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THE CONSTRUCTION OF MEANINGS FOR TREND IN ACTIVE GRAPHING

ABSTRACT. The development of increased and accessible computing power has been a major agent in the current emphasis placed upon the presentation of data in graphical form as a means of informing or persuading. However research in Science and Mathematics Education has shown that skills in the interpretation and production of graphs are relatively difficult for Secondary school pupils. Exploratory studies have suggested that the use of spreadsheets might have the potential to change fundamentally how children learn graphing skills. We describe research using a pedagogic strategy developed during this exploratory work, which we call Active Graphing, in which access to spreadsheets allows graphs to be used as analytic tools within practical experiments. Through a study of pairs of 8 and 9 year old pupils working on such tasks, we have been able to identify aspects of their interaction with the experiment itself, the data collected and the graphs, and so trace the emergence of meanings for trend.

KEY WORDS: data handling, graphing, trend, spreadsheets.

BACKGROUND

One of the most significant characteristics which distinguishes contemporary living from that as recent as fifty years ago is the central importance of information. In particular, there is a great emphasis placed upon the presentation of data in various forms as a means of informing or persuading. The development of enormously increased and accessible computing power has been a major agent of this change. Software tools, such as databases and spreadsheets, owe their existence to the need for the collation of raw data and for the meaning of that data to be communicated through compact images including diagrams, graphs and charts.

There is an assumption in the news media that such images are transparent, in the sense that the reader will gain immediate understanding of their message, but within mathematics education concerns have been raised on this issue.

Reading a diagram is a learned skill; it doesn't just happen by itself. To this point in time, graph reading and thinking visually have been taken to be serendipitous outcomes of the curriculum. But these skills are too important to be left to chance. (Dreyfus and Eisenberg, 1990.)

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More recently, Carraher et al. (1995) have explored the interpretation of graphs presented in the media concerning significant events (a political election) by adults with relatively little schooling. They observed how everyday knowledge is used in this interpretation in ways that may be quite different from the intentions of the statistician producing the graph. Nemirovsky has also explored extensively the ways in which students' intuitions about physical phenomena may influence their reading of the graphs which represent them (e.g., Nemirovsky and Rubin, 1991; Noble and Nemirovsky, 1995).

A number of research studies from both mathematical and scientific perspectives (for example, Kerslake (1981) reporting on the *Concepts in Secondary Mathematics and Science* project, Johnson and Welford (1988) reporting on the findings of the Assessment of Performance Unit, Sharma (1993) reporting on the *Strategies and Errors in Secondary Mathematics* project) have revealed relatively low levels of graphing skills amongst secondary school pupils, and highlighted particular areas of difficulty. Such studies typically distinguish skills in the interpretation of graphs from those needed to construct graphs. Two studies with lower secondary pupils in Britain and North America (Swatton and Taylor, 1994; Padilla et al., 1986) show relatively low success rates in interpretative skills (interpolation and particularly reading relationships between variables), compared to higher rates in some construction skills (such as plotting points). Donnelly and Welford (1989) found a range of difficulties including the tendency for weaker pupils to over-interpret the information. These findings serve to strengthen the widely-held perception of interpretation skills as being of a higher order than construction skills.

However, there is an alternative explanation for these very poor levels of performance in the interpretation of graphs. Traditionally much emphasis has been placed by teachers, and school mathematics schemes, both on the conventions of graphical representation and on neat presentation. Drawing neat, detailed graphs by hand is time consuming, particularly for young children with limited motor skills, even when the intellectual demands of the content are relatively low. We suggest that this time factor, combined with the relatively high profile given to graphical representation as a *topic*, has led to the production of graphs frequently being seen as the end-point, indeed the purpose, of the task, with little attention focused on interpretation (beyond rather superficial 'reading' of data), or on the use of graphs as problem solving tools. We have characterised this traditional approach to graphical representation as *Passive Graphing* (Pratt, 1995), indicating the passive use of graphs for illustration and low-level interpretation.

Just as the increased use of computers has changed the demand for, and access to, information, the use of computers in education has the potential to revolutionise the ways in which children learn graphing skills. We suggest that a mastery of graphing requires three separate, though clearly related, capabilities:

- an understanding of how to interpret and use graphs,
- a knowledge of the conventions and technicalities of graphs, such as the use of scale, and
- the practical skills required to produce graphs by hand.

In a conventional classroom situation where graphs are drawn by hand, it is difficult to separate the physical and intellectual demands of producing graphs, from the intellectual demands of interpreting them. The demands of learning conventions and technicalities are high, and this, together with the time required to produce graphs by hand, has led to attention being focused on a limited range of types of graph, particularly in primary schools. It is easy to make the assumption that the experience of drawing graphs, and 'knowing' the conventions are necessary prerequisites for being able to interpret graphs effectively.

Preliminary studies using a computer-based pedagogy with primary school pupils (aged eight to eleven years) have led us to challenge both this conventional view of graphing, and assumptions generally made about the range of graphs which are appropriate for children of this age. When graphs are produced on the computer, there is no need for the child to have the practical skills required to produce graphs, or to be explicitly taught the conventions of graphing. Attention can be focused directly on the interpretation of the graph, and we have observed relatively high levels of success in interpretative skills shown by young pupils working with graphs generated from spreadsheets (Ainley, 1995; Pratt, 1995). This leads us to suggest that interpretive skills in graphing may be less conceptually difficult than they appear in other studies. We find support for this view in a study reported by Bryant and Somerville (1986), who claim that children as young as six to eight years old do not find the spatial demands of plotting and reading points difficult when they are freed from the need to focus on construction skills by being provided with ready made line graphs.

In a recent review of a number of studies related to young children's graphical interpretation skills, including the use of motion sensors and other real-time datalogging devices, Phillips (1997) offers further support for this view, concluding that there is evidence of a "surprising proficiency" demonstrated by some young pupils. These pupils are "capable of a wide range of operations with graphs that include . . . the use of scattergraphs to see a trend".

Furthermore, we have collected some evidence which suggests that children working on tasks which involve the interpretation of graphs with the computer also acquire significant insights into the conventions and technicalities of graphing (Ainley, 1995). We conjecture that by allowing the development of interpretation skills *before* explicit teaching of the conventions and technicalities of graphing and the practical skills of drawing graphs, the computer may allow us to re-evaluate the progression traditionally applied to graphing skills. The reversal of the more conventional order that generating graphs precedes their interpretation is a specific case of a more general notion, which Papert (1996) refers to as the *power principle*:

> *... or "what comes first, using it or 'getting it'?" The natural mode of acquiring most knowledge is through use leading to progressively deepening understanding. Only in school, and especially in [school mathematics education] is this order systematically inverted.* (p. 98)

Access to the use of a computer for producing graphs may fundamentally change the ways in which graphs can be used in the classroom, and this in turn raises questions about how the role of graphing may be perceived by children. We suggest that, as with the availability of calculators in arithmetic (Shuard et al., 1991), the possibility of producing graphs with a computer does not simply remove the need for traditional pencil and paper skills, nor allow results to be achieved more quickly. It opens up new topics and new ways of learning. Before looking more closely at the pedagogic approach that we have used, we will identify some of the features of the spreadsheet environment that we see as significant in providing resources for children's learning.

The Pedagogic Contribution of Spreadsheets

There are a number of features in the spreadsheet environment which have particular relevance to the learning of graphing skills. It is clear from our own experience and that of other researchers that there are costs as well as benefits.

The ease with which many different kinds of graphs can be produced
A spreadsheet offers the facility to produce and modify the details of a range of graphs very quickly. This allows children to experiment with producing many different graphs, most of which would be difficult to produce by hand. However, the computer does not consider the sense or the purpose of the graph. This may seem obvious, but is not always apparent to novice users. Provided that the appropriate quantity and type of data has been highlighted, the computer will produce a graph, but it provides no feedback as to whether the graph makes sense. This may cause difficulties for chil-

dren when they use the computer for exploring graphical representation. It appears from our initial observations (Pratt, 1995) that children intuitively apply different criteria from mathematicians or scientists when deciding on the 'best' graph to use. They focus on the superficial appearance of the graph rather than on how easy it is to read or whether it is meaningful, a phenomenon also reported by Ben-Zvi and Friedlander (1996). Organising the layout of data on the spreadsheet so that it can be graphed efficiently emerges as an important analytic skill.

The Nature of Graphs Produced with a Spreadsheet

Graphs produced with a spreadsheet have a number of characteristics which differ significantly from graphs produced with pencil and paper.

- Spreadsheet graphs are dynamic, in the sense that their size and proportions can be altered by dragging the corners of the graph window (although not in the sense of allowing direct manipulation of the graph, as in some other applications). Unless the user has specified otherwise, the scales shown on the axes may change as the graph is distorted.
- Spreadsheet graphs can be created interactively: once the graph has been created, it will change as data is changed on the spreadsheet. It may be possible for the teacher to exploit the potential of this level of interactivity to help children develop their understanding of scale. However, Goldenberg (1987) and Yerushalmy (1991) have shown how the children can be misled by illusions that arise out of the way in which computers present the graphs on screen.
- The appearance of the spreadsheet graph can be changed through menus which control the scales on the axes, the orientation, the style of markings and labels, and so on.

These features may give a sense of control to an experienced user, but can make novices feel less secure. Other features may initially seem problematic: for example, the computer will by default choose a scale which is convenient for fitting the graph to a standard window, but which may not be one that is appropriate or easy to use. In some software, the default setting is for graphs to be drawn showing only the horizontal gridlines.

A New Pedagogic Approach: Active Graphing

We have argued that spreadsheets offer the various characteristics, listed above. In problematising them, we wish to emphasise that in practice particular characteristics may not be experienced, leaving them as mere potentials. Nor is the nonrealisation of the potentials in using a spreadsheet simply due to the mental state of the individual child. The design of educational tools and pedagogic approaches must recognise the influence of the setting in which the tools are to be used.

Several authors have discussed the notion of the transparency of a resource, roughly speaking the ease with which a child might make sense of mathematical ideas through the use of the resource. Lave and Wenger (1991) discuss the notion of the *transparency* of a technology as existing with respect to some purpose, tied to cultural practice. They argue that transparency should not be viewed as a feature of the artifact itself but as something to be achieved through specific forms of participation. Research by Meira (1998) on the use of instructional devices supports the notion that transparency emerges through use. An implication of this way of thinking is that it is necessary not only to analyse the characteristics of the tool, but also to consider the setting in which it will be used.

One aspect of the setting is the task given to the child; this task is likely to shape how the tool is used and therefore what that tool means to that child. Indeed, Nemirovsky and Noble (1997) argue that tools must necessarily have a "lived-in space" where the tool is experienced. They argue that it is experience of using the tool that defines the tool for that individual.

Influenced by a framework in which children's appreciation of graphs is likely to be shaped by both the nature of those graphs and the way in which they were used, we also analysed the setting in which our exploratory work with young children had taken place. Our analysis led us to identify a number of features in both the use of the spreadsheet, and the setting of tasks,[1] which seemed to contribute to children's success in the interpretation of graphs.

- *Immediacy*: because graphs can be produced very quickly, the collection of data, the tabulated recording and the graph are brought into close proximity. Research evidence from studies of data-logging projects (Nachmias and Linn, 1987; Mokross and Tinker, 1987; Brasell, 1987) supports our view that this proximity is important both in children's developing understanding of the conventions of graphing, and in their ability to interpret complex graphical representations.
- *A familiar and/or meaningful context*: this allows children to understand and feel ownership of the data.
- *The presentation of a complete image*: this encourages children to take a holistic view, rather than focussing on the separate processes involved in drawing the graph.
- *The use of a number of similar graphs*: this allows children to focus on differences and common features, encouraging discrimination. Chan-

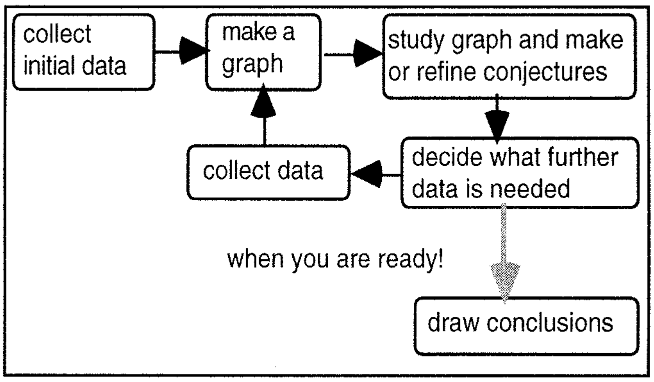


Figure 1. A schematic for the Active Graphing approach.

ging the appearance of the graph by dragging (as described above) adds to this experience.

- *A clear purpose for using the graph to solve a problem*: this enables children to experience active use of graphs as problem solving tools rather than seeing them as illustrations.

These features have been incorporated into a pedagogic approach, *Active Graphing*, which preliminary studies have shown to be successful in introducing the use of scattergraphs to primary school children (aged 8–11 years) (Pratt, 1994, 1995). The process of Active Graphing is shown diagramatically in Figure 1. The key feature is the iterative use of the graph as an analytic tool during an ongoing experiment. Children enter data directly into a spreadsheet as they experiment; for example, children exploring the flight of paper spinners might record the wing lengths and time of flight. Rather than a more conventional sequence of completing data collection, graphing the data and interpreting the graph, children are asked to make a scattergraph after recording a few experiments. They are then encouraged to discuss what they see, make conjectures about the effects of changing one variable (such as wing length), and decide what further experiments they need to do to test these conjectures. The sequence of experimenting, recording data and discussing graphs is repeated until they are able to decide on the best wing length to make a good spinner.

Meanings for Trend

Our initial research with Active Graphing raised a number of issues that have been explored further in this study, designed to allow us to observe children more systematically as they engaged with Active Graphing tasks. The study aimed to explore how children construct meanings for aspects of graphing. In particular, we were concerned to investigate the role played by children's interactions within and across three domains, experiment (E), data (D) and graph (G). During any interactive period, the domain of interaction (DoI) is identifiable by analysis of the children's activity on and off the computer. The term 'activity' is used here to mean the whole range of the children's responses to the tasks, including physical actions, input to the computer and their discourse. We shall see in the later analysis incidents where the focus of the activity changes from the numerical data in the spreadsheet to the graph generated from the data and to the ongoing experiment. Each transition involves a shift in the DoI. Indeed, the discussion sometimes involves a comparison or connection across two DoI's, for example when a child relates the numerical data with features of the graph.

It is useful to imagine these three domains of interaction in a triangular relationship (the EDG triangle), such that interaction within a DoI takes place at the vertex of the EDG triangle whilst interaction across any two DoI's occurs on the corresponding edge. The EDG triangle provides a visual metaphor for discussing how the balance of interactions in the three DoI's may change as the children's experience of active graphing increases. In this paper we discuss the findings of this study in relation to children's construction of meaning for *trend* within graphs of experimental data: other aspects of the study have been reported elsewhere (Ainley et al., 1998a, b).

An epistemological analysis of trend, in the context of this study, suggests the following sub-elements to the concept of trend:

- a dependency in bivariate data (correlation),
- a continuing visual relationship between the succession of points (linearity)
- a potential for unrealised data to lie between already available data (interpolation),
- a potential for unrealised data to lie beyond the extremes of already available data (extrapolation),
- a connection between the dependency in the data and the relationship between the experimental variables (interpretation).

AIMS AND APPROACH

The aim in this study was to observe children engaged in an Active Graphing approach in order to:

(i) categorise the nature of children's activity underpinning learning about trend, as defined by the five sub-elements (correlation, linearity, interpolation, extrapolation and interpretation),
(ii) trace those types of activity across the three DoI's in the EDG triangle, and
(iii) model how the components of the Active Graphing approach might contribute to the patterns observed in (i) and (ii).

By pursuing the three aims above, we hoped to shed light upon the relationship between the Active Graphing approach and children's emergent thinking about trend.

The study was carried out with a Year 4 class (8–9 year-olds) who had not had any previous exposure to the Active Graphing approach, or to the use of scattergraphs, although they were familiar with using spreadsheets, and with working on open-ended problems in mathematics and science. We worked with the class teacher to plan a teaching programme of four tasks that fitted broadly into themes in her planning for the term. These were carefully designed to incorporate key features that are central to Active Graphing:

- a problem which was sufficiently intriguing to gain the children's interest,
- a relatively simple experimental element in which the effects of changing one variable could be measured with reasonable accuracy, and
- some scope for children to make decisions about the design of the experiment, so that they felt ownership of the task and its outcomes.

The tasks are described in detail in the following sections. Work on each task was led by the class teacher and ran in the classroom for one week, generally with a two-week gap between tasks. For pragmatic reasons, the class was organised in two halves, each working on the task in pairs or small groups for a period of up to two hours on alternate days: thus each group worked for 2 sessions on each task. Four girl/boy pairs (two from each half of the class) were selected for close observation.

The researcher worked in close association with the teacher as a participant observer. In her role as researcher, she recorded the children's activity while working on each task through field notes, audio tapes and examples of their work saved on disc. At the end of the week's work on each task there was a plenary session in which groups presented their work to the whole class, and data from these sessions was recorded in a similar way. The researcher recorded regular informal interviews in which the children reviewed their work to date and discussed their plans for the next stage. As a participant observer she also took on some aspects of the role of teacher in response to her observations of children's progress. On some occasions this involved informing the class teacher of the children's situation, and inviting her to intervene. On other occasions, when the class teacher was occupied with other groups, the researcher stepped into her role, and intervened *as a teacher* in the children's work.

The data collected consists of the recorded sessions and interviews, the children's work (spreadsheets and graphs) and field notes. The first level of analysis involved reviewing the audio recordings in conjunction with field notes to identify sections for transcription. This selection focussed on significant learning incidents of various lengths, and at this stage categories of such incidents began to emerge. It was necessary to take into account tensions in the interplay of the roles of teacher and researcher at this stage, with some conversations between the children and the researcher being regarded as teaching incidents (Ainley, 1999). In the analysis such sections of transcripts were regarded in the same way as those involving the class teacher: that is, they were valuable in providing the context of other discussion, but not necessarily valued as indicative of the children's spontaneous activity.

A second level of analysis involved the integration of the selected transcriptions, the field notes and examples of children's work to produce extended narrative accounts, describing the work of each pair on each task. Individual and collective analysis of these accounts by the research team consolidated the categorisation of significant learning incidents, based on two different kinds of criteria. One strand of analysis involved identifying incidents in which the children's activity focussed on a single DoI (E, D or G), and those in which the activity linked DoI's (for example E-D, or D-G). This notation is used to indicate that the children were working with the first DoI, but also talking about the second. This analysis enabled us to look at the pattern of the children's activity across different DoI's. A second strand of analysis focussed on the identification of patterns in the children's activity, categories for which emerged at various stages of data analysis. Later, we present illustrative examples of three types of activity, which we saw as significant in children's construction of meaning for trend: *shape-spotting*, *normalising*, and *feature-spotting*.

The Tasks

The four tasks used in the study were called *Bridges*, *Display Area*, *Helicopters* and *Sheep Pen*. Although they were presented in this order in the classroom, the tasks naturally form two pairs: *Bridges* and *Helicopters* having some different characteristics from *Display Area* and *Sheep Pen*.

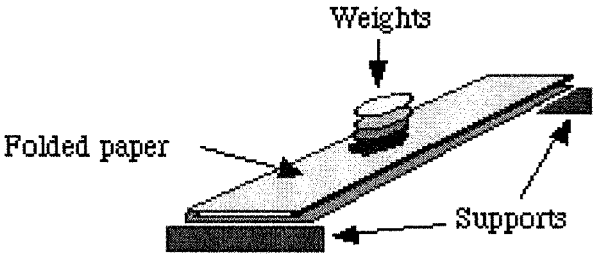


Figure 2. The Bridges apparatus.

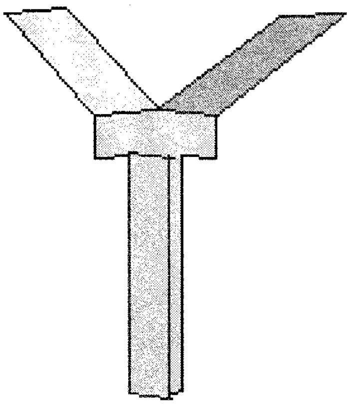


Figure 3. A 'helicopter'.

In *Bridges*, the children were challenged to construct a bridge from folded paper that could hold a marble egg. In her introduction to the task the teacher set up the problem by saying that another teacher had claimed that this would be impossible. The bridges had to be constructed according to the same basic pattern (see Figure 2), although groups could choose where to place the supports. The children experimented by making bridges with different numbers of folds, and testing how much weight each could hold by gradually placing more weights on the bridge until it collapsed.

In *Helicopters* the children worked with simple paper spinners (Figure 3), and were challenged to investigate how the design parameters (e.g. length of wings, length of tail) would influence their flight. Each group explored changing one parameter, experimenting by dropping each 'helicopter' from a chosen height and measuring the time it took to reach the ground. The challenge was to find the 'best' spinner, that is, the one that had the longest time of flight.

The idea for a third task, *Display Area*, arose as part of a project about Tudors in which the children had looked at miniature portraits. The problem was to make a rectangular frame from a 75 cm length of ribbon, which could hold as many miniatures as possible: i.e. the children had to find the rectangle with maximum area for a perimeter of 75 cm. The children worked initially on this task by pinning a length of ribbon on to a display board to make the frame, then measuring the length and width of the frame and entering these results on the spreadsheet. With help, they set up a third column on the spreadsheet with a formula to calculate the area of the frame.

In *Sheep Pen*, the fourth task, the children were given the problem of designing a rectangular sheep pen using 39 m of fencing, to be set against a wall, that would hold as many sheep as possible. In other words, they had to find the maximum area for a rectangle in which one length and two widths total 39 m. They modeled this using straws cut to 39 cm, bending the straw to make a 'pen', measuring its width and length, and entering these on the spreadsheet. As in *Display Area*, they used a third column to calculate the areas of the pens. In both tasks, the children produced graphs of the width (or length) of the rectangle and its area. Both tasks produce similar graphs, in which the maximum value is found from a parabola.

The last two tasks, *Display Area* and *Sheep Pen*, differed in a number of ways from the first two. The first pair of tasks involved variables that could be counted (the number of folds) or measured (time, weight, length). In the second pair of tasks, one of the variables (area) could not be measured directly but could be calculated by the spreadsheet from other measured variables (length, width).

There was a second important difference. In the first pair of tasks, the children only had access to experimental data. In the second pair, the mathematical relationships between the measurements (in particular between the width and the length of the rectangles) were accessible to the children and amenable to algebraic modelling (e.g. $l = \frac{75-2w}{2}$ for *Display Area*, and $l = 39 - 2w$ for *Sheep Pen*). It was therefore possible to identify the rules connecting width and length, and translate these into a formula on the spreadsheet to *generate* data. A more detailed analysis of this aspect of the study can be found in Ainley et al. (1999).

The Teaching Programme

Work on each of the four tasks was organised in a similar way, following the class teacher's normal pattern. Each task was introduced in a whole class session, which involved posing the problem, brain-storming by children about the variables involved and how these might be measured, opportunities for predictions about the outcomes, and instruction about practical organisation of time and resources. The children then worked in pairs or small groups, with half the class working on the task at any time because of resource constraints. During this period the teacher monitored the work of all the groups, including those who formed the research focus, responding to requests for help, and also intervening to resolve disputes and to extend children's thinking. Occasionally she would intervene to give specific instruction, on either a technical or a mathematical issue, but more often such points would be dealt with in plenary sessions, when groups had the opportunity to report back on their work. Such sessions always occurred at the end of the period of work on an task, but were also convened at other times, as the teacher felt appropriate. The teacher and the children were familiar with this style of working in all areas of the curriculum. As the teaching programme progressed, the teacher explicitly drew on experiences from previous tasks as each new challenge was introduced.

ILLUSTRATIVE EXAMPLES OF SIGNIFICANT LEARNING INCIDENTS

Despite the differences between the two pairs of tasks, many of the same themes emerged in our analysis of the children's work. These themes are identified through the nature of the children's activity, and particularly the discourse between children and with the researcher or teacher and were identified post hoc.

Shape-Spotting Activity

Children's first response to seeing scattergraphs was often to interpret them in a 'pointwise' way. By this we mean *looking at* particular points on the graph. We have observed children describing a graph as simply "up, down, up and down". Such interactions, taking place on the G vertex, appear superficially to have little to do with the construction of meaning for trend, although the identification and reading of specific points is clearly an important skill. However, we also observed the spontaneous emergence of a different response to the graph, which we have called *Shape-spotting*. At times the children were fascinated by the visual impact of the graph and attracted towards the identification of familiar shapes (e.g. triangles, circles) and pictures (e.g. of animals, houses). In the following illustrative extract, three children La, Da and Cl discuss the latest graph of their *Helicopter* data (Figure 4).

(All data excerpts are taken from the extended narrative accounts of each pair's work. The first person refers to the researcher, who is the second named author. In the transcriptions of dialogue, pauses in speech

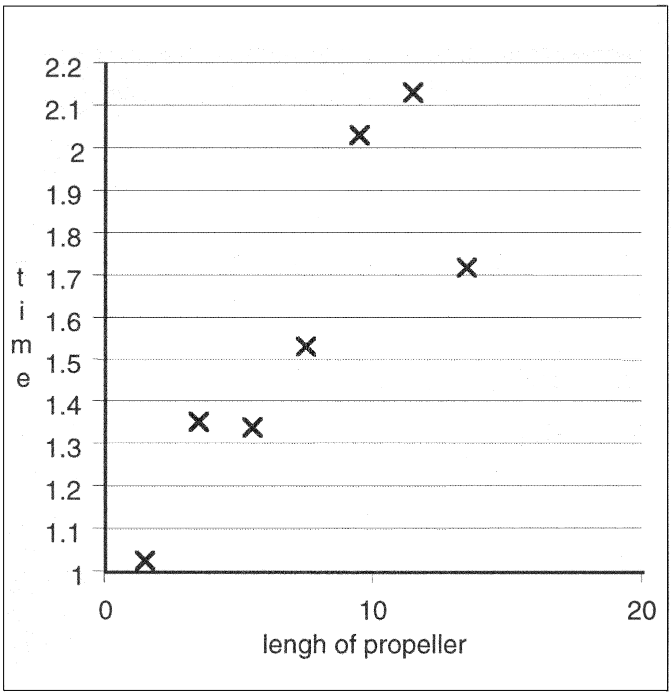


Figure 4. “It goes like a ramp.”

are indicated by three dots (. . .), but no attempt has been made to quantify the length of these. Square brackets [ ] indicate a section of speech which was irrelevant to the main discussion, or too unclear to be transcribed. *Italicised* comments are taken from field notes.)

1. **La:** It’s going like in a pattern.
2. **Cl:** It’s in steps . . . this is a really slopey step and it’s going like that.
3. **La:** And then it looks as if it’s going like a ramp. It first goes like that and then it goes like that, in a ramp.
4. **Da:** It’s going like that, I think. And then it’s going to go down there.
5. **Res:** Can you talk to me a bit more about this?
6. **La:** I think it goes like mmm *(indicating an upward movement)* and then down again. If we do high ones it could still go up.
7. **Cl:** Looks like some sort of animal lying down.
8. **Da:** It looks like a lion. There is the tail and the head. There is its body.

Such shape-spotting was often transient and clearly meaningless in relation to the experiment (though the children would sometimes invent humorous stories which made fictional links to the experiment). Nevertheless, this

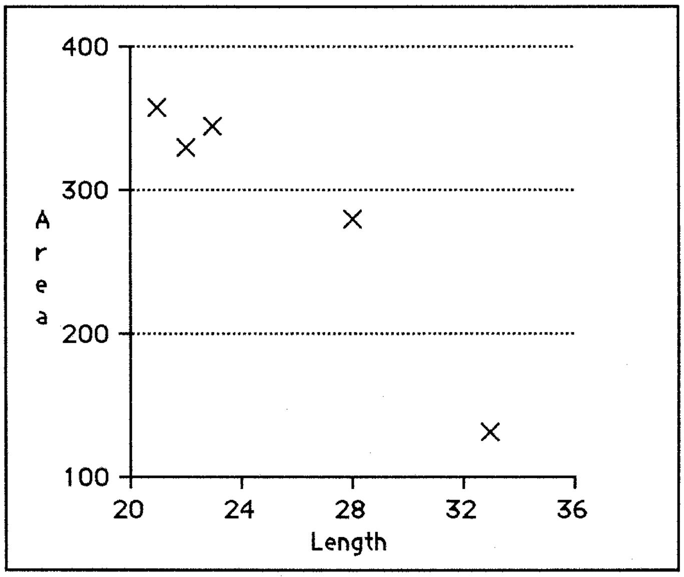


Figure 5. "It's a hill."

type of shape-spotting contrasts with the point-wise reading of the graph described earlier. In order to see the ramp (line 3) or the lion (line 8), the children need to be *looking through* the points. Looking through involves stressing the significance of some points whilst ignoring the relevance of others. Although the basis of that stressing and ignoring may be arbitrary and unrelated to the experiment, it is nevertheless a process that underlies an appreciation of at least one sub-element of trend, linearity. Shape-spotting also demands the invention of new points not amongst the already available data, as in interpolation and extrapolation. Let us emphasise that we do not claim that in shape-spotting the children were in any sense aware of linearity, interpolation or extrapolation. Both looking at points and looking through the points can at best be regarded as early steps towards a fuller construction of trend.

In their work on *Display Area*, La and Da also saw shapes in their graph, but their shape-spotting here is perhaps a little more sophisticated.

9. **Res:** Okay so what is this graph saying *(Figure 5)*?
10. **La:** It's a hill.
11. **Da:** It's like a mountain there.
12. **La:** I think it's gonna come down again.
13. **Da:** and go back to nought.

When they describe their graph (Figure 5) as a ‘hill’ (line 10) or a ‘mountain’ (line 11), La and Da seem to be referring not only to the shape they can see (essentially a downward slope), but also imaginatively predicting what the shape of the graph will be when they have added more data. La’s statement that “it’s gonna come down again” (line 12) appears to refer to the (imagined) left hand side of the graph, indicating her increasingly clear sense that it is legitimate to extrapolate from the trend she has identified. It is significant that Da makes an implicit link between the graph and the data (or perhaps to the experiment) in his statement that the graph will “go back to nought” (line 13).

Normalising Activity

We use the term *normalising* to refer to an activity we observed in a number of different situations, in which the children were unhappy with the appearance of the graph and wanted to modify it towards some perceived norm. Gaps in data were perceived as abnormal and so children normalised by collecting more data to fill in the gaps. Discrepancies in linearity were another common source of normalising, manifesting itself in children checking their data and repeating trials of their experiment. The class teacher soon came to recognise normalising incidents as fruitful opportunities to talk about experimental accuracy and fair testing.

In the following extract, we observe a straightforward normalising incident in which Ta and El were concerned that a particular point did not seem to fit the rest of the emerging pattern in their graph for the *Bridges* task (Figure 6).

14. **Res:** What are these results showing and how do you feel about them?
15. **Ta:** They’re not that good, because on here it should have been *(pointing to the crosses for 4 and 7 folds)*, on the sevens, and fours it should have been bigger . . . number . . .
16. **Res:** Right, which one are you pointing at? This one here, for seven?
17. **Ta:** Because look, here, the four folds *(i.e. the position of the cross)*, and its only the same as two . . .
18. **Res:** You’re right, I wouldn’t be very happy about this either
19. **Ta:** And for three it’s twenty seven
20. **Res:** Right, so what do you think about going on about this?
21. **Ta:** Try it again.
22. **Res:** You would like to try the four again? OK. *(To El)* Ta says that he doesn’t feel very happy about this four result here, because he said that the cross is falling.
23. **El:** Yeah.

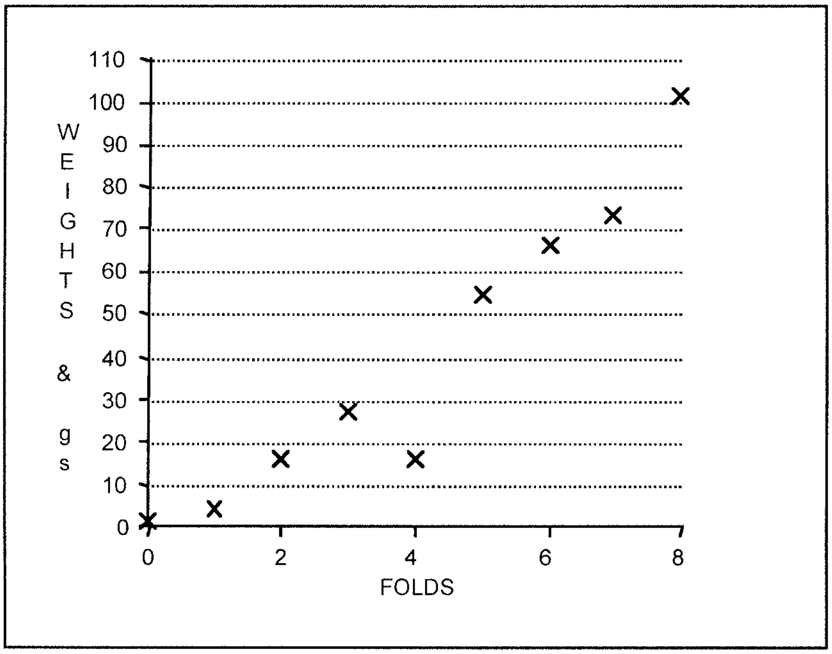
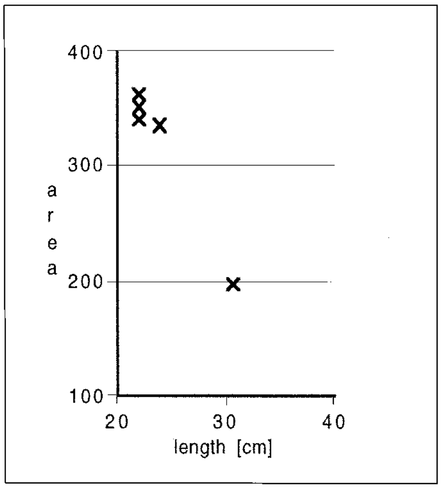


Figure 6. "The cross for 4 is falling."

24. **Res:** Where exactly would you expect it to be?
25. **El**: Forty . . . it should be about here *(points to (4, 40)).*
26. **Res:** Ok you're pointing at in between the results for 3 and 5.
27. **El:** Yeah.

Ta is clearly looking at the graph as a whole, looking *through* the individual points to identify a trend. This enables him to identify points that do not appear to fit the trend (lines 15, 17 and 19). El also uses her sense of the trend to predict where one of these points should be if it is to fit (line 25). Although they are looking at the graph, their discourse involves the graph and the data, particularly as Ta justifies his claim about the cross for four folds (line 15). Such incidents usually led the children to repeat that part of the experiment, in the hope of getting a result that fitted their predictions.

Our second example illustrates a more complex use of normalising, triggered by an anomaly which appeared early in Cl and Ch's work on *Display Area*. Their graph (Figure 7) had three crosses in a vertical line, representing three rectangles with the same length but different widths, thus giving different areas. As the perimeter is fixed, this is clearly impossible. The teacher recognised this situation, and took the opportunity to intervene to encourage them to think about this problem. Because



width (cm)	length (cm)	area
14	24	336
6.5	30.5	198.25
16	22	352
15.5	22	341
16.5	22	363

Figure 7. Cl and Ch’s graph and data.

the mathematical model in this task was accessible to the children, the class teacher knew that normalising could be the trigger for consideration and articulation of the relationships between the variables, rather than further experiment. Her attention was needed by another group, and so the researcher took on the role of supporting the children’s thinking.

28. *The teacher is called away.*
29. **Ch:** Can we measure this one again to see what that actual one is . . . it’s sixteen and a half yes delete them two.
30. **Res:** How are you going to choose which one to delete?
31. *The children discuss this, but are unsure what to do next.*
32. **Res:** Can you tell me how much this is going to be? If this is twenty two how much is the other? . . . How would you find it?
33. **Cl:** Let’s measure.
34. **Res:** Can you do it in your head? . . . How would you find it out?
35. **Cl:** Twenty two add twenty two is . . . that’s forty four then you err then you have to try and make seventy five.
36. **Res:** OK, so how do you make seventy five from forty four?
37. **Ch:** Forty four . . . ohh I think I get it. What you do is
38. **Cl:** Twenty-one.
39. **Res:** Thirty-one.
40. **Ch:** Oh yeah.
41. *The children check this calculation.*
42. **Res:** So then these two things would be thirty-one. What would each be?

43. **Cl:** Fifteen times two equals thirty, sixteen times two equals thirty two.
44. **Res:** We have thirty one though, OK?
45. **Cl:** I don't know.
46. **Res:** It's very close what you are saying. If we have, this is twenty two and this twenty two and this is forty four so these two are thirty one, both of them, so how much each.
47. **Ch:** Divide it
48. **Cl:** Ahh so it's fifteen and a half.
49. **Res:** Excellent! So this is how to choose which one.
50. **Cl:** Now I get it! That one is right but these two aren't.

In this example, Cl and Ch began by attempting to normalise the graph in accordance with the logic of the experimental situation by checking their measuring (lines 28 to 33). Through the guidance of the teacher/researcher, a realisation emerged that it was possible to calculate the width of a rectangle given the length and the perimeter. In order to normalise the graph, their attention, and their discourse, was drawn to the data and they began to work on the mathematical relationship between width and length (lines 34 to 50).

In the first incident, Ta demonstrated, through his normalising, an awareness of interpolation. In the second example, an explicit appreciation of the mathematical dependence between the two variables (correlation) emerged. Cl and Ch went on to use the rule they had discovered as a correcting procedure, applied to experimental data. Although they were aware of the utility of their rule for checking existing data, they did not yet see the potential for generating new data.

Feature-Spotting Activity

The activity we have called *feature-spotting* is related to shape-spotting in that both demand a visual awareness of pattern in the graph. Shape-spotting takes place on the G vertex of the EDG triangle, whereas feature-spotting connects aspects of the visual appearance of the graph to the experiment. Children sometimes connected particular points on the graph with incidents in the experiment; so, for example, they might identify one point as 'the one I dropped', or 'the one that hit the chair on the way down'. Feature-spotting became more sophisticated when larger grain features of the graph were involved. In the following extract, Ta, El and Je discuss with the researcher the graph that they have produced from the *Helicopters* task (Figure 8). They have conducted a large number of trials with different wing lengths, but difficulties in timing the flights accurately has meant that the graph is rather 'messy'. They are able to describe some trends in

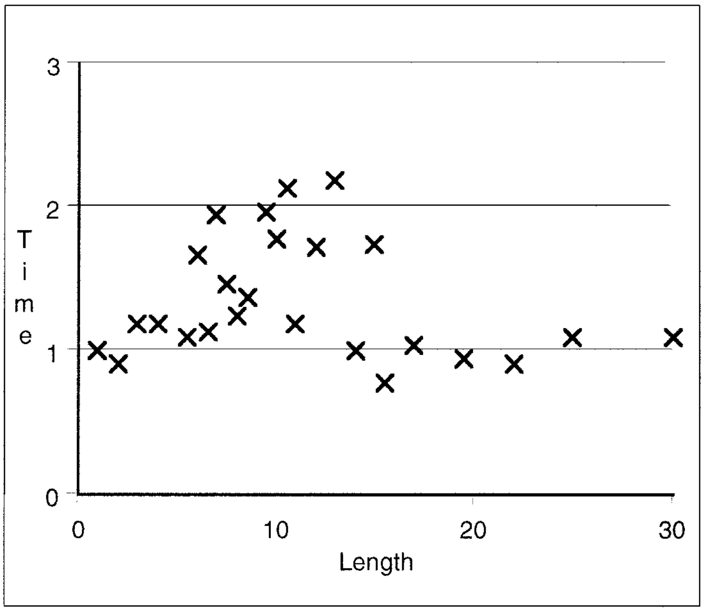


Figure 8. Ta and El's scattergraph.

the graph, but the complexity of the image, and of the children's choice of language mean that the researcher finds it necessary to intervene frequently to help them articulate their ideas.

51. *Ta and El begin by describing how the left side of the graph goes up and the right side is "all on the same line". They carry on to describe how the graph goes up and down.*
52. **Ta:** They are going up instead of down because there *(pointing to the right-hand side of the graph)* they are going down.
53. **El:** Sort of . . . they are climbing like that and they are going like that.
54. **Ta:** I think the under 10, 15, under 15 are going are the best.
55. **Res:** Okay, so less than 15 and more than . . . what is this?
56. **Ta:** Actually 12 and 9 they are all coming up.
57. **Res:** Yes, and then what's happening?
58. **Ta:** Over 12 they are all going down.
59. **Res:** They are going down and becoming more or less the same lower down, okay?
60. **Ta:** They are just falling instead of spinning.
61. **Res:** Oh, you say falling instead of spinning.
62. **Ta:** Yeah, going straight down.

63. **Res:** Right, and they are going fast because they are going down fast. Then the flight is quite short, isn't it? They don't stay up in the air for a long time, do they?
64: **El:** No, they are just falling down.
65: **Res:** Is that what we are seeing here *(pointing to right-hand side of graph)*?
66. **El:** These are like staying in the air – longer sort of time.
67. **Res:** Are these staying longer?
68. **El:** Well, not longer but they are staying in the air the same sort of time.
69. **Je:** They are all the same, aren't they?

In this extract, we can trace how the discourse shifts between the features of the graph they are looking at, and the flights of the spinners which they are recalling, as the vocabulary of 'going up and down' is used for both. Ta's first comment that 'They are going up instead of down' (line 52) clearly refers to the crosses on the graph. Later his 'Yeah, going straight down' (line 62) must refer to the flight of the spinners, but between these two comments he says 'Over 12 they are all going down' (line 58), which could describe the crosses, or the spinner, or both. Je's final question is also interesting. El has been talking about the spinners (lines 64, 66 and 68). Yet, when Je says 'They are all the same' (line 69) she seems to have switched back to focusing on the crosses (or perhaps the times they represent), since the spinners themselves were quite different in appearance because of their wings. The ambiguity in the language between graphical features and the experiment supports a movement along the G-E edge of the EDG triangle.

Our second example of feature-spotting comes from the fourth task the children undertook, *Sheep Pen*. Their activity on this task was rather different from previous ones. Although physical materials were offered (art straws cut to length to model the fencing), many of the children, as a result of their experiences with *Display Area*, soon abandoned the practical activity in favour of using a spreadsheet formula to model it (Ainley et al., 1998a, 1999). Two pairs of children (Ta and El, Je and Ph) both used such a formula to give the length of a sheep pen for any width they chose. However, the graphs they produced looked rather different because of the scale, and because they had entered slightly different values for the width. In the following extract, the researcher encourages them to compare their graphs.

70. *I ask the two pairs to compare their graphs. (Figures 9 and 10).*
71. **Ph:** Ours is a little skinny one, yours is a big, fat podgy one.
72. **Res:** Do we think the graphs are similar?

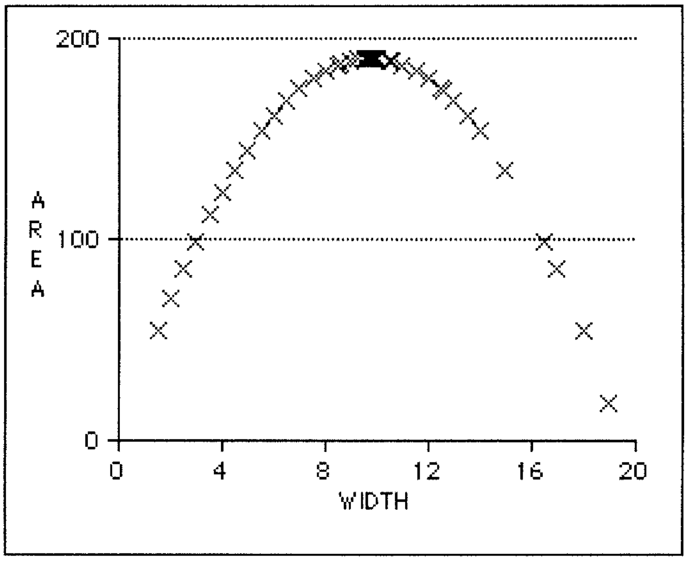


Figure 9. Ta and El's sheep pen graph.

73. **Ph:** Yes, sort of.
74. **Je:** That's thin . . . that's fat and that's thin and . . .
75. **El:** That one doesn't go back to the end . . . it goes like . . . *(Figure 9).*
76. **Res:** Why do you think it's that?
77. **Je:** Yours stops there and not go to the bottom *(Figure 9).*
78. **Res:** Why do you think it is so?
79. **Ta:** They have chosen lower numbers than ours.
80. *Children point at examples of lower numbers.*
81. **Res:** Is there anything special about the crosses, you know, on this graph? *(Figure 10)* Do you think they are like evenly spread or do you think they are squashed at all?
82. **Ph:** At the top they're really squashed [*unclear*] and at the bottom they're really spread out.
83. **Res:** Right, *(to Ta)* what do you think about your graph then? Do you think they are squashed in some area?
84. **Ta:** Yeah, here *(pointing to Figure 9).*
85. **Res**: Why do you think this is so? Why do you think in some areas?
86. **Ta:** We got the same.
87. **Je:** On the 190s . . . two the same.
88. **Res:** Aaah!
89. *I ask Je whether their crowded data around 190 in Figure 10 is there for the same reason. Ph says they worked around 190 to try to find the maximum.*

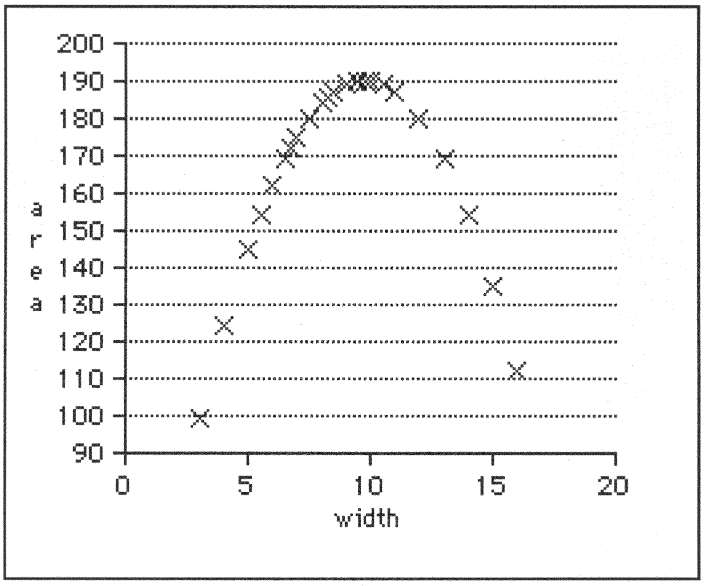


Figure 10. Je and Ph's sheep pen graph.

The discussion in this extract differs from the previous example, since in G-E discourse children are not referring to an actual experiment (as they never carried this out) but to experiments with their computer models of the situation. Nevertheless, we can see evidence of the children *looking through* features of the graph to describe features of the (virtual) experiment. As in the previous example, the referents for their comments are frequently ambiguous, encompassing the graph, the experiment and occasionally the data. When Ta says that 'They have chosen lower numbers than ours' (line 79) he seems to be using 'lower' to describe both the positions of the crosses on the graph, and the size of the numbers used for the width. In fact it is Ta and El who have used the lower values for the width: Ta is misled by the appearance of the graph.

The children are also very aware of the differences (lines 71,74, 75 and 77) and the similarities of the two graphs (lines 82, 84, 86 and 87). Ph describes the appearance of one graph vividly ("they're *really* squashed", line 82); Je and Ta support this observation. Both of them talk about having '[two] the same' for one of the values (lines 86 and 87), that is two different widths giving the same area. Ph goes on confidently to say that this is because they 'worked around' 190 (line 89) to see if they could find a higher value – higher both as a value for the area and as a position on the graph. However, because of the scale of the graphs, it is now impossible for the children to spot the highest crosses, and so their attention is more

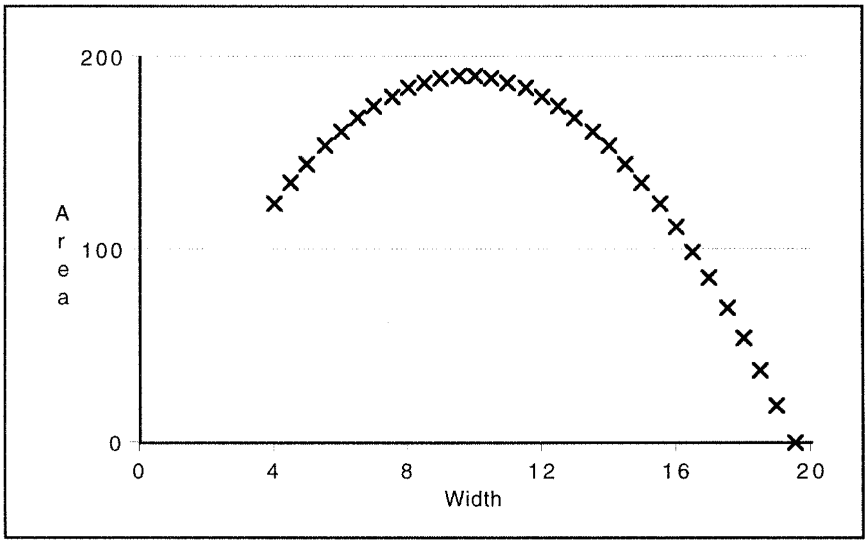


Figure 11. 'This hill here . . . '.

focused on the spreadsheet data. In a final example from the same task, La and Da are working systematically with their spreadsheet model of the sheep pens. They have used a formula to increase the values of the width by 0.5 each time, but are still not satisfied that they have found the real 'top of the hill' on their graph (Figure 11).

90. **Res:** *(you said)* we don't really need to make them, we can look at the graph and look at the data and make decision. Let me ask you something . . . you said . . . you pointed at the top of this hill here okay, that somewhere there is the maximum we can have, okay? And we also found that 190 is the best we have already.
91. **La:** Yes that's 190 *(pointing to the graph)*.
92. **Da:** And that' s not a square *(referring to the sheep pen).*
93. **Res:** But you think we can go even bigger than that?
94. **La:** We could go higher.
95. **Res:** How can we go higher?
96. **La:** Aaahh!
97. **Da:** I think we could do 20 now.
98. **La:** Where is 190? So we could do 9 10 . . .
99. **Da:** 29.
100. **La:** No because we've done halves . . . and we've done wholes.
101. **Res:** Yes.
102. **La:** (Mumbling tentatively) we can do quarters.

103. **Res:** Say it again!
104. **La:** We can try quarters.
105. **Res:** Because as you say we have done all the whole numbers and we have done all the halves.
106. **La:** So now we do quarters.

La identifies a feature of the graph – the flattened top of the hill – and realises that they could 'go higher' (line 94). Her comment seems to refer both to the shape of the graph, and to the values of the area that they will produce (E/D). Her understanding of the way that their data is being generated, and of what this means in the virtual experiment, leads her to suggest a strategy of trying values in between those they already have i.e. changing the formula to increase the width by 0.25 each time. This is what she means in saying: 'We can try quarters' (line 104). This strategy does prove successful in making the 'hill' go higher.

It is clear in both of these examples from *Sheep Pen*, the last of the four tasks, that the children's confidence and skill in reading scattergraphs has increased to the point where they move effortlessly in their discussion between the graph, the data and the problem situation. Interpolation of a (new) maximum value between the two existing highest values illustrates a sophisticated sense of trend in the graph. Feature-spotting, in contrast to shape-spotting, demands interpretative skills in making sense of the data through reference to the experiment.

Summary

Through the examples in this section we have illustrated the following aspects of the children's activity:

- When children's activity is focussed on the graph, *shape-spotting*, as distinct from pointwise reading, was seen as a significant category of activity, since it indicates attempts to look through, rather than at, the individual points.
- Two further activities, *normalising* and *feature-spotting*, were identified. We see both of these as emergent meanings for trend, characterised by shifts in attention from the graph to the data and/or the experiment.
- *Shape-spotting*, *normalising* and *feature-spotting* appeared and reappeared intermittently, rather than following a particular sequence, presumably cued by surface features of the activity. However, as children progressed through the programme of tasks, their awareness of trend and interpretation of graphs generally became more fluent.

DISCUSSION

The aim of this study was to elaborate how the children's thinking about trend emerged during Active Graphing, through monitoring and categorising the children's activity in relation to their use of the three DoI's of Experiment, Data and Graph. We set out earlier three specific aims and will structure the discussion by examining the data against each of these aims in turn.

Aim 1: To Categorise the Nature of Children's Activity

Our analysis has identified and illustrated different activities which children engage in during the active graphing approach: *shape-spotting*, *normalising* and *feature-spotting*. These three activities all indicate children's developing skills in *looking through* the individual points presented on the graph to observe regularities in the data and relationships between the experimental variables. Of these activities, shape-spotting may initially appear unrelated to interpretation and use of the graph, but we have come to see this as signaling a move away from *looking at* the data in a pointwise manner. Thus shape-spotting can be used as an opportunity for intervention to draw children's attention towards making links across DoI's through discussion of regularities.

There is a dialectic relationship between the activities of normalising and feature spotting: successful normalising has a pay-off in terms of accuracy, supporting the construct and notion of trend, and making it easier to spot features in the graph. As the sense of trend becomes more firmly established, the identification of perceived abnormalities in the graph becomes more efficient, supporting further normalising.

However, our data also shows the intermittent nature of sense-making as children learn to read and interpret the graphs efficiently, indicated by the appearance and re-appearance of shape-spotting, normalising and feature-spotting throughout the programme of four tasks. In our final example in the previous section, taken from the fourth task, La shows perhaps the most sophisticated interpretation skills we observed, but she and Da still talk about their graph as a 'hill'. We did not see the clear changes where one type of activity was replaced by another, as they worked through the programme. Rather, the developing sophistication of their constructions of trend were indicated by greater levels of fluency in their discourse.

Aim 2: To Trace Those Types of Activity Across the Three DoI's in the EDG Triangle

In our analysis it became clear that much of the children's activity in response to all four of the tasks involved interaction not with a single DoI, but linking across the domains in different combinations. A good illustration of this is given in the example of feature-spotting described earlier, relating to a scattergraph produced during work on *Helicopters* (see Figure 8). Here we saw the children's discourse moving seamlessly between descriptions of the graph in front of them, and references to aspects of the experiment they had previously carried out. Nemirovsky (1998) terms such 'talking and acting without distinguishing between symbols and referents' as *fusion*, which he sees as an expression of fluency in symbol use (in which he includes graphical symbols). In this and many other examples of feature-spotting we observed fusion in the children's discourse as they constructed connections between their intuitions and observations of the experiment and features of the scattergraph. Of the five sub-elements of trend, at least three (interpolation, extrapolation and interpretation) exist on edges of the EDG triangle, demanding connections between the D and G vertices or between E and G vertices, and sometimes articulated across all three DoI's. The two examples of *feature-spotting* taken from the *Sheep Pen* task (related to Figures 9, 10 and 11) are particularly clear illustrations of how the children learned to make connections between the different DoI's. In these, the children's activity links all three DoI's, as they use the appearance of the graph to conjecture that a different treatment of the data will produce the solution to the experimental problem.

Aim 3: To Model How the Components of the Active Graphing Approach Might Contribute to the Patterns in 1 and 2 above

The Active Graphing approach supported interaction across the DoI's by allowing the juxtaposition of Experiment, Data and Graph in two senses. They were juxtaposed in time, as data could be entered as soon as it was collected and graphs could be produced quickly. They were also juxtaposed in space, as the computer could be used wherever the experiment was carried out, and graph and data could be shown on the same screen.[2]

We believe that the purposefulness of an Active Graphing task, the focus on an overall problem to be solved, draws the children into the need to make sense of the graph, and offers opportunities to understand the *utility* of graphs as analytical tools. As the children engage in normalising and feature-spotting, an awareness of a dependent relationship between the two variables in question emerges. Thus, making sense of the graph supports an understanding of the experimental situation, and of the problem solution. As the graph is used as an analytic tool within the ongoing experiment, children are given the opportunity to appreciate the utility of graphs, and to develop familiarity with the conventions and technicalities of their construction as they repeatedly read and interpret them.

The Active Graphing tasks each contained an explicit purpose: to find the strongest bridge, the best helicopter, the largest display area or largest sheep pen. At the same time, the tasks were designed in such a way that the explicit purpose became transparently connected with the utility of spreadsheet graphs in analysing the experiment. This linking of purpose and utility rendered the process of making sense of the graphs meaningful. In presenting the four tasks, we distinguished between one pair (Bridges and Helicopters) from the other (Display Area and Sheep Pen) because the latter incorporated a mathematical model and the potential to make use of spreadsheet formulas. Superficially, one might have expected the latter pair of tasks, which demanded the use of algebra, to seem somehow less "real" to the children. In practice, the four tasks were treated by the children in much the same way. The meaningfulness seemed to stem from the linking of purpose and utility rather than the mathematical content.

It would be reasonable to ask whether spreadsheets are intrinsic to the active graphing process. Could not ready-made graphs be provided on paper? Environments that provide opportunities for children to engage in sense-making activity need to allow children to test out and sometimes invalidate conjectures that stem from their current understandings of the problem situation. A teacher could not anticipate the range of conjectures that even one child might wish to explore. The spreadsheet environment enables a degree of control to be passed over to the child and we see this as essential in supporting the child's sense-making activity.

ACKNOWLEDGEMENT

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NOTES

[1] Although the term 'task' has a more authoritarian connotation than we would wish to imply, it is used here to distinguish the task set from the children's activity in response to it.

[2] In fact the children were using portable machines which allowed the children to keep their machine physically close to the experiment facilitating the active graphing process.

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