{\text{John } \boxed{\text{ate the broccoli}} \text{ wet}}$

Unfortunately, most syntacticians would not be too happy with either of these proposals; for reasons we cannot discuss here they would prefer a structure like:

(21) John $\boxed{\text{ate } \boxed{\text{the broccoli}} \text{ wet}}$

This structure is claimed to be still ambiguous: either John could be wet or the broccoli. According to these assumptions the ambiguity is neither lexical nor strictly structural, much as the referential ambiguity of the prounoun in

(22) John aß den Broccoli. Er war nass
John ate the broccoli. He/it was wet

Here, the pronoun (*er*) can refer both to John and to the broccoli without there being any structural ambiguity. Likewise, it is assumed that *wet* contains a hidden semantic relation (also called subject relation) connecting *wet* with either *John* or *the broccoli*. But whether or not this relation is also encoded in the syntactic tree is theory dependent; at least there are theories that do not necessarily express it as part of the syntactic structure.

In conclusion, then, what counts as a structural ambiguity depends on how much structure we are willing to approve and how semantic relations are encoded in syntactic structure. In general, however, the term "structural ambiguity" is used in its broadest sense, so that all kinds of intra-sentential grammatical relations count as "structural", even if they are not reflected in constituent structure. According to this terminology, the relation between the predicate *wet* and its subject can be construed in two different ways, so that the ambiguity **IS** "structural" in the broad sense of the term. In contrast, there is arguably no structural ambiguity in (22) because the semantic ambiguity does not depend on the intra-sentential grammar of positioning the predicate *wet* in relation to its subject.[12]

EXERCISE 1:
Discuss the semantic differences in your understanding of the *when*-clause in the following sentences:

(23) a. Fred will realize that Mary left when the party started
b. Fred will realize that Mary left when the party starts

EXERCISE 2:
Discuss the ambiguity of (24) and draw different structures for the two readings:

(24) a. John realized that Mary left when the party started
b. John said the man died yesterday

[12] This may become more clear by comparison with a language like German that allows two different constructions:

(i) a. Johann sah seine Frau nackt
John saw his wife naked
b. Johann sah nackt seine Frau
John saw naked his wife
c. *Johann nackt sah seine Frau

Whereas (i-a) is still ambiguous, (i-b) is unambiguous, indicating that a semantic relation (the relation of *predication*) is encoded in syntax in a particular way.

EXERCISE 3:
What are the different structures for (25) and how can the ambiguities be paraphrased?

(25) a. a table of wood that was from Galicia
b. the girl with the hat that looked funny
c. the girl and the boy in the park

Conclusion

The upshot of the foregoing discussion is a certain parallelism between syntax and semantics. In a sense, then, we have "explained" semantic ambiguity by reducing it to syntactic ambiguity. In many cases, the ambiguity is one of attaching a constituent either to a higher or to a lower box. In fact all the examples in the exercises above are of that type. The ambiguity is similar to the one in (26):

(26) ten minus three times two
a. $10 - 3 \times 2$
b. $(10 - 3) \times 2$

(26-a) corresponds to low attachment of $\times$ 2, and (26-b) corresponds to high attachment.[13]

But of course it is insufficient to discover syntactic ambiguities; the real work that ultimately explains why the sentences have different meanings has not yet been done. This should become clear by looking at a simple example where syntactic ambiguity alone does not suffice to induce semantic ambiguity: Arguably, $x + y - z$ is not semantically ambiguous between $(x + y) - z$ and $x + (y - z)$ because the result is always the same number. It is the particular semantic rules combining the boxes (the constituents in a syntactic structure) that ultimately do the job, but no such rules have been stated yet. This is precisely what we are going to do in Section 5.

[13] The analogy with mathematics is not that far fetched as it seems: it can be shown (cf. Scheepers (2009)) that subjects who had to calculate one of the two formulas above solved the natural language completion tasks with examples like

(i) The pensioner complained about the content of the fliers that…

in different ways. The completions task can be solved by attaching the relative clause beginning with *that* either high (referring to *the content*) or low (referring to *the fliers*). High relative clause attachment was more frequent after having solved (26-b) (and vice versa). The result shows that the mathematic equation had a priming effect on the perception of natural language structure.

3.2 Scope and Syntactic Domains

In this section we will take a closer look at the concept of scope. Semanticists often have basic intuitions about the semantic relations between particular words or phrases, and it is these relations that are relevant for the ambiguity to arise. Let us discuss this in some detail by looking at (27):

(27) The doctor didn't leave because he was angry

One simple paraphrase is the following:

(28) The doctor stayed because he was angry

The alternative second paraphrase is more involved:

(29) The doctor left and he was angry, but the latter was not the reason for the former

Or perhaps more clearly:

(30) The doctor left for some reason, but not because he was angry

Observe that the two paraphrases really say different, in fact, contradictory things: In one the doctor left, in the other, he didn't. Since only one of them can be true (in the same situation), it follows from (12) that the paraphrases have different meanings.

It seems to be obvious that there is no lexical ambiguity involved; on the other hand, ambiguity of structure is much less self-evident. So: how, if at all, does this ambiguity correlate with different structures?

Intuitively, in one reading the *because*-clause states the reason for the doctor's staying. In the other reading, it states the reason for the doctor's leaving, and it is denied that this is the true reason for his leaving. We therefore say that in the first reading—call this the high attachment reading (for reasons that will become obvious in a minute)—the staying, i.e. the expression *didn't leave*, is in the domain of the *because*-clause. In the second reading—call this the low attachment reading—the domain of the *because*-clause does not comprise the negation, only the verb *leave* is in its domain.

Let us now focus on the role of negation in this example, analysing the ambiguity from a different perspective: in the low attachment reading, the causal relation is denied, and we say that the *because*-clause is in the domain of *didn't.* This becomes particularly clear in paraphrase (30) where the negation *not* immediately precedes *because.*

By contrast, in the high attachment reading, the domain of the negation is restricted to the verb *leave.*

Taking these observations together, we suggest the following structures: [14]

[14] A viable alternative to (31-a) is (i) with the *because*-clause attached even higher to the main clause:

(31) a. The doctor $\boxed{\boxed{\text{didn't leave}}\ \boxed{\text{because he was angry}}}$

b. The doctor $\boxed{\text{didn't}\ \boxed{\text{leave because he was angry}}}$

It remains to explain the notion of a domain. This can be done by giving a precise definition:

(32) X is in the **syntactic domain** of Y if and only if X is contained in the smallest box that contains Y.

Going back to (31-b), the *because*-clause is in the syntactic domain of *didn't* and the verb alone is in the domain of the *because*-clause. By contrast, the box containing negation in (31-a) does not contain the *because*-box, thus the *because*-clause is not in the domain of the negation.[15]

In semantics, the notion of a domain corresponds to that of **scope**; it is claimed that the ambiguity is basically one of scope. In the case at hand, the negation either has scope over *leave* only, or it has scope over *because*, too, in which case it it is the causal relation that is negated. Likewise, the *because*-clause has either scope over *leave* only, or it has scope over the negation and the verb. The semantic notion of scope thus corresponds to the syntactic notion of a domain; the guiding principle that connects our semantic intuition with syntactic structure is:

(33) ***The Scope Principle:***
If α has scope over β then β is in the syntactic domain of α.

The syntactic domain was defined in (32), repeated as:

(i) $\boxed{\boxed{\text{The doctor didn't leave}}\ \boxed{\text{because he was angry}}}$

Both analyses satisfy the restriction that the negation is in the domain of the *because*-clause.

[15]Returning to the last footnote and the structure proposed there, the definition in (32) urges us to look at the *smallest* box containing negation; this box is shown in (i):

(i) $\boxed{\boxed{\text{The doctor}\ \boxed{\text{didn't leave}}}\ \boxed{\text{because he was angry}}}$

But since the smallest box is contained in the one shown in (i) of footnote 14 and since the larger box already excludes the *because*-clause, it is not really necessary to consider the more fine grained structure. We will henceforth ignore many structural details that are irrelevant for the case in point.

(34) X is in the **syntactic domain** of Y iff X is contained in the smallest box that contains Y.[16]

Accordingly, in the mathematical example in (26), multiplication in (26-a) expressed by × is in the scope of substraction expressed by −, whereas − is in the scope of × in (26-b). The ambiguity of (26) is resoved by the use of brackets and by other special notational conventions. Instead of brackets, we could have used boxes:

(35) a. $\boxed{10 - \boxed{3 \times 2}}$

b. $\boxed{\boxed{10 - 3} \times 2}$

Of course we could also have used brackets in our linguistic examples, but boxes are much easier to read.[17]

The notion of scope is essentially a semantic notion, but as such it is difficult to explain in precise terms. Basically, it reflects which operations come first when calculating the meaning of an expression. Detecting an ambiguity therefore requires finding two different orderings in which certain elements of the same clause (particular words or phrases) can be considered when calculating the meaning of an expression. In many, but, as we will see, not in all cases, these different orderings go hand in hand with syntactic ambiguities.

Instead of attempting to give a definition of scope, let us further exemplify the notion by applying it to another standard situation that illustrates the intended range of applications. To see how the notion is put to work, take (25-c) repeated as:

(36) the girl and the boy in the park

The syntactic ambiguity is one of attachment: assume that *in the park* is attached to *the boy*:

(37) the girl and $\boxed{\text{the boy in the park}}$

The crucual semantic ambiguity results from the relation between the term that expresses coordination (*and*) and what is coordinated. In (37), the conjunction *and* has scope over the predicate *in the park*, so that intuitively only the boy must be in the park.

[16]Readers with some background knowledge in syntax should notice that the notion of a domain is also called c-command in Generative Syntax: X is in the domain of Y if and only if Y c-commands X.

[17]Even more transparent are tree representations as they are used in Generative Grammar. In this book we don't want to commit ourselves to any syntactic theory and so we even refrained from using trees, which have no place in other types of syntactic theories.

A more elaborate structure is (38):

(38) $\boxed{\boxed{\text{the girl}}\ \text{and}\ \boxed{\text{the boy in the park}}}$

Alternatively, *in the park* could syntactically attach to the entire conjunction, which means that *in the park* has scope over the conjunction *and*. This is shown in (39):

(39) $\boxed{\boxed{\boxed{\text{the girl}}\ \text{and}\ \boxed{\text{the boy}}}\ \boxed{\text{in the park}}}$

Under this construal, the boy and the girl must both be in the park. A paraphrase for this reading would be something like:

(40) the girl and the boy who are in the park

Note that the intended reading could also be expressed by

(41) the boy and the girl in the park

But (41) is not a good paraphrase (for any reading) because it is itself ambiguous. A good paraphrase uses the same critical expressions and is known to be semantically and syntactically unambiguous.[18]

3.3 Syntactic Domains and Reconstruction

In this section we will discuss a number of further scopal ambiguities which at first sight do not seem to be based on a syntactic ambiguity but which nevertheless are often explaned in terms of syntactic structure. The case in point can best be illustrated by reference to the syntax of German; similar but more complex versions of the phenomenon also exist in English, which will be discussed at the end of this section.

Let us first discuss a case in point from a semantic perspective. Consider:

(42) Beide Studenten kamen nicht
Both students came not

[18]Of course, if A is a paraphrase of B, A and B should be synonymous. But more often than not synonymy is hard to get by. For example, the paraphrase we attempted in (40) seems to work fine because it uses the verb *be* and its plural morphology which unambiguously makes it clear that both the girl and the boy are in the park. Unfortunately, however, the finite verb also bears tense information (the present tense in (40)). But this additional piece of meaning is not contained in the original expression. Strictly speaking, therefore, the paraphrase does not meet the criterion of synonymy. But this failure seems tolerable, because (40) is still a good approximation if no ideal paraphrase can be found.

This sentence is ambiguous, it can either mean

(43) Reading A: neither of the two students came

or it can mean

(44) Reading B: not both of the students came (one of them came).

The second reading requires support from intonation: a rise on *beide* and a fall on *nicht*. It is easy to verify that if A is true, B is false, and if B is true, then A must be false.

Discussing this ambiguity in terms of scope, let us first identify the crucial elements that induce the ambiguity. These seem to be the negation *nicht* and the determiner *beide*. Reading A is characterized by *beide Studenten* having semantic scope over *nicht*, whereas the reverse holds for reading B. In a syntactic structure like (45), however,

(45) $\boxed{\boxed{\text{beide Studenten}}\ \ \boxed{\text{kamen nicht}}}$

the negation is in the syntactic domain of *beide Studenten*, but not conversely, therefore it seems we only get reading A. Reading B is not what we see immediately in the structure. The only way to get the intended scope relations in syntax would be the boxing in (46):

(46) $\boxed{\boxed{\text{beide Studenten kamen}}\ \ \text{nicht}}$

This structure, however, is otherwise unmotivated and incompatable with the syntax of German, as it seems to predict that negation can be attached to an entire clause. But this is not borne out, as can be seen from the ungrammaticality of (47):

(47) *beide Studenten sind gekommen nicht
both students have come not

The correct way to say this would be:

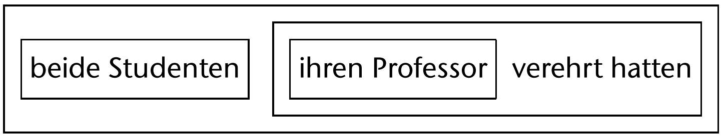
(48) beide Studenten sind $\boxed{\text{nicht gekommen}}$

Therefore it seems that we are stuck: the existence of reading B seems to contradict the scope principle (33).

However, taking a closer look at the syntax of German will reveal that this contradiction is only apparent. Let us ask how the structure of (42) is generated. There are two leading assumptions that guide our analysis. One is that German is a so-called SOVlanguage. This means that the unmarked word order in German subordinated clauses is: Subject precedes object precedes verb(s). An example would be

(49) beide Studenten ihren Professor verehrt hatten
both students their professor worshipped had

the structure of which is (50):[19]

(50)


The second assumption concerns main clauses. In German, all such clauses have the finite verb in second position, which is why German is also called a V2-language. In a V2-language, any one of the major constituents (subject, objects, verbs or adverbs) may precede the verb:

(51) a. Beide Studenten hatten ihren Professor verehrt
b. Ihren Professor hatten beide Studenten verehrt
c. Verehrt hatten beide Studenten ihren Professor

All these constructions are well-formed main clauses in German.

The crucial link between the SOV-property and the V2-property is the assumption entertained in Generative Grammar that the V2-property is the result of two movement operations: starting with an SOV-structure and the finite verb in the final position, V2 is derived by moving the finite verb into the second position (the upper arrow in (52)) and another constituent (any of S, O, or V) into the first position (one of the lower arrows in (52)):

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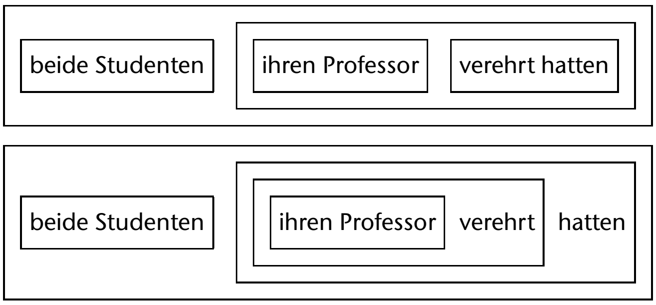
[19]Note that a more fine-grained structural analysis could add more boxes, as illustrated in

(i) a.
b.


Both structures are okay, but the additional boxes are irrelevant for the argument. In general it is our strategy to omit all boxes that add superfluous structure.

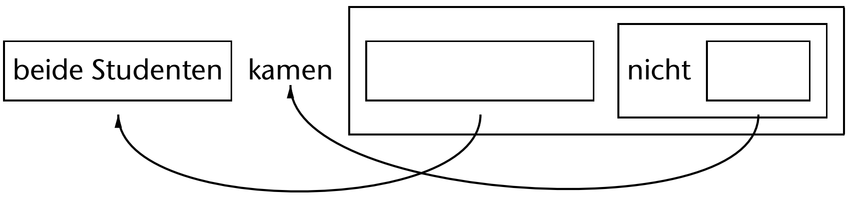
(52) S O V V_{fin}

Returning to sentence (42), we assume that the subject *beide Studenten* has been moved into the first position (also called the pre-field position) by a syntactic movement rule (traditionally called "topicalization"). In order to derive a syntactic ambiguity it will turn out crucial to ask: what is the structure *before* movement? In fact, there are two possibilities, as attested in dependent clauses:

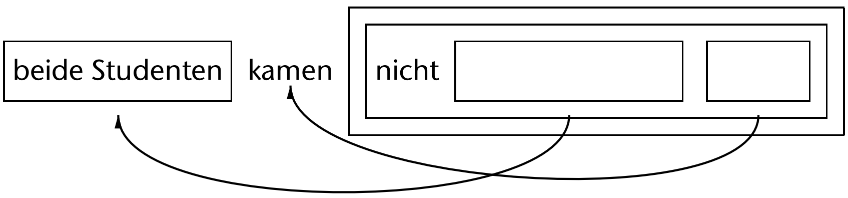
(53) a. (dass) beide Studenten nicht kamen
(that) both students not came
b. (dass) nicht beide Studenten kamen
(that) not both students came

Both clauses are grammatical and differ in meaning: In (53-a), the negation is in the scope of *beide Studenten*, which corresponds to reading A, whereas *beide Studenten* in (53-b) is in the scope of negation. The structural ambiguity thus amounts to the two possibilities shown in (54) and (55):

(54)



(55)



Given these structures we now see that we could indeed represent reading B, if only we were allowed to semantically interpret *beide Studenten* in the position occupied before movement. This way, we can avoid a violation of the scope principle, because moving back the subject into the position of its original box, the subject is **reconstructed** into the syntactic domain of the negation. In consequence, the negation can have scope over the subject, as desired.

If we don't reconstruct and take (42) at its surface value, the subject still has scope over negation. Intuitively, only the verb is negated. But now look at the domain of negation in (54): as it turns out, the verb is not in the domain of negation, because it has been moved into the V2-position! Here the solution is again reconstruction: The verb has to be interpreted semantically in the empty box where it originated from. In general, this kind of movement of a single word (so called head-movement) always requires reconstruction, whereas reconstruction of phrases is optional.

The above explanation can be generalized and applied to many more examples that allow for an ambiguity precisely because a certain movement has taken place. Let's take movement of an object into first positions as an example. Most speakers would agree that

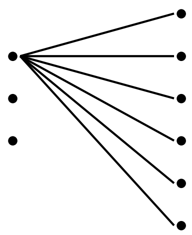
(56) jeden Schüler$_{accusative}$ lobte genau ein Lehrer$_{nominative}$
every pupil praised exactly one teacher

has two readings:

(57) a. Reading *A*: For every pupil there is exactly one teacher who praised him
b. Reading *B*: There is exactly one teacher who praised every pupil

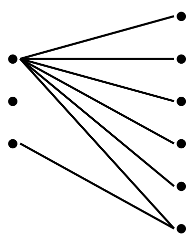
In order to see how the meanings differ, consider first a situation with three teachers and six pupils. The relation of praising is represented by a line:

(58) teacher pupil



In such a situation both reading *A* and reading *B* are true: every pupil is praised, and there is only one teacher who is praising. In consequence, (58) does not suffice to disambiguate the situation. But now consider (59):

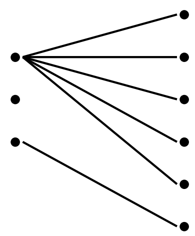
(59) teacher pupil



In this situation (57-b) is still true because the additional teacher does not praise every student (but only one), so there is still exactly one teacher who does. On the other hand, (57-a) is false because there is one student who is praised by more than one teacher.

Next, consider (60):

(60) teacher pupil



In this setting (57-b) is false because no teacher praises all of the pupils. On the other hand, each pupil is praised by a teacher, and no one is praised by more than one teacher, hence (57-a) is true.

We have thus shown that the construction is ambiguous, and we can now relate the ambiguity to movement.

(61) jeden Schüler lobte $\boxed{\text{genau ein Lehrer}}$ $\boxed{\boxed{\phantom{\text{genau ein Lehrer}}}\ \boxed{\phantom{\text{lobte}}}}$

Reading *A* is the one where the subject is in the scope of the object. This reading corresponds to the (surface-)structure shown in (61). The second reading is the one where the object is in the scope of the subject. This reading can be derived from (61) by "reconstruction", ie. by moving the object back into the original position in the domain of the subject.

Summarizing so far, we have shown that certain ambiguities arise as consequences of movement. We would expect that in a structure that does not involve movement, no ambiguity arises. And in fact, subordinate sentences with SOV-structure as in (62), where no movement has taken place, do not exhibit any ambiguity of the sort we discussed above; the sentence is perceived as unambiguous in German, with *exactly one teacher* having wide scope over *every pupil*:

(62) ich glaube, dass genau ein Lehrer jeden Schüler lobte
I believe that exactly one teacher every pupil praised

(63) ...dass $\boxed{\boxed{\text{genau ein Lehrer}}\ \boxed{\boxed{\text{jeden Schüler}}\ \text{lobte}}}$

Thus, the embedded clause is unambiguous, having only reading *B* in (57-b).

So far it seems that ambiguities of this kind are a perculiarity of German. To some extent, this is true, in as far as German but not English is a V2 language. The fact that English is not can easily be demonstrated by the ungrammatical V2 constuctions in (64) (the star indicates ungrammaticality in the intended reading with *their professor* as the object and *both students* as the subject of *worship*):

(64) a. *Both Students had their professor worshipped
b. *Their professor worshipped both students
c. *Worshipped had both students their professor

However, English still has residual V2 in *Wh*-questions like

(65) a. Who had worshipped his professor?
b. Who had both students worshipped?

We might then ask whether residual V2 gives rise to the same kind of ambiguity, and in fact it does, although the data is somewhat more involved. Consider:

(66) How many dogs did everyone feed?

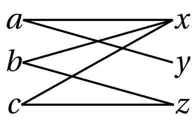
There are at least two readings of (66), which can be paraphrased as follows:[20]

(67) a. For which number n does it hold that n dogs were fed by everyone?
b. For which number n does it hold that everyone fed n dogs?

There is a subtle difference here, best explained by way of describing a situation in which two different answers to the question could be given.

Suppose there are three persons a, b, and c and three dogs x, y, and z. Assume further that a fed x and y, b fed x and z, and c fed x, y, and z.

(68) persons dogs



On one reading, then, the answer is "one" because x is the only dog fed by everyone. According to the other reading, the answer is "two", because everyone fed two dogs.

Let us pin down the difference in terms of scope. What are the crucial scope inducing elements involved? By comparing the above paraphrases, we see that in one, namely (67-a), the expression *everyone* (a so-called quantifier) is in the scope of the numeral n (or the expression n-dogs), whereas the reverse holds in (67-b). More schematically, the

[20]Some speakers might also get a third reading, called the pair-list reading, which can be paraphrased as:

(i) For which person x and which number n does it hold that x fed n dogs

This reading will be ignored in the discussion of (66).

situation can be presented as in (69):

(69) a. How many n: everyone fed n dogs
b. How many n, n dogs: everyone fed

(69-b) corresponds to the surface order of (66), whereas in (69-a), *n* dogs appears in the object position of *feed.* This has been described in the literature as a case of partial reconstruction: Its not the entire *wh*-phrase *how many dogs* that reconstructs to the object position, but only a part of it. We thus see that reconstruction is also operative in English, though in a slightly different way: Many semanticists would insist that the *how-many*-part should still be the top of the structure, with everything else in its domain. Therefore reconstruction can only be partial, but it can still be exploited to derive the ambiguity.

EXERCISE 4:
Try to account for the ambiguity of:

(70) Genau 5 Bücher hat jeder gelesen
Exactly 5 books has everyone read

Observe that *jeder* is the subject. Give two paraphrases; analyse the two readings of this sentence by describing two types of situations that make the paraphrases true and false.

EXERCISE 5:
Do the same with:

(71) Genau 5 Bücher hat keiner gelesen
Exactly 5 books has noone read

3.4 Logical Form

3.4.1 Reconstruction

We have seen in the previous subsections that certain ambiguities can arise even if there is no difference in the syntactic structure as such. Nonetheless the structures we discussed above already determine an ambiguity if it is assumed that the starting position of movement is somehow part of the syntactic representation and that semantic interpretation is done either at the landing site of movement or at the position before movement took place. The latter choice was called reconstruction and in many popular theories it is assumed that the material that has been moved is moved back into the original position where it can be interpreted in accord with the Scope Principle. The resulting structure, which departs from the surface structure after reconstruction, is also called the **Logical Form** of a sentence. A subcase of structural ambiguity then arises if a sentence structure can be interpreted with respect to different Logical Forms. Examples already discussed above are the following:

(72) Beide Studenten kamen nicht

a. LF_1: $\boxed{\text{beide Studenten } \boxed{\text{nicht kamen}}}$

b. LF_2: $\boxed{\text{nicht } \boxed{\text{beide Studenten kamen}}}$

(73) Jeden Schüler lobte genau ein Lehrer

a. LF_1: $\boxed{\text{jeden Schüler } \boxed{\text{genau ein Lehrer lobte}}}$

b. LF_2: $\boxed{\text{genau ein Lehrer } \boxed{\text{jeden Schüler lobte}}}$

Recall that head movement (i.e. the movement of the verb into V2-position) is always reconstructed, therefore the LFs do not coincide with an input structure. Otherwise, however, LFs may correspond to what we see, in particular when movement didn't apply. The general idea is that each sentence has a Logical Form that disambiguates by providing a syntactic representation of semantic scope in line with the scope principle. The level of LF is thus intended as an unambigous syntactic representation in accord with the following principle:

(74) *The LF Scope Principle:*
At the level of LF, an element α has scope over β if and only if β is in the domain of α.

Thus far we had no difficulty with providing LFs for the sentences discussed. In case of an attachment ambiguity, the surface order itself is ambiguous, that is, the same linear structure is assigned two different surface structures and these structures may also count as the respective LFs. However, there are a number of problematic cases, the most important one of which will be discussed in the next subsection, where there is no obvious link between scope and syntactic domains.

3.4.2 Quantifier Raising

The following sentence (75) has a marked and an unmarked reading, paraphrased in (76):

(75) A student read every book

(76) a. For some student it holds that he read every book
b. For every book there is a (possibly different) student who read it

The marked reading is not that easy to get (perhaps it helps to put some stress on *student*); it is paraphrased in (76-b). The unmarked reading (76-a) is also called the linear reading because the scope of the quantified phrases *a student* and *every book* in corresponds to the linear precedence relation of the phrases in (75) in the obvious way. In contrast, the marked reading is called the reverse reading, because as the paraphrase reveals, the linear ordering and the scope relations in the paraphrase (76-b) reverses the linear order of quantifiers in the original sentence (75). It seems to be a peculiarity of the English language that some speakers seem to accept such a reading; the literal translation of (75) into German is unambiguously linear.[21]

However, in some situations, the reverse reading, although being somewhat marked, seems to be the only one that makes sense. Consider the following example:

(77) Ein Tisch berührte jede Wand
a table touched every wall
'a table is tangent to every wall'

The linear reading would require that there is (at least) one table that is in contact with all walls. Now, imagining a room with four walls, this situation seems extremely unlikely; therefore, after some reflection, most speakers would agree that the reverse reading (78) is a possible reading for (77).

(78) Each wall is tangent to a table

Cf. also

(79) A mirror borders every wall (to reflect the image of an incredibly honed athlete on each of their surfaces ...)

The problem for the only salient reading is that there seems to be nothing in syntactic structure that would point to a syntactic ambiguity; the structure seems to be unambiguously (80) for English and something like (81) (simplified) for German:

(80) $\boxed{\text{a mirror } \boxed{\text{borders } \boxed{\text{every wall}}}}$

[21] The reason for this seems to be that German word order allows to place the accusative object to the left of the nominative subject, which allows for a linear correspondance with scope relations.

(81) ein Tisch berührte jede Wand

Likewise, the structure of (75), namely (82),

(82) a student read every book

does not permit for neither syntactic not lexical ambiguity. This is indeed a severe problem that lead to very different solutions in different theories. In a theory that strictly correlates syntactic domains with scope, the problematic reading would require a structure like this:

(83) a. a student read every book

b. a mirror borders every wall

But for reasons of syntactic analysis, such a structure is not an option for a language like English (nor for any other language).

A popular solution consists of deriving structures similar to (83) by brute force. Many linguists thus propose a syntactic movement operation that effectively generates a new structure (a new Logical Form) by applying a process that takes (82) as input and then moves the object into a new position in which *a student* is in the domain of *every book*. This movement rule is called **Quantifier Raising (QR)**. Depending on whether QR operates to the left or to the right, the resulting structures are (84) or (85):

(84) every book a student read

(85) a student read every book

Both (84) and (85) can be LFs for the problematic marked reading of (75).

At this point it should be mentioned that linear order is irrelevant at the level of LF. This is because crucial notions like scope and syntactic domain are not linear notions: they are purely hierarchical and all that matters is whether one box is contained in another. This notwithstanding, it has become common practice to assume that QR goes to the left, in accordance with conventions in mathematical logic and the fact that in the unmarked case, the scope inducing element precedes the scope dependent element in natural language. The problematic reading under discussion seems to be an exception to that rule, but the structure in (84) would rectify the imperfection by simply putting the object in front of the remainder of the sentence.

3.4.3 Opaque and Transparent Readings

The following sentence is ambiguous in both German and English:

(86) Gertrude sucht ein Buch
G. is-looking-for a book

One might imagine two different situations that could truthfully be described by (86).

(87) There is a certain book (Gertrude's sister requested as a Chrismas present) that Gertrude is looking for

The other situation would be:

(88) Gertrude tries to find a present for her sister which should be a book (but she has no particular book in mind)

This ambiguity seems to be related to the different ways the indefinite article *a* can be used. The reading of *a book* we paraphrased as *a certain book* is called the specific reading of the indefinite NP, whereas the reading in which the identity of the book does not matter is called the unspecific reading.

Linguists in the tradition of Richard Montague (1973) have analysed this difference as a difference in scope. As usual we first have to identify the two elements whose scope converts in the LFs of an ambiguous sentence form. One of them is the indefinite phrase *a book*. The only second element involved is the verb. In fact, only certain verbs allow for the unspecific reading; there is no such ambiguity in *Gertrude found/read/destroyed a book*. Exceptional verbs like *seek, owe, look-for, worship* that allow for the unspecific reading are also called *opaque verbs* (and the reading is called **opaque**) whereas ordinary verbs and the repective reading are called **transparent**.

Suppose we paraphrase *seek* as *try to find*.

(89) Gertrude tries to find a book

(89) exhibits the same kind of ambiguity as (86). Since *find* is a transparent verb, the only source of the ambiguity is the verb *try*.

(90) Gertrude $\boxed{\text{tries to } \boxed{\text{find } \boxed{\text{a book}}}}$

We thus established that *a book* is in the domain and in the scope of the opaque verb (*try, seek, owe* etc.). In the specific, transparent reading, the indefinite should not be in the scope of that verb. In the tradition of Montague it is assumed that the transparent reading is the result of QR:

(91) $\boxed{\text{a book } \boxed{\text{Gertrude } \boxed{\text{tries to find / seeks } \boxed{}}}}$

Note that this account of the ambiguity relies on a syntactic operation (QR) and that it cannot explain why it exists in most languages of the world, whereas the scope ambiguity we discussed in the last subsection (also accounted for by QR) seems to exists only in a minority of languages.

EXERCISE 6:
Explain the ambiguity in:

(92) My brother wants to marry a Norwegian

*3.4.4 Hidden Structure**

The following mind teasing ambiguity requires a sophisticated logical analysis; hence the machinery used for paraphrasing them will contain expressions like “if then”, “only”, and the variable *x*.

(93) I know what I’m saying

The two readings can be paraphrased as in (94) or more formally in (95):

(94) a. I only say what I know (to be true)
b. I am totally aware of what I am saying

(95) a. if I say x then I know x
b. if I say x then I know that I say x

(95-a) paraphrases the only-truth-teller whereas (95-b) is the total control guy.

We may then ask how this ambiguity can systematically be derived and whether or not it can be described as a structural ambiguity.

As it turns out, there are indeed two different syntactic constructions that can account for the ambiguity. We may distinguish between two types of complements of *know*: one is an indirect question, the other is called a free relative clause. Let us start with the latter by considering a sentence like

(96) Ich esse was du kochst
I eat what(ever) you cook

The constituent introduced by *was* / *what(ever)* is sometimes (misleadingly) called a *concealed question*, but in fact our semantic intuition tells us that (96) contains no question at all. Rather, we would suggest (97) as a paraphrase:

(97) Ich esse alles (das) was du kochst
I eat all (that) what you cook

(97) makes it clear that was du kochst modifies the object pronoun das (*what*) which refers to the thing to be eaten. The construction therefore contains a relative clause, attached to the optional element put into brackets in (97). If the pronoun is missing, the relative clause is also called "*free*" because it seems to lack a head noun. This terminology notwithstanding, *what you cook* is often analysed as a relative clause which is attached to an empty head, as shown in (98):

(98) a. Ich esse $\boxed{\text{Ø } \boxed{\text{was du kochst}}}$

b. I eat $\boxed{\text{Ø } \boxed{\text{what you cook}}}$

This is interpreted at the level of LF as something like:

(99) For any x: if you cook x then I eat x

For the purpose of our discussion, it is immaterial how this interpretation comes about (most semanticists seem to believe that the larger box containing the empty operator undergoes QR at LF). The point here is that, by analogy, we also get a free relative clause structure for (93), and a semantic interpretation that parallels (99):

(100) a. I know $\boxed{\text{Ø } \boxed{\text{what I say}}}$

b. For any x: if I say x then I know x

Now observe that (100-b) can also be paraphrased as "When I say something, I know that it's true", and this is almost identical to what we proposed as a paraphrase in (94-a) above.

Let us next turn to an alternative syntactic parse. This is much simpler; according to this analysis, the *what*-clause is plainly a complement of *know*. Let us illustrate with a simpler example:

(101) I know $\boxed{\text{who came to the party}}$

This complement is called an **indirect question** (with no QR being involved this time). Adopting a semantic analysis of indirect questions proposed by Hintikka (1962), we may assume that (102) is a good paraphrase for the meaning of (101):

(102) If some individual x came to the party, then I know that x came to the party

Observe that in this analysis the complement of *know* is no more an indirect question, but an ordinary *that*-clause.[22]

By analogy, it now follows that (93) can be paraphrased as (103):

(103) If x is something I am saying, then I know that I am saying x

I thus claim that I am aware of what I am saying, and this is precisely the paraphrase we offered above in (94-b).

We have thus shown that the two syntactic analyses are mirrored in the semantics and that a syntactic ambiguity predicts a semantic one: Since each construction brings along its own semantics, we get different interpretations, despite a superficial identity of expressions.[23]

[22]This is an important step in the analysis of questions, indirect or otherwise. In Chapter 6 we will see how to handle expression like *knowing that* which take *that*-clauses as complements. Given this, it is easy to reduce the semantics of questions to that of *that*-clauses and other components (like the conditional), as illustrated in (102).

[23]One of the requirements for an ambiguity to be purely structural was identity of lexical material in terms of identical meaning. For the above ambiguity to be purely structural, we therefore must require that the expression *was*/*what* in the indirect question have the same meaning as in the free relative clause. Whether this holds is a matter of the semanic analysis of these types of constructions and it depends on how different theories handle these expressions. Unfortunately, most theories would assign different meanings, hence the ambiguity is not purely structural.

3.4.5 Summary

Semanticists love to analyse ambiguities, and we as beginners can also profit enormously from this obsession. Why?

- The study of ambiguities may give you a rough idea of what semanticists are concerned with and consequently, what semantics is about.
- Ambiguities can be revealed only by disambiguation, which forces the student of semantics to consciously reflect on meaning and on how complex meanings emerge.
- Ambiguities also provide for a testing ground for theories: if we know that a certain construction should be ambiguous but the theory fails to predict that (e.g. if we only get opaque readings), it is in need of further elaboration or revision.

Ambiguities also tell us something about Logical Form and the relation between so-called overt syntax on the one hand (the surface syntax that forms the input to phonology) and covert syntax on the other (the structures called Logical Form, which serve as the input for semantics). However, not all semanticists accept Logical Forms that depart too much from the surface; hence the study of LF is not necessarily a silver bullet for the the study of semantics.

As you might have experienced, it's not all that easy to describe an ambiguity, and it's even more difficult to detect its reason or source. Nonetheless, giving an account of such intuitions about ambiguity is precisely what a semantic theory is about: such a theory should be able to explain the fact that we can understand these sentences in different ways. Thereby, we hope to account for the more general ability of humans to understand sentences. Any theory (primarily and correctly) dealing with unambiguous cases should, to the extend that it is correct, also be capable of explaining the more complex ambiguous cases.

Moreover, disambiguations call for a precise language, a language that does not itself allow for ambiguities. It has become common practice to paraphrase sentences of natural language using notions of mathematical logic and set theory. These notions will be introduced in the chapters to come.

EXERCISE 7:
Another example, known as Russell's ambiguity, discussed in Russell (1905), is this:

(104) Ich dachte Ihre Yacht ist länger als sie ist
I thought your yacht is longer than it is

In one reading, my belief is contradictory: it's impossible that my yacht is longer than it (in fact) is, and therefore it is highly implausible that I entertained a belief in such a contradiction. In the natural reading, however, no such contradiction arises. Try to paraphrase this reading and analyse the ambiguity in terms of scope of the *als/than*-clause

EXERCISE 8:
Consider:

(105)	Vor	20 Jahren	waren	die	Professoren	noch jünger
	Ago	20 years	were	the	profs	even younger

(105) can mean something very trivial, namely this:

(106) For each professor it holds that 20 years ago he was younger than he is today.

This is a self-evident truism, people simply get older, and because (106) is so obviously true, it is most probably not the intended meaning. Rather, one wants to say something like

(107) The average age of a professor twenty years ago was lower than the average age of a professor nowadays.

This assertion makes much more sense, it is not a self-evident truism (but probably wrong). Observe that in the trivial reading the people we are talking about are the same in the comparison, we compare each individual's age in time, whereas in the non-trivial reading we are comparing two entirely different groups of people, namely the professors today and the professors 20 years ago. Moreover, in this reading we are talking about the *average* age of professors. More technically speaking something like the "generic" professor, the proto-type of a typical professor, is involved. This concept of **genericity** (☞Generizität; ☞Generic mood) is an additional aspect and an additional complication.

Discuss how the structural aspects of the ambiguity can be accounted for and how the extra bits of meaning may creep into the meaning of the sentence. In order to solve the second part it suffices to identify the lexical trigger for the generic reading: Which expression is potentially ambiguous between a generic and an "ordinary" meaning?

COMMENT: If the paraphrases we have given in (106) and (107) come close to the different Logical Forms, the example shows that these paraphrases may contain semantic material not explicitly expressed by the meaning of the original sentence, although it must implicitly be contained in it. Hence, the example suggests that the "distance" between a surface expression and its LF may be surprisingly big. It is a non-trivial task for the semanticist to show how to bridge this gap in a systematic, non-ad-hoc way.

4 Introducing Extensions

In the last chapter we argued that many ambiguities can be traced back to an ambiguity of syntactic structure. The general principle that motivates such a move is that sentence meaning not only depends on the meaning of individual words but also on syntactic structure, ie. the way these words are put together in syntax. For example, two unambigous words can be arranged in different orders, as in

(1) a. Fritz kommt
Fritz is-coming

b. Kommt Fritz
Is-coming Fritz

Whereas the verb-second structure in (a) is normally interpreted as a declarative sentence, the verb-first structure in (b) is interpreted as a yes-no-questions. The two arrangements lead to different meanings, although the lexical material is the same (and there is no ambiguity of scope involved here).

The strategy we will pursue in what follows is to take the meanings of words and then combine them alongside and in tandem with the syntactic structure. Such a procedure is called **compositional** and the principle behind it is this: the meaning of a complex expression is fully determined by its structure and the meanings of its constituents. Once we know what the parts mean and how they are put together, we have no more leeway regarding the meaning of the whole. This is the principle of compositionality, which can be traced back to the German philosopher ☞**Gottlob Frege (1848-1925)** and has become a central assumption in contemporary semantics:

(2) *Frege's Principle of **Compositionality**:*
The meaning of a composite expression is a function of the meaning of its immediate constituents and the way these constituents are put together.
(cf. ☞Frege-Prinzip))

It thus follows that not only do the meanings of the words determine the meaning of the whole; it also holds that the meaning of a complex expression can only depend on the meaning of its **immediate** constituents (the largest boxes contained in the box that contains the complex expression, also called daughter nodes in Generative Grammar), together with the specific syntactic combination involved. Hence syntactic structure is all the more important for any calculation of meaning. Each constituent must be assigned a meaning on the basis of the meaning of its immediate constituents. Immediate constituents may themselves be complex, having immediate constituents of their own. This way, the procedure matches the recursiveness of syntax: it must also be **recursive**. The recursiveness of semantics, then, explains why it is possible to understand sentences we might never have heard before.

4.1 Psychologism

Given Frege's Principle, it is necessary to have a clear concept of the meaning of a word. In Chapter 2. we approached this question by considering *relations* between meanings: two meanings can be synonymous, incompatible or hyperonymic, etc. We did not, however, develop a clear concept of the meaning of an individual word as such.

When learning a new word, we learn how to combine a certain pronunciation, its phonetics and phonology (Lautgestalt) with its meaning. Thereby, a previously meaningless sequence of sounds like schmöll becomes vivid, we associate with it the idea of someone who isn't thirsty any more. In that case, one might be tempted to say that the **meaning** of an expression is the idea or conception (Vorstellung) a speaker associates with its utterance.

A number of objections have been raised against such a "**psychologistic**" notion of meaning, particularly by the founding fathers of modern "logical" semantics (see the historical remarks below):

- **Subjectiveness**: Different speakers may associate different things with a single word at different occasions: such "meanings," however, cannot be objective, but will rather be influenced by personal experience, and one might wonder how these "subjective meanings" serve to communicate between different subjects.

- **Limited Coverage**: We can have mental images of nouns like *horse* or *table*, but what on earth could be associated with words like *and, most, only, then, of, if,* ...?

- **Irrelevance**: Due to different personal experiences, speakers can have all sorts of associations without this having any influence on the meaning of an expression.

- **Privacy**: The associations of an individual person are in principle inaccessible to other speakers. So, again, how come they can be used for interpersonal communication?

In view of these considerations, many authors concluded that we need a more objective notion of meaning.

Suppose you have just learned the meaning of schmöll. What you have acquired is not only associations, but also the facility to apply the expression in an appropriate way: you might refuse a glass of orange juice because you are schmöll. You say: "Danke, ich bin schmöll" (Thanks, I'm schmöll). Given that your communicative partner has somehow acquired the same meaning, this common behavior is based on the following assumptions:

- each partner has learned the meaning of an expression in a similar way, most frequently by **reference** to the kinds of things, events, properties etc., that the expression is intended to **denote**: we refer to horses (or pictures of horses) when we make a child learn the word *horse*; we smile when we teach the word *smile*, we refrain from drinking when we are schmöll, etc.

- each partner wants to convey information in a way that guarantees the content of the message to be identical for both the speaker and his audience; otherwise, misunderstandings were the rule rather than the exception

- each partner is capable of extracting certain abstract meanings from the use of certain words like *and* which do *not* have a depictive meaning.

The first aspect of this notion of meaning captures the fact that by using words we can refer to things in the "outside world", i.e. in our environment; this is an objective feature of the word in relation to the world, called the **reference** (Sachbezug) or the referential meaning of an expression.

The second aspect of communication is that, while speaking, there is some flow of information that may change the mental state of the listener in a specific way, depending on what has been said (and of course how it has been said, but as we discussed before, this is not part of the literal meaning). In other words, an utterance is useful because it can change the **state of information** the listeners are in.

Simplifying somewhat, we may say that any description of the semantics of an expression involves two aspects: a referential one that enables us to refer to things by using linguistic expressions — this will be called the **extension** (Extension, Sachbezug) of an expression — and another aspect that deals with the information conveyed, which will be called the **intension** (Intension) of an expression. In this Chapter we deal only with extensions, we come back to intensions in Chapter 6.

Historical Remark

In this text, we adhere to the tradition of logical semantics, which was originally designed (at the end of the nineteenth century) in an attempt to make the language of mathematics more precise. As it turned out, the methods developed there proved to be flexible enough to be also applicable to the semantics of natural language.

The most important pioneers of logical semantics were the philosophers ☞ Gottlob Frege (1848-1925) and ☞Bertrand Russell (1872-1970), who both worked on the foundations of mathematics at the end of the 19th century. Interestingly, both authors considered natural language too irregular to be rigorously analysed with the logical methods they developed; their primary interest in this respect was the development of a language **not** like natural language in that it should not contain any ambiguities. Nonetheless, their influence on modern linguistics, notably that of Frege's article *On Sense and Reference* (Über Sinn und Bedeutung, 1892) and Russell's *On Denoting* (1905) (= Russell (1971)) cannot be underestimated. However, the conceptual tools and methods of logical semantics have not been fully and rigorously applied to natural language before the late 1960s, perhaps most significantly influenced by the work of the US-logician Richard Montague (1930-71).

The distinction between *extension* and *intension* is similar to Frege's use of the terms *Bedeutung* and *Sinn*; it originates from the work of Rudolf Carnap (cf. Carnap (1947), = Carnap (1972)). The term ☞*intension* should not be confused with the homophone *intention*, there is no relation whatsoever between the terms.

Who is who in logical semantics?

4.2 Simple Extensions

For some expressions of natural language it is fairly obvious that they refer to things or persons, for others a little bit of reflection is necessary to find an appropriate extension, and for a few there seems to be no reference at all. Let us look at some examples:

(3)
- — Tübingen, Heidelberg, Prof. Arnim v. Stechow, Ede Zimmermann (**proper names** (Eigennamen))
- — the president of the US (**definite descriptions** (Kennzeichnungen))
- — table, horse, book (**nouns** (Nomina))
- — bald, red, stupid (**adjectives** (Adjektive))
- — nobody, nothing, no dog (**negative quantifiers** (negative Quantoren))

Proper names and descriptions are the simplest cases. *Tübingen* and *Heidelberg* clearly refer to certain cities in Germany. What *nobody* and *nothing* refer to appears mysterious; adjectives and nouns are somewhere in between: a noun like *table* does not refer to a particular table, nonetheless a certain reference is recognizable. In what follows we will try to find suitable kinds of objects (sometimes of a very abstract nature) to serve as the reference of different types of expressions of natural language. These objects will be called the **extensions** of the respective expressions.

It should be noted that we say that a proper name like *Heidelberg* refers to the city of Heidelberg. One may object that it is not the name itself but the language user that refers to Heidelberg when uttering the name. However, while we do not deny that reference is a pragmatic relation holding between persons (referrers) and things (referents), we can see no harm in using the same term for a semantic relation between (referring) expressions and things (extensions), as long as the two are not confused. In particular, we do not mean to prejudge the notoriously difficult issue of the interdependence of these two relations, which has played a major role in 20th century philosophy of language. For us, the equivocation is but a matter of convenience. The reader should note, though, that the near-synonym *extension* does not support the same kind of ambiguity; it is expressions, not speakers, that have *extensions*.

Let us now look at the above examples more closely. As already said, names and definite descriptions denote individuals. However, there is a difference between the two kinds of expression. The relation between a name and its bearer – its referent – is a completely conventional one: whoever has been baptized Arnim von Stechow *is* Arnim von Stechow. The act of baptizing is one that establishes a *linguistic* connection between a name and an individual. In contrast, the referent of a definite description cannot be determined by just looking at linguistic facts and conventions. For instance, Barack Obama is not *the president of the US* by linguistic convention, but because he has been elected by the American voters. The reference of the definite description *the president of the US* is the individual that, at a given time, happens to be president of the US. The determination of the reference thus depends on what the facts in a particular situation are. Since Obama is the president today (in 2011), the description denotes Obama when used today, whereas it denoted George Bush in 2008. Thus, the reference (extension) of a description depends on time and circumstances. This is not true for names: once being Arnim von Stechow means always being Arnim von Stechow. But being the president does not mean always being the president.

First of all, the relation between a name and its bearer is purely conventional: what the name refers to, i.e. what its extension is, only depends on certain communicative (i.e.: linguistic) conventions as they are established in the course of a christening. With definite descriptions, matters are different. Although who happens to be the referent of *the president of the US*, is partly a matter of linguistic convention – what precisely, the definite article; the noun *president*; the partitive construction etc. mean – but only partly so; for a large part, it is up to the American voters to decide, and their decision does not bear on linguistic convention; after all, they decide who is going to *be* their president, not (just) who is going to be called *Mr President*. In fact, after at most two elections, the extension of the description *the president of the US* is going to change, but linguistic conventions will stay, and the description will keep its meaning. Quite generally, one may say that, over and above linguistic conventions, the extension of a definite description depends on (mostly) extra-linguistic facts. Accordingly, the extension of a description depends on situation and context. This situational dependence is a general trait of extensions, which will accompany us throughout the rest of this text. Names (and some other expressions), which are not affected by it, can be seen as a limiting cases (of null dependence), to which we will return in Chapter 6.

As a second difference, definite descriptions, unlike names, do not always have referents in the first place. This may be illustrated by an example made famous (in linguistics and philosophy) by Bertrand Russell, who brought it up at a time well after *La Grande Nation* had turned into a republic: **the present king of France**. The example shows that extra-linguistic facts may not only bear on who or what a description refers to; in some situations the expression may fail to have any referent at all. The point is that the lack of reference at that time is not the result of linguistic convention, but the result of the French Revolution:

This failure to refer to anything rarely happens with proper names. After all, we cannot give names to places, persons, or things and then discover that they do not exist. If anything like this ever happened, we would have to conclude that something in the act of baptizing went seriously wrong. As a case in point, some 19th century astronomers hypothesized a planet they called *Vulcan* in order to explain irregularities in the planetary path of Mercury; only later did they find out that no such planet exists. It would thus seem that the name *Vulcan* had been coined illegitimately, and—despite appearances—never had a referent.

For the next few chapters, we will pretend that all referential noun phrases – names, descriptions, and pronouns – refer to individuals, which may thus serve as their extensions. Hence we will ignore both emtpy descriptions like *the present king of France* and somewhat neurotic names like *Vulcan*, returning to empty descriptions in Chapter 7.

What both proper names and definite descriptions have in common, then, is their reference to **individuals** (Individuen). From now on, this term will be used as a technical term applying to anything linguistic expressions like names, descriptions, or pronouns refer to. On the other hand, common nouns like *king* or *table* do **not** refer to individuals, they show what is sometimes called **multiple** or **divided reference** (they are "Gemeinnamen or Allgemeinnamen") in that they potentially relate to more than one individual of a kind. Instead of saying that such terms have more than one extension, we take their extensions to be **sets** of individuals. Thus, e.g., the extension of the noun *table* is the set of all tables.

Sets play an important role in semantics. The notion derives from mathematics, namely from set theory (as you might have guessed). A set is an abstract collection of things or distinct objects; it is completely determined by its **elements**, the members of the set. Thus, if we are speaking of the set of cities, each city is an element of this set, and this is all we can find in there: it's only cities in there.

In order to name a set, we can list its members; this is most often done using curly brackets. E.g. the set of cities can be listed by specifying a list like

(4) {Madrid, Venice, Berlin, Tübingen, Rome ...}

The order of elements here is immaterial. Now, to express that Berlin is a city, we formalize this by saying that Berlin is an **element** of the extension of city. This is written as

(5) Berlin $\in$ {Madrid, Venice, Berlin, Tübingen, Rome ...}

Of course this only works for small sets. It is impossible for us to give a complete list of German cities, but in principle, this could be done, and has been done, cf. ☞Städteverzeichnis Deutschland.

Note that the denotation of *city* depends on some sort of convention, namely that a place can only be called a city if it has got a certain forensic title, its town charter (Stadtrecht). It also depends on the facts of the world whether or not a settlement got that title. Moreover, things may change in time: what is a city now may not have been a city 100 years ago, or may loose its city status by becoming part of a larger city. Thus, the extension of *city* depends on the facts in the world.

This we observed already with definite descriptions. The common feature of descriptions and common nouns is explained by the fact that descriptions contain common nouns as constituents. E.g., the description *the largest city in Germany* contains the common noun *city*, whose extension may vary. Consequently, the extension of the description can vary as well (e.g. the denotation could have been different before and after reunification). Likewise, the extension of *the king of France* is nowadays empty (i.e., there is no king of France) because the denotation of the (complex) noun *king of France* is the empty set. Note that the extensions of the expressions *king of France* and *king of Germany* are identical (at the time of writing), both denote the empty set. Yet it is clear that the meaning is different; the extension only describes a limited aspect of the meaning.

It has been proposed that the extension of each common noun (at a given time) is a set. Granted that these extensions depend on the facts, and given that our factual knowledge might be limited, it follows that we sometimes simply don't know the extension of a word. That is, we do not always know which elements exactly make up the extension of a given word like *table*. But in practice and in theory (and as far as linguistic theorizing is concerned) this lack of knowledge is less important than one might think. First, one should bear in mind that the actual extension of a word should not be confused with its meaning. Hence, not knowing its actual extension does not imply not knowing its meaning. The fact that we do not know all the details of the world we inhabit has nothing to do with our linguistic conventions and abilities. Second, not knowing the actual extension does not imply that we are unable to decide (on demand) whether a given entity is a table or not: of course we can apply the notion to things we have never seen before and whose existence we didn't know anything about. This implies that we are endowed with ways to determine the extension of a word without knowing it in advance.

Thirdly, in scientific inquiry we are often entitled to abstract away from certain insufficiencies. E.g., the meaning of almost any noun is vague: there can always be borderline cases. Consequently, the extensions can be vague, too. Although it would be possible to capture this in a vague set theory (called theory of "fuzzy sets") we may well ignore this additional complication. This does not imply that vagueness is always unimportant: for example, we might be uncertain where to draw a line between *sphere* (Kugel) and *ball* (Ball, Kugel), yet the word *ball pen* translates into German not as Ballschreiber, but as Kugelschreiber. And a ball in American football would hardly be called a Ball by a German soccer player (it's probably called an egg). So for some intents and purposes, it is important where to draw the line between Ball/*ball* and Kugel/*sphere*, but for the calculations we will be making in the present introduction such differences will play no role, and hence will be ignored.[24]

[24]Other complications arise with nouns like *milk* which are called **substance words** or **mass nouns** (Stoffnamen, Massennomen) because they refer not to individuals but to bits and parts of a substance. We might then say that any bunch of molecules that makes up a quantity of milk is an element of the

Apart from nouns, adjectives can be said to denote sets, too. Again, though, numerous questions arise: e.g., color terms like *red* and *green* are notoriously vague: Where exactly is the borderline between *red* and *green*? For the time being, these difficulties will be ignored.

Sets can also be used as extensions of intransitive verbs. For example, the verb *sleep* has as its extension the set of all sleepers, which is of course the set of sleeping individuals. The sentence

(6) John is sleeping

can be said to be true if and only if the individual John is an element of that set.

For transitive verbs, however, we get into difficulties. Take the verb *kiss* as an example. Intuitively, in a sentence like

(7) John kisses Mary

two individuals are involved. The two are connected by the relation of kissing. The relation of kissing thus applies to the **pair** consisting of John and Mary. This is an **ordered pair**, which is why (7) is different from

(8) Mary kisses John

Let us introduce a notation for ordered pairs: a pair is enclosed in angle brackets $\langle a, b\rangle$ with a the first element of the pair and b the second element. Note that although the set $\{a, b\}$ is the same as $\{b, a\}$, this does not hold for ordered pairs: the pair $\langle a, b\rangle$ is different from $\langle b, a\rangle$ (unless $a = b$).

We might say, then, that (7) holds if and only if the pair ⟨John, Mary⟩ is an element of the **relation** of kissing, whereas (8) is true if and only if the pair ⟨Mary, John⟩ is such an element. For this to make sense, we must assume that *kiss* denotes a relation, and that relations are sets of ordered pairs. This is precisely how the notion of a relation is formalized in mathematics.

(9)
a. *sleep* (= schlafen): the set of sleepers
b. *kiss* (= küssen): a relation between kissers and kissees, ie. the set of pairs $\langle x, y\rangle$ such that x kisses y.
c. *give* (= geben): a three-place relation, a set of triples.

The notion of a triple should be obvious: whereas a pair is a sequence of two elements, a triple is a sequence of three elements. We may thus summarize our descriptions of certain extensions in the following table.

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milk-set. This method raises a number of questions we cannot discuss here; let us therefore ignore these kinds of expressions and turn to expressions we can easily handle in terms of sets.

(10)

type of expression	logical type of extension	example	extension of the example
proper name	individual	Fritz	Fritz Hamm
definite description	individual	die größte dt. Stadt[25]	Berlin
noun	set of individuals	Tisch	the set of tables
intransitive verb	set of individuals	schlafen	the set of sleeping individuals
transitive verb	set of pairs of individuals	essen	set of pairs ⟨eater, eaten⟩
ditransitive verb	set of triples of individuals	schenken	set of triples ⟨donator,recipient,donation⟩

Recall that our list in (3) also contains the negative expressions *nobody, nothing*, and *no dog*. We have not attempted yet to describe their extension; this will be done in Section 5.4.

4.3 Truth Values as Extensions of Sentences

Looking at the verbs in (9), one may detect an ordering, the so-called **valency** of verbs: 1-place verbs only need a subject; 2-place verbs (called transitive) need a subject and an object; 3-place verbs are called ditransitive: they require a subject, a direct object, and an indirect object. Corresponding to these types of predicates there are three-place tuples (triples), two-place tuples (pairs) and one-place tuples (individuals). The generalization here is that predicates can be represented by sets of n-place tuples. So there is a simple connection between the valency of a verb and the n-tuples in its extension: the higher the former, the longer the latter. We thus arrive at the following observation:

(11) *Parallelism between valency and type of extension:*
The extension of an n-place verb is always a set of n-tuples.

What is remarkable about this parellelism is the fact that it does not only hold for lexical expressions, it holds for complex expressions as well. E.g. *walk* is a one-place predicate, and so is *walk slowly*, therefore *walk slowly* will also denote a set 1-tuples. *slice* is two-place, and so is *slice slowly/carefully* etc. Moreover, by adding an object to a two-place relation, e.g. adding *the salami* to *slice*, we get *slice the salami*, which itself only requires a subject. This implies that adding an object turns a two-place relation into a one-place predicate. Likewise:

(12) give (3-place)
give a book (2-place)
give a book to the student (1-place)

(13) I give a book to the student (0-place)

[25] the largest German city

The last step in (12) suggests that one-place predicates are one-place "relations"; this terminology might seem somewhat counterintuitive, since relations are normally conceived of as two-place. But there is nothing wrong with extending the terminology this way, and this is indeed standard practice in mathematics.

What might be even more puzzling is the step from (12) to (13); the qualification "0-place" is in fact intended to suggest that the valency of a sentence is zero and that sentences are nothing but zero-place verbs or zero-place relations. This is quite remarkable, but still somewhat mysterious, unless we know how to deal with zero-place relations.

Speaking of sentences as of 0-place verbs might be felt as undue terminological hardship. Perhaps a more intuitive conception is to replace the notion of an n-place verb with that of a sentence with n gaps. Thus, a transitive verb is a sentence with 2 gaps, an intransitive verb is a sentence with 1 gap, and a sentence is a sentence with no gaps. The connection would then be that a sentence with n gaps denotes a set of n-tuples.

Let us illustrate this in a table, taking (14) as a sample sentence:

(14) Der Papst zeigt dem Präsidenten den Vatikan
the Pope shows the president the Vatican
'The Pope shows the Vatican Palace to the president'

(15)

verb or verb phrase	valency	extension
zeigt	3	set of all triples ⟨a,b,c⟩ where a shows b to c
zeigt dem Präsidenten	2	set of all pairs ⟨a,b⟩ where a shows b to the president
zeigt dem Präsidenten den Vatikan	1	set of all 1-tuples ⟨a⟩ where a shows the Vatican to the president

We might then continue in the following way:

(16)

sentence	valency	extension
Der Papst zeigt dem Präsidenten den Vatikan	0	set of all 0-tuples ⟨⟩ where the Pope shows the Vatican to the president

To see what (16) amounts to, we first need to ask what a zero-tuple is. Why, obviously, it's a list of length zero! Mathematicians assume that such lists exists. Of course, there is only one such list, the empty list. For the sake of simplicity, we identify it with the empty set:

(17) There is precisely one zero-tuple, viz. the empty set $\emptyset$.

As a consequence, the set of all zero-tuples contains the empty set $\emptyset$ as its one and only element; in other words (or symbols), $\{\emptyset\}$ is the set of all zero-tuples.

According to (16), the extension of (14) comes out as the set of all 0-tuples, where the Pope shows the Vatican to the president. What set is this? That depends on the facts:

- Suppose that the Pope does not show the Vatican to the president. Then the set would have to be empty; for if there were a 0-tuple where the Pope shows the Vatican to the president, then the latter would be the case – contrary to what we assumed. Hence, if (14) is false, the set determined in (16) is the empty set.

- If, on the other hand, the Pope does show the Vatican to the president, the set in (16) would contain all 0-tuples there are, because it would be true (though awkward) to say of any such 0-tuple that it is a 0-tuple, where the Pope shows the president the Vatican. Hence, if (14) is true, the set determined in (16) is the set of all 0-tuples, i.e the singleton $\{\emptyset\}$.

We thus conclude that (14) has one of two possible extensions depending on whether or not it is true: if it is, we get $\{\emptyset\}$; if not, we have $\emptyset$. Our next step is to note that this does not only work for the particular sentence under discussion. It works for all sentences the same way! That is, if a sentence is true, its extension is $\{\emptyset\}$, and this holds for all true sentences. This means that all true sentences have the same extension, namely $\{\emptyset\}$. Likewise, all false sentences have the same extension, namely the empty set $\emptyset$. These two sets are also called **truth values**. In logic and semantics, they are also represented by the letters T and F or by the numbers 1 and 0:[26]

(18) *Frege's Generalization:*[27]
The extension of a sentence S is its truth value, i.e. 1 if S is true and 0 if S is false.

Recall that the extension of an expression was called its reference (Sachbezug). It should have become clear by now that the extension of a sentence (its reference), being its truth value, cannot be identified with its meaning, or otherwise all true sentences would be

[26]Incidentally, the identification of 0 and 1 with $\emptyset$ and $\{\emptyset\}$, respectively, is in line with the following standard set-theoretic construction of the natural numbers, the *von Neumann ordinals* (after the Hungarian mathematician John von Neumann (1903-57)):

(i)
$0 := \emptyset$
$1 := \{0\} = \{\emptyset\}$
$2 := \{0,1\} = \{\emptyset,\{\emptyset\}\}$
$\ldots$
$n+1 := \{0,1,\ldots,n\} = n \cup \{n\}$

Note that $1 = \emptyset \cup \{0\}$ and $2 = 1 \cup \{1\}$ so that each natural number contains its predecessor as an element *and* as a subset. The notion of a subset and the relation of union denoted by $\cup$ will be explained in the next Chapter.

[27]The generalization, and the very term *truth value* (Wahrheitswert)—though not the identification of truth values with numbers—go back to Frege (1892).

synonymous. But we already remarked above that there is more, namely the information conveyed by a sentence, its intension, that contributes to the meaning of a sentence. Before going into intensions, let us see what we can do with simple extensions. The basic question we want to answer is this: how can we determine the extensions of phrases and sentences, given the extensions of words?

5 Composing Extensions

The *Principle of Compositionality* stated in Chapter 4 goes a long way toward explaining how speakers and hearers are able to use and understand expressions they have not come across before: starting with the smallest 'atoms' of syntactic structure, the words or morphemes provided by the lexicon, the meanings of ever more complex expressions can be determined by combining the meanings of their parts. Hence the language user only needs to learn and know the meanings of the lexical expressions and the ways in which they are combined.

The meanings thus determined in turn may serve to relate to the extra-linguistic world around us in ways not accessible by lexical meanings alone. Insects are cases in point. They rarely have names[28] and usually cannot be referred to by lexical expressions other than pronouns. However, even a nameless bug long gone and far away can be singled out by a definite description like *the creature that bit me in my left earlobe half a year ago*. And compositionality explains how this is possible: the lexical meanings of the parts combine into the meaning of the entire description, which in turn determines a certain animal (provided there is one that fits the description). Now, whatever this meaning is, it somehow encodes information that suffices to determine a particular nameless insect—reference to which thus becomes possible by a suitable composition of lexical meanings.

Although in general the extensions of complex expressions are determined by compositionally determining their meanings first, it turns out that more often than not, there is a more direct way. It is a remarkable fact about language that in many (though not all) cases, the referent of an expression can be determined by combining the *extensions* of its parts in a compositional way. In this Chapter we will look at a variety of such cases, some of which will also help to find out what the extensions of particular expressions are in the first place. Only thereafter, in Chapter 6, will we look at the limitations of the composition of extensions and the nature of meaning in somewhat more general terms.

[28] A. A. Milne's beetle Alexander (http://blog.ewanscorner.com/2010/07/alexander-beetle/) may be an exception, but then again it may also be a piece of fiction...

5.1 Connectives and Truth Tables

Identifying the extensions of (declarative) sentences as truth values has important—and somewhat surprising—consequences for the semantic analysis of complex sentences. For it turns out that, in certain cases, the extension of a complex sentence is entirely determined by the extensions of its immediate parts. (1) is a case in point:

(1) $\boxed{\text{Harry is reading}}$ and $\boxed{\text{Mary is writing}}$

Under the (disputable) assumption that the conjunction (*and*) and the two boxed sentences form the immediate parts of (1), we may observe that the truth value of the entire sentence is fully determined by the truth values of the latter: if either of them is false, then so is (1); otherwise, i.e. if both are true, (1) is as well. In a similar vein, we observe that the truth value of (2) also depends on the extensions of the boxed parts:

(2) $\boxed{\text{Harry is reading}}$ or $\boxed{\text{Mary is writing}}$

In the case of (2), the whole sentence is true as long as one of the boxed sentences is; otherwise, i.e. if both are false, then so is (2). Hence the extensions of coordinated sentences like (1) and (2) depend on extensions of the sentences coordinated, in a way that is characteristic of the respective conjunction. These dependencies can be charted by means of truth value charts, so-called *truth tables*:

(3)

Harry is reading	*Mary is writing*	(1)
1	1	1
1	0	0
0	1	0
0	0	0

(4)

Harry is reading	*Mary is writing*	(2)
1	1	1
1	0	1
0	1	1
0	0	0

(3) and (4) show the possible distributions of truth values of the sentences coordinated, and the effect they have on the truth value of (1) and (2), respectively. In both cases this effect may be thought of as the output of a certain operation acting on the inputs given by the constituent sentences. As the reader may immediately verify, the operation described in (3) always outputs the maximum of the input values, whereas the one in

(4) uniformly yields their minimum.[29]

Using standard notation of formal logic, these operations may be indicated by the symbols '$\wedge$' and '$\vee$'. Hence the truth value of (1) can be written as '$p \wedge q$', where p and q are the truth values of its constitutent sentences; similarly '$p \vee q$' denotes the truth value of (2).

Since truth values are the extensions of sentences, (3) and (4) show that the extensions of coordinations like (1) and (2) only depend on the extensions of the coordinated sentences and the coordinating conjunction. Let us denote the extension of an expression A by putting double brackets '$[\![\ ]\!]$' around A, as is standard in semantics. The extension of an expression depends on the situation s talked about when uttering A; so we add the index s to the closing bracket. Generalizing from the above examples to any declarative sentences *A and B*, we thus get:

(5) $[\![A \text{ and } B]\!]_s = [\![A]\!]_s \wedge [\![B]\!]_s$

(6) $[\![A \text{ or } B]\!]_s = [\![A]\!]_s \vee [\![B]\!]_s$

(5) and (6) show in what way the extension of a sentence coordination depends on the extensions of the sentences coordinated and the choice of the coordinating conjunction. The latter, it would seem, contributes a specific combination of truth values. It is therefore natural and, indeed, customary to regard this contribution itself as the extension of the respective conjunction. In other words, the extension of *and* (in its use for coordinating declarative sentences) is the combination $\wedge$ of truth values, as charted in (3). Similarly, the extension of *or* may be identified with the operation depicted in (4). We thus arrive at:

(7) $[\![\text{and}]\!]_s = \wedge$

(8) $[\![\text{or}]\!]_s = \vee$

(7) and (8) allow us to compositionally derive the truth values of (1) and (2), respectively, directly from the extensions of their immediate parts, without first determining their meanings:

(9) $[\![(1)]\!]_s$
$= [\![\text{Harry is reading}]\!]_s \wedge [\![\text{Mary is writing}]\!]_s$ cf. (5)
$= [\![\text{Harry is reading}]\!]_s\ [\![\text{and}]\!]_s\ [\![\text{Mary is writing}]\!]_s$ by (7)

(10) $[\![(2)]\!]_s$

[29] Of course, this description turns on the identification of the truth values with the numbers 0 (= false) and 1 (= true), which we have treated as a matter of convention and convenience (although it could be further motivated). Incidentally, the combination described in (3) coincides with the (ordinary arithmetical) multiplication of truth values, again conceived of as numbers; and the one in (4) boils down to subtracting the product from the sum!

$= [\![\text{Harry is reading}]\!]_s \vee [\![\text{Mary is writing}]\!]_s$ cf. (6)
$= [\![\text{Harry is reading}]\!]_s\ [\![\text{or}]\!]_s\ [\![\text{Mary is writing}]\!]_s$ by (8)

It is easily seen that the third lines in (9) and (10) each derive from their predecessors by application of the equations (7) and (8) respectively. If they still look unfamiliar, this may be due to the fact that—just like '$\wedge$' and '$\vee$'—the notations '$[\![\text{and}]\!]_s$' and '$[\![\text{or}]\!]_s$' both stand for arithmetical operations that can be applied to truth values. And it is precisely this application to the truth values of the sentences conjoined that delivers the truth value of the entire coordination. According to (9), then, the extension of (1) is obtained by applying the extension of one of its parts, viz. *and*, to the extensions of the other two parts; and similarly for (2). Hence in both (1) and (2) the extension of the entire expression is determined by the extensions of its (immediate) parts in the same way, i.e. by applying the extension of one of them to the extensions of the others.

Let us now see how the truth values of simple sentences like the ones conjoined in (1) and (2) can be obtained from the extensions of their parts.

5.2 Subject and Verb

Once the extensions of sentences have been identified as truth values, it is not hard to see how extensions can be combined in accordance with Frege's Principle:

(11) *Extensional Principle of Compositionality:*
The extension of a compound expression is a function of the extensions of its immediate parts and the way they are composed.

Note that we slightly modified Frege's principle: the difference from the general principle of compositionality stated earlier is that we now take into account only the *extensions* of the expressions involved (rather than their meanings). This is a simplification to which we return in Chapter 6.

Let us now apply the principle to a simple sentence like

(12) Paul schnarcht
Paul snores
'Paul is snoring'

The immediate parts are *Paul* and *schnarcht*. The extension of the proper name is the individual Paul, the extension of *schnarcht* is the set of snoring individuals (at a certain time in a certain situation). Can we determine a truth value by looking at these extensions? Yes, we can: Having adopted the framework of set theory, we only need to say that the sentence is true if the extension of *Paul* is an element of the extension of *schnarcht* (= the set of individuals snoring). If Paul is not an element of that set, the sentence is false. The same is true for *Tim schnarcht*. And for *Paul schläft* (Paul is sleeping), *Tim* *schläft* etc. Therefore, a general pattern is at work here:

(13) The extension of a sentence of the form "proper name + verb" is the truth value 1 if the extension of the proper name is an element of the extension of the verb; otherwise its extension is 0.

In a similar way we can 'calculate' the extensions of simple clauses whose subject is a not proper name but a definite description or any expression whose referent is an individual. For descriptions, an English example is the following:

(14) The president of the USA walks

The only thing our truth conditions require is this: Whoever should turn out to be the president (e.g., Barack Obama, at the time of writing), check if this individual, namely the extension of 'the president of the US,' is among the individuals walking (at the time of writing). If so, the sentence is true, if not, the sentence is false. We thus have:

(15) $[\![$the president of the USA walks$]\!]_s = 1$ if $[\![$the president of the USA$]\!]_s \in [\![$walk$]\!]_s$, otherwise $[\![$the president of the USA walks$]\!]_s = 0$.

Generalizing from these examples, we state the following rule of extensional composition:

(16) The extension of a sentence of the form "referential subject + verb" or "referential subject + verb phrase" is 1 if the extension of the referential subject is an element of the extension of the verb (phrase); otherwise, its extension is 0.

In (16) the qualification *referential* is meant to apply (at least) to proper names and definite descriptions, but not quantificational phrases such as *jedes Kind*, to which we will turn in due course.

5.3 Verb and Object

Before doing so, let us look at sentences with transitive verbs and referential objects. As before, we would like to be able to calculate the extensions of sentences of the form "subject + verb + object":

(17) Paul liebt Mary
Paul loves Mary

We already know the extensions of the names, namely the individuals called Paul and Mary, and we also know the extension of *liebt*, which is a set of n-tuples, namely the set of pairs $\langle x, y \rangle$, such that it holds that x loves y. For (17) to be true it, must therefore be the case that the pair ⟨Paul, Mary⟩ is an element of the extension of *liebt*.

(18) $\llbracket \text{Paul liebt Mary} \rrbracket_s = 1$ if and only if $\langle \llbracket \text{Paul} \rrbracket_s, \llbracket \text{Mary} \rrbracket_s \rangle \in \llbracket \text{liebt} \rrbracket_s$

At this point, however, we encounter a difficulty: (18) requires us to consider $\langle \llbracket \text{Paul} \rrbracket_s, \llbracket \text{Mary} \rrbracket_s \rangle$, ie. the pair consisting of the subject and the object. However, when looking at the syntactic structure of (18), it turns out that the subject and the object do not form a constituent. Rather, we learned that in order to derive (17) two transformations were needed, namely topicalization and verb-second movement. In order to simplify things a bit, let us assume that these movements are undone, so that we now have to interpret the still simplified structure in (19):

(19) $\boxed{\text{Paul } \boxed{\text{Mary liebt}}}$

Equivalently, the structure for English would be (20):

(20) $\boxed{\text{Paul } \boxed{\text{loves Mary}}}$

It is easy to see that the pair consisting of the subject and the object mentioned in (18) still does not form a constituent in either structure. Is that a problem?

Well, yes and no. "Yes," because the rule (18) does not conform to the Principle of Compositionality, and "no" because it's easy to find a way around the problem so as to still conform to the principle. Taking (11) seriously means that the extension of the sentence, its truth value, must be calculated from its *immediate* constituents. These are Paul and the embedded box. The latter is often called a verb phrase, abbreviated as VP. Accordingly, we first have to calculate the extension of the VP *loves Mary*/*Mary liebt* before we can determine whether the sentence S is true or false. The main question therefore reduces to the problem of assigning extensions to the VPs in (19) (or (20)). The problem seems to be that our original encoding of transitive verbs as two-place relations does not fit with the syntax.

The good news is that it is quite easy to overcome this problem. The key to an understanding of the method of compositional interpretation is to look at the set-theoretic objects that correspond to the syntactic categories. In the above example, we thus ask what type of denotation the VP has, and we've already seen in the last section that this must be a set. But which set? Of course this must be the set of Mary's lovers. So Paul loves Mary if and only if Paul is an element of the set of Mary's lovers, which is the extension of the VP.

Now the only remaining problem is to determine that set on the basis of the relation *love*. But again, this is easy. We only have to look at those pairs in $\llbracket \textit{liebt} \rrbracket_s$ (or $\llbracket \text{love} \rrbracket_s$) whose second member is Mary. These are the tuples whose common property is that someone loves Mary. Now out of this set of pairs, we collect all the first members and let them form a new set. This of course is the set of Mary's lovers.

What remains to be done is to state a general rule that interprets complex verb phrases along these lines. Before turning to this rule, observe that our old rule (13) should be revised. Previously, we only looked at combinations of a subject and a verb. But according to standard terminology, both complex VPs and single intransitive verbs are VPs, they both function as the main predicate of a sentence. This implies that our old rule can be replaced by the more general rule (21):

(21) The extension of a sentence of the form "proper name + VP" is the truth value 1 if and only if the extension of the proper name is an element of the extension of the VP; otherwise the sentence is false.

For intransitive verbs, the extension of the VP is simply the extension of the verb. For complex VPs containing a transitive verb and its object, we need a new rule:

(22) The extension of a VP of the form "verb + proper name" (or "proper name + verb" for German) is the set of all individuals such that any pair consisting of that individual and the extension of the proper name is an element of the extension of the verb.

This sounds complicated enough, so let us demonstrate the method by way of a small example. Assume the relation of loving is represented by the following n-tuples:

(23) Let the relation *love* be the following set of 2-tuples:
$\{\langle a,b\rangle,\langle b,c\rangle,\langle b,d\rangle,\langle b,e\rangle,\langle d,d\rangle,\langle e,d\rangle\}$

Note that this extension is purely stipulated, any other two-place relation would do the same job. Technically speaking, what we did by stipulating a certain (arbitrary) extension like (23) is to assign a particular **interpretation** to the word *liebt* (or rather the stem form *lieb-*); any other set of pairs would also be an interpretation and would do the same job. The idea is that instead of having to specify the real extensions in the real world, which is often impossible because we lack knowledge about the real extensions, we can simply choose a representation that goes proxy for the real extension. Any such collection of extensions is also called a **model** for a language. Thus, a model contains all extensions of all expressions of a language, of course, stipulated extensions, as the entire model is only stipulated. In our case, the very small model contains just the extension of the predicate *love* and the extension of proper names.

Let us assume that in our model (23) d is the extension of Dorothy, and a is the extension of Albert. Is it true (in our model) that Albert loves Dorothy? Of course we could inspect (23) directly by looking for a pair $\langle a,d\rangle$. But this is not the point of the exercise. What we want to calculate is the extension of the VP *love Dorothy*, so we first look at all pairs containing d as an object. This is

(24) $\langle b, d\rangle, \langle d, d\rangle$, and $\langle e, d\rangle$

Now each of b, d, and e in (24) loves Dorothy; putting them into a set we get the set of Dorothy's lovers:

(25) $\{b, d, e\}$

This is the extension of the VP. Is Albert an element of that set? No. Therefore the sentence *Albert loves Dorothy* is false in this model.

Now, after having seen an example, you should return to (22); the condition sounds rather complicated, but we hope that by having gone through an example, its content has become much clearer. Nonetheless, the wording of the rule still looks cumbersome. We can, however, simplify (22) enormously if we are allowed to use a little bit of set theory. Some bits of terminology and notation will be introduced in the following digression.

Digression into Set Theory (1)

As already explained above, a set is any collection of objects. There are a number of ways to characterize sets. According to one such method we simply list the members of a set, as we already did in an earlier example:

(26) {Madrid, Venice, Berlin, Tübingen, Rome ...}

The dots here say that this characterization is not quite complete.

Another way to characterize sets is by stating a property that is common to all and only the members of a set. This is a property that qualifies an individual as a member of that set. For this one normally uses a special notation:

(27) a. $\{x : x$ is a city in Europe$\}$
b. $\{x : x$ is a natural number that can be divided by 5$\}$

The letter x is called a variable. As usual, variables function as a kind of place holder standing in for objects without specifying which. The colon following the variable x is read 'such that'. The whole expression now reads: "the set of those x such that x is ...". The use of these auxiliary symbols is highly conventionalized; as variables one normally uses letters from the end of the alphabet. Thus, instead of saying (28-a), we will also write (28-b):

(28) a. Let A be the set of all cats
b. A := {x : x is a cat}

Read (28-b) as “A is defined as the set of all x such that x is a cat”. Hence $A = [\![\text{cat}]\!]_s$. Note that $A = \{x : x \in A\}$; in fact, this equation holds for any set A.

End of digression

Given these conventions let us return to rule (22). By applying the set notation and also the bracket notation for extensions, we can simplify (22) considerably:

(29) $[\![\text{verb + proper name}]\!]_s := \{x : \langle x, [\![\text{proper name}]\!]_s\rangle \in [\![\text{verb}]\!]_s\}$

Now, this looks quite easy again, as it is, and therefore we will continue to use this kind of notation when stating further rules for the combination of extensions.

Before going on, let us refine the rule in two ways. First we should mention the fact that the rule as stated presupposes that the verb precedes the object. As we have seen, the reverse is true in German, so that the required rule now looks as follows:

(30) $[\![\text{proper name + verb}]\!]_s := \{x : \langle x, [\![\text{proper name}]\!]_s\rangle \in [\![\text{verb}]\!]_s\}$

It is often assumed that the order of constituents is immaterial for semantic considerations.

Another detail concerns the fact that we presuppose that the verb is indeed a transitive verb. Let us make this a little bit more explicit by assuming a special category for transitive verbs, say TV. Then (30) can be reformulated as:

(31) $[\![\text{proper name + TV}]\!]_s := \{x : \langle x, [\![\text{proper name}]\!]_s\rangle \in [\![\text{TV}]\!]_s\}$

The category TV makes it explicit that the verb denotes ordered pairs, as we already assumed above. The extension of combining a TV with a proper name is a set (not a truth value!), i.e. the extension of an intransitive verb.

The advantage of employing the method above can be seen from the following problem. Assume we have a three place verb, that allows three proper names as arguments. A sentence like *(that) Paul introduces Bernadette to Geraldine* can be expressed as (32):

(32) (dass) Paul Bernadette Geraldine vorstellt

On the assumption that the syntactic structure is (33),

(33) dass $\boxed{\text{Paul } \boxed{\text{Bernadette } \boxed{\text{Geraldine vorstellt}}}}$

Let us try to develop a semantics for (33). The new challenge is that this time the verb denotes a three-place relation (a set of 3-tuples) rather than a set of pairs. How can we design a semantics for this case?

The key for a simple solution lies in the fact the when combining a ditranstive verb like *vorstellt* with its first objekt *Geraldine* we reduce the arity of the verb by one and we then get a two place relation, namely one that denotes in the same way as TVs do. This means that once we have a rule that applies to the innermost box, we get a TV and we can then go on with the rule we already have, namely with (31). That is, we only need one additional rule for ditransitive verbs, call them DTVs. You are asked to make this rule explicit as an exercise.

Thus far, complex VPs contain a verb and an object. Other complex VPs contain a predicate that selects an adjective as in (34):

(34) Paul is dead

Assume that *dead* denotes the set of dead individuals. What (34) says, then, is that Paul is an element of that set. It seems, then, that the verb *be* in this context has no meaning at all. This is exactly what (35) says:

(35) $[\![\text{is + adjective}]\!]_s := \{x : x \in [\![\text{adjective}]\!]_s\}$ $(= [\![\text{adjective}]\!]_s)$.

Later on we will also describe rules for ad-nominal adjectives in expressions like *the old man*. Before doing so, however, we have to discuss the semantics of quantifiers. This will also comprise finding an extension for mysterious negative expressions like *no man* or *nothing*.

EXERCISE 9:
Try to formulate the semantic rule that takes a proper name plus a DTV so that the result is the denotation of a TV.

5.4 Quantifiers

The grammatical subject of the following sentences is neither a proper name nor a description:

(36)
a. Jeder Student schnarchte
   Every student snored
b. Eine Frau schnarchte
   A woman snored
c. Keine Fliege schnarchte
   No fly snored

There is no relevant difference here between German and English, so it might be easier to continue our dicussion with reference to English. In order to exclude unwanted (generic) readings, we switched to the past tense of the verb, but this detail is otherwise immaterial and will also be ignored in what follows.

The problem already mentioned earlier is that the subject expressions do not seem to refer to anything in particular and that, therefore, these expressions cannot have an extension. Fortunately, however, Frege found a way to treat these quantifying phrases as abstract objects that do have an extension. It is clear that the extensions of nouns and verbs are sets: For example, in a normal situation during the night the predicate *snore* might contain a lot of individuals, during daytime and in particular during this lecture in the present situation the extension should rather be the empty set. Let us ignore details of this sort. Even if we don't know the extension precisely, we can say *under what conditions* these sentences are true and false. We know that the extension of the entire clause is a truth value; so the task is this: given the two sets corresponding to the predicate *snore* and the predicate *student*, what is the role of *every* in determining the truth value of the sentence *every student snores*?

The trick here is to think about what must be the case for the sentences to become true **in terms of a comparison between the extensions of the two predicates**. Conceived of this way, we may say that

(37) "Every student snores" is true if and only if the set of snoring entities contains the set of students.

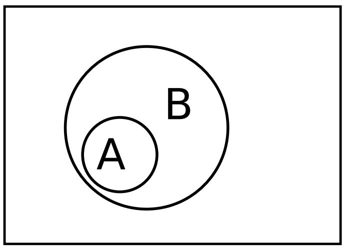
This sort of "containment" is a set theoretical notion which is called the **subset** relation. It's useful at this place to do a little bit of set theory again.

Digression into Set Theory (2)

When every member of a set A is also a member of a set B, we call A a **subset** of B. This is formally written as $A \subseteq B$. The notation already suggests that for $A \subseteq B$ to hold it is not necessary that the sets are different. If every member of A is also a member of B this does not exclude the possibility of A and B having exactly the same members.[30]

We also say that if A is a subset of B, then B is a **superset** of A. These relations are often visualized by using so called ☞**Euler-diagrams**, as shown in (38):

(38)



Taken together, the subsets of a given set A again form a set called its *power set* and

[30]The notation $\subseteq$ is actually composed out of $\subset$ and $=$, which suggests that $\subset$ is the *proper* subset relation, whereas $A \subseteq B$ means A is either a proper subset of B or equal to B. According to the **extensionality** axiom of set theory, A and B are identical if and only if $A \subseteq B$ and $B \subseteq A$.

written as $\wp(A)$.

In order to describe the composition of meanings in terms of set theory, we will introduce two simple yet important operations on sets. Both take pairs of sets as input and yield another set as output.

The first is **union** of sets. The union of A and B is defined as the set whose members are precisely the members of A together with the members of B. These are just the objects which are elements of A or B (or both). The notation for this is $\cup$:

(39) $A \cup B = \{x : x \in A \text{ or } x \in B\}$

Intersection of two sets is the second operation; it's written as $\cap$ and defined as:

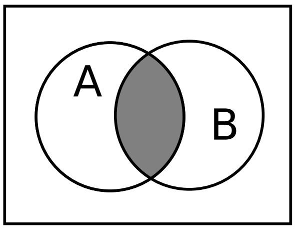
(40) $A \cap B = \{x : x \in A \text{ and } x \in B\}$

Intersection produces a set whose members are just the members of both A and B. As an example, take the sets of socks and shirts. Since nothing is both a shirt and a sock the intersection is the empty set. As another example, the set of red entities and the set of socks. Then the intersection of these sets are the set of red socks.

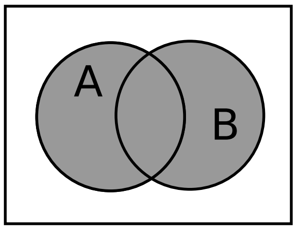
Note that if A is a subset of B, then $A \cup B = B$ and $A \cap B = A$.

Both the union and the intersection of sets are often visualized by using Euler diagrams:

(41) a.



b.



In each diagram, the shaded region represents the set that result from performing either operation to A and B. (a.) is meant to represent intersection, (b.) union.

IMPORTANT NOTE: The big disadvantage of this method is that the diagrams illustrate the workings of union and intersection by representing, as it seems, only a special case (non-empty intersection, an overlap of A and B) suggesting that A and B have specific properties (the existence of A's not being B's and vice versa) they would not have in general, e.g., if A is a subset of B, in which case the intersection of A and B would be A, the union would be B. This outcome is not directly represented in the above graphics, but of course it follows from the definition of union and intersection. Therefore Euler diagrams are not really suited to *define* these operations, although they may nicely illustrate how the definitions work in the situations depicted by the diagrams.

It is also important to realize that, while union and intersection are *operations* on sets, subsethood (or supersethood, for that matter) is not, because it does not output anything when applied to a pair of sets A and B. Rather, subsethood either holds or does not hold between A and B; it is a *relation* rather than an operation. The difference between set-theoretic relations and operations is analogous to that between arithmetical operations (like addition and multiplication) that output numbers, and arithmetical relations like $<$ and $\neq$, which hold between pairs of numbers. The algebraic analogies between arithmetic and set theory, which go deeper than we can indicate here, most clearly show in the structure of powersets, which is also known as *Boolean Algebra*, from which derives the term *Boolean operations*, which we will use to comprise set-theoretic intersection and union.[31]

End of digression

Let us now return to (37) and reformulate the truth conditions by using set-theoretic notation:

(42) a. (36-a) is true if and only if the extension of *student* is contained in the extension of *snore*; that is: the set of students is a **subset** of the set of snoring entities. — Formally: $[\![\text{student}]\!]_s \subseteq [\![\text{snore}]\!]_s$.
b. (36-b) is true if and only if the extension of *woman* and the extension of *snore* have a common element; that is: the set of women and the set of snoring entities **overlap**. — Formally: $[\![\text{woman}]\!]_s \cap [\![\text{snore}]\!]_s \neq \emptyset$.
c. (36-c) is true if and only if the extension of *fly* and the extension of *snore* have no common element; that is: the set of flies and the set of snoring entities are **disjoint**. — Formally: $[\![\text{fly}]\!]_s \cap [\![\text{snore}]\!]_s = \emptyset$.

Make sure that you understand these conditions and convince yourself that they are intuitively correct. Having stated (42), it seems that we are done.

However, this is not quite right. First observe that in (42) we are talking about particular sentences. What we want to formulate, however, is a general rule. So what we want so say is something like:

(43) a. $[\![\text{every + noun + VP}]\!]_s = 1$ if and only if $[\![\text{noun}]\!]_s \subseteq [\![\text{VP}]\!]_s$.
b. $[\![\text{a + noun + VP}]\!]_s = 1$ if and only if $[\![\text{noun}]\!]_s \cap [\![\text{VP}]\!]_s \neq \emptyset$.
c. $[\![\text{no + noun + VP}]\!]_s = 1$ if and only if $[\![\text{noun}]\!]_s \cap [\![\text{VP}]\!]_s = \emptyset$.

So far, so good. But although we've made some progress, the rules in (43) still do not conform to the Principle of Compositionality.

There are in fact two problems to be solved. The first is that we have not yet defined an extension for the quantifier expressions *every*, *a*, and *no* themselves. That is, we've

[31]The term derives from the English mathematician George Boole (1815–1864), one of the founding fathers of modern logic.

not yet said what $[\![\text{every}]\!]_s$ is. The second problem is to harmonize (43) with syntactic stucture.

Turning to the first problem, observe that the quantifiers in (43) compare the extensions of two sets and therefore describe a relation between them: one might say that *every* is the subset relation, *a* is the common element relation and *no* is the relation of disjointness. Thus, we can indeed assign some sort of extension to each of the quantifiers in (42), namely a particular relation between sets.

(44) $[\![\text{every}]\!]_s$ is the set of pairs $\langle$X,Y$\rangle$ such that $X \subseteq Y$;
$[\![\text{a}]\!]_s$ is the set of pairs $\langle$X,Y$\rangle$ such that $X \cap Y \neq \emptyset$;
$[\![\text{no}]\!]_s$ is the set of pairs $\langle$X,Y$\rangle$ such that $X \cap Y = \emptyset$

So conceived, we can state the semantic analysis of the sentences in (36) as in (45):

(45) $\langle [\![\text{student}]\!]_s, [\![\text{snore}]\!]_s \rangle \in [\![\text{every}]\!]_s$
$\langle [\![\text{woman}]\!]_s, [\![\text{snore}]\!]_s \rangle \in [\![\text{a}]\!]_s$
$\langle [\![\text{fly}]\!]_s, [\![\text{snore}]\!]_s \rangle \in [\![\text{no}]\!]_s$

However, it is obvious that the ordered pair is not a constituent in the syntactic analysis. In order to get into the position to tackle this problem it's worthwhile again to to do a little bit of set theory.

Digression into Set Theory (3)

The only additional ingredient needed is that sets may themselves have sets as their members. Here are some examples:

(46) $\{\{a,b\},\{b,c\}\}$
$\{\{a,b,c\}\}$
$\{a,b,c,\{d,e\}\}$
$\{\{a\},\{a,b\},\{a,b,c\}\}$
$\{\emptyset\}$
$\{\emptyset,\{\emptyset\}\}$[32]
$\{\{a,b,c\},\{a,b,c,d\},\{a,b,c,e\},\{a,b,c,d,e\}\}$

Note that the number of elements in these sets are 2, 1, 4, 3, 1, 2, and 4 respectively. Note also that in the last example, all sets contained in this set are supersets of $\{a,b,c\}$. If our model contains exactly five individuals, namely a,b,c,d,e, the last set described in (46)

[32]This set has two members, namely the empty set and the set containing only the empty set. Such sets may look weird, but they do play an important role in mathematics. In fact, this very specimen happens to be the set-theoretic surrogate of the natural number 2, according to von Neumann's construction (cf. footnote 26). Why, if it were not for sets having themselves sets as their members, the notion of a set would be totally boring and irrelevant for mathematics (and linguistics).

consists of *all* supersets of its smallest element, $\{a, b, c\}$. Using the notation introduced earlier, this set can also be described as shown in (47):

(47) $\{X : \{a, b, c\} \subseteq X\}$

The variable X does NOT range over individuals, rather, its values must be sets. The general convention is to use capital letters from the end of the alphabet as variables standing in for sets, whereas small letters are used as variables for individuals. Thus, $\{X : \{a, b, c\} \subseteq X\}$ is the set of all supersets of $\{a, b, c\}$

End of digression

With this in mind, let us return to the analysis of *every student snores*. The syntactic structure is unproblematic:

(48) $\boxed{\boxed{\text{every student}}\ \text{snores}}$

What is needed is an extension for the subject *every student*. Assume that the extension of *student* is {a,b,c}. Then (48) is true if this set is a subset of the *snore*-extension. Or equivalently, $[\![\text{snore}]\!]_s$ must be a superset of $[\![\text{student}]\!]_s$. But if X is a superset of Y, it is an element of all supersets of Y, that is

(49) $Y \subseteq X$ if and only if $X \in \{Z : Y \subseteq Z\}$

Utilizing this equivalence for our puzzle, it is follows that

(50) $[\![(48)]\!]_s = 1$ if and only if $[\![\text{snore}]\!]_s \in \{\text{Z}: [\![\text{student}]\!]_s \subseteq Z\}$

(50) allows us to derive an extension for the subject: Comparing (50) with (49) suggests that $\{Z : [\![\text{student}]\!]_s \subseteq Z\}$ is the extension of *every student*.

Assuming so, we have reached our first goal: we found an extension for the immediate constituents of the sentence. It only remains to combine this extension with the verb. For this to work properly we state the following rule:

(51) The extension of a sentence of the form “quantifying subject + verb” or “quantifying subject + verb phrase” is 1 if and only if the extension of the verb (phrase) is an element of the extension of the quantifying subject; otherwise, its extension is 0.

Given that all intransitive verbs are VPs, a more symbolic version of (51) is this:

(52) $[\![\text{quantifying-subject + VP}]\!]_s = 1$ if and only if $[\![\text{VP}]\!]_s \in [\![\text{quantifying-subject}]\!]_s$

Now that we've analysed quantifying subject expressions, it's necessary to make one further important step. Above expressions like *every man* were assigned an extension and this extension played a crucial role in (52). Now observe that in (52) we explicitly mention a syntactic relation in speaking of subject expressions. However, there is nothing in our semantic theory that makes special reference to the grammatical relation of being a subject. What we really want to say is that the phrase *every man* has the same extension regardless of whether it is a subject or an object or anything else.

We therefore shift from grammatical relations to grammatical categories. In terms of categories, the quantifiers *every, a,* and *no* are called **determiners** and the complex expression *every/a/no man* is called a determiner phrase **DP**.

With this in mind, we can now reformulate (52) in a more adequate, maximmaly simple way as:

(53) $[\![\mathrm{DP} + \mathrm{VP}]\!]_s = 1$ if and only if $[\![\mathrm{VP}]\!]_s \in [\![\mathrm{DP}]\!]_s$

It still remains to analyse the internal structure of the DP *every student.* It is clear that the following must hold:

(54) $[\![\text{every} + \text{noun}\ ]\!]_s = \{X : [\![\text{noun}]\!]_s \subseteq X\}$

But (54) does not show how the extension of the whole is composed from the extension of *every* and the extension of the noun. Recall that the extensions are defined as in (44). We may now apply the same trick we already exploited with transitive verbs in (29):

(55) $[\![\text{every} + \text{noun}]\!]_s = \{X : \langle [\![\text{noun}]\!]_s, X\rangle \in [\![\text{every}]\!]_s\}$

Thus, *every* + noun denotes the set of X that are supersets of the noun denotation, because *every* denotes the superset relation.

Of course, what works for *every* also works for other quantifiers:

(56) a. $[\![\text{a} + \text{noun}]\!]_s = \{X : \langle [\![\text{noun}]\!]_s, X\rangle \in [\![\text{a}]\!]_s\ \}$
b. $[\![\text{no} + \text{noun}\ ]\!]_s = \{X : \langle [\![\text{noun}]\!]_s, X\rangle \in [\![\text{no}]\!]_s\ \}$

Generalizing still further, all determiners belong to the same syntactic category D and they all denote relations beween sets, so that the three rules stated in (55) and (56) can now be collapsed into one:

(57) $[\![\mathrm{D} + \text{noun}]\!]_s = \{X : \langle [\![\text{noun}]\!]_s, X\rangle \in [\![\mathrm{D}]\!]_s\ \}$

Moreover, as we will see later, the rule does not only work for nouns being lexical items. The complement of a determiner need not be a simple noun, we also have complex expressions like *every tall man, a former student, no brother of mine* etc. All these expressions belong to the same syntactic category, they are common noun phrases often abbreviated as **NP**. Single nouns are also NPs (this is why the category is recursive: Adding PPs or adjectives does not change the category, and therefore the process of adding can go on ad infinitum). As it will turn out, the extension of all NPs is the same as that of simple nouns; all of them denote sets. We can therefore generalize (57) still further, finally ariving at the following rule:

(58) $[\![\mathrm{D} + \mathrm{NP}]\!]_s = \{X : \langle [\![\mathrm{NP}]\!]_s, X\rangle \in [\![\mathrm{D}]\!]_s\}$

Before closing this section, let us briefly discuss the denotation of German *nichts* (nothing). This is a DP that can be paraphrased as *no object*. It thus follows that the extension of *nichts* is:

(59) $[\![\text{nichts}]\!]_s = \{X : [\![\text{object}]\!]_s \cap X = \emptyset\}$

But now, if the set of entities comprises all the things there are, then

(60) $[\![\text{object}]\!]_s \cap X = X$

Therefore, the only X that gives the empty set as a result is the empty set itself, so that

(61) $[\![\text{nichts}]\!]_s = \{\emptyset\}$

It thus follows that the extension of *nichts* is not nothing (*nichts*), but a set with an element that turns out to be the empty set.

EXERCISE 10:
What is the extension of the DP *etwas* (= something)?

5.5 Names as Quantifiers

In this section we show how to solve a problem that comes up frequently in semantics. To illustrate, recall the rule for combining subjects with predicates. You may have noticed that we actually had to stipulate two different rules: One rule that applies to names and descriptions, saying that the $[\![\text{subject}]\!]_s$ must be an element of the $[\![\text{predicate}]\!]_s$ (cf. (13)), and another one which applies to quantifying DPs, saying that the $[\![\text{predicate}]\!]_s$ must be an element of the $[\![\text{quantifying subject}]\!]_s$ (cf. (51)). This duality seems a little bit strange, and one might wonder whether it is really necessary to maintain such an asymmetry. Ideally, there should be only one rule here, despite the two distinct modes of semantic composition.

The standard solution in such cases is this: Instead of simply identifying the extensions of proper names and descriptions with their referents—the individuals bearing the names or fitting the descriptions—we treat them like quantifying DPs whose extensions are sets of sets. Which sets could do the job? Following the strategy of analysis pursued in the previous section, the extension of a name would have to consist of the extensions X of all those (possible) VPs that can form a true sentence with that name in subject position. Thus, the extension of *snore* is in the extension of *Paul* (as a quantifier) just in case it counts Paul (the bearer of the name *Paul*) among its elements. In general, then, the extension of *Paul* is the set of sets X such Paul is an element of X. Intuitively, this is the set of all the properties Paul has. Saying now that Paul snores amounts to saying that the extension of *snore* is an element of that set of properties of Paul's.

Now that proper names are DPs we can eliminate the rule for names. This solves our problem, because there is only one rule now for the combination of subjects with predicates. Assume that the extension of Paul is defined as in (62):

(62) $[\![\text{Paul}]\!]_s = \{X : \text{Paul} \in X\}$

The following calculation shows that we get the correct truth conditions (as is common in mathematics, we abbreviate "if and only if" as "iff"):

(63)

$[\![\text{Paul schnarcht}]\!]_s$ is true iff	(by application of rule (53))
$[\![\text{schnarcht}]\!]_s \in [\![\text{Paul}]\!]_s$ iff	(by (62))
$[\![\text{schnarcht}]\!]_s \in \{X : \text{Paul} \in X\}$ iff	(by set theory)
Paul $\in [\![\text{schnarcht}]\!]_s$ iff	(by the meaning of schnacht)
Paul $\in \{y :$ in s, y snore$\}$ iff	(by set theory again)
Paul snores in s	

It remains to account for definite descriptions. These, like names, denote things in the universe, not sets. But unlike names, we do not have a finite list of definite descriptions in the lexicon. We cannot assume, as we did in (63), that there is a complicated lexical entry for each description. Rather, we presuppose that there are semantic rules that allow us to pick out an individual that satisfies the description. Given that individual, we now have to apply a rule that shifts the extension of the description (an individual) to the extension of a DP (a set of sets). Such rules that replace the extension of an expression with a more complex one, are called **type shifting** rules, because they change the set-theoretic type of the extension without affecting its substance.[33] In the case at hand, the pertinent type shift is known as *Montague Lifting* and can be stated as in (64): [34]

(64) $\text{LIFT}(a) := \{X : a \in X\}$

To illustrate, assume that the extension of a proper name is an individual and that its syntactic category is PN (= proper noun). Let us encode the syntactic category of a meaning as a superscript on the double brackets. Then

[33]The pertinent notion of a *type* will be made more precise in Section 5.7.

[34]The term alludes to Richard Montague, who introduced (62) and (64), writing LIFT(a) as a^*.

(65) $[\![\text{Paul}]\!]_s^{PN}$ = Paul.

Application of type shifting now yields the DP-meaning of *Paul*, as defined in (62):

(66) $\text{LIFT}([\![\text{Paul}]\!]_s^{PN}) = \{X : [\![\text{Paul}]\!]_s^{PN} \in X\} = \{X : \text{Paul} \in X\} = [\![\text{Paul}]\!]_s^{DP}$

In a similar way, we may now convert the extension of a definite description, say the pope, into the more complicated extension of a DP: [35]

(67) $\text{LIFT}([\![\text{der Papst}]\!]_s) = \{X : [\![\text{der Papst}]\!]_s \in X\} = \{X : \text{the pope in } s \in X\} = [\![\text{der Papst}]\!]_s^{DP}$

Given this modification, the unified rule that applies to all kinds of subject-predicate combinations alike reads as follows:

(68) The extension of a sentence of the form "DP + VP" is 1 iff the extension of the VP is an element of the extension of the DP; otherwise, the sentence denotes 0. More formally: $[\![\text{DP} + \text{VP}]\!]_s = 1$ iff $[\![\text{VP}]\!]_s \in [\![\text{DP}]\!]_s$.

5.6 Boolean Combinations of Extensions

In this section we will bring together a number of issues discussed on the fly in previous sections. Our principal aim is to account for one of the many attachment ambiguities encountered in Chapter 3.

First recall from our second *digression into set theory* (on p. 73) that we illustrated the set-theoretic operation of intersection with examples like *red socks*: given that both the noun *sock* and the adjective *red* have sets as their extensions—the set of socks and the set of red objects, respectively—, the idea was that the combination of adjective and noun is the intersection $[\![\text{red}]\!]_s \cap [\![\text{sock}]\!]_s$. Keeping in mind that the term *noun phrase* (NP) covers both simple nouns like *sock* as well as complex expressions like *red sock*, we may thus formulate the underlying rule as in (69):

(69) If α is the denotation of a noun phrase NP and β is a set that is denoted by an adjective A, then $[\![\text{A} + \text{NP}]\!]_s = [\![\text{A}]\!]_s \cap [\![\text{NP}]\!]_s = \alpha \cap \beta$.

Note that the rule is **recursive** in that it allows us to apply it to already complex NP, as in *stinking red sock*, *old stinking red sock*, *beautiful old stinking red sock*, etc.

The above semantic combination of extensions by intersection also applies in other syntactic contexts, e.g. when NPs are modified by relative clauses or by prepositional clauses, rather than adjectives, as in *woman from Berlin*. Let us discuss this more closely.

[35] In case the reader wonders what (67) says about pope-less situations: we will return to this question in Chapter 7.

To begin with, it is natural to assimilate assign to a preposition like *from* a binary relation as its extension, viz. the set of pairs $\{a, b\}$, where (person or object) a is, i.e. comes or originates, from place b. Given this assimilation of (some) prepositions with transitive verbs, we may then employ the same semantic mechanisms to combine them with their complements, thereby arriving at:

(70) $[\![\text{from Berlin}]\!]_s = \{\text{x : x is from Berlin}\}$

Then the extension of the modified noun phrase can be obtained by intersection:

(71) $[\![\text{woman from Berlin}]\!]_s = [\![\text{woman}]\!]_s \cap [\![\text{from Berlin}]\!]_s = \{x : x \text{ is a woman}\} \cap \{x : x \text{ is from Berlin}\} = \{x : x \text{ is a woman and } x \text{ is from Berlin}\} = \{x : x \text{ is a woman from Berlin}\}$

As an illustration, let $\{a, b, c\}$ be the set of women and $\{a, x, y\}$ the persons from Berlin in s. Then accordng to (71), $[\![\text{woman from Berlin}]\!]_s = [\![\text{woman}]\!]_s \cap [\![\text{from Berlin}]\!]_s = \{a, b, c\} \cap \{a, x, y\} = \{a\}$.

Before putting this formalization to work in the analysis of ambiguities, a short remark on modification by adjectives is in order. Above, we stated the rule for predicate adjectives on the assumption that the adjective denotes a set of things and we also illustrated that the mechanism is recursive. Because $A \cap B = B \cap A$ this implies that the order of adjectives is immaterial. But this is not necessarily the case, many adjectives do not denote simply sets. Consider a small model, consisting of a row of circles and boxes: ○□□□○□○□. Now assume that some boxes and circles are painted black: ●□□■●□○□. Pick out the rightmost black circle. This is the 5th element in the row. Next pick out the black rightmost circle. But now we are stuck. The rightmost circle is the 7th element in the row. However, this one is not black. So the description does not denote. The reason for this is that the adjective *rightmost* does not simply denote the set of "rightmost" things; rather it contains an implicit superlative that makes the order of adjectives relevant.

Other adjectives, even quite normal ones, pose similar difficulties. Consider the following sentences:

(72) a. Jumbo is a small elephant
b. Jumbo is a big animal

Our semantics for these sentences predicts that (72-a) implies that Jumbo is small (and an elephant) and (72-b) implies that Jumbo is big (and an animal; for more details it will be necessary to analyse the verb *is* more closely. This will be done in Section 5.9). Intuitively, both sentences can be true at a time, so Jumbo is both small and big—a contradiction. The reason is that adjectives like *small* and *big* require a standard of comparison (someone is small *for an elephant*, but not small *for an animal*) and this is not yet expressed in the semantics developed so far.

There are similar problems with other adjectives like the ones in (73):

(73) a. an alleged murderer
b. the former president

As we cannot go into the details here, the interested reader may consult Heim and Kratzer (1998) for further discussion.

5.6.1 Calculating an Attachment Ambiguity

Let us now look at an attachment ambiguity like

(74) the women and the children from Berlin

In Chapter 3 ambiguities of that sort were analysed as

(75) $\boxed{\text{the women}}$ and $\boxed{\text{the children from Berlin}}$

vs.

(76) $\boxed{\boxed{\text{the women and the children}}\ \text{from Berlin}}$

Let us first analyse (76). The crucial task is to supply for an extension for *the women and the children.*

At this point it will help to recall the semantic operation called set union defined in (39) on page 73. So far, this operation did not play any role in the above analyses. But now, set theoretic union comes in handy. Intuitively, *the women and the children* form a set that contains all of the children plus all of the women. As the former is {x : x is a child} and the latter is {x : x is a woman}, we accordingly have:

(77) $[\![\text{the women and the children}]\!]_s = \{x : x \text{ is a child}\} \cup \{x : x \text{ is a woman}\} = \{x : x \text{ is a child or } x \text{ is a woman}\}$

If this is correct, then we can derive a number of consequences. First, the semantics of *and* in this context is different from that of the sentential connective symbolized as $\wedge$. This is obvious, because we do not coordinate sentences or truth values. The sematics of *and* when combining plural DPs must therfore be set theoretic union.

The next consequence we can derive from (77) is that

(78) $[\![\text{the women}]\!]_s = [\![\text{woman}]\!]_s$ and $[\![\text{the children}]\!]_s = [\![\text{child}]\!]_s$

This is somewhat mysterious; we will return to the point at the end of the section in 5.6.2.

The third consequence from all this is that we can now account for the ambiguity. Semantically, *and* is represented as ∪ and predicate modification via attachment of a prepositional phrase is represented as ∩. We can therefore symbolize the ambiguity schematically as:

(79) a. $\boxed{\boxed{\text{A} \cup \text{B}} \cap \text{C}}$ (= high attchment of C = (76))

b. $\boxed{\text{A} \cup \boxed{\text{B} \cap \text{C}}}$ (= low attchment of C = (75))

In order to show that the ambiguity is "real" we have to construct a model such the (79-a) and (79-b) indeed have different denotations. Let us assume therefore that

(80) $A = [\![\text{child}]\!]_s = \{a, b, c\}$
$B = [\![\text{woman}]\!]_s = \{x, y, z\}$
$C = [\![\text{from Berlin}]\!]_s = \{a, x, y, d, e\}$

a. High attachment:
$A \cup B = \{a, b, c, x, y, z\}$
$\{a, b, c, x, y, z\} \cap C = \{a, b, c, x, y, z\} \cap \{a, x, y, d, e\} = \{a, x, y\}$

b. Low attachment:
$B \cap C = \{x, y\}$
$A \cup \{x, y\} = \{a, b, c\} \cup \{x, y\} = \{a, b, c, x, y\}$

As $\{a, b, c, x, y\} \neq \{a, x, y\}$ we have shown that the construction is really ambiguous.

Let us now come back to the mysterious equations in (78). At first sight, it seems that the determiner *the* and the plural morphology are semantically vacuaous. But this seems extremely unplausibel, as there are clear semantic difference between:

(81) a. John cares for children
b. John cares for the children
c. John cares for the child

Thus, both the plural and the definite article contribute to meaning.

The surprising effect in the case at hand is that the semantic contribution of pluralization and that of the determiner seem to cancel each other. To see this, we must briefly go into the semantics of plural NPs.

5.6.2 Plural NPs

The one big problem we must come to grips with is the denotation of plural objects. In principle, all plural NPs denote sets of sets. For instance, the denotation of *men* is the set of all non-empty sets of men. This is the set of all non-empty subsets of the extension of *man*. The same way, the plural of *child* denotes all (non-empty) sets of children, etc.

You might wonder why the denotation of a plural noun is allowed to contain elements that are not at all plural, since it also contains sets with only one element. The reason for this can be derived from inspecting the following dialogue:

(82) a. Do you have children? Yes, I have a daughter
b. No, I don't have children

In (82-a), the answer is affirmative, although I might have only one child. But if *children* always referred to two or more of them, then I would be entitled to reply (82-b) in case I only have one child. As this is absurd, it makes more sense to include the one-child case into the definition of *children*. On the other hand, in plain sentences like (83) (uttered out of the blue),

(83) I have children

we *do* understand *children* as referring to more than one. The choice between singular and plural phrases must therefore be regulated by considerations that belong to pragmatics.

Plural denoting sets can be quite large and not easy to handle. Therefore we did a little bit of cheating here: we ignored the plural morphology entirely and tried to keep with the denotations we already have at hand. But once given the more complicated objects we can now describe the semantic effect of the definite article *the* when combined with a plural object. Its function is to pick out the largest set of the NP-extension. E.g, assume that $[\![\text{child}]\!]_s = \{a, b, c\}$, then $[\![\text{children}]\!]_s = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$ and $[\![\text{the children }]\!]_s = \{a, b, c\} = [\![\text{child}]\!]_s$. This way, the effect of pluralization and the semantics of the article neutralize each other.

Once given plural objects like *the children* we now would have to adjust all the remainder of the grammar in order to integrate the new objects in combination with predicates, i.e. in sentences like *the children hide behind the trees*. This requires a more complicated semantics for the pluralization of *hide*, but also for the two place relation *behind*. This could be done in a very systematic way that resembles type shifting as discussed in Section 5.5, but we will refrain from going into any detail here.

5.7 Type-driven Interpretation

In section 5.5 we mapped the simple extension of an individual to the more complicated one of a quantifier. This method, also known as **type shifting**, is an elegant way of unifying rules of semantic composition. Instead of distinguishing the two subject-predicate rules (16) and (51), we may now do with (51) alone.

(16) The extension of a sentence of the form "referential subject + verb" or "referential subject + verb phrase" is 1 if and only if the extension of the referential subject is an element of the extension of the verb (phrase); otherwise, its extension is 0.

(51) The extension of a sentence of the form "quantifying subject + verb" or "quantifying subject + verb phrase" is 1 if and only if the extension of the verb (phrase) is an element of the extension of the quantifying subject; otherwise, its extension is 0.

However, this unification comes with a price: extensions, which are meant to determine reference, only do so in a rather roundabout way. After all, proper names merely refer to their bearers, whereas their type-shifted extensions encode the latter by set-theoretic means. The same point applies to definite descriptions, personal pronouns, and other referential expressions once they undergo type shifting. Hence while type shifting decreases the number of rules at the syntax/semantics interface, it tends to create complex and artificial extensions. However, this trade-off between the complexity of extensions and the number of rules can be evaded by having the semantic rule itself decide which combination of extensions is appropriate:

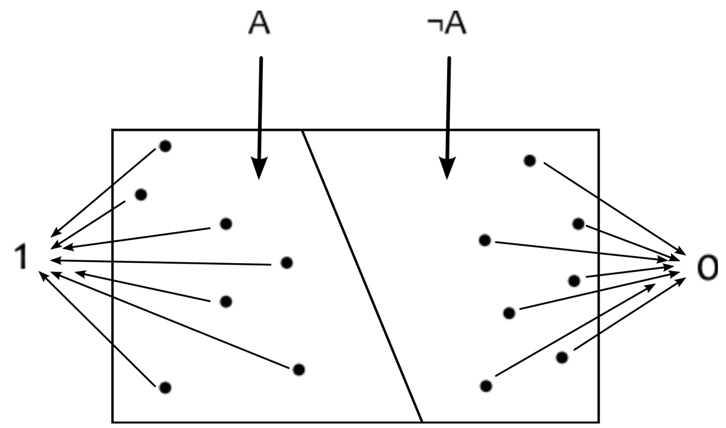
(84) The extension of a sentence of the form "subject + verb (phrase)" is 1 if and only if EITHER: (a) the extension of the subject is an element of the extension of the verb (phrase); OR: (b) the extension of the verb (phrase) is an element of the extension of the subject. Otherwise, the extension of the sentence is 0.

To avoid distracting complications, we assume that only the extensions of quantifying DPs contain any sets as their members, but neither the extensions of referential subjects nor those of predicates do. Hence for sentences with referential subjects, option (b) fails, and thus (84) boils down to (16): the sentence is true just in case (a) is. Similarly, for sentences with quantifying subjects, option (a) fails, and thus (84) boils down to (51): the sentence is true just in case (b) is. Taken together, then, (84) boils down to the combination of (16) and (51).

This unified combination opens up an interesting perspective on the syntax/semantics interface in general. Loosely speaking, (84) is self-regulating in that it has the extensions of the expressions combined decide on the way they are combined. General mechanisms of self-regulating semantic combination have been developed in the tradition of **type-driven interpretation**, where different kinds of extensions are assigned specific labels known as *types* that determine appropriate modes of combination. In the following we briefly indicate the basic tools and mechanisms. For further details, we refer the reader to the pertinent literature, and especially the standard textbook by Heim and Kratzer (1998).

The method is based on a particular way of encoding sets in terms of so-called *characteristic functions* that explicitly distinguish membership and non-membership by truth values. To see how this works, let us assume we are given a fixed domain of individuals which taken together, form a (non-empty) set U. Then any subset A of U is *characterized* by a function f that assigns 1 to A's members and 0 to all other individuals in U:

(85)



The arrows depict a relation between individuals and truth values that can be defined in set-theoretic terms, as a set of ordered pairs. What makes this relation a *function* is the very fact that each member x of U is associated with, or *assigned*, precisely one object, the *value* (for x); what makes it a *characteristic* function is the fact that the value always happens to be a truth value; what makes it the characteristic function *of* the set A is the fact that the truth value assigned to any given $x \in U$ happens to be 1 just in case x is a member of A. In general, the characteristic function of a set of individuals consists of all pairs $\langle x,1\rangle$ where $x \in A$ plus all pairs $\langle x,0\rangle$ where x is an individual but not a member of A. It is easy to see that, no matter what we take to be our universe U, its subsets stand in a one-one correspondence to their characteristic functions: the empty set corresponds to the function that assigns 0 to all members of U; the singletons $\{x\}$ correspond to functions that assign 0 to all individuals but x; etc. Given this correspondence, sets may as well be replaced by their characteristic functions—which is precisely what happens in type-driven interpretation.

In order to arrive at a self-regulating system of extensions, the latter are classified according to their *types*. If the extension of an expression is an individual (as in the case of names and definite descriptions), it is said to be of type e; if it is a truth value (as in the case of declarative sentence), it is said to be an extension of type t; if it is the characteristic function of a set of individuals, then its type is $\langle e, t\rangle$, thus indicating that it assigns a truth value to each individual.[36] So after replacing sets by their characteristic functions, intransitive verbs and verb phrases turn out to have extensions of type $\langle e, t\rangle$, and so do common nouns. Now, since the extensions of quantifier phrases are sets of predicate extensions, it is not hard to guess what their type would be, viz. $\langle\langle et\rangle, t\rangle$. In fact quite generally, $\langle a, b\rangle$ is the type of functions that assign values of type b to *inputs* of type a.

Replacing sets by their characteristic functions has immediate consequences for semantic combination. Simple sentences consisting of subject and predicate are cases in point. If the subject is referential, its extension is an individual to which the predicate extension assigns a truth value. Obviously, this truth value is the truth value of the sentence; it is 1 just in case the subject extension is in the set characterized by the extension of the predicate, which is the truth condition of a predicational sentence. Similarly, if the subject is quantificational, its extension assigns a truth value to the predicate extension. Again, this truth value is the truth value of the sentence; it is 1 just in case the predicate extension is in the set characterized by the extension of the subject, which is the truth condition of a quantificational sentence. Thus in both cases, the truth value of the sentences ensues as the value the extension of one of the constituents assigns to the extension of the other one. In mathematical parlance, determining the value of a function is called *applying* it. Hence, in both kinds of subject-predicate combinations, the relevant semantic combination turns out to be *functional application*, the only difference being in which of the two constituents provides the function; and it is precisely this piece of information that is encoded by the types of their extensions. For if the subject is referential, its extension will be of type e and thus fits the type of the predicate extension, $\langle e, t\rangle$, in that the former forms the input to the latter. Otherwise, i.e. if the subject is quantificational, its extension is of type $\langle\langle e, t\rangle, t\rangle$, to which the predicate type $\langle e, t\rangle$ forms the input. Hence, it is easy to decide in which way the extensions of the two constituents, subject and predicate, combine by just looking at the types of their extensions: if one is of the form $\langle a, b\rangle$, where the other is of the form a, then the former gets applied to the latter to output the truth value of the sentence.

Type-driven interpretation now generalizes the strategy of reading off the pertinent semantic combination from the types of the extensions involved. To this end, functions are also used to replace relational extensions like those of transitive and di-transitive

[36] e abbreviates the philosophical term *entity*. The notation originates with Montague's 1970 *Universal Grammar*, but the classification itself can be traced back to Bertrand Russell's work on the foundations of mathematics at the beginning of the 20th century. Type-driven interpretation was developed in the 1980s in extension of Montague's work.

verbs, using a technique known as Currying.[37] Instead of going into the general mechanism, we just look at an example, the extension of the transitive verb *kiss* (in a given situation), which we have taken to be a set of ordered pairs $\langle x, y\rangle$ of kissers x and kissees y. Let us imagine a situation in which John and Mary kiss and Jane also kisses John (though not vice versa), and no one else is involved in any kissing. Hence the extension of *kiss* contains precisely three pairs of persons. Currying now turns this relation into a function that can be combined with the extension of the (referential) object by functional application. The idea is that this function assigns to any object extension the corresponding extension of the ensuing verb phrase. Thus, e.g., John will be assigned the (characteristic function of) the set of persons that kiss him, i.e. Mary and Jane, whereas Mary will be assigned the (characteristic function of) the singleton set containing John; etc. We leave it to the reader to check that the Curried version of the extension preserves all information about who kisses whom. Since the Curried extension assigns characteristic functions (of sets of individuals) to individuals, its type is going to be $\langle e, \langle e, t\rangle\rangle$. And once again, when it comes to combining the verb with its (referential) direct object e, the types reveal that their extensions combine by applying the extension of the former to that of the latter.

In a similar (though not exactly parallel) way, determiner extensions also get replaced by functions whose rather complicated type turns out to be $\langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle\rangle$. We will not go into this here but merely point out that at the end of Currying day, many syntactic constructions (or branchings) come out as combining expressions with matching types of the forms $\langle a, b\rangle$ and a, ready to be combined into extensions of type b by functional application. Hence, if all goes well, we end up with a single rule of (extensional) semantic combination applicable to any syntactically well-formed binary structure [A B]:

(86) If $[\![\mathrm{A}]\!]_s$ is of some type $\langle a, b\rangle$ and B is of type a, then: $[\![[\mathrm{A\ B}]]\!]_s$ is the result of applying $[\![\mathrm{A}]\!]_s$ to $[\![\mathrm{B}]\!]_s$. Otherwise, $[\![[\mathrm{A\ B}]]\!]_s$ is the result of applying $[\![\mathrm{B}]\!]_s$ to $[\![\mathrm{A}]\!]_s$.

The rule presupposes that in the second case too, the extensions of A and B match in type. Doubtlessly, a lot of work would have to be done to ensure this. However, we will not pursue this interpretation strategy any further, but once again invite the reader to consult the relevant literature.

5.8 Quantifier DPs in Object Position*

Let us tackle a problem that arises with transitive verbs and quantifying expressions in object position. Looking back to the rule that combines a verb and an object ((22) and its variants discussed in section 5.3), we are in trouble: The rule seems to work only for names and descriptions, but not for quantifying objects as in (87).

[37] The term alludes to the American mathematician Haskell Curry (1900–1982); in German the technique is also known as Schönfinkelei, after the Russian mathematician Moses Schönfinkel (1889–1942).

(87) Paul loves every girl

This is because our semantics of transitive verbs expects an individual as the extension of the object, not a set of sets, as quantifying DPs actually denote. Fortunately, there are several ways out of this embarrassment, a couple of which we will now look at.

The most straightforward strategy of interpreting (87) is by tampering with its syntactic structure. To see how this works, let us briefly consider a paraphrase:

(88) Every girl is loved by Paul

Obviously, (88) and (87) have the same truth conditions. Moreover, there is nothing mysterious about the contribution the quantificational DP makes to arrive at these truth conditions; after all, it is in (surface) subject position and thus can be analyzed according to our rule (53) above. Now, by and large, (87) and (88) consist of the same lexical material.; it is just arranged in different ways.[38] The reason why, unlike (87), (88) presents no obstacle to semantic analysis, is that in it the material is arranged in such a way that the quantifier, being its subject, is an immediate part of the sentence.

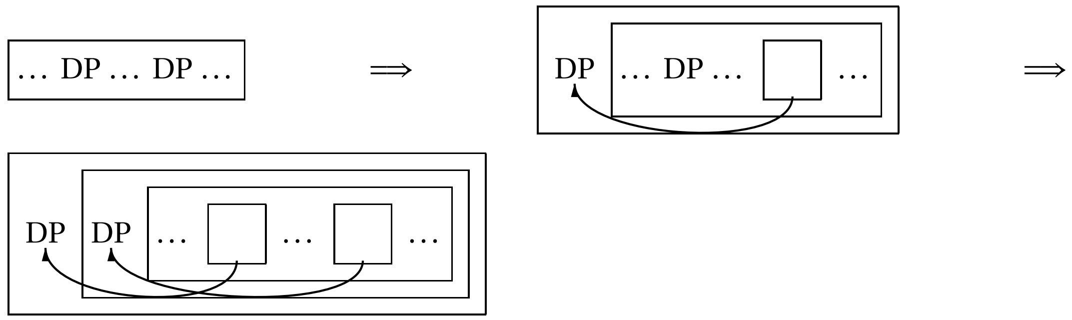
5.8.1 Quantifier Raising

There are different methods that aim at interpreting the quantifier as an argument of being loved by Paul. A simple strategy for escaping our embarrassment has it that, for the purposes of semantic analysis, (87) must be carved up in the same way as (88), i.e. it must be assigned an LF that consists of the quantifier and the remainder of the sentence. This remainder, then, would have to receive an extension, too—viz., the extension of the remainder of (88), i.e. the VP *is loved by Paul*:

(89) $[\![\text{Paul loves every girl}]\!]_s = \left[\!\!\left[\ \boxed{\text{every girl}}\ \ \boxed{\text{Paul loves \_\_}}\ \right]\!\!\right]_s = 1$ if and only if $[\![\text{is loved by Paul}]\!]_s \in [\![\text{every girl}]\!]_s$

The middle term in the above equation indicates (the extension of) a syntactic structure that derives from the original (surface) structure of (87) by an operation we already discussed in section 3.4.2, namely **Quantifier Raising**, or **QR**, for short. The exact formulation and nature of this syntactic transformation will not concern us here. Roughly, QR transforms a given structure by moving a quantifying DP upward to the left of a sentence, leaving a gap in its original position. The operation may be applied more than once within the same structure:

[38] We are deliberately glossing over some details here, like the passive morphology and the preposition *by*.

(90) 

Instead of using arrows, a co-indexing convention sees to it that each upwardly moved DP is connected with the gap it leaves behind; the convention also replaces the empty box with a place holder (a variable) that is coindexed with its matching DP:

(91) $\boxed{\text{DP}_x \ \boxed{\text{DP}_y \ \boxed{\ldots\ x \ldots\ y \ldots}}}$

In the case at hand, QR produces the following LF:

(92) $\boxed{\text{every girl}_y \ \boxed{\text{Paul loves } y}}$

Given its purpose as syntactic input to interpretation, we better make sure that the result (92) of applying QR is compositionally interpretable. As it turns out, this is not obvious. To be sure, (89) shows that the extensions to be combined at top level are the same as in (88); in other words, we have been assuming that not only the quantificational DPs but the remainders, too, are semantically equivalent:

(93) $[\![\text{Paul loves } y]\!]_s = [\![\text{is loved by Paul}]\!]_s$

Following the strategy of section 4.2, the right term in (93) is a set of individuals:

(94) $[\![\text{is loved by Paul }]\!]_s = \{y : \text{in } s,\ y \text{ is loved by Paul}\}$

Since any individual y is loved by Paul just in case Paul loves y, we may reformulate the latter term so as to avoid the passive voice:

(95) $\{y : \text{in } s,\ y \text{ is loved by Paul}\} = \{y : \text{in } s, \text{ Paul loves } y\}$

Our chain of equations (93)-(95) thus adds up to:

(96) $[\![\text{Paul loves } y]\!]_s = \{y : \text{in } s, \text{ Paul loves } y\}$

Now this looks pretty systematic; in fact, it is: quite generally, the extension of a sentence with a (single) gap can be characterised as the set of individuals that **satisfy** the sentence in the sense that a true sentence results once the gap is filled with a name for that individual.[39] Given (96), the complete truth conditions amount to saying that the extension described there is an element of the extension of the quantifying DP. Thus, it follows that:

(97) $\left[\!\left[\boxed{\text{every girl}_y} \ \boxed{\text{Paul loves } y} \right]\!\right]_s = 1$ if and only if (iff)
$\{y : \text{in } s, \text{Paul loves } y\} \in [\![\text{every girl}]\!]_s$ iff
$\{y : \text{in } s, \text{Paul loves } y\} \in \{X : [\![\text{girl}]\!]_s \subseteq X\}$ iff
$[\![\text{girl}]\!]_s \subseteq \{y : \text{in } s, \text{Paul loves } y\}$ iff
$\{x : \text{in } s, x \text{ is a girl}\} \subseteq \{y : \text{in } s, \text{Paul loves } y\}$ iff
Every z that is a girl in s is such that Paul loves z in s =
Every girl in s is such that in s Paul loves her =
Paul loves every girl in s

One advantage of the QR-strategy is that it is very general, not only applying to direct objects but to all sorts of positions in which quantifying DPs may occur. Thus, as readers are invited to verify for themselves, the following sentences can be treated along the same lines as (87):

(98) a. The assistant showed the shop to *a customer*
b. Mary looked behind *every door*
c. The janitor saw *no one* leave the building
d. A raindrop fell on the hood of *every car*

Moreover, since QR may apply to a given sentence more than once, it can also be used to deal with multiple quantification, which we will not go into, however. Moreover, QR also comes in handy in the analysis of so-called *bound pronouns*:

(99) Every boy hopes that he will pass the test.

In its most obvious reading, the pronoun in the embedded clause in (99) stands in for the schoolboy(s) quantified over in the main clause.[40] However, this does not mean that it is short for the quantifier; for the following sentence certainly means something entirely different:

[39] Its systematicity notwithstanding, this characterisation is a far cry from a compositional interpretation of gappy structures. In fact, we will defer this task to an appendix, because it involves formal techniques that go far beyond the basic issues addressed in this text.

[40] Note that in (99), *he* may also refer to a person that had been mentioned before. We are not concerned with this reading here, which would call for a separate analysis (a simpler one, actually).

(100) Every boy hopes that every boy will pass the test.

In order to capture the intended reading of (99) (and distinguish it from (100)), one may, however, treat the pronoun as if it were a gap left behind by the quantifier that would have to be raised in the first place:

(101) $\boxed{\text{every boy}_y \ \boxed{y \text{ hopes } \boxed{\text{that } y \text{ will pass the test}}}}$

Note that this structure contains two gaps—one for the original position of the subject, one for the pronoun. Once dissected and indexed in this way, the sentence (100) comes out as true if the extension of the doubly gapped part is an element of the extension of the quantifier:

(102) $[\![\text{Every boy hopes that he will pass the test}]\!]_s = 1$ if and only if $\{y\text{: in } s\text{, } y \text{ hopes that } y \text{ will pass the test}\} \in [\![\text{every boy}]\!]_s$

Unfortunately, we are not in a position to justify these truth conditions beyond this point, partly because of technical obstacles mentioned in Fn. 39, partly because (99) involves the kind of construction (clausal embedding) only to be addressed in the next chapter. Even so, we hope that (102) does have some plausibility if only as a sketch.[41]

5.8.2 In situ *Interpretation*

QR is not without alternatives (and not without its own problems). In fact, our original example (87) could also be handled in a variety of ways that do not involve a rebracketing of the syntactic input. We will briefly go into one of them, and then leave the subject altogether.

To arrive at a so-called ***in situ* interpretation** of the VP in (87), let us return to the satisfaction set (96) that served as the extension of the LF (92). Since we are after the extension of the VP, this set may appear irrelevant—the more so because it depends on the extension of the subject, Paul. With a different name in subject position—*John*, say—we would have had a different satisfaction set, viz.:

(103) $\{y : \text{in } s\text{, John loves } y\}$

In general, whenever we replace the subject in (87) with a name or description of an individual x, the corresponding satisfaction set—the extension of the remainder after

[41] Actually, as it stands, (102) does not bear scrutiny for a third reason: the binding mechanism introduced in the appendix does not square with the (simplified) semantics of clausal embedding of the next chapter: the latter misses the so-called *de se* aspect implicit in the pronoun, i.e. the fact that it relates to the students quantified over as being seen from their own perspectives (their selves, as it were) …

moving the object DP—would be:

(104) $\{y : \text{in } s, x \text{ loves } y\}$

And in general, such a replacement sentence would be true if this set (104) were an element of the extension of the object, $[\![\text{every girl}]\!]_s$. Now, the extension that we are after, $[\![\text{loves every girl}]\!]_s$, consists of precisely those individuals x that make (87) true if the subject *Paul* is replaced by (a name of) x. Hence that extension collects all those individuals x whose corresponding satisfaction set (104) is an element of $[\![\text{every girl}]\!]_s$. We may thus characterise the extension of the VP in the following way:

(105) $\{x\text{: } \{y : \text{in } s, x \text{ loves } y\} \in [\![\text{every girl}]\!]_s\}$

Note that (105) can be defined in terms of the extension of the transitive verb:

(106) $\{x\text{: } \{y : \langle x, y\rangle \in [\![\text{love}]\!]_s\} \in [\![\text{every girl}]\!]_s\}$

Crucially, now, (106) shows how the extension of the VP is composed of the extensions of its immediate parts, the verb *love* and the object *every girl.* As it turns out the combination is independent of the particular example and generalizes to (almost) arbitrary quantificational DPs in direct object position. We thus arrive at the following compositional alternative to the above QR treatment:

(107) If TV is a transitive predicate and DP its (quantifying) object, then $[\![\text{TV} + \text{DP}]\!]_s := \{x : \{y : \langle x, y\rangle \in [\![\text{TV}]\!]_s\} \in [\![\text{DP}]\!]_s\}$.

The same rule also works for much simpler sentences like *Paul loves Mary*, if we assume that *Mary* is type shifted, ie. has the same logical type as quantificational DPs. Then, according to (107), being loved by Paul must be one of the properties Mary has, which is exactly how it should be.

5.8.3 Discussion

Obviously, the semantic combination (107) avoids the compositionality issue mentioned in connection with QR because it avoids any gaps and co-indexing in the syntactic input. This may be seen a price worth to pay for the somewhat opaque and *ad hoc* character of (107). However, it should be noted that it is also less general and principled than the QR strategy in that it only applies to DPs in direct object position. For different environments (e.g. indirect objects and ditransitive verbs), additional rules would have to be formulated in the spirit of (107). While in many cases these rules may be subsumed under a general pattern, more complex phenomena like multiple quantification and bound pronouns appear to require more involved techniques. At the end of the day, then, the *in situ* strategy might turn out to be too restrictive, and if extended to other cases more complicated than it appears from (107) alone.

On the other hand, although the mechanism of QR can be made precise quite easily, it leads to massive overgeneration in generating too many readings. An example may illustrate the point:

(108) His mother loves every boy

(108) cannot have the same reading as

(109) Every boy is being loved by his mother

with the pronoun *his* being interpretated like a gap. But this would be predicted by applying QR in the manner introduced above.

Finally, consider:

(110) A man ate an apple from every basket

Applying QR to *every basket* would yield a reading that is paraphrased in (111):

(111) For every basket there is an apple and a man who at that apple

Such a reading (with different men and different apples for each basket) is intuively unavailable, but nonetheless generated by QR. Likewise, in more complex situations, QR generates multiple ambiguities, many of which are intuitively unavailable. It follows that QR must be subjected to severe restrictions that seem to have an *ad hoc* character in many cases. The tricky and complicated part here is to find a principled way to rule out unwarranted applications of QR. As already announced, we will leave it at that, inviting the readers to consult the literature listed in the Further Reading part at the end of the book.

EXERCISE 11:
Give a precise account of *Paul loves Mary* by explicitly stating each step of the calculation of truth conditions with respect to the following model: $[\![\text{Paul}]\!]_s = p$, $[\![\text{Mary}]\!]_s = m$, $[\![\text{love}]\!]_s = \{\langle a,b\rangle, \langle p,p\rangle\}$.

EXERCISE 12:
Design a new *in situ* rule for quantifying objects of three-place verbs, exemplified by *every boy* in:

(112) John $\boxed{\boxed{\boxed{\text{bought}}\ \text{every boy}}\ \text{a toy}}$

EXERCISE 13:
Describe a model for which (113-a) is true and (113-b) is false:

(113) a. Mary kisses a doll
b. Mary kisses every doll

Now calculate the truth conditions according to (107), thereby stating a formal proof that the model does what it is supposed to do.

5.9 The Verb ***to be***

One of the most vexing problems in the analysis of natural language is the verb *be* (also called the copula). Above we considered sentences like

(114) Paul is smart

and we decided that *is* has no meaning of its own. However, there are other uses of *is*, attested in the following examples:

(115) a. Obama is the president of the USA
b. Paul is a nerd

In (115-a), *Obama* and *the president of the USA* each denote an individual, so it is obvious that the semantic content of *is* is the identity relation =. In this case, *is* is not meaningless but denotes the set of all pairs $\langle x,x\rangle$. It thus follows that *is* expresses two different verbs, depending on whether it combines with an adjective or a definite description (or a proper name).

(115-b) represents still another case. Here we are combining *is* with a DP whose grammatical function is that of a so-called predicative noun (Prädikatsnomen). Recall that we have already calculated the meaning of *a nerd* as the set of all (non-empty) X that overlap with the set of nerds. But this type of denotation does not easily fit with the denotation of the subject. It seems, then, that we need still another type of *is* that combines a quantifying DP with a subject. Such a denotation for *is* has indeed be proposed in the literature, but it is rather complex.[42]

A simpler solution would be to postulate that in such constructions neither the copula *be* nor the indefinite article *a* have any meaning of their own. Instead of postulating

[42]The interested (ambitious) reader might try to check that the following relation between individuals x and DP-denotations Q does the job:

(i) $[\![\text{is}]\!]_s = \{\langle x,Q\rangle : \{x\} \in Q\}$

This analysis of the copula can be traced back to W. V. O. Quine's 1960 book *Word and Object*, a classic in philosophy of language and logical semantics.

another ambiguity of *is* we now postulate one of *a*. Thus, in ordinary constructions, *a* still denotes a quantifier, but in predicative constructions it does not. These constructions are syntactically characterized as combining an indefinite DP with verbs like *be* or *become*. Some evidence for this alternative way of treating the indefinite article as vacuous can be drawn from the fact that the German analogue *ein* in these contexts is optional. Thus, the following pairs of sentences are identical in meaning:

(116) a. Mindestens ein Deutscher ist ein Weltmeister
at least some German is a world champion
b. Mindestens ein Deutscher ist Weltmeister

(117) a. Jeder Weltmeister ist ein Medalliengewinner
every world champion is a medal winner
b. Jeder Weltmeister ist Medalliengewinner

The optionality of *ein* thus suggests that *ein* in these constructions is as meaningless as the copula verb *ist*. If so, $[\![\text{nerd}]\!]_s = [\![\text{a nerd}]\!]_s = [\![\text{is a nerd}]\!]_s$, and the rule for combining a subject with a VP may apply as usual.

6 Intensions

Extensions are that part of a theory of meaning that ensures that linguistic expressions can be used to refer to entities in the world. It should be obvious from the previous chapter that this cannot be the whole story about meaning; for the meaning of an expression cannot be equated with its extension: if this were so, all true sentences would have the same meaning (and so would all false sentences). In the present chapter, we will fill in some of the gaps deriving from the extensional analysis of meaning.

6.1 Intensional Contexts

In this section we will show how the compositionality of meaning provides additional motivation for going beyond extensions. So far we have been relying on extensional compositionality, i.e. the assumption that the extension of an expression is a function of the extensions of its (immediate) parts. As it turns out, however, this principle cannot hold in full generality: in order for compositionality to work throughout language, a broader concept of meaning is called for. This part of the theory of meaning is concerned with the *information* conveyed by a sentence.

Consider the following two sentences:

(1) a. Hamburg is larger than Cologne
b. Pfäffingen is larger than Breitenholz

It so happens that both sentences are true[43], which means that they have the same extension, and both extensions can be calculated from the extensions of the names and the relation *larger than* (the set of pairs $\langle x, y\rangle$ where x is larger than y). But now consider so-called *propositional attitude* reports, i.e. sentences that tell us something about the information state of a person:

(2) a. John knows that [Hamburg is larger than Cologne]
b. John knows that [Pfäffingen is larger than Breitenholz]

There is no reason to doubt that the extensions of the words *John, knows,* and *that* – whatever they may be – are exactly the same in (a.) and (b.). Moreover, no structural ambiguity is detectable. We also know that the embedded sentences (i.e. the sentences in (1)) have the same extensions. Now, regardless of what exactly the extensions of *know* and *that* are, given the Principle of Extensional Compositionality, we can infer that the extensions of the more complex sentences in (2) must also be identical, simply because the extensions of (1-a) and (1-b) and those of all other relevant lexical items and constructions are the same. But now we face an obvious dilemma. It is surely not the case that anyone who knows that Hamburg is larger than Cologne also knows that Pfäffingen is larger than Breitenholz. In particular, assume that John knows (1-a) but not (1-b). Consequently, (2-a) is true, whereas (2-b) is false in the same situation, despite the fact that the extensions are the same. In fact, our theory of extensional compositionality predicts identical truth values, contrary to fact. What went wrong?

It seems intuitively clear that the complement (the object) of a verb like *know* cannot be a truth value. If this were the case then any piece of knowledge would imply omniscience. This means that extensional compositionality fails in a context like ... *know that*... . In plain words: the principle cannot hold in full generality; there seem to be exceptions. Such environments in which extensional compositionality fails are called **intensional contexts**. If we embed an expression (e.g., a subordinate clause) in an intensional context, then the contribution of the embedded expression cannot be its extension. But what else could it be?

6.2 Propositions

The difference in truth conditions between the sentences in (2) seems to be due to the state of information John is in. More precisely, what the sentences claim is that John's state of information comprises the information expressed by the embedded sentences. But the embedded sentences convey different information, they report different facts. Hence the truth value of the entire sentence depends on the information expressed by the embedded sentence. In semantics, the technical term for this information is the

[43] Pfäffingen and Breitenholz are districts in the municipality of Ammerbuch; ☞http://de.wikipedia.org/wiki/Ammerbuch#Gemeindegliederung.

proposition (Proposition) expressed by the sentence. The truth values in (2) may differ because the propositions expressed in (1) do.

What is a proposition? What is the information contained in, or conveyed by, a sentence? To answer this question consider the following sample sentences:

(3) 4 fair coins are tossed

(4) At least one of the 4 tossed coins lands heads up

(5) At least one of the 4 tossed coins lands heads down

(6) Exactly 2 of the 4 tossed coins land heads up

(7) Exactly 2 of the 4 tossed coins land heads down

(3) is the least informative of the five sentences, because it does not tell us anything about the result of the tossing. The other sentences are more informative in this respect. (4) is less informative than (6), though; and (7) is more informative than (5). Presupposing that each coin either lands heads up or down (thereby excluding a third possible outcome of the tossing), (6) and (7) are equally informative. Whether (4) and (5) are also equally informative depends on our understanding of "informative": in a certain sense, both contain the same *amount*, or *quantity*, of information. But qualitatively, they are of course totally different. To see this, assume that all four coins land heads up. Then (4) is true, but (5) is false. According to the so-called **most certain principle of semantics**, repeated in (8),

(8) If a sentence A is true but a sentence B is false in the same situation, then A and B cannot have the same meaning.

we must conclude that although the *amount* of information might be the same, the meanings are still different. And so are the propositions expressed by the two sentences, if the propositions are what makes up the contribution of the embedded sentence in an attitude report. To see this consider John again:

(9) John knows that at least one of the 4 tossed coins lands heads up

(10) John knows that at least one of the 4 tossed coins lands heads down

If (4) and (5) expressed the same proposition, then a substitution argument along the lines of the previous section would show that (9) and (10) coincided in their truth values—which they do not have to. Hence, to be on the safe side, we better make sure that the propositions expressed by (4) and (5) differ even though the sentences do seem to carry the same *amount* of information. In other words: when it comes to propositions, it is the *quality* of information that counts, rather than quantity.

Tossing coins is reminiscent of probabilities, and this is not accidental. As one can easily verify, the quantitative informativity of these sentences corresponds to the probability of a certain event taking place. This probability can be measured by counting the positive outcomes in relation to all possible cases. Moreover, a sentence A is *more* informative than a sentence B if the *number* of cases that make A true is *smaller* (!) than the *number* of cases that make B true.

On the other hand, the *qualitative* differences between the sentences depend on the *kind* of situations that make sentences come out true or false. Thus, the qualitative difference between (4) and (5) consists in the fact that the positive cases, i.e. the cases that make the respective sentence true, are not the same, although their number may be. Generalizing from the example we may say that two sentences that are qualitatively equally informative apply to exactly the same cases; their **informational content** can be identified with the set of cases that make the sentence true. This content is usually identified with the proposition:

(11) The **proposition** expressed by a sentence is the set of possible cases of which that sentence is true.

Returning to the coin tossing scenario, let us chart the possible cases according to the outcome for each of the four coins c1-c4 (arranged in the order of tossing, say):

(12)

assignments = possible cases	sentences c1	c2	c3	c4
1	1	1	1	1
2	1	1	1	0
3	1	1	0	1
4	1	0	1	1
5	0	1	1	1
6	1	1	0	0
7	1	0	1	0
8	0	1	1	0
9	1	0	0	1
10	0	1	0	1
11	0	0	1	1
...	...			
15	0	0	0	1
16	0	0	0	0

EXERCISE 14:
We leave it to the reader to fill in the missing rows in (12).

The table just lists all possible outcomes of a tossing; each of the sixteen different rows represents one type of situation. Now, the informational content of, and hence the proposition expressed by, the sentences (3) to (7) can be identified with the following sets of possibilities:

(13) a. At least one coin lands heads up = {1-15}
  b. At least one coin lands heads down = {2-16}
  c. Exactly 2 coins land heads up = {6-11}
  d. Exactly 2 coins land heads down = {6-11}
  e. Exactly one coin lands heads down = {2-5}

EXERCISE 15:
Continue:

(14) a. Exactly one coin lands heads up = ?
  b. c3 lands heads down = ?
  c. All coins land heads down = ?

It would now seem that the propositions expressed by (13-c) and (13-d) are identical. But this is not really so. In order to fully capture the meaning of the sentences, we need to consider more possibilities. Above, we simply didn't take enough cases into account; we only looked at scenarios with exactly 4 coins being tossed. In a situation with five (or more) coins being tossed, the set of outcomes to be considered is much larger, although the quantitative amount of information expressed by the two sentences still remains the same. In such a situation, the proposition expressed by the sentences differ, which can easily be checked by imagining a possible situation in which (13-c) is true and (13-d) is false.

The point just made is much more general, which becomes clear when we turn to sentences like:

(15) a. 4 coins were tossed when John coughed
  b. 4 coins were tossed and no one coughed

Again, (15) shows that, in general, we need to distinguish more cases —not only according to the outcome of coin tossings but also according to who happens to be coughing. Hence, it would seem, we need to add more and more columns to the table—one for each potentially coughing person—thereby dramatically increasing the number of rows, i.e. cases, and their length. Since the examples (15) were quite arbitrarily chosen, it is clear that the number of cases needed to distinguish any two non-synonymous sentences by the cases they apply to, must be very large indeed. And the cases themselves must be differantiated in quite varied and arbitrary ways. In fact, the cases needed appear unlimited in their fine-grainedness and scope, thus amounting to complete descriptions of arbitrary **states of affairs**. Any such complete description must take into account anything we can conceive of: the number of hairs on my head, the names of all my ancestors, the position of all atoms in our world etc. Such a completely specified possible state of affairs is also called a **possible world**. In effect then, we can say that only sufficiently many cases can capture all potentially relevant aspects of (literal) meaning. We therefore identify the meaning of a sentence with a sufficiently large set of possible worlds, namely the set of **all** possible worlds of which the sentence is true.[44]

In sum, then, **propositions are sets of possible worlds**. And possible worlds are highly specific, completely specified (and very big) situations where every possible case is determined. As Wittgenstein (with whom (11) originates), puts it: "die Welt is alles, was der Fall ist" (*The world is all that is the case*).[45]

The switch from situations to worlds induces a change in notation. From now on, we use w (for world) rather than s (for situation) as an index on the extensions $[\![\ldots]\!]$. By definition, then, $[\![S]\!]_w$ is the denotation or extension of sentence S in a world w, namely its truth value in w. We may now reformulate (11) as follows:

(16) The **proposition** expressed by a sentence is the set of possible worlds of which that sentence is true.

(17) A sentence S is true of a possible world w if and only if $[\![S]\!]_w = 1$.

(18) By $[\![S]\!]$ we mean the proposition expressed by S:
$[\![S]\!] := \{w : [\![S]\!]_w = 1\}$

Hence it follows that

(19) A sentence S is true of a possible world w if and only if $w \in [\![S]\!]$.

Or symbolically:

(20) $[\![S]\!]_w = 1$ if and only if $w \in [\![S]\!]$.

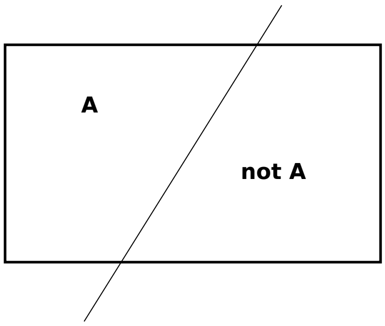
This is basically the definition of Wittgenstein (1921) (= Wittgenstein (1922)) and Carnap (1947) (= Carnap (1972)). Adopting Wittgenstein's terminology, we may say that all possible combinations of circumstances (Sachverhalte) make up a **Logical Space** (logischer Raum) and each sentence cuts this space into two parts: in one part of which

[44]The attentive reader must have noticed that we say of a sentence that it is true or false *of*, rather than *in*, a world (or case, or situation). This is to avoid the unwelcome impression that, in order to be true or false, a sentence needs to be uttered in that world (or case, or situation). As a matter of fact, it does not even have to exist; or it may exist as a sound form carrying a different meaning.

[45]This is the first sentence in ☞Ludwig Wittgenstein's (1889–1951) *Tractatus logico-philosophicus*, arguably the origin of (16).

the sentence is true, in the other of which the sentence is false.

(21)



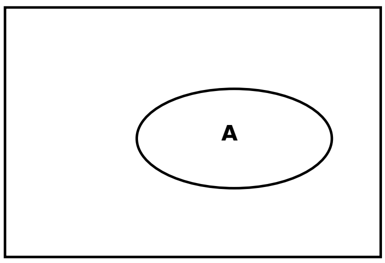
(21) cuts the Logical Space represented by the rectangle into two, the A-worlds and the not-A-worlds.

Actually, Wittgenstein identified possible worlds (his *possible states of affairs*) with sets of certain *elementary sentences* of an idealized language of science; Carnap followed him, but was more liberal as to the choice of language. The idea is that the sentences determining a world constitue a *state description* (Carnap's term) that leaves no (describable) facts open. Later advocates of Logical Space proposed to recognize possible worlds as independently given objects. Nevertheless there is considerable disagreement among philosophers about the ontological status of Logical Space: should we imagine possible worlds as abstract units of information or should we conceive of them as made up of atoms, in analogy to our real world? The most extreme (and provocative) position has been taken by the American philosopher David Lewis (1941–2001) in his 1986 book *On the Plurality of Worlds*.[46] Luckily, semantic analysis can proceed without resolving these difficult matters. So we will leave it at that.

Propositions as Sets

Note that we represented a proposition inside the box that displays a Logical Space in (21) by a region produced by a cut; in fact, however, the shape of that region plays no role when considered in isolation; we could equally well represent the divisision between worlds (the A-worlds and the non-A-worlds) as the difference between the region inside A and a region outside A in a drawing like (22):

(22)



[46]Cf. also ☞http://www.zeit.de/1999/44/199944.lewis1_.xml.

Suppose A is the proposition that S, for some sentence S. Inside A are the facts that make S true, this corresponds to ⟦S⟧. Outside A we have the facts that hold when ⟦not S⟧ is true. It's immaterial whether or not all A-worlds are in one circle, they need not occupy a connected region and could be dispersed in any regions of the Logical Space.

As a side effect of defining propositions as sets, we can now define the proposition "A and B" as an operation on sets, namely the intersection of A with B. This gives us a neat semantics for the conjunction of sentences in terms of the propositions they express:

(23) $[\![S_1 \text{ and } S_2]\!] := [\![S_1]\!] \cap [\![S_2]\!]$

That is, the set of worlds where "S_1 and S_2" holds is precisely the intersection of all worlds of which S_1 is true and all worlds in which S_2 is true. Likewise, disjunction expressed by "S_1 or S_2" corresponds to set-union; we leave this for the reader to verify.

6.3 From Propositions to Intensions

Let us now define the notion of intension. Previously, we identified the intension with the informational content of a sentence. We will now show that the intension of a sentence S is practically the same as the proposition expressed by S, but for reasons that will become clear in a minute our definition of intension is a little bit more involved.

Our starting point is again the box in (22) which we mean to depict Logical Space—a very large, possibly infinite set of possible words W, among them our actual world. Using a method encountered in connection with type-driven interpretation (cf. Section 5.7), the proposition in (22) can also be represented as a table in which every possible world is assigned a truth value:

(24)

world	truth value
w_1	1
w_2	0
w_3	1
...	...
w_n	0
...	...

This table represents the characteristic function of a proposition, i.e. a function assigning truth values to possible worlds. We will now declare the characteristic function of the proposition expressed by a sentence to be the *intension* of that sentence. For example the intension of the sentence

(25) Barschel wurde ermordet
Barschel[47] was murdered

is a function whose inputs are possible worlds and that assigs the value 1 to every world in which Barschel was murdered, and 0 to any other world.

The intension of a sentence shows how its truth value varies across Logical Space. Since the truth value of a sentence is its extension, we may say that *its intension is its extension as depending on the world.* The most interesting aspect of this characterization is that it carries over from sentences to arbitrary expressions like nouns, verbs, adjectives etc., the extensions of which we determined in the previous two sections. As we saw there, what the extension of a given expression is, depends on the situation. Having replaced situations with points in Logical Space, we can now define the intension of any expression E to be its extension as depending on the world, i.e. as the function that assigns to any world w in Logical Space the extension of E at w. As in the case of sentences, such functions may be represented by tables matching worlds (in the left column) with extensions (to their right). The following tables (27)-(29) indicate what the functions may look like in the case of the expressions in (26):

(26) a. The president snores
b. the president
c. snores

(27)

world	extension
w_1	1
w_2	0
w_3	1
...	...
w_n	the truth value 1 just in case the president in w_n snores in w_n
...	...

(28)

world	extension
w_1	George
w_2	George
w_3	Hillary
...	...
w_n	the person who is president in w_n
...	...

(29)

[47] Uwe Barschel (1944-1987) was a German politician (prime minister of Schleswig Holstein) who had to resign under scandalous circumstances (comparable to the Watergate affair) and who was found dead in the bathtub of his hotel room a few days after his resignation. The circumstances of his death could never be clarified. Cf. http://www.jurablogs.com/de/was-weiss-helmut-kohl-ueber-barschels-tod.

world	extension
w_1	{George, Hillary, Kermit}
w_2	{Bill, Hillary}
w_3	{Bill, George, Hillary}
…	…
w_n	the individuals who snore in w_n
…	…

Table (27) characterizes the proposition expressed by (26-a). Table (28) shows how the extension of the description (26-b) varies across Logical Space: whereas George is the president in worlds w_1 and w_2, Hillary holds this office in w_3. Similarly, table (29) shows how the extension of the intransitive verb (26-c) varies across Logical Space. Hence the three functions indicated in tables (27)-(29) represent the intensions of the expressions in (26). It is important to realize that the values assigned by the intension in (27) are always truth values, whereas those assigned in (28) are individuals, and the intension described in (29) assigns sets of individuals to words. Of course, this is as it should be: the extensions of the expressions differ in type (to adapt a term from Section 5.7), and their intensions must reflect these differences in the values they assign. On the other hand, all intensions have the same input—the worlds of Logical Space.[48]

The upshot is that we may define the intension of an expression α as a function that assigns an extension to each possible world. More formally, we can build up intensions from extensions in the following way:

(30) The intension of α, written as $[\![\alpha]\!]_i$, is that function f such that for every possible world w, $f(w) = [\![\alpha]\!]_w$.

According to (30), the intension of a sentence S is a function that assigns to S one of 1 and 0 depending on a possible world w. If the value of that function is 1 for a world w, then S describes a fact of w. If not, S does not describe a fact, and we say that S is false of w.

6.4 Composing Intensions

The above discussion has revealed that the concept of an intension applies to arbitrary expressions—as long as we can describe their extensions. Intensions can now be combined in order to form new intensions of complex expressions, in accordance with the

(31) *Principle of Intensional Compositionality:*

[48] This is actually a simplification to be withdrawn in Chapter 7, where different expressions are seen to call for different inputs to their intensions. Still, the difference is independent of the type of expression and its extensions.

> The intension of a complex expression is a function of the intensions of its immediate parts and the way they are composed.

Let us illustrate this with the following example:

(32) Paul schläft
Paul sleeps
'Paul is sleeping'

The intension of (32) is a function that assigns to any world the truth-value 1 if Paul is sleeping in that world, and false otherwise. How can this be calculated on the basis of the intensions of *Paul* and *schläft*? The intension of *schläft* works analogously to the intension of *president* above: it is a function assigning the set of sleeping entities to each possible world. What is the intension of *Paul*? In reality, the name refers to a particular person, namely Paul.[49] But what about other possible worlds? Could someone else have been Paul? Hardly. Of course, another person could have been *called* "Paul", but calling her or him Paul wouldn't make this person Paul. Paul could have had another name, but he would still be Paul. When considering the possibility that Paul's parents almost called him "Jacob", we would have to say that Paul (and not Jacob) could have gotten another name different from the one he actually has. Thus, with the name "Paul" we always refer to the same person—regardless of what this person would be called in other circumstances.[50] We conclude from this that the intension of *Paul* looks like this:

(33)

world	entity
w_1	Paul
w_2	Paul
w_3	Paul
...	Paul
w_n	Paul
...	Paul

This table of course reveals a certain redundancy because the extension of the name does not depend on any of the worlds. Still, for systematic reasons we do assume that all

[49]Of course, there are millions of people called "Paul". The convention is that if in a given situation only one person comes to mind, then we can use the name to refer to that person. In a text-book context, however, no such specific context is given. Here the convention is that *Paul* should really be understood as Paul_i, which the index i disambiguating between the millions of Pauls by saying to which of them the expression is intended to refer.

[50]This interesting observation about names was overlooked in many philosophical discussions prior to the pioneering work of Saul Kripke (1972) (= Kripke (1993)). Alas, it has also been disputed, notably by the Dutch semanticist Bart Geurts, who brought up the following type example: *If a child is christened 'Bambi', then Disney will sue Bambi's parents.*

names, like all other expressions, have an intension which determines the extension at every possible world. The only difference from other expressions is that this is a *constant* function that yields the same individual for each possible world.

How can we combine these intensions in order to get a complex intension? This is quite simple. We only have to compute the extensions for every possible world. Having done so we get another table, which for each world w contains a row with the value 1 if and only if $[\![\text{Paul}]\!]_w \in [\![\text{schläft}]\!]_w$. This new table is again an intension, i.e. a function that assigns truth values to each possible world. This way, the combination of intensions is reduced to the combination of extensions which has already been described in Section 5. It's only that the results of this computation for each possible world now make up a new intension, a function that assigns to each world w a truth value, namely the result of calculating the extension in w. The new intension is thus calculated in a "pointwise" manner, with the "points" being the extension in each point in the Logical Space (i.e. each possible world).

Let us now, after this long detour, return to our analysis of:

(34) a. John knows that [Hamburg is larger than Cologne]
b. John knows that [Pfäffingen is larger than Breitenholz]

It is clear by now that the embedded sentences express different propositions. The Principle of Intensional Compositionality (31) says that the intensions of the embedded sentences are responsible for the difference in meaning of the entire sentences. Hence, the key to the solution of our problem must be that the object of *know* (the *that*-clause) is an intension (rather than an extension, i.e. a truth value).

What, then, is the extension of the verb *know*? Earlier we assumed that the extension of a transitive verb is a set of ordered pairs of individuals. For a verb like *know*, however, it ought to be clear by now that we need a relation between an individual (the subject) and a proposition (representing the meaning of a sentence). In other words, for in (34) to be true, the extension of the verb *know* would have to contain the pair consisting of Paul and the intension of the embedded sentence. Quite generally, it would have to consist of pairs of persons and (characteristic functions of) propositions.

Now, given that the embedded sentences in (34) are not synonymous, we can easily imagine a possible world of which (34-a) is true but (34-b) is false, so that John might know one proposition but not the other. In other words, what we proposed above as the truth conditions for *Johann weiß dass p* (= *John knows that p*) is the following:

(35) $[\![\text{Johann weiß dass } S]\!]_w = 1$ iff $\langle [\![\text{Johann}]\!]_w, [\![S]\!] \rangle \in [\![\text{weiß}]\!]_w$.

Now, if p and p' are different propositions it might well be that $\langle [\![\text{Johann}]\!]_w, \text{p} \rangle \in [\![\text{weiß}]\!]_w$ and $\langle [\![\text{Johann}]\!]_w, \text{p'} \rangle \not\in [\![\text{weiß}]\!]_w$.

To arrive at (35), we still need a rule for constructing the extension of the VP weiß dass S:[51]

(36) $[\![\text{attitude verb + object clause}]\!]_s := \{x : \langle x, [\![\text{S}]\!]\rangle \in [\![\text{attitude verb}]\!]_s\}$

Using (36), we may go on with our extensional subject-predicate treatment (16) from Chapter 5 and derive (35). We leave this to the attentive reader, who will have noticed that (36) is rather similar to (29)–also from the previous chapter—, where a transitive verb was combined with its object (in that order, which is not an issue here). Of course, this similarity is not coincidental. After all, both rules are dealing with transitive verbs and their complements, which only happen to be of a different kind. However, there is another, more interesting difference between the clausal case (36) and its nominal analogue (29): whereas the latter had the object feed its extension to the extension of the VP, the complement clause contributes its intension. This, then, is how the compositionality challenge from Section 6.1 is finally solved: whereas normally, i.e. in extensional environments, the extension of a compound expression can be obtained by combining the extensions of its parts, in intensional contexts one of the parts contributes its intension instead. It should be obvious that this does not affect the overall nature of the process of composing meanings, i.e. intensions: as in the extensional environment considered at the beginning of this section, (36) allows a pointwise calculation of the intension of the VP, by determining its extension world by world. But whereas both the verb and the object clause contribute their intensions, only part of the verb intension–viz. its extension–is relevant when it comes to determining the extension of the VP at a given point; in this respect the verb behaves like any constituent in an extensional environment. The complement clause, however, always, i.e. at each point (possible world), contributes its full intension, which is what makes the construction intensional.

EXERCISE 16:
We leave it to the reader to reformulate (36) in analogy to the rules in section 5.

Summarizing so far, all we did was replace the extension of the complement of *know* with its intension. Therefore the extension of the complex sentence is not calculated on the basis of the extensions of its parts, but in the case at hand on the basis of the intension of the sentential complement. As will be seen in connection with clausal connectives in Section **??**, most connectives are not truth-functional, which means that their interpretation is based on the intensions of their complements, rather than their truth values. This also applies to all verbs and adjectives that take sentences (or infinitives) as

[51] Since the complementizer *dass* does not seem to have a meaning of its own, we treat it as part of the attitude verb; this is done purely for convenience.

their complement.[52]

As another case in point, consider (37):

(37) Paul seeks an exciting German detective story

The problem is the following: assume that the actual extension of *exciting German detective story* is actually the empty set, so there is no exciting German crime novel. Assume further that there is no cheap French Bordeaux wine either. Then the two predicates have the same extension, namely the empty set. Nonetheless we cannot conclude from (37) that (38) holds.

(38) Paul seeks a cheap French Bordeuax wine

This, however, should be the case if the Principle of Extensional Compositionality were applicable. From this we must conclude that the verb *seek* creates an intensional context, turning the object of *seek* into something intensional (the different intensions of the properties mentioned above). This may then also explain the ambiguity of examples like the one discussed in section 3.4.3.[53]

6.5 Intensions and Sense Relations

When talking about sense relations in section 2.3 we had not yet introduced sets. From a set theoretical point of view, however, it is easy to see that many sense relations can be represented as set theoretical relations between extensions. For example, the hyponymy relation between *man* and *human being* can be represented as a subset relation between the extensions of $[\![\text{man}]\!]_w$ and $[\![\text{human being}]\!]_w$. One is tempted, therefore, to say that a noun phrase A is a hyponym of a noun phrase B if and only if $[\![\text{A}]\!]_w \subseteq [\![\text{B}]\!]_w$.

However, we did not present it that way because there is a potential danger of misunderstanding: the correspondance between sense relations and relations between sets needs further qualification. For it could well be that $[\![\text{A}]\!]_w \subseteq [\![\text{B}]\!]_w$ without A being a hyponym of B. For example, assume that each professor is an adult, though *adult* is not a hyponym of *professor*. The fact that there is no hyponymy here reveals itself in that it is not *inconceivable* that there are younger than adult professors. Hence sense relations are not a matter of extension. Rather they are a matter of Logical Space. Since it is conceivable that some professors are non-adults, this is reflected in Logical Space in that some possible worlds are inhabited by underage professors. Although it is clear that such worlds are purely hypothetical (Denkmöglichkeiten) the mere existence of such worlds also blocks inferences. For example, from

[52]An exception is adjecives (and corresponding verbs) like those occuring in *it is **true** that …, it **holds** that …*, etc.

[53]An explanation of the ambiguity along these lines can be found in Quine (1960) (= Quine (1980)).

(39) Saul Kripke is a professor.

we cannot *logically* infer

(40) Saul Kripke is an adult.

although in our world this is more than probable (it's a practical inference, not a logical one). The reason is that it is at least conceivable that Saul became professor before reaching the age of adults (18 years in Germany). This means that the set of possible worlds that validate (39) is not a subset of possible worlds that represent (40).

On the other hand, from

(41) This is a bus.

we can validly infer

(42) This is a vehicle.

The proposition (41) is a subset of (42), and this is so because this time there is no possible world in (41) that is not contained in (42). And this is so because there is no possible extension of *bus* that is not contained in the extension of *vehicle*. In consequence, when depicting sense relations as relations between sets, we must always additionally keep in mind that these relations between extensions must hold in all possible worlds:

(43) A is a hyponym of B if and only if $[\![\mathrm{A}]\!]_w \subseteq [\![\mathrm{B}]\!]_w$ **in all possible worlds** w.

In fact, sense relations are *intensional* relations, and it is for this reason that we felt reluctant to represent them as a mere relation between extensions.

We also demonstrated above that there is a close connection between inferences between sentences and sense relations. Let us look at another example. Usually it is assumed that *cat* and *dog* are incompatible. Two propostions are incompatible if and only if they do not share a common world, and likewise the sense relation of incompatibility can be represented by two extensions that do not share an element. For the sense relation to hold, this non-overlap must hold in every possible world. Among others, the following inferences should be valid:

(44) Fido is a dog
$\vDash$ Fido is not a cat

(45) Fido is a cat
$\vDash$ Fido is not a dog

Valid inferences are marked by $\vDash$ in classical logical notation; an inference is an entailment relation between sentences. The line containing $\vDash$ expresses the inference or the conclusion; the premisses of the inference are the sentence(s) above $\vDash$. In general, if Σ is a set of sentences, then we write

(46) Σ
$\vDash \beta$

if and only if β is true in all possible worlds that make all sentences in Σ true; that is, if and only if there is no possible world in which all sentences of Σ are true and β is false.

Returning to (45), *cat* and *dog* are natural kind terms and it is in agreed among philosophers that if something is a cat in a possible world, it is a cat in every possible world and therefore could not be a dog, and vice versa (see eg. Kripke (1972) and Putnam (1975) for an extensive justification). Hence, the set of cats and the set of dogs are disjoint, as are the set of worlds that satisfy (45) and (46). In terms of disjointness, there is again a parallelism between sense-relations and inference relations.

In fact, it is often assumed that we only have intuitive access to sense relations of this sort via inference relations between propositions. If certain inferences hold, this justifies the existence of certain sense relations. This way, the intuitive evaluation of inferences may take priority over that of sense relations: the justification of certain sense relations procedes via that of inferences, and these in turn rest on our intuitions about conceivable possible worlds (or at least conceivable situations; but after all, worlds are only large situations and nothing prevents very small worlds (cf. ☞www.bussongs.com/songs/its_a_small_world_after_all_short.php).

6.6 Sentence Semantics and Lexical Semantics

The above remarks suggest that sentence semantics and the formal study of meaning in truth-functional semantics also provides the ground for the study of the meaning of words. In a sense, however, this conclusion is somewhat premature. Consider the case of *cat* and *dog* again. From sentence semantics we've learned that each extension is a set. So the intension of *dog*, eg., is a function that assigns to each possible world a certain set. Of course we have required that this be the set of dogs. But we also argued that we do not know the extension in a large enough situation, let alone in our world or even in a possible world. In fact we can only *stipulate* certain extensions for possible worlds. Therefore, in a formal semantics the interpretation functions are not by themselves restricted; it is normally assumed that any such function can do the job, and that restrictions must be stipulated.

But this also means that in a formal model (i.e. a function that specifies the intensions of all expressions of the language under consideration), no special restrictions on the intensions of lexical items hold *per se*, except for the intensions of logical expressions like *and, or, all*, etc. This in turn means that without any further restrictions the semantic model tells us nothing about sense relations; rather, at the outset all lexical items are interpreted independently from each other and no such relations are automatically built into the model.

This is rather bad news. The formal models we develop cannot be claimed to be models for natural semantics unless we specify sense relations. Practically, this is done only if necessary for particular descriptive purposes, by stipulating so called **meaning postulates**. These are sentences of the language under investigation that are assumed to be valid in all models for natural language; the postulates exclude certain possible worlds from being conceivable, and they thereby restrict the interpretation function. For example,

(47) No cat is a dog

is a reasonable meaning postulate in every model for natural language. That is, (47) considered as a meaning postulate must be true in all possible worlds. This of course also restricts possible intensions: (47) excludes all extensions where the noun denotations overlap (in which case (47) would be false).

So the disappointing message is that at the end of the day sentence semantics tells us almost nothing about the meaning of individual words. In particular, it cannot in full answer the question "What Is Meaning?" (although the title of a semantics introduction by a prominent semanticist might suggest otherwise).

On the other hand, semanticists generally do not bewail this unfortunate state of affairs. Why not? For one thing, sentence semantics may still provide the tools for the analysis of interesting lexical items, for example modal verbs like *must*, *can*, *ought*, *might*, and others. (cf. eg. Lewis (1973), Kratzer (1977), or Kratzer (1981)). As a simple example for the kind of semantic analysis we have in mind, consider the verb *know*: we already analysed its extension as a relation between a subject and a proposition. Imagine one utters (48-a) truthfully. One could not at the same time deny the truth of (48-b)

(48) a. Mary knows that Bill snores
  b. Bill snores

It would thus be a contradiction to say

(49) #Mary knows that Bill snores, but Bill doesn't snore

Therefore (48-a) entails (48-b). This is specific to the verb *know*; the inference does not hold with a verb like *believe*:

(50) Mary believes that Bill snores
  $\not\models$ Bill snores

It is fully consistent to say:

(51) Mary believes that Bill snores, but (in fact) Bill doesn't snore

We therefore seem to have missed something in our account of the meaning of *know*. One possible way to refine our analysis is to say that we have to restrict the interpretation function for *know* in such a way that the inference from (71) to (48-b) becomes valid. This could be done in the following way:

(52) For all individuals x, for all propositions p, and for all possible worlds w: if $\langle x, p\rangle \in [\![\text{know}]\!]_w$, then $w \in p$.

This says that if p is known (by x) in a world w, then this world w is in p, which means that p is true in w. Therefore we can infer (that) p from x knows that p:

(53) x knows that p
$\vDash p$

This is a crucial restriction on the interpretation function; otherwise, without the restriction, the inferences would be invalid (contrary to fact) and we would have missed one aspect of the meaning of *know*.[54] But this entailment is not due to any sense relations of the classical type. Rather it concerns the relation between sentences and lexical items, and it is for this reason that sense relations are only a very restricted and limited area of semantic relations. It is for this reason that sentence semantics is much more general than lexical semantics and that the methods developed for sense relations do not carry over to sentence semantics, let alone to a more general semantic theory.

Another reason why semanticists do not bother much about sense relations is this: even when restricting ourselves to the comparatively simple task of finding the correct type of extension for certain syntactic classes of expressions, there are all kinds of problems of a non-trivial nature that remain. Eg., recall from Section 5.6 the semantics of adjectives and the problems exemplified in:

(54) a. Every small elephant is a small animal
b. Every big midget is a big entity
c. Every alledged murderer is a murderer

Any of these sentences would come out as always true, if adjectives are boolean intersective operations. As already discussed, this cannot be correct, but the relevant issue now is that it's not that our model contains too many possible worlds: restricting the set of possible worlds will not help for the above examples as long as the semantics for

[54]There are further conditions that one might impose; the pioneering work in this area has been done by Jaakko Hintikka (1962). These conditions on the interpretation functions all have the effect of rendering certain entailments valid. The study of the relation between these valid inferences and the conditions we can impose on the interpretation functions is sometimes called **Intensional Logic**.

adjectives is intersection. Again, something has gone fundamentally wrong here, and it's not always easy to fix the problem.[55] There are many similar question, eg. when considering comparatives, the semantics of modal verbs and the interaction of modals with comparatives (as in ambiguous sentences like *he eats more than he was allowed to*).

In general, many problems still await for a solution, even with only moderate aims in mind. In particular, these problems concern grammatical categories (comparatives, mood, tense, causality etc.), rather than the content of run of the mill lexical items like *walk, steal* or *president.* As an example we will discuss the semantics of tense in the next subsection.

6.7 Tense, Time and Logic*

In this section we will briefly comment on an additional component of propositions that has hitherto be ignored. Suppose I utter

(55) I have a beard

And twenty minutes later, I say

(56) I don't have a beard

This sounds like a contradiction, but in fact it isn't. For suppose I shaved between the utterances. Then both sentences should be true. But of course they are true only at the time of utterance. This reveals that time is an important issue in our reasoning with sentences. In fact, even Frege, who was mostly concerned with mathematical, hence eternal truths, acknowledged that reference to time should be part of any proposition.[56]

Technically this is usually achieved by making interpretations time-dependent. Previously, interpretations assigned an extension to every possible world, now interpretations assign extensions to every pair ⟨w,t⟩ consisting of a possible world w and a moment of time t (or sometimes a time interval). We thus assimilate moments of time (or time intervals) to possible worlds in that assignment functions (which determine the extensions) not only depend on a world w but also on a time t. This is needed in order to express something like:

(57) a. $[\![$It will be the case that p$]\!]_{w,t}$ = 1 iff there is a moment of time t' after t, such that $[\![\text{p}]\!]_{w,t'} = 1$.

b. $[\![$It was the case that p$]\!]_{w,t}$ = 1 iff there is a moment of time t' before t, such that $[\![\text{p}]\!]_{w,t'} = 1$.

[55] A locus classicus where these problems were discussed is Kamp (1975).

[56] "Alle Bestimmungen des Orts, der Zeit, u.s.w. gehören zu dem Gedanken, um dessen Wahrheit es sich handelt." From Frege (1893), p. xvii.

It is tempting to analyze sentences like (58):

(58) John will come

by analyzing their paraphrase (59) along the above lines.

(59) It will be the case that John comes

Analyses along these lines have been proposed in philosophical logic, mainly relying on **translations** of sentences into some logical notation, presupposing that the meaning of the formulae (their semantic interpretation) and that of their natural language analogues coincide. To a linguist, however, this is not satsifactory. For one thing, linguists would not be content to merely find some translation that seems to work on intuitive grounds alone. What they would like to see is an explicit translation procedure that starts off with natural language syntax, rigorously applies transformational processes, and finally ends up with the formulae proposed by philosophers, logicians or mathematicians on intuitive grounds. Moreover, and more often than not, linguists detect inadequacies in intuitive formalizations, due to a mismatch between the proposed formal system and natural language. Let us briefly discuss two examples.

(60) Everyone will win

According to the strategy sketched and relying on the above analysis (57-b), we would expect that this sentence comes out as meaning that there is a future time at which (61) holds:

(61) Everyone wins.

This, however, is not the most natural reading of (60). For then at some moment in the future it would have to be the case that everyone wins *at that moment.* So there must be several simultaneous winners—a rather implausible situation. Hence the preferred reading is that for every person there will be a (different) time t' after t so that that person is *the* winner at t'. In other words, in the preferred reading *everyone* has wider scope than *will.* This fact reveals that we have to analyze the internal structure of a sentence in order to capture the fact that *will* has wide scope over *win*, but not over *everyone*. Likewise, a sentence like

(62) Everyone went to school

suggests that we even have to look into the morphology of words in order to pick out the parts that are relevant for semantic interpretation.

As a second example, imagine a family going on a holiday; they were just leaving town in their car when Mary says:

(63) I didn't turn the stove off.

The information conveyed by the sentence should make Arthur turn around and drive back home again. Arthur, a famous logician, quickly paraphrases (63) as (64) and interprets it according to (57-b):

(64) It is not the case that at some moment in the past it holds that Mary turns the stove off

But this amounts to saying that Mary *never* turned off the stove before now. This is obviously false and therefore cannot be the intended meaning of the sentence. Arthur then concludes that he might have misconstrued the scope of negation, so the next formula he tries is this:

(65) For some moment in the past it does not hold that Mary turns the stove off

This looks much better: At some moment in the past it is not the case that Mary turns the stove off. However, thinking about the literal meaning of (65) it turns out that this is trivially true: it is obvious that there might be indefinitely many moments where Mary did other things than turning the stove off. So either the sentence is trivially false, or it is trivially true. In neither interpretation would the content of the sentence have the intended effect of making the logician drive home.

The example reveals that natural language does not work the way some logicians have predicted. This is an important insight. It tells us something about natural language that we might not have found out without any attempt of formalization. The example shows that something is going wrong in the way we conceive of tense, and that truth conditions like (57-b)—crucial in the formulation of formal languages of tense logic—are too simplistic.

However, it is not obvious which lesson is to be drawn from the example. An educated proposal for an intuitively correct paraphrase is this:

(66) At some relevant time interval before the utterance time (immediately before leaving the house) it's not the case at any moment within that interval that Mary turns the stove off.

The relevant interval mentioned in (66) (the relatively short interval before leaving) has been called the **reference time** (Betrachtzeit) by the German philosopher ☞Hans Reichenbach (1891-1953). He distinguished between reference time, **utterance time** (Äußerungszeit) and **event time** (Ereigniszeit). The reference time in the above example is partly determined by pragmatics, so the basic insight here is that tensed sentences can only be dealt with in a system that leaves some room for pragmatic considerations. However, there is a restriction on which time intervals can be relevant which is expressed by the grammatical tenses: The future tense expresses that the reference time is located anywhere on the time line after the utterance time, and the past tense expresses that the reference time lies anywhere before the utterance time. The event time then still has to be related to the reference time, this is done by a grammatical system called *aspect.* A sentence like

(67) John had slept for three hours when Jill came in

then expresses that the event time of sleeping is an interval (of three hours) that occured *before* the reference time of Jill's entering. In (63), however, aspect plays no role, so it follows that the event time is located somewhere within the reference time.

Reichenbach's analysis suggests that any system that treats tense operators as simple operators on propositions (as we did in our formal language above) cannot account for tense and aspect in natural language. He proposes that tense expresses a relation between moments of time and time intervals (rather than a relation between propositions, as in our formal language). The reference time is often pragmatically determined, but can also made explicit by the use of adverbials, as did the adverbial *when*-sentence in (67). This also explains the awkwardness of (68):

(68) *John slept tomorrow

The adverbial *tomorrow* says that the time of reference is the day after the time of utterance; on the other hand, the past tense contained in *slept* says that the reference time is before the utterance. This is a plain contradiction, which explains the unacceptability of (68). Cf. Reichenbach (1947) and Rathert (2004) for more on tense and aspect in logic and grammar.

6.8 From Intensions to Extension and Back Again

We end this chapter by taking a closer look at the distinction between extension and intension. As we mentioned at the beginning, there is an obvious reason why extensions cannot be, or adequately represent, meanings: as far as sentences are concerned, there are only two of them. Intensions clearly fare better in this respect: after all, there are infinitely many of them.

Another reason why extensions cannot stand in for meanings is that someone who learns the meaning of an expression does not automatically know its extension. Again, sentences may illustrate this point. Even if your German (or English) is perfect, you will not know the truth value of all German (English) sentences. (25) is a case in point: of course we all know its meaning, but its extension is known to only very few people (who have every reason to keep this a secret). However, do intensions fare any better in this respect too? In fact, does't anyone who knows the intension of (25) also have to know its extension? After all, given a table with truth values and possible worlds, we only have to pick out the actual world in order to arrive at the truth value. This is certainly so, but then how do we know which of the worlds is our actual one? In Logical Space, the input column to our intensions, possible worlds, are given as maximally detailed and specific states of affairs. Yet even if we know that any case in which Barschel committed suicide is one in which he was not murdered, we do not know which of these cases corresponds to what actually happened. And, to be sure, even if we did know he committed suicide, this would still not put us in a position to pick out the actual world. If we did, we would be omniscient. For there remain infinitely many details in the world we inhabit that we do not know about and that distinguish it from other points in Logical Space.

Knowing the intension of a sentence therefore does not imply knowing its truth value. It only involves knowledge of which *hypothetical* states of affairs would make it true. And this may well be an adequate criterion for understanding a sentence: "Einen Satz verstehen, heißt, wissen was der Fall ist, wenn er wahr ist" (*To understand a proposition means to know what is the case if it is true*; Wittgenstein 1921, Tractatus 4.024). Adopting this slogan we agree that the intension of a sentence exhausts its meaning, whose core thus turns out to be informativity.

The fact that intensions correspond to meanings, does not leave extensions without interest or importance, though: the extension results from the intension as its value for the actual world; it is thus what the intension is all about. However, in order to find out what the extension of an expression is, one must know enough about reality to identify it. So there is a road from intensions to extensions, and those who know enough about the actual world can see where it leads. This observation on the relation between extension and intension is captured by a famous slogan usually attributed to Frege: *Sense determines reference.* In our notation and terminology,[57] it boils down to the following general principle holding for any expression A and possible world w:

(69) *Frege's Assumption:*
The extension of A (at w) is determined by its intension $[\![A]\!]$, viz. as the value that $[\![A]\!]$ assigns to w.

Frege's Assumption thus paves the way from intension to extension. But is there also a way back? In a sense, there is. This is so because, by definition, the totality of possible extensions of an expression makes up its intension. We thus obtain a principle mirroring (69), which may be seen as one of Carnap's most fundamental (and most ingenious) contributions to semantic theory:

(70) *Carnap's Assumption*:
The extensions of A varying across Logical Space uniquely determine the intension of A, viz. as the set of pairs $\langle w, [\![A]\!]_w \rangle$ matching each world w with the extension of A at w.

[57] Some terminological clarification will be given at the end of the current section.

It should be noted that, while (69) has a unique intension determine a unique extension, (70) has the (unique) intension depend on the totality of extensions across Logical Space. The latter dependence is obviously much more involved than the former. In other words, the way from intension to extension is simpler than the way back. However, the former requires knowledge of all pertinent facts, the latter only purely conceptual knowledge of Logical Space.[58]

In terms of a corny metaphor, we have thus seen the extension and the intension of an expression are two sides of the one coin–its meaning; somewhat more precisely, the upshot of the assumptions above is that they represent two different but interdependent functions of meaning–reference and information. However, there is more to be said about the relation between the two. They also interact in complex ways when more than one expression is at stake. This aspect of the theory of extension and intension comes out most clearly in the way compositionality works. In Section 6.1, we had seen that the Principle of Extensional Compositionality fails when it comes to so-called *intensional contexts*. However, this does not mean that we have given up on compositionality altogether. In fact, in Section 6.4 we saw that intensions do behave compositionally. In effect, we have thus argued for the:

(71) *Principle of Intensional Compositionality*:
The intension of a compound expression is a function of the intension of its immediate parts and the way they are composed.

Now, given the near identification of intension and meaning suggested above, (71) comes out as a variant of (72), and is thus hardly a surprise:

(72) *Principle of Compositionality of Meaning*:
The meaning of a compound expression is a function of the meaning of its immediate parts and the way they are composed.

However, closer inspection of the way compositionality actually works in connection with intensional contexts, reveals that matters are more complex than these last two slogans suggest. Let us reconsider the original case of propositional attitude reports. In order to escape the embarrassment of substitution to which the Extensional Principle of Compositionality had led, we followed Frege's strategy of feeding the intension of the complement clause into the compositionality process where its extension (i.e. its truth value) failed. This was a strictly local repair in that we only replaced the extension of the embedded sentence by its intension, and left the rest of the compositional contributions

[58]Whereas the relation postulated in (69) is one of functional application (applying the intension to an input world), (70) employs an operation known as *functional abstraction*. (69) and (70) are brought out most clearly in Montague's *Intensional Logic*, where the two routes are expressed by logical operators known as Cup and Cap.

untouched: it is the *extension* of the verb phrase that is determined by the *intension* of the embedded clause and the *extension* of the attitude verb. This local shift from extension to intension allowed us to integrate the analysis into the composition of extensions which we have taken to work as before. It was only afterwards that we noted that this overall strategy of combining extensions and, if need be (i.e. in intensional contexts), intensions conforms to (71). However, it seems that the compositionality process itself is better described by the following principle, which brings out more clearly that intensional contexts are treated by a kind of repair strategy:

(73) *Frege's Principle of Compositionality*:
The extension of a compound expression is a function of the (a) extensions or (b) intensions of its immediate parts and the way they are composed.

The principle, which is given in a rather imprecise form here (but could be made precise in an algebraic setting), is to be understood as allowing (a) and (b) to co-occur in the following sense: the extension of a compound expression may be determined by combining (a) the extension of one of its parts with (b) the intension of another one–just as we had it in the case of propositional attitude reports. It should be noted that (73) does not say that intensions are only invoked as a last resort, i.e. when extensions lead to substitution problems; this, however, is the way in which the principle is usually applied.[59]

As pointed out at the end of Section 6.4, the strategy of determining extensions of expressions from the extensions and/or intensions of their parts guarantees that intensions behave compositionally. In other words, (73) implies (71). One may wonder, though, whether (73) actually says anything over and above (71), or whether it only splits up intensional compositionality into two different cases, viz. (a) extensional vs. (b) intensional constructions.[60] There is, however, a slight difference between the very general principle (71) and Frege's more specific strategy (73) in that the latter, but not the former imposes a certain amount of homogeneity on the combinations of intensions: if the extension of an expression B remains stable across two worlds w and w', then according to (73), so does its contribution to the extension of a larger expression A at those worlds; however it would still be in line with (71) if the intension of A differed across w and w'. Although no natural combinations of intensions with this property are known to us, their very possibility and the fact that they are ruled out by (73) shows that Frege's strategy does restrict the possible combinations of intensions. [61]

[59] With notable exceptions, among them Montague (1973), where a default mechanism has expressions in argument positions always contribute their intension, whether or not extensional substitution fails.

[60] Note that a construction is already intensional if one of the parts contributes its intension, whatever the contribution of other part(s). As a matter of fact, syntactic environments that are intensional with respect to *all* (immediate) parts appear to be extremely rare–if they exist at all.

[61] Here is an artificially constructed example of a grammatical modifier Φ embedding sentences B (of a

Some historical and terminological remarks on the origin of the distinction between extension and intension may be in order. Even though we attributed some of the principal concepts and assumptions to Frege, his original writings do not conform to the terminology employed here. In particular, Frege used the German noun Bedeutung (in ordinary contexts translated as "meaning") as a technical term for reference and extension. According to this terminology, a sentence "means" (= bedeutet) its truth value—which is more than just a bit confusing; we thought this kind of a warning is appropriate for those who want to read Frege (1982) in the original; the English translations make up for the confusion by philological inaccuracy, rendering Bedeutung as "reference" or "denotation". In any case, most of what we have said about extensions and how to find them, goes back to Frege's work. However, and more importantly, while Frege's Bedeutung fully corresponds to our "extension", his Sinn ("sense") does not truly match what we have been calling the "intension" of an expression: a sense (in Frege's sense) is not a function whose values are extensions. It is is not at all easy to explain (and understand) just what exactly Fregean sense is, and we will not even try so here but refer to the extensive literature on Frege's work.[62] We would still like to point out that our loose identification of Fregean senses with intensions is not completely arbitrary; they do have a lot in common. To begin with, even though senses are not functions, they do determine extensions relative to facts, thereby justifying the attribution of (69) to Frege. Moreover, the way extension and intension interact compositionally, is precisely as Frege had de-

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non-existing language, to be sure):

(*) $[\![\Phi B]\!]_w = 1$ if and only if $[\![B]\!]\,(w) = [\![B]\!]\,(w_0)$,

where w_0 is the actual world. Here Φ is not an expression on its own but indicates a particular kind of embedding (realized by some morpho-syntactic process like inversion, say). Hence the embedding is a unary construction, with B being the only immediate part of ΦB. (This aspect of the example is merely to avoid unnecessary distractions.) It is easy to see that (*) conforms to the Principle of Intensional Compositionality: the intension of the function F an expression ΦB is determined by the function F that assigns to any (characteristic function of a) proposition p the intension of the (characteristic function of) the set of worlds in which p has the same truth value as in the actual world; applying F to $[\![B]\!]$ then results in the intension described by (*).

On the other hand, (*) is not in line with (73), as can be seen from considering a particular sentence B_0 that expresses the proposition which is only true at the actual world. (Again, this assumption is made for convenience and dramatic effect; the reader is invited to find a more natural example.) Now, clearly, the the truth value of a sentence of the form ΦB, cannot be determined from the the truth value of its only immediate part B: at any non-actual world w, B_0 has the same truth value as a contradictory sentence $\perp$, but according to (*), ΦB_0 comes out false whereas $\Phi\perp$ is true. However, the truth value of ΦB, cannot be determined by merely looking at the intension of B either: ΦB_0 is true at w_0 and false at any other world, but the intension of its sole immediate part B is always the same.

[62]The account in Stepanians (2001), Chapter 7, is particularly recommendable. We should also mention that there are more differences between Frege's and Carnap's distinction than we have mentioned here, mostly due to the fact that Frege's underlying notion of a function was not the set-theoretic one, which he explicitly rejected. Again, we have to refer the interested reader to the literature.

scribed his Bedeutungen and Sinne at work–with the (somewhat confusing) twist that he also used the term Bedeutung for the compositional contribution a part makes to the extension of the whole (compound) expression. Keeping this in mind, it is certainly fair to attribute (73) to Frege, too.

Although the terms have had some currency among philosophers of language and logicians before, the distinction between extension and intension as we have been using it, ultimately goes back to Carnap (1947) (= Carnap (1972)), who had already pointed out the connection with, and differences to, Frege's distinction. Consequently, this approach to meaning has been dubbed "Frege-Carnap semantics". It has become popular among linguists ever since Montague (1973) showed how it can be generalized beyond the restricted phenomena analyzed by its founders. However, the approach is not without its rivals, even within the tradition of analytic philosophy. In particular, Bertrand Russell had criticized Frege as needlessly complicating matters by splitting up meaning into two dimensions. Instead, he developed a framework for semantic analysis that is based on one kind of semantic value only. As it turns out, the two approaches are intertranslatable, preferences for one or the other being largely a matter of taste; cf. **?** for details.

7 Presuppositions

7.1 The Definite Article

In Section 4.2 we noted that referential expressions do not always have referents, Russell's classical description *the present king of France*, being a case in point. However, when it came to composing extensions and to constructing intensions, we chose to ignore this complication. In the present chapter, we will revise this decision and scrutinize the very notion of a referential gap. In doing so, we will find that referential gaps are part of a more general phenomenon. But before we get to this, we will address the seemingly simple question of whether failure of reference implies lack of extension. As it turns out, this is not obvious. To achieve clarity on this point, let us look at an empty description in the context of a sentence—the original example from Russell (1905):

(1) The present king of France is bald.

Given the political facts of our actual world at the time of writing, the subject of (1) has no referent. As a consequence, (1) cannot be true. It is thus tempting to conclude (as Russell did) that the sentence is *false*, with 0 as its extension. Interestingly, this conclusion has repercussions on the question of what the extension of the subject in (1) is: if the truth value of the sentence is to be derived by the Extensional Principle of Compositionality (= (11) from Chapter 5, repeated below for the readers' convenience), then the extension of the subject cannot be its referent—simply because it has no referent.

(2) *Extensional Principle of Compositionality:*
The extension of a compound expression is a function of the extensions of its immediate parts and the way they are composed.

The fact that the subject of (1) has no referent may suggest that it has no extension either. However, it may be assigned an extension if is treated as a quantifier. This extension would have to be specified so that (1) does come out false. Given the interpretation of sentences with quantificational subjects, this means that it must not contain the extension of the predicate of (1). In fact, it should not contain the extension of *any* predicate, because *no* sentence with the same subject as (1) should come out as true. We thus venture the following **quantificational analysis** of the extension of the latter at our actual world w^*:

(3) $[\![\text{the present king of France}]\!]_{w^*} = \emptyset$

(3) only captures the extension of the definite description under scrutiny at the actual world. In order to account for its *intension*, a more general equation must be found, covering its extensions across Logical Space. Given an arbitrary possible world w, two cases need to be distinguished according to whether *the present king of France* has a referent: if not, then (3) carries over from w^* to w:

(4) $[\![\text{the present king of France}]\!]_{w} = \emptyset$

If, on the other hand, the French do have some person k_w as their king in a given world w, the extension sought can be constructed following the strategy laid out in Section 5.5, as the set of all sets (of individuals) that contain k_w as a member.

(5) $[\![\text{the present king of France}]\!]_{w} = \{X : \text{k}_w \in X\}$, where k_w is the only member of $[\![\text{present king of France}]\!]_{w}$.

Both cases, (4) and (5), may be captured at a fell swoop—and in a variety of ways:

(6) $[\![\text{the present king of France}]\!]_{w} =$
$\{Y\text{: for some } x,\ [\![\text{present king of France}]\!]_{w} = \{x\} \text{ and } x \in Y\} =$
$\{Y\text{: for some } x,\ [\![\text{present king of France}]\!]_{w} = \{x\} \text{ and } [\![\text{present king of France}]\!]_{w} \cap$

$Y \neq \emptyset$ } =
{Y: for some x, $[\![\text{present king of France}]\!]_w = \{x\}$ and $[\![\text{present king of France}]\!]_w \subseteq Y$ }

We leave it to the reader to verify that the above chain of equations is correct.

According to (6), then, a predicate extension Y is a member of the extension of Russell's famous description if it meets two conditions: (i) that the extension of the noun phrase *present king of France* be a singleton; and (ii) that the latter be a subset of Y. Obviously, (i) is a degenerate condition in that it is independent of Y: if one set meets it, any set does; and if some set does not meet it, none does. In this second case, condition (ii) does not play a role because (i) alone makes the extension of the description literally empty, ultimately leading to the falsity of sentences like (1). We already noted this effect in connection with (4), which is a special case of (6).

By treating the subject of (1) as quantificational, (6) manages to assign an extension to it in a uniform way and independently of whether it has a referent. Moreover, (6) can be extended to a fully compositional interpretation, combining the extensions of *present king of France* with those of the definite article.[63] In fact, the latter still need to be adapted to fit our quantificational analysis of definite descriptions. Recall that we already analyzed the extensions of determiners as relations between two sets (of individuals). In the case at hand, the sets related are the extensions X of *king of France* and of Y of *[is] bald*. According to (6), the relation expressed by *the* holds if and only if Y meets the above-mentioned conditions that (i) X is a singleton and (ii) a subset of Y. Generalizing from the particular predicate, we may thus characterize the extension of *the* (at any world w) as follows:

(7) $[\![\text{the}]\!]_w = \{\langle X, Y\rangle : \text{for some } x,\ X = \{x\} \text{ and } x \in Y\} =$
$\{\langle X, Y\rangle : \text{for some } x,\ X = \{x\} \text{ and } X \cap Y \neq \emptyset\} =$
$\{\langle X, Y\rangle : \text{for some } x,\ X = \{x\} \text{ and } X \subseteq Y\}$

Condition (i) can be formulated in a variety of ways, too. In particular, instead of quantifying over the members of the nominal extension X, one may restricts its **cardinality** $|X|$, i.e. the number of its elements: $|X| = 1$. Moreover, it is customary to split up this cardinality condition into two parts: (i-a) that X has at least one members; and (i-b) that X has at most one member. We will follow this convention and, from now on, adopt (8) as our "official" quantificational analysis of the definite article:

(8) $[\![\text{the}]\!]_w =$

[63]The basic strategy of analyzing definite descriptions as quantifiers goes back Russell's criticism of Frege's naive analysis (see below), against which Russell (1905) brought forward a number of arguments—some good, some debatable, some confused. The compositional formulation of Russell's theory of descriptions goes back to Montague (1973).

$\{\langle X, Y\rangle : |X| \geq 1 \text{ and } |X| \leq 1 \text{ and } X \subseteq Y\}$

Since condition (i-a) amounts to there being members of X, it is also known as the **existence condition**; and (i-b) is called the **uniqueness condition**.

Using (8), (6) can be derived by the extensional combination of determiners D and noun phrases N from Chapter 5, repeated (and notationally adapted) here:

(9) $[\![D + N]\!]_w = \{Y : \langle [\![N]\!]_w, Y\rangle \in [\![D]\!]_w\}$

Although (8) is adequate when applied to sentences like (1), problems arise once we replace the noun phrase *present king of France* by something more mundane:

(10) The table is dirty.

Sentences like (10) can be uttered truthfully, it seems, even if the extension of *table* in the world (or situation) talked about contains more than one table. For instance, a customer in a coffee bar may use (10) and thereby talk about the table she and her friends sit at, notwithstanding all the other tables in the place. However, the cardinality condition (i) seems to rule out such a situation. More precisely, it is the uniqueness condition (ii-b) $|X| \leq 1$ that appears to be violated in this case, whereas the existence condition (ii-a) $|X| \geq 1$ is obviously satisfied.

There are at least two plausible ways in which the quantificational analysis of the definite article can be upheld to cover such cases; both shift part of the burden of explanation to pragmatics:

- One may employ pragmatics to see to it that, in the situation described, (10) is understood as relating to a small situation with respect to which the uniqueness condition does hold (because there is only one table in that situation, viz. the one closest to the speaker).

- One could add a further, contextually determined condition (of salience or pertinence) to (8) which in the situation at hand, is only satisfied by the table the speaker talks about.

We will not decide among these options because the issue is largely orthogonal to the topic of the current chapter.[64] But then, however the issue is resolved, the uniqueness condition will continue to be part of the quantificational analysis of the definite article, on a par with the existence condition and condition (ii). However, the parity is

[64] A lot has been written about the subject. We only mention two sources corresponding to the two options, viz. Reimer (1998) and Neale (1990). Moreover, definite descriptions allow for a number of usages that appear to escape the quantificational analysis for independent reasons. We cannot go into these matters here and recommend Heim (1991) for a decent survey.

deceiving. To see this, it should first be noted that there are limits to the above maneuver of circumventing the uniqueness condition. Consider our coffee bar scenario again. Although a guest sitting at a table may use (10) to convey something true (or false) about the table she is sitting at, the manager of the establishment cannot do so—not if he is standing behind the counter, talking to one of the temps and there simply is no obvious candidate for the table he would be talking about. And, more to the point: if the manager did utter (10), his utterance would not be false but somewhat inappropriate, likely to provoke a reaction like *Hey, wait a minute—there are 15 tables out there, which one are you talking about?*, rather than *No, sir, you're wrong: there is more than one table here.*[65] A similar reaction may ensue if someone were to use our initial Russellian sentence (1): one would not judge the utterance as plain false but rather as misguided, based on wrong presumptions. On the other hand, if our coffee bar guest had said (10) even though her table was all spick and span, her utterance may have been rejected as false. In other words, there is an asymmetry between condition (i)—existence and uniqueness—on the one hand, and condition (ii) on the other: if one of the former fails, the utterance it is somewhat beside the point; if (ii) fails, it is merely false.

While this asymmetry may or may not be within reach of a pragmatically enriched quantificational analysis of definite descriptions, it is a direct consequence of the so-called **naive** analysis of definite descriptions.[66] According to it, an expression like *the table* has no extension in case there is no table or more than one; otherwise, its extension is its referent (as we had it in the previous chapters). Therefore, the intension of the expression *the king of France* must be a partial function: it does not have a value for a world w in case there is no (pertinent) table in w (**failure of existence**), or more than one (**failure of uniqueness**). In other words, the semantics of *the* introduces a gap in the intension of *the table* in case the extension of the predicate *table* is not a singleton. But then, by the principle of extensional compositionality (see above), the whole sentence also lacks an extension, that is, it does not have a truth value in w. This way the gap in the extension of a certain expression propagates or is inherited to the extension of the entire expression.

Truth-valueless sentences cannot be used to make assertions. Asserting a sentence like (1) therefore requires that the extension of *present king of France* is a singleton. Hence, according to the naive analysis, condition (i) is a prerequisite for the sentence (1) to have a truth value. Such prerequisites are called **presuppositions**.[67]

[65]In fact, in an influential paper, Kai von Fintel (2004) has proposed the appropriateness of *Hey, wait a minute*, as opposed to *No*, as a diagnostic of (denial of) presuppositional content.

[66]The naive analysis goes back to Frege (1892) and was one of the targets of criticism of Russell (1905).

[67]The term **presupposition** is a translation of Frege's notion Voraussetzung; the re-translation of *presupposition* into German as used by linguistics is not Voraussetzung, but Präsupposition. We will not attempt to give a precise definition of the term. From what we have said, it may refer to properties of:

(i) a. sentences and/or their parts

(1) is thus said to have an **existence presupposition** and a **uniqueness presupposition** (Einzigkeitspräsupposition), both of which only concern the extension of the noun phrase *present king of France*. Taken together the uniqueness and the existential presupposition are the combined presuppositions of (1), induced by the determiner *the*.

One difference between the naive and the quantificational analysis descriptions comes out in sentences like:

(11) The present king of France is not bald

The naive analysis would obviously attribute a truth value gap to (11), inherited from the referential gap of the subject. And indeed, at first blush, (11) seems just as odd as (1). However, the sentence can also be understood as saying something trivially true; after all the present king of France *cannot* be bald because France is no longer a monarchy. One way to arrive at this reading is by posing an ambiguity in the negation and assume that, apart from reversing the truth value (if there is one), it may also turn a truth-valueless sentence into a false one ("weak negation"). We will not go into this possibility here but only mention that it has its limits when it comes to sentences containing more than one definite description.[68]

The quantificational analysis predicts (11) to be true if construed as the negation of (1), i.e. if the negation outscopes the subject at the relevant level of syntactic analysis (i.e., at the level of Logical Form). On the other hand, if the subject may also take wide scope, its existence condition would be in force whereupon (11) comes out as false. Hence, as indeed Russell already pointed out, the two different usages of (11) can be explained in terms of structural ambiguity. This analysis has not been accepted by all linguists though.

To summarize, the important properties of the definite article *the* and its analyses are the following:

Under the naive analysis, (1) and its negation (11) can be true or false only if condition (i), expressed in (12), is true.

(12) There is (exactly) one King of France

b. statements or utterances of sentences
c. speakers when making statement or utterances
d. propositions

Any of these except (d.) can be found in Frege's original writing.

[68] The following one, due to Hans Kamp (1980), is a case in point://

My dog did not bite your cat—I don't even have a dog//

The problem is that the presupposition introduced by the object—that the hearer has one cat—does not seem to be affected by the negation. Neither the weak nor the strong (truth-value-reversing) construal of negation capture the fact that the above sentence still seems to presuppose that the addressee owns a cat: the former would propagate the truth-value gap, while the latter would be consistent with the addressee's not having a cat.

That is, (12) is an entailment of (1) and it is also entailed by (11). Under the quantificational analysis, (12) is also an entailment of (1), and there is an analysis of (11) which still entails (12). However, there is a difference between the two analyses: On the naive analysis, the existence and uniqueness presuppositions follow from a sentence A containing a definite description and from its negation. On the quantificational analysis, the presuppositions also follow from A, but they do not follow from the logical negation of A (this would be inconsistent with two-valued logic) but they only follow if negation is applied to the predicate of A, not to its subject. In a technical sense, then, the Fregean notion of a presupposition has no counterpart in Russell's two-valued analysis.

7.2 More on Entailments and Truth Value Gaps

Even if the two-valued, quantificational analysis of the definite article turns out to be superior to the naive, Fregean approach, the latter is of some principled interest to semantics. For it seems that something like a Fregean presuppositional analysis may be applicable to a variety of other phenomena, distinct from the definite article. In this section, we will look at some of them.

As a point of departure, recall our analysis of the verb *know* in Section 6.6. The important fact to be accounted for there was the entailment shown in (13):

(13) John knows that Berta is sick.
$\vDash$ Berta is sick.

The validity of the inference was garanteed by a condition on the interpretation of *know*: For any world w the following must be true in any model:

(14) If x knows p in w, then w is an element of p (i.e. p is true in w).

This is a property of the interpretation function I(**know**) which assigns an intension to the verb *know*. It cannot be a property of verbs like *believe* because the inference in (15) is invalid.

(15) John believes that Berta is sick.
$\nvDash$ Berta is sick.

There are more verbs that behave like *know* in this respect, among them *remember* and *manage*:

(16) John remembers that Berta is sick.
$\vDash$ Berta is sick

(17) Berta managed to become sick.
$\vDash$Berta became sick.

On the other hand, many other verb, including *try* and *seem*, pattern with *believe*:

(18) Berta tried to become sick.
$\not\models$ Berta became sick

(19) It seems that Berta is sick
$\not\models$ Berta is sick

Verbs which, like *know, remember, manage, regret,* ..., entail their complements, are called **veridical**. Veridical verbs are thus unlike verbs like *believe*, *seem*, and *try*, which leave the truth of their complements open. We say that, in the case of veridical verbs, the inference to the truth of the complement is licensed by the lexical item *know* (*remember, manage*, ... (but not by *believe, try, seem* ...) – and thus **lexically triggered** because it is the meaning of the (lexical) verb that licenses the conclusion.[69]

Now, the relevant observation that will motivate the remainder of this section is the following:

(20) John didn't know that Berta was sick.
$\models$ Berta was sick.

Like (13), (20) seems to be a valid inference. If I truthfully utter that John didn't know p, I must believe that p is true. Even when I say

(21) I didn't know that Berta is sick

I express that I take it to be true that Berta is sick.Veridical verbs like *know*, which entail the truth of their complement even if they are negated, are called **factive**. It appears that veridical verbs tend to be factive in general, but there are a few notable exceptions, among them *prove* and *show*. In order to capture factivity, we need a new meaning postulate similar to the one stated in (14):

[69]Curiously, in some cases the meaning of the verb interacts with the kind of complement it takes. Compare the following inferences:

(i) John forgot that Berta wanted to come
$\models$ Berta wanted to come

(ii) Berta forgot to come
$\not\models$ Berta came

The verb *forget* is only veridical when it takes a *that*-complement, not when taking an infinitival complement; on the contrary, it implies that the complement is false:

(iii) Berta forgot to come
$\models$ Berta didn't come

(22) If x does not know p in w, then w is an element of p (i.e. p is true in w).

But now, taking (22) and (14) together, it follows that p must be true of any world in which x knows p, and that p must also be true of any world in which x does not know p. If knowing and not knowing were the only possibilities, it would now follow that p must be a *tautology*, i.e. true throughout Logical Space. But surely the sentence *Berta is sick* is not. So what went wrong?

Of course, one of the premisses above must give way, and it seems that the most plausible way out is to permit **truth value gaps**: If p is false, then it is neither true nor false to say of someone that (s)he knows p. If in fact Berta is not sick, then neither the sentence *John knows that Berta is sick*, nor the sentence *John doesn't know that Berta is sick* can be uttered truthfully. Of course, this is quite analogous to, and in the spirit of, Frege's presuppositional analysis: as the use of a definite description presupposes the existence and uniqueness of its referent, so the use of a factive verb presupposes the truth of its complement.

As another illustration, consider the following inferences:

(23) a. Berta managed to make a phone call
$\models$ Berta made a phone call
b. Berta didn't manage to make a phone call
$\models$ Berta didn't make a phone call

Since these inferences seem to be valid, we now get into a similar predicament as before. If there are only two truth values and if A implies B, and not A implies not B, then A and B would have to be synonymous: according to (23-b), the (doubly negated) conclusion of (23-a)—i.e. Berta's making a phone call—would have to imply the (doubly negated) premise of (23-a). Again, this is clearly wrong for the sentences above: *Berta managed to make a phone call* reports some effort on Berta's side to make the call—perhaps due to recovery from illness, or general stress. And she would also have taken this effort if she didn't manage to get sick. So both premises have truth value gaps in case Berta did not make any effort to make a call.

EXERCISE 17:
The verb *regret* has the same presupposition as *know*. Explain why it is weird to say (24-a) while this kind of awkwardness does not show up in (24-b):

(24) a. #I don't know that Berta is sick
b. I don't regret that Berta is sick

(25) a. John regrets that Berta is sick
b. John does not regret that Berta is sick

The admittance of truth value gaps thus allows us to solve our problem: the presupposition is not a tautology, because it can be false just in case its trigger sentence lacks a truth value. Furthermore, the premises and the conclusions in (25) are not equivalent, because the premises might lack a truth value while the conclusions do not, other truth values being equal. Moreover, truth value gaps allow us to *define* the notion of presupposition:

(26) Definition:
If a sentence A entails a sentence B, and if the negated sentence "not A" also entails B, and if B is not a tautology, then B is called a **presupposition** of A.

According to this definition, *Berta took some effort to get sick* is a presupposition of (25). Likewise, *that Berta is sick* is a presupposition of *John knows/doesn't know that Berta is sick*.

EXERCISE 18:
Prove the following theorems:

(27) Theorem 1:
If B is a presupposition of A, then $[\![A]\!] \subset [\![B]\!]$ and $[\![\text{not } A]\!] \subset [\![B]\!]$.

(28) Theorem 2:
If B is a presupposition of A, and if $[\![B]\!]_w \neq$ T, then $[\![A]\!]_w$ is neither true nor false.

(29) Theorem 3:
If A and B have the same presuppositions, then A lacks a truth value if and only of B does.

Truth value gaps create an additional complication in our semantics. Recall that above we defined the intension of a sentence as a function from possible worlds to truth values characterizing the proposition it expresses, i.e. the set of possible worlds of which it is true. This simplification is no more possible as soon as presuppositions enter the scene. In order to describe that B is a presupposition of A in the way we did a minute ago, we have to say (a) that B is true in all A-words, (b) that B is true in all not-A worlds, and that (c) B is false in all worlds where A has no truth value. By this last step the intension of A divides Logical Space into three (disjoint) parts: the **positive intension**, where A is true, the **negative intension** where A is false, and a remaining set of worlds where A has no truth value. This tripartition would get lost if we identified the intension of a sentence with the set of possible worlds of which it is true.

We therefore have to return to the idea that intensions are *functions* from worlds to truth values, but we now add the possibility that these functions may be partial, i.e. undefined for some worlds. Being undefined of course means that the sentence has no truth value in that world, which in turn should imply that some of its presuppositions are *violated* in that world. Violation of presuppositions therefore leads to undefinedness, or partiality.

As the phenomenon of presupposition is ubiquitous in semantics, partiality is not restricted to sentence meanings (intensions),. For example, whether I say *I like him* or *I don't like him*, whatever refers to the pronoun *him* will, under normal circumstances be an animate male being; similarly, the pronoun *her* is confined to females. This seems to be part of the meaning of these pronouns, and, in fact, that part of the meaning is a presupposition. When saying *I don't like him*, I cannot thereby deny that the referent of *he* is male.

Arguably, something similar happens if someone told you:

(30) #The table knows me

Presumably, you would have great difficulties in telling (or finding out) whether or not (30) is true. It's simply non-sensical, because *knowing* requires a conscious subject. In philosophy, violations of presupposition of this kind have been called **category mistakes**; in the case at hand, the speaker using (30) may be accused of erroneously putting a piece of furniture in the category of conscious beings. In linguistics, the analogous terminology is that a **selectional restriction** of the verb *know* has been violated. Indeed it would seem that the factivity of *know* could also be construed as the result of a selectional restriction to the effect that the object clause expresses a true proposition.

By the same reasoning, suppose we want to describe the meaning of

(31) John is hungry

where I happened to give my TV-set the name *John*. Then sentence (31) does not make much sense, unless we try to understand *hungry* in a metaphorical sense. Confining ourselves to literal meaning, we would nonetheless say that *hungry* requires that the subject is animate. If it is not, the literal extension should be undefined. But if we were to stick to the old truth conditions, this would not come out correctly, as the sentence would come out false.

Now recall how Russell derived the presupposition of the definite article. This was basically achieved by tampering with the scope of negation. But in the case of *know* discussed above, it is not so clear how the presupposition could escape the scope of negation; after all, the complement of *know* is in the syntactic domain of the negation. And what is worse, the presupposition induced by *manage* is not even the complement of *manage*. Rather, it must be described semantically as a property of the lexical item *manage*. There is no way to apply Russell's method to this kind of presupposition. Neither could this work for any of the selectional restrictions discussed above.

In sum then, selectional restrictions may be analyzed in terms of presuppositions;

conversely many presuppositions triggered by lexical items and in relation to their arguments may be due to selectional restriction. If this is so, our semantics faces a major complication, since almost any predicate comes with selectional restrictions and therefore leaves room for undefinedness. E.g., a predicate like *glatzköpfig* (= *bald*), should divide the universe of entities into three domains: the set of people who can be meaningfully asserted to be glatzköpfig, those who can meaningfully be denied to be glatzköpfig, and the remainder who do not have a head at all. This means that, like propositions, predicate extensions cannot be sets: for each world and almost any predicate we have to assume a positive extension yielding the value 1 when applied to an x, a negative extension yielding the value 0 when applied to an x, and finally yielding undefined when applied to the remaining x's.

As a result of these complications, truth conditions need to be split up into conditions for truth and for falsity, with respect to a positive and a negative extension. Thus, (31) receives the truth value 1 iff John is an element of the positive extension of the VP; its truth value is 0 iff John is an element of the negative extension of the VP; and it is undefined in all other cases. As interpretation functions are partial functions, and VP-denotations can themselves be complex, being composed from smaller building blocks, truth conditions become more complicated in intriguing ways, as evidenced by some of the examples discussed below.

7.3 Presupposition and Assertion

It is common to explain the notion of presupposition by appealing to a contrast between what is asserted and what is presupposed. Here are some representative examples:

(32) Susan will be late again
   a. Presupposition: Susan has been late in the past
   b. Assertion: Susan will be late

(33) Susan won't be late again
   a. Presupposition: Susan has been late in the past
   b. Assertion: Susan won't be late.

(34) Susan stopped drinking
   a. Presupposition: Susan drank (for an indefinite period of time)
   b. Assertion: At a certain point in time and for a while, Susan didn't drink

(35) Susan didn't stop drinking
   a. Presupposition: Susan drank (for an indefinite period of time)
   b. Assertion: At a certain point in time and for a while afterwards, she drank

By definition, then, the presupposition is neither asserted nor negated, but is "*communicated*" in both an affirmative assertion and the respective negative assertion. Given this **pragmatic** distinction between assertion and presupposition, semantic theory would have to characterize (and calculate formally) two propositions: the proposition stated by an utterance and the one presupposed by it. However, attractive though it may be, this idea runs into serious problems. In the above examples one may be able to tease apart what is *asserted* from what is *presupposed*. But, in general, it is not possible to spell out the asserted part without also mentioning the presupposed part of the "communicated" message. As an example, consider one of Frege's examples:

(36) Whoever discovered the elliptic form of the planetary orbits died in misery

There are several presuppositions involved here, among others that planetory orbits are elliptic. Let us concentrate on the following two propositions:

(37) a. Someone discovered the elliptic form of the planetary orbits
  b. Someone died in misery

Clearly, (37-a) is a presupposition of (36), presumably triggered by the free relative construction (*whoever*), while (37-b) is the assertion made by (36). This seems straightforward. However, without anything else being said, the above account of the assertion vs. presupposition distinction is unable to express that the predicates of the two sentences in (37) must be instantiated by the same person. That is, whoever died in misery must be the one who discovered the elliptic form of planetary orbits. At first blush this may look like a simple requirement on the subjects of (37-a) and (37-b), viz. that they be co-referential. However, the common subject is a quantifier, and as such does not even have a referent![70]

The problem has been discovered by Lauri Karttunen and Stanley Peters; in a number of papers during the seventies of the last century they developed a highly technical and sophisticated theory of presuppositions, but, at the end of the day, had to admid that they were unable to solve the problem. Before quoting from their original work (Karttunen and Peters (1979), p. 53), it must be pointed out that in the literature at that time, presuppositions were often treated as a special case of so-called *conventional im-*

[70]An additional problem can be identified in (i):

(i) Someone stopped drinking
  a. Presupposition: Someone drank (for an indefinite period of time)
  b. Assertion: At a certain point in time and for a while, someone didn't drink

Here again it is absolutely crucial to identify both "someones", but in addition, the temporal relation between assertion and presupposition is crucial too: the time at which the presupposition holds must be immediately before the time the assertion is about, but this relation cannot be expressed by considering presupposition and assertion in isolation.

plicatures, a notion that goes back to the work of Paul Grice (cf. Grice (1967, 1975, 1981)). With this in mind, the following citation clearly states the point:

> NOTE: One problem with these rules is that they do not assign the correct conventional implicatures to sentences such as *Someone managed to succeed George V on the throne of England.* What the rules given here predict is (correctly) that this sentence is true iff someone succeeded George V to the throne and (incorrectly) that it conventionally implicates that it was difficult for someone to do that. This is unsatisfactory because the implicature just stated is true (you or I would have found it extremely difficult), but the sentence is in fact an odd thing to say precisely because it conventionally implicates a falsehood—namely that George V's successor had difficulty ascending to the throne. What our rules as stated lack is any way of linking the choice of a person who is implicated to have difficulty to the choice of a person who is asserted to have succeeded. We expect that this deficiency will be remedied through further research, but we note here that the task is not a trivial one. [...] the problem arises directly from the decision to separate what is communicated in uttering a sentence into two propositions. In particular, it exists in connection with the notion of conversational implicature and also with any theory of presupposition that separates these from truth conditions (i.e. does not treat them simply as conditions for having a determinate truth value).

Arguably, this interdependence of assertion and presupposition may also be found in examples like Russell's king. If the presupposition is that there is a unique king, how can we formulate the assertion (i.e. the truth conditions) in isolation? *This* king is bald? So, who does *this* refer to? Perhaps the simplest way of framing the assertion is (38):

(38) He is bald

In our above treatment we followed Russell and carefully avoided this complication by formulating the predication condition (c) as a quantified statement. However, the anaphoric formulation (38) (which, incidentally, is very common in the semantic literature) seems more straightforward and intuitive. Likewise, it seems that the simplest way of dealing with assertion and presupposition in (39) is using an anaphoric pronoun:

(39) John doesn't regret that Bill died
  a. Presupposition: Bill died
  b. Assertion: John doesn't regret it

So, due to the anaphoric nature of the pronoun, the assertion cannot be formulated independently from the presupposition. We thus see that it is impossible to treat presupposition and assertion *as seperate entities* in an insightful way.

More recent theories of presupposition rely heavily on this anaphoric relation (cf. in particular van der Sandt (1992)). We cannot do justice to these developments in this introduction; nonetheless we will make an attempt to capture the spirit of these theories by trying to sketch a framework that combines truth conditions that involve anaphoric relations with the identification of presuppositions as they function in discourse.

7.4 Presupposition and Discourse

We have seen that presuppositions are part and parcel of semantics in that they are triggered by semantic properties of lexical items or syntactic constructions. Nonetheless there is a pragmatic side to presuppositions: if a sentence S presupposes p then, in general, S can only be used in a conversation if both the speaker when uttering S, and the hearer when understanding S, take p for granted at that point of the conversation. For example, if someone utters

(40) It was (not) Martha who died

he assumes that the hearer somehow takes the truth of *someone died* **for granted**. In general, the participants in a conversation share quite a few assumptions; and some expressions (like *regret*) require that certain propositions are part of this conversational background. In pragmatics, the background assumptions of an ongoing conversation, are known as the **common ground** (Stalnaker (1973, 1978)). If what a sentence presupposes is not part of the common ground, the utterance is more often than not felt weird or misleading. This is particularly obvious when the utterance is in conflict with what one of the participants believes or knows, provoking a reaction that goes beyond simply negating its content. For example, if I am convinced that Paul is alive and well, and then Mary tells me that he died years ago, I won't put her right before contradicting her: "That's wrong: I only saw him yesterday...", etc. However, if Mary had chosen to presuppose, rather than assert, the same piece of information, e.g. by saying:

(41) Ringo doesn't know that Paul died years ago

it does not suffice to object *that's wrong* (or *(41) is false*), because that would be understood by Mary as claiming that Ringo *knows* that Paul died, thereby still implying that we all know him to be dead. Since this is not what I want to say, I must utter something that makes it clear that Mary and I—as the participants in the conversation—do not share a common ground—e.g., "Hey, wait a minute: Paul's not dead, I only saw him yesterday".[71] What has to be denied, then, is precisely a presupposition.

[71] Cf. footnote 65 above.

This means that presuppositions must be part of the common ground. In other words, if any successful utterance of a sentence S presupposes that a proposition p is part of the common ground, then p is a presupposition. And conversely, if I assert a proposition A successfully (without provoking weird consequences) and if p is not part of the common ground before and after the utterance of A, then p is not a presupposition of A.

This account of presupposition elaborates on a model of discourse semantics that has already been addressed in our motivation for intensions. Recall that we arrived at our characterization of propositions as sets of possible worlds by accounting for *information* in terms of permitting and excluding states of affairs. We will now define the notion of *new* information relative to a given common ground of a conversation.

Let's assume that the common ground Γ is the set of possible worlds that are compatible with the discourse participants' convictions at a certain point of a conversation. This means that if the truth of a proposition p is taken for granted, then p is entailed by Γ (i.e. Γ is a subset of p). As this holds for any such p, it follows that Γ is the intersection of all propositions that are believed to be true by all of the participants of the conversation. Given Γ, we may now describe the effect of uttering a sentence S in that context. If the assertion is successful and not objected to by the participants of the conversation, the effect will be that the proposition A is added to the common ground. This effect is a gain of information and will therefore result in a smaller set of possibilities, with all the former possible states of affairs incompatible with S now being excluded from the common ground. That is, the common ground is modified or **updated** by removing the worlds in which S is false and by keeping the worlds in which S is true. Formally, we can describe the effect of uttering S as resulting in a modified conversational background Γ' such that

(42) $\Gamma' = \Gamma \cap [\![S]\!]$,

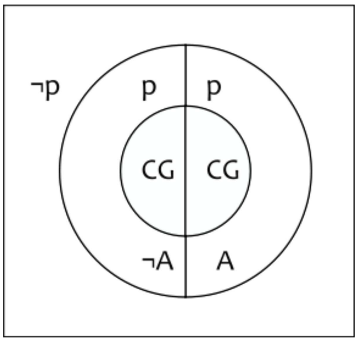
where $[\![S]\!]$ is the positive intension of S, i.e. the set of worlds of which S is true. We may now relativize informativity to a given context with a conversational background Γ. A proposition p is **informative** w.r.t. Γ iff $\Gamma \cap p \neq \Gamma$; otherwise, i.e. if $\Gamma \subseteq p$, p is **uninformative** in Γ.

What does all this imply for the notion of a presupposition? In general, if a speaker asserts a sentence containing an element that triggers a presupposition, and if the assertion is accepted by updating a common ground, then this may proceed successfully only if the common ground Γ to be updated already contains the presupposition. In other words, the presupposed content of an utterance must be *uninformative* with respect to Γ in that it is already entailed by Γ.

Put another way: if we define the **context change potential** of a proposition p as the difference between Γ and Γ' (for any Γ), then it follows that the potential assertion made by S is this difference, whereas the presupposition is already entailed by Γ. The presupposition must be part of the common ground, the assertion (the proposition that properly adds to this) must be new.

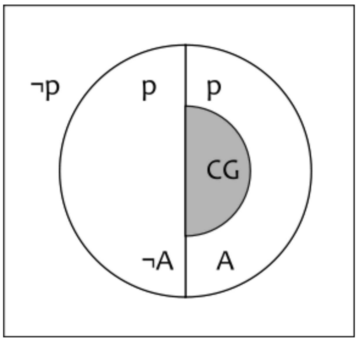
Let us visualize the situation in an Euler diagram.

(43)



The presupposition p contains all the words in which A can be true or false. Moreover, the presupposition follows from the common ground so that $\mathrm{CG} \subseteq p$. The effect of uttering A is to eliminate all worlds incompatible with CG. The resulting common ground is shown in (44):

(44)



If we conceive of propositions as effects on common grounds (as the information added to a common ground), as has become standard practice in discourse semantics, the proposition A can be characterized as that function, that maps CG on CG′, for any CG.

Let us now address the problem discussed at the end of the last section: there we observed that the presupposition and the statement cannot be teased apart so easily. Nonetheless it is possible to describe that part of an assertion that is not presupposed in a simple fashion while at the same time avoiding the pitfalls described above. As a concrete example, consider

(45) Pete realized that the street was wet (= A)

Now, the communicated proposition **minus** the presupposition can be characterized as

(46) If the street was wet, Pete realized that the street was wet

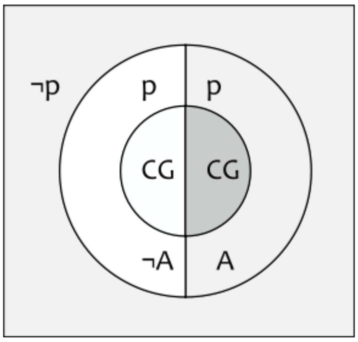
Or alternatively,

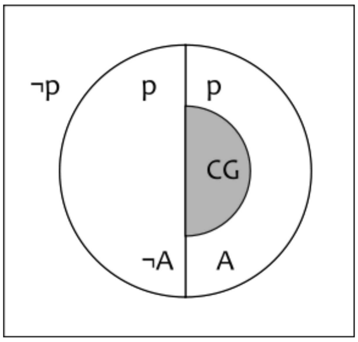
(47) If the street was wet, Pete realized it (= $p \rightarrow A$)

This is so because we can take the presupposed part for granted as (48-a) and adding (48-b) it follows by modus ponens that (48-c):

(48) a. The street was wet
b. If the street was wet, Pete realized it
c. Pete realized that the street was wet.

Note that we do not claim that (48-b) is what has been said by uttering (48-c). (48-b) only adds the truth conditions that must be added in order to get (48-c) given the presupposition, i.e. a common ground containing (48-a). It seems to us that the term *assertion* in this context is misleading, or at least rather vague: Which of the propositions in (48) is "asserted"? This seems to be a matter of mere terminology only. If the "asserted part" is the one that makes up the context change potential, then (48-b) is the shaded region in (49), which, when added to CG, yields (50), which is exactly the same modified common ground CG′ as shown in (44).

(49)


(50)


It thus holds that

(51) $[\![\mathrm{CG}]\!] \cap [\![\mathrm{A}]\!] = [\![\mathrm{CG}]\!] \cap [\![\mathrm{p} \rightarrow \mathrm{A}]\!]$.

Now the remarkable thing is that the conditional statements enables us to see how the presupposition is related to the rest of the sentence; in particular, it allows us to make the connection between presupposition and the entire proposition in a way that solves the problem we described at the end of the last section. Reconsider:

(52) Someone stopped smoking

Above we were in trouble to state that the one having smoked is the same who has now spopped smoking. But this connection is easily captured in the conditional (53):

(53) If a certain person has been smoking before, then *he* now stopped smoking

Likewise, the one who is bald, is the King, as formulated in (54):

(54) If there is exactly one King of France, *he* is bald

And finally, implicative verbs also permit to refer to the presupposition with a pronoun:

(55) If p then John knows/realized *it*

These examples reveal that there is a general method to tease apart "assertion" and presupposition without representing them as seperate entities. In fact, the presupposition is part of the communicated message in a twofold way: it is entailed by the entire utterance and it enters into a characterization of what is old information and what is new with respect to a common ground, if only as the antecedent of a conditional statement. This way of dealing with presuppositions therefore solves the problem that "assertions" cannot be represented independently of presuppositions, and it also reveals a kind of anaphoric relation to what is presupposed, which is a basic building block of more recent theories of presupposition (cf. van der Sandt (1992)).

7.5 Accomodation

The model of conversation sketched above characterizes presuppositions as being part of the common ground. Unfortunately, there are some obvious exceptions to that. Suppose I arrive late at a party and my excuse is one of the following:

(56) a. Sorry, I had to find a parking space for my car
b. Sorry, I had to find a baby sitter for my youngest daughter

There are a number of inferences involved here: (56-a) implies that I have a car; (56-b) allows to infer that have a daughter and in addition that I have more than one daughter, otherwise the superlative *youngest* would not be appropriate. We might try to negate (56-b) by saying:

(57) Wolfgang wasn't late. This time, he didn't have to find a baby sitter for his youngest daughter

But the inferences still hold. So we may want to say that they are presuppositions triggered by the possessive pronoun and by the superlative of the adjective. Given what was said above, one would thus expect that these presuppositions need to be part of the common ground. However, it is clear that (56) can be uttered appropriately and successfully even in case the host of the party does not know that I have a daughter. This seems to be a problem, or a lacuna, in the theory sketched so far; some amendment is called for.

The point to be made is that in many cases the presupposition of a sentence may convey new information indeed. A case in point is cited by Karttunen (1974) who found the following example in an official MIT bulletin about the spring 1973 commencement:

(58) We *regret* that children cannot accompany their parents to commencement exercises

The point of (58) is not only to remind one of a well-known fact; it is intended to convey new information (namely that, regrettably, children are not admitted) to anyone who didn't know before.

How can we reconcile this with the model of conversation developed above? The point is that the presupposition is not part of the common ground, but yet the utterance of (58) is successful. How can this come about?

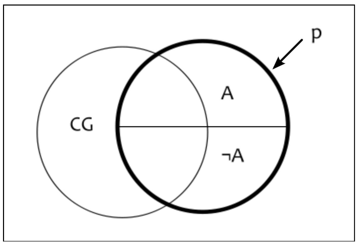
The answer is that the hearer tacitly adjusts his version of the CG so as to make it compatable with the speaker's. This process has been dubbed **accomodation** (cf. Lewis (1979)). The point is that the compatability with the CG is required to interpret *S* correctly, but that our theory does not require this compatability to exist before the utterance *S* has been made. In other words, an utterance may change the CG in a number of different ways, one of them being that, as a prerequisite of accepting (or rejecting) *S* in the current discourse, one also has to accomadate the CG.

The point has been emphasized in an unpublished paper by Kai von Fintel (2000), from which we quote the central paragraph:

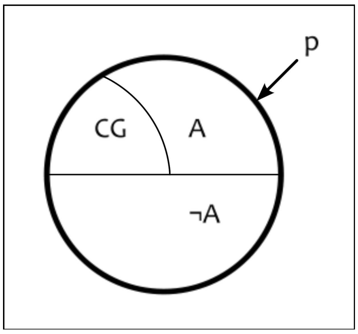
> Our examples are clearly cases where the presupposed proposition is not in the common ground <u>prior to the utterance</u>. But note that this in fact is not what the common ground theory of presupposition says, at least not once we look very closely at what it tries to do. We saw that sentence presuppositions are requirements that the common ground needs to be a certain way for the sentence to do its intended job, namely updating the common ground. Thus, the common ground must satisfy the presuppositional requirements <u>before the update can be performed</u>, not actually <u>before the utterance occurs</u>.

Graphically, this can be illustrated as a transition from (59) to (60):

(59)



(60)



As a result of uttering *A* the common ground is adjusted so as to be compatible with both *A* and its presupposition.

In many cases this kind of adjustment is performed routinely, as in the above cases of accommodation. There are other cases where presuppositions cannot be accommodated so easily, e.g. in uttering (61) out of the blue:

(61) Paul will come to the party as well

The *as well*-part presupposes that someone else who is relevant in the context will come to the party, but it does so without giving a clue as to who that other person may be. But this is crucial for *as well* to function properly: it requires that this person has already been identified in the CG. There is no way to fix the CG without further information, so that (61) will lead to a breakdown of communication, at least temporarily.

7.6 Presupposition Projection

Suppose that John has no wife. Now consider:

(62) a. Either John is away or John's wife is away
b. Either John has no wife or John's wife is away

(63) a. If John is away, his wife also is
b. If John has a wife, his wife is away

(64) a. John is away and his wife also is
b. John has a wife and (but) his wife is away

The a-sentences straightforwardly presuppose that John has a wife, they are weird in the context given, but this doesn't seem to apply to the b-sentences. Uttering these sentences does not presuppose that the CG already contains as an established fact that John is married. Although this fact nonetheless follows from the truth of (64) (because it is asserted in the first conjunct), this does not hold for (62) and (63). So one may well ask what the truth conditions for these sentences are.

Frege held that failure to satisfy a prepupposition leads to truth-value gaps so that the fact that John has no wife should lead to a gap for the sentence *John's wife is away*, which, due to compositionality, affects the entire sentence, which also should lack a truth value (by extensional compositionalit of *either or*, *if … then*, and *and*). Intuitively this is correct for the a-sentences, but a wrong prediction for (62) and (63) which may still be true or false, whereas (64-b) is presumably judged false because the first conjunct is.

This compositionality problem is known as the **projection problem** for presuppositions. The question is: under which conditions can presupposition survive and "project" to the entire sentence, and which conditions determine that such a projection is "blocked"? Of course, one of the defining characteristics of presuppositions is that they are not blocked in negated sentences. But negation is just one context—what about conjunction, disjunction and implication, as illustrated in the above examples? It seems that it can't be the truth values, i.e. the extensions themselves, that determine the presuppositional behaviour, but that it is rather some intensional relation between the sentences that makes the difference.

The data above exemplify that in certain syntactic environments, a certain presupposition does not project. It is readily seen that this holds for all sorts of presupposition in these contexts; the phenomenon is quite general:

(65) a. John came and Mary came too
b. Either John didn'come, or Mary came too
c. If John came, then Mary came too

(66) a. John insulted Mary and regretted it
b. Either John didn't insult Mary, or he regretted it
c. If John insulted Mary, he regretted it

Many different explanations for the observed failure of presupposition projection have been given in the literature, cf. the brief survey in Levinson (1983) pp. 191ff (= Levinson (1990) pp. . In this introduction we will sketch a treatment that has been put forth in a paper by Irene Heim (1983). Her point of departure is discourse semantics as discussed above.

Recall that an utterance of A in a context CG is well-formed iff CG implies A's presuppositions. One may therefore define:

(67) CG + A is well-formed iff CG is a subset of A's presupposition(s) and if so, CG + $[\![A]\!]$ = CG $\cap$ $[\![A]\!]$.

This works fine as long as the presuppositions can be defined in an unproblematic and local way, i.e. as triggered within their minimal clause. However, as we have seen above, it does not work for complex clauses because we do not have determined the presuppositions yet, if any, of any of the complex sentences. But in the framework of discourse semantics this can be done in a straightforward way, based on the presupposition(s) of the constituent sentences.

Let us first consider the simple case of a conjunction $A \wedge B$. The basic intuition is simply that we first add A to a common ground CG, which yields a modified context CG$'$, and we then add B to CG$'$ which yields a doubly modified CG$''$. More formally:

(68) $\text{CG} + (A \wedge B) := (\text{CG} + A) + B$

This already explains the difference between (64-a) and(64-b). In (64-b) we add as A a sentence without presupposition. The result is a common ground CG$'$ that implies A. But then, A is a presupposition of B. At the point of adding B it is indeed the case that the presupposition of B is part of the modified common ground CG$'$. But then, CG$'$ + B is alway defined, so that we always get a well-defined common ground CG$''$. As there is no way of being undefined, this means that the complex sentence has no presupposition.

Next, consider (64-a). Adding A is unproblematic. However, A is logically independent from B, in particular, it does not imply a presupposition of B. Hence, adding B to CG$'$ may indeed fail, if John has no wife in CG. Hence the entire sentence does have the presupposition that he does have a wife.

Having defined conjuction, it remains to account for negation. This again is straightforward:

(69) $\text{CG} + [\![\neg A]\!]_s = \text{CG} \backslash (\text{CG} + A)$

We first add A to the common ground, which by definition guarantees that the presupposition be satisfied. Then we subtract the result from CG, which accounts for the negation.

Once having defined negation and conjunction, we can express implication and disjunction in the usual way. Thus, an implication $A \rightarrow B$ is equivalent to $\neg(A \wedge \neg B)$. By definition, then, it holds that

(70)
$$\begin{aligned}
&\text{CG} + (A \rightarrow B) = \\
&\text{CG} + \neg(A \wedge \neg B) = \\
&\text{CG} \setminus (\text{CG} + (A \wedge \neg B)) = \\
&\text{CG} \setminus ((\text{CG} + A) + \neg B) = \\
&\text{CG} \setminus ((\text{CG} + A) + (\text{CG} + A) \backslash ((\text{CG} + A) + B)))
\end{aligned}$$

By eliminating redundancies this simplifies to:

(71) CG \ ((CG + *A*) \ ((CG + *A*) + *B*))

As in the case of conjunction, the stepwise addition of *A* and *B* guarantees that *A* can introduce and satisfy the presupposition of *B* so that the entire clause does not require its presupposition already to be implied by CG. The remaining calculations involving negation guarantees that the truth conditions come out correctly.

Finally, let us look back to (62-b), repeated as (72-a). This is logically equivalent to (63-b) in (72-b):

(72) a. Either John has no wife or John's wife is away
b. If John has a wife, his wife is away

Therefore, the same mechanism applies as before. Note that this dynamic way of dealing with disjunction also explains why (73) is okay:

(73) Either John has no wife or she is away

The use of the pronoun seems to be a problem because it cannot refer back to *no wife*. But as has become clear from the above analysis, the equivalence of (73) with (74)

(74) If John has a wife, she is away

makes it clear that the existence of an antecedent for *she* in (73) is guaranteed by evaluating the second clause in the context of the first, as required by (71).

We cannot do justice here to the vast amount of literature on presuppositions, cf. eg. wikis on ☞**presupposition triggers** and the Further Reading Section. We should stress at this point, however, that due to Strawson's influence presuppostions in the literature are predominantly described as a pragmatic phenomenon, whereas we tried to make it clear that presuppositions are lexically triggered and therefore belong to the lexical meaning of expressions, that is, its semantics.

EXERCISE 19:
Determine, and try to explicitly account for, the presuppositions in (75):

(75) a. John too is smoking again
b. If I were rich, I would retire

8 Further Reading

[The further reading part still has to be written. here are a few suggestions:]

In general you will need a little bit more syntax in order to access the semantic literature. For most purposes, an elementary introduction to Generative Grammar as e.g. Aarts (2001) will do.

Other elementary but concise introductions are Kearns (2000) and Heim and Kratzer (1998).

We recommend Hodges (1977) as an easily accessable introduction to logic.

The seminal work on Quantifier Raising is May (1977), cf. also Heim and Kratzer (1998) for extensive justification.

Additional influential papers that reflect on the semantic vs. pramatic status of presuppositions include Donnellan (1966), Stalnaker (1974), and Kripke (1977). The formally most sophisticated analysis of semantic presupposition as context chaning potential is Beaver (2001).

9 Solutions to Exercises

SOLUTION TO EXERCISE 1, page 28:

(1) a. Fred will realize that Mary left when the party started
    b. Fred will realize that Mary left when the party starts

The when clause is in the past tense in (a), whereas the main clause is in the future tense. A sentence like *Fred will do X when the party started* is incongruent, therefore the *when*-clause belongs to the embedded clause *Mary left*, which is also in the past tense and agrees with the tense of *left*.

In (b) it would be incongruent to say *Mary left when the party starts*. Therefore the when-clause cannot belong to the embedded clause but must modify the main clause.

SOLUTION TO EXERCISE 2, page 28:

(2) a. John realized $\boxed{\text{that Mary left when the party started}}$
    b. John $\boxed{\text{realized } \boxed{\text{that Mary left}}\ \boxed{\text{when the party started}}}$

(3) a. John said $\boxed{\text{the man died yesterday}}$
    b. John said $\boxed{\text{the man died}}$ yesterday

SOLUTION TO EXERCISE 3, page 29:

(4) a table of wood that was from Galicia

(5) a. a table (made) of Galician wood
b. a wooden table from Galicia

(6) a. a table of $\boxed{\text{wood that was from Galicia}}$

b. $\boxed{\text{a table of wood}}$ that was from Galicia

Remark: most standard semantic theories have difficulties with interpreting the structure in (6-b). Semanticists would rather assume the following structure:

(7) a $\boxed{\boxed{\text{table of wood}}\ \text{that was from Galicia}}$

The reason for this cannot be discussed here and we deliberately ignore the difficulty.

⊛⊛⊛⊛⊛⊛

(8) the girl with the hat that looked funny

(9) a. the girl with $\boxed{\text{the hat that looked funny}}$

b. $\boxed{\boxed{\text{the girl with the hat}}\ \boxed{\text{that looked funny}}}$

Here again, many semanticists would prefer different structures:

(10) a. the $\boxed{\text{girl with the}\ \boxed{\text{hat that looked funny}}}$

b. the $\boxed{\boxed{\text{girl with the hat}}\ \text{that looked funny}}$

Again, we cannot discuss the difference, but cf. the next example.

⊛⊛⊛⊛⊛⊛

(11) the girl and the boy in the park

(12) a. the girl and $\boxed{\text{the boy in the park}}$

b. $\boxed{\boxed{\text{the girl and the boy}}\ \boxed{\text{in the park}}}$

Note that in (b) the boy and the girl both are in the park, so that a paraphrase would be

(13) the boy and the girl who *are* in the park

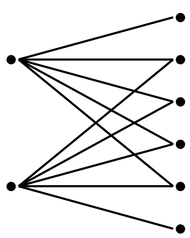
This means that the *who*-clause has to modify a plural entitiy formed by conjunction. It would therefore not be possible, as suggested in the alternative analyses in (10) to attach the modifying expression only to a noun which does not display any plural morphology. The plural comes in conjoining the boy and the girl, therefore attachment cannot be to a noun in this case, and perhaps more generally so.

SOLUTION TO EXERCISE 4, page 40:

(14) Genau 5 Bücher hat jeder gelesen
Exactly 5 books has everyone read

Consider the following situation:

(15) people books



In one reading saying that

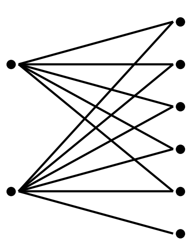
(16) there are exactly 5 books such that everyone read them

the sentence is false, since only 4 books were read by everyone. The other reading, saying that

(17) everyone read exactly five books

is true in this situation. Next add one line in the above model:

(18) people books



Then (16) is true in (18) but (17) is false because one person read 6 books.

SOLUTION TO EXERCISE 5, page 40:
Assume 2 people and ten books. Assume one has read nothing and the other has read five books.

The two readings are:

(19) a. there are exactly 5 books such that no one read them (true)
b. noone read exactly five books (false)

Now assume that the person who read 5 books reads a sixth's book. Then (19-b) is true (19-a) is true.

SOLUTION TO EXERCISE 6, page 45:
In one reading of *My brother wants to marry a Norwegian*, he has thinks: in case I should marry, my wife should be a Norwegian. In the second reading I know that my borther wants to marry Linnea, a girl from Norway. In this situation the sentence is true even if my brother does not know that she is Norwegean.

SOLUTION TO EXERCISE 7, page 48:
The structural amgiguity can roughly be captured by assuming that the comparative *than*-clause is either attached low or high. Low attachment to the comparative *longer* as in (20-a) represents the implausible reading where the object of thought is a contradiction; in that case the *than*-clause is in the scope of *thought*:

(20) a. I thought $\boxed{\text{your yacht is longer than it is}}$
b. $\boxed{\text{I thought } \boxed{\text{your yacht is longer}}}$ than it is

The high attachment reading in (20-b) can be paraphrased as

(21) I thought that your yacht is longer than its actual length in the real world

(21) makes it clear that the *than*-clause is not in the scope of *think*.

SOLUTION TO EXERCISE 8, page 49:
Structurally, the ambiguity can only be accounted for if we assume that there is a non pronounced than-clause. The ambiguity would be the same as in the last exercise, as far as structure is concerned. The additional generic aspect in the more plausable reading derives from the definite article *der/the*. This is particularly clear for German, as the generic reading can also be expressed without it, cf.

(22) a. Der Löwe ist ein Säugetier
The lion is a mammal

b. Löwen sind Säugetiere
Lions are mammals

The sentences in (a) and in (b) are semantically equivalent.

SOLUTION TO EXERCISE 9, page 71:
The only additional rule needed to account for

(23) (dass) Paul Bernadette Geraldine vorstellt
(that) Paul Bernadette Geraldine introduces
'(that) Paul introduces Bernadette to Geraldine'

is one that combines the three place predicate vorstellt with its indirect object *Geraldine*:

(24) $[\![\mathrm{PN} + \mathrm{DTV}]\!]_s = \{\langle x, y\rangle : \langle x, y, [\![\mathrm{PN}]\!]_s\rangle \in [\![\mathrm{DTV}]\!]_s$

The result is a two place relation that behaves like a transitive verb TV.

SOLUTION TO EXERCISE 10, page 78:

(25) $[\![\text{something}]\!]_s = \{X : [\![\text{object}]\!]_s \cap X \neq \emptyset\} = \{X : X \neq \emptyset \text{ and } X \subseteq [\![\text{object}]\!]_s\}$

SOLUTION TO EXERCISE 11, page 94:
Type shifting of *Mary* yields $\{X : m \in X\}$. We then have to apply rule (107) on p. 93 to:

(26) $[\![\text{John kisses Mary}]\!]_s = [\![\text{John}]\!]_s \in [\![\text{kisses Mary}]\!]_s$
$= [\![\text{John}]\!]_s \in [\![\mathrm{TV} + \mathrm{DP}]\!]_s = j \in \{x : \{y : \langle x, y\rangle \in [\![\mathrm{TV}]\!]_s\} \in [\![\mathrm{DP}]\!]_s\}$
$= \{y : \langle j, y\rangle \in \{\langle a, b\rangle, \langle p, p\rangle\}\} \in [\![\mathrm{DP}]\!]_s = \emptyset \in [\![\text{Mary}]\!]_s$

This is false, since the empty set is not an element of $\{X : m \in X\}$.

SOLUTION TO EXERCISE 12, page 94:

(27) If DTV is a transitive predicate, then $[\![\mathrm{DTV} + \mathrm{DP}]\!]_s := \{\langle x, z\rangle : \{y : \langle x, z, y\rangle \in [\![\mathrm{TV}]\!]_s\} \in [\![\mathrm{DP}]\!]_s\}$.

SOLUTION TO EXERCISE 13, page 95:
Assume that $[\![\text{kiss}]\!]_s = \{\langle m, a\rangle, \langle m, c\rangle\}$ and $[\![\text{doll}]\!]_s = \{a, b\}$. Then $\{y : \langle m, y\rangle \in [\![\mathrm{TV}]\!]_s\} = \{a, c\}$. Is this an element of $[\![\text{every doll}]\!]_s$? Since this set contains all supersets of $\{a, b\}$ and b is not in $\{a, c\}$, the anser is 'no'. Thus, *Mary kisses every doll* is false. Now consider

the extension of *a doll*. This is all sets with an non-empty intersection with *doll*. Is being kissed by Mary an element of that set? Since $\{a, c\}$ and $\{a, b\}$ have an element in comman, the answer is 'yes', and the sentence is true.

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