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AN APPROACH FOR DEVELOPING AN OPTIMAL DISCOUNT PRICING POLICY*

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This paper addresses the problem of why and how a seller should develop a discount pricing structure even if such a pricing structure does not alter ultimate demand. The situation modeled is most appropriate where the seller's product does not represent a major component of the buyer's final product, where the demand for the product is derived, or where the price is only one of many factors considered in making a purchase decision.

A model of buyer reaction to any given pricing scheme is developed to show that there exists a unified pricing policy which motivates the buyer to increase its ordering quantity per order, thereby reducing the joint (buyer and seller) ordering and holding costs. As a result, the seller is able to reduce its costs while leaving the buyer no worse off and often better off.

The model is extended to handle variable ordering and shipping costs and situations where the seller faces numerous groups of buyers, each having different ordering policies. Finally a case study is presented explicitly showing how the proposed pricing policy can be applied to the situation of a large seller selling to a number of different buyer groups.
(PRICING; MARKETING)

Although there is a substantial literature on how a *buyer* should react to a quantity discount schedule [see for example Hadley and Whiten 1963 and Chase and Aquilano 1977], there is no established theory as to how a *seller* should develop a discount pricing structure. Most discussions of discounts implicitly or explicitly assume that discounts are given by the seller in response to pressures from large buyers. However, there are numerous instances where a large seller offers discounts to a large number of smaller buyers who lack the economic power to demand such a price discount. Examples are a tire manufacturer selling to a number of privately-owned retail outlets that stock and subsequently sell the manufacturer's product to the ultimate consumer, a large electronic components manufacturer selling to a large number of small distributors, and a cleaning supplies manufacturer selling its products to a large number of small firms who consume the product during their manufacturing process. One possible explanation for discounts in such situations is that the seller is using the discount structure to maximize its profits by modifying the buyer's ordering policy.

We demonstrate the technical feasibility of such an explanation by constructing a pricing scheme which minimizes the total transaction costs associated with ordering, shipping and inventorying the seller's product without requiring any central planning between the buyers and seller.[1] Moreover, we show that the pricing scheme results in both parties being at least as well off as if there were with no discounts. We start with the premise that the market between buyer and seller is in equilibrium. By this we mean that the seller has established some list price, P_0, for the product where this price is independent of the quantity ordered by the buyer, and that the buyer, based on this price, has determined (i) its (annual) demand, D, for the product and (ii) the optimal order quantity Q^*, the size of the order placed each time. The question then is, "Can the seller devise a pricing discount schedule to induce the buyer to change its ordering policy so that (a) the buyer is no worse off, and (b) the seller's profits are increased, even if the total annual demand does not change?"

More specifically, we will show that it is possible to design a unified pricing policy $p(Q)$ where the average price p of the seller's product is a monotonically decreasing function of Q, the amount ordered by the buyer. Moreover, we will show that such a policy provides an economic incentive to the buyer to order in quantities greater than Q^*. This enables the seller to convert more quickly a noninterest generating asset

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[1] Of course it is also necessary to determine the legality of giving different quantity discounts to different buyers. However, as pointed out by Crowther (1967) and shown explicitly in our subsequent development, the proposed pricing schedule is based on cost considerations and therefore legal restrictions should not impede implementation.

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(inventory) into a interest generating asset (cash) and also reduces the number of times the buyer places an order in a given period of time.[2] This latter factor reduces both the buyer's and seller's annual ordering costs. It is, in fact, these reduced annual ordering costs that allow the seller to develop a pricing scheme which increases its profits while still being able to compensate the buyer for incurring the extra holding costs.

These holding costs and ordering costs can be surprisingly high. For example, current estimates for a salesman making a sales call indicate that the direct average cost of taking an order exceeds $100 (*Marketing News*, 1982). This is in addition to the cost of the paper work associated with the order. Average shipping costs per unit often decrease with increases in quantity shipped, thus considerable savings also can be realized by increasing the shipment size. Finally given the recent increases in interest rates it is surprising that no established pricing schemes exist which explicitly take into account the cost savings to the seller associated with speeding up the sale of the product.

The basic ideas used in this paper originated from a suggestion by Crowthers (1967) and formalized by Dolan (1978). However the specifics of our analysis differ from Dolan's work in a number of ways. First, the model proposed by Dolan assumes the seller has only one buyer (or at least all buyers have identical inventory parameters) and the costs for each buyer are constant. In reality, sellers often have many buyers with different costs and final demand for their end product. We incorporate such reality into our analysis.

Second, Dolan provided no means by which the seller could specify the incremental savings to be passed on to the buyer. This is an important limitation, since the buyer may not be willing to take the risk of changing its order quantity unless there are economic incentives for it to do so. Finally, and probably most importantly, Dolan was not able to derive a pricing scheme which guaranteed the seller maximum profits.

It should be noted that the pricing scheme outlined allows the seller to modify the buyer's behavior without increasing channel conflict. In this way, it is akin to the work of Jeuland and Shugan (1983) and McGuire and Staelin (1982), although as will be discussed subsequently, the assumptions these authors make on how the buyer's behavior is modified are considerably different from the approach taken herein. Also our approach is in marked contrast to that taken by some sellers (especially in contractual vertical channel systems) who find it in their best self-interests to transfer their inventory holding costs to the buyer by "forcing" the buyer to carry more inventory than the buyer believes is best for itself. For example, automobile dealers often complain about cars being forced on them by automobile manufacturers (White 1971). Such practices can lead to conflict between the two agents. In contrast, the proposed pricing policy explicitly insures that the buyer's total costs are not increased, thereby enabling the seller to increase its profits without increasing channel conflict.

The rest of the paper is organized as follows. First a pricing policy that insures the two agents reach the joint profit maximizing solution is developed for the one manufacturer-one group of homogeneous buyers problem. The procedure allows the seller to determine the split in increased profits between itself and each of the buyers. We next extend the model to allow for variable ordering and shipping costs. Then the solution is generalized to the situation of a seller facing numerous groups of buyers, each having different ordering policies. Finally we provide a case study which explicitly shows how the proposed pricing policy can be applied to the case of a large seller, selling to a number of different buyer groups.

The Model

Assumptions

We assume the seller and all buyers' inventory policies can be described by simple EOQ inventory models based on deterministic demand, no backlogs, no stockouts, and deterministic lead times to determine their inventory policy. In making such an assumption we realize that some firms may have more complex inventory policies. However, the EOQ model normally is a good representation of a firm's actions. Moreover, this model is the one used by many firms for determining their inventory levels. Also, the EOQ assumption could be relaxed by incorporating a more complex inventory policy, although it is not clear that such complexity would be merited.

A second assumption of the model is that the seller has knowledge of the holding and ordering costs governing the buyer's inventory policy. This is not a very restrictive assumption, since the holding and ordering costs can be inferred from the ordering behavior of the buyer prior to the development of any new pricing policy. Moreover, the proposed procedure can be supplemented with a sensitivity analysis to estimate the

[2] A similar suggestion has been made by Ingene and Levy (1982) with respect to retail credit sales. In their analysis, they are interested in converting credit card sales into more liquid cash sales.

effects of making errors in inferring these costs. (See the results section for an example of such an analysis.) Third, we assume the seller is price competitive and that his competitive market price to all buyers, independent of the quantity ordered, is P_0. We ignore any competitive price reaction to the seller's discount policy and in addition assume that demand is not increased by the lowering of price. Although such an assumption possibly limits the applicability of our analysis, this assumption allows a first step which may be extended later to more complex models. There are still a number of industries where such an assumption is a good first approximation. In particular this assumption should hold in situations where the seller's product does not represent a major component in the buyer's final product (e.g., an electric component for a durable good), where the buyer is the ultimate consumer and has a fixed requirement for the product (e.g., cleaning supplies) or where the price is only one of the many factors considered in making the purchase decision. Such situations often arise in industrial selling. Thus, our price discount policy is motivated by cost justifications versus any attempt to increase market share by lowering the price. This assumption is a major difference between our model and the models of Jeuland and Shugan (1983) and McGuire and Staelin (1982). In both of these latter cases, it is assumed that final demand is altered by the change in the wholesale price $p(Q)$. Thus in these latter situations the losses in revenue associated with decrease in price are reduced at least in part by the increase in demand brought about by the lower final price.

Another ramification of our emphasis on cost savings versus increasing sales is that annual demand for the seller's product is not changed. As a result, the seller does not have to alter its optimal production schedule, a schedule which is based on demand and production set-up costs, neither of which are modified under our assumptions. In fact, the only factors which are modified are (a) the buyer will incur extra holding costs associated with the extra quantity ordered each order, (b) the buyer and seller both will incur fewer costs associated with ordering and processing an order since the buyer orders less often and (c) the seller will convert the noninterest earning asset into an interest earning asset more quickly. Another implication of the constant demand assumption is that it is possible to limit attention to one period. We acknowledge that expectation about exogenous factors, such as increasing interest rates and increasing prices, may cause buyers to deviate from their annual purchase rate. However we believe this representation of demand is a good first approximation of reality. Moreover, it reduces the complexity inherent with a multi-period model.

Finally, we assume the buyer's holding costs are greater than the seller's benefits associated with receiving payment for its product earlier. Although this is a necessary assumption, it would seem to be nonrestrictive, since capital costs are only one component of the holding costs and in any case the seller generally has lower holding costs due to economies of scale.

Notation

We build on the notation used by Dolan (1978). For the one seller-one group of homogeneous buyers case, define the following variables.

$\bar{B}(Q)$: Buyer's annual cost excluding the cost of the product associated with purchases from the seller in lot sizes Q.

$B(Q)$: Buyer's annual augmented cost, i.e., $\bar{B}(Q)$ plus the cost of the product, associated with purchases in lot sizes Q.

$\bar{S}(Q)$: Seller's annual cost excluding the cost of the product associated with purchases by the buyer in lot sizes Q.

$J(Q)$: $\bar{B}(Q) + \bar{S}(Q)$, the joint cost function for both agents.

H_b: Buyer's cost of holding a unit in inventory for one year.
H_s: Seller's cost of capital per year.
A_b: Buyer's fixed cost to place an order of any size.
A_s: Seller's fixed cost to process an order of any size.
D: Buyer's annual demand rate for the product sold by the seller.
Q: Size of orders placed by the buyer.
P_0: The list price to the buyer before quantity discounts.

One Seller-One Group of Homogeneous Buyers Model

We first solve the one seller-one group of homogeneous buyers problem with ordering and holding costs assumed to be constant. The assumption of one group is relaxed in a later section. The problem is solved by explicitly considering only one buyer. It should be noted, however, that the solution is applicable only to situations where a seller faces a group of homogeneous buyers, since otherwise the seller's average inventory level would be modified by changes in the one buyer's ordering policy.[3]

Using the above notation and assuming that the final demand is constant over time, the buyer's cost function per year, excluding the cost of the product, as a function of the quantity ordered is

$$\bar{B}(Q) = (\text{ordering cost}) + (\text{holding costs}) \quad \text{or} \quad \bar{B}(Q) = \frac{D}{Q}A_b + \frac{Q}{2}H_b.$$

Similarly the seller's costs per year are

$$\bar{S}(Q) = \begin{array}{l}\text{order processing costs} - \text{decrease in capital costs due to}\\ \text{buyer purchasing the units} \quad \text{or}\end{array}$$

$$\bar{S}(Q) = \frac{D}{Q}A_s - \frac{Q}{2}H_s.$$

It follows that the joint cost function $J(Q) = \bar{B}(Q) + \bar{S}(Q)$ is

$$J(Q) = \frac{D}{Q}(A_b + A_s) + (H_b - H_s)\frac{Q}{2}. \tag{1}$$

Based on the assumption that the buyer uses an EOQ rule to determine its order quantity, the buyer would order

$$Q^* = [2A_bD/H_b]^{1/2} \tag{2}$$

each time it orders. However, this ordering quantity does not minimize the joint cost function. To see this, differentiate (1) and solve for Q^{**}, the order quantity that minimizes the joint cost function of the buyer and the seller. This yields

$$Q^{**} = \left[\frac{2(A_b + A_s)D}{(H_b - H_s)}\right]^{1/2}. \tag{3}$$

Since the numerator in (3) is greater than in (2) and the denominator in (3) is less than in (2), Q^{**} must be greater than Q^*. Also since $J(Q)$ had a minimum at Q^{**}, $J(Q^{**}) \leqslant J(Q^*)$, which implies that $\bar{B}(Q^{**}) - \bar{B}(Q^*) \leqslant \bar{S}(Q^*) - \bar{S}(Q^{**})$, or the

[3]Explicitly we assume that the pattern of demand faced by the seller is unaltered by changes in the quantity ordered by the buyers. This will occur if the seller is distributing its product to a large number of buyers who place orders independently of each other. Such an assumption was not explicated by Dolan (1978) or Crowther (1967).

decrease in the seller's costs are more than the increase in the buyer's costs when the ordering quantity is increased from Q^* to Q^{**}. It is this difference that provides the seller with the economic means to motivate the buyer to operate at Q^{**} by passing back some of these savings to cover the extra costs associated with placing larger orders.

This analysis, however, says nothing about how to design a pricing discount scheme to motivate the buyer to operate at Q^{**} or how the gains are shared between the two agents. The sharing of the gains should depend entirely on the environment prevailing in the channel.

Consider two extreme cases:

(a) The two agents get together and decide among themselves to operate at Q^{**} and bargain on the sharing of the gains. The determination of how they split these gains depends on the relative bargaining power of these two cooperative members.

(b) The seller sends the buyer a price list that is a function of the quantity ordered and the buyer communicates to the seller the quantity it wishes to order (i.e., the transaction is done at "arms length"). Even though the seller has no evident power in this situation, the seller is the dominant player in this game since it can use the information on the buyer's reaction to its price list to develop the pricing strategy which maximizes its (i.e., seller's) profits. Thus, even though the buyer is allowed to behave in a way which maximizes its profits, it does so *conditional* on the seller's pricing structure.

Although there may be some situations where case (a) mirrors reality, we analyze (b) in this paper because we believe it is the more representative case. Consequently, we next develop a pricing policy which allows the seller to increase its profits and still insures that a cost-minimizing buyer will order Q^{**}.

Model Development

First define a price discount schedule to have the following structure: the seller charges price P_0 for any order $Q \leqslant Q_0$, price P_1 for any quantity Q greater than Q_0 but less than or equal to Q_1, price P_2 for any quantity greater than Q_1 but less than or equal to Q_1, and so on down to price P_m, where $P_0 > P_1 > P_2 > \cdots > P_m$. Let p be the *average unit* price over all items purchased. Then it is easy to show that a plot of the average price p versus the quantity purchase Q for the above defined price discount schedule would have the shape as shown in Figure 1.

Dolan has shown that using the above price discount schedule, it is impossible for the seller to motivate the buyer to order Q^{**} with just one price break (i.e., $m = 1$) and still increase its profits. Consequently, we take the approach of approximating a discrete price discount schedule made up of many price breaks with a continuous price-quantity relationship $p(Q)$. The function used should fit the above average price structure closely if not exactly and also have the following properties.

1. $p'(Q)$, $p''(Q)$ both exist.

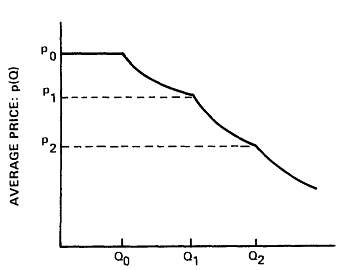

FIGURE 1. Example of Average Price versus Quantity for a Quantity Discount Schedule.

2. A cost minimizing buyer will find that it is in its best self-interest to order in lot sizes of Q^{**} given the price list $p(Q)$.

3. The augmented cost, $S(Q)$, at Q^{**} incurred by the seller under the discount structure $p(Q)$ should be less than with no discounts, i.e., $S(Q^{**}) < \bar{S}(Q^*)$, where $S(Q) = (P_0 - p(Q))D + \bar{S}(Q)$, $Q > Q^*$.

One relationship that satisfies the above conditions is:

$$\begin{aligned} p &= P_0 \exp[-a(Q - Q^*)] \quad &\text{for} \quad Q \geqslant Q^*, \\ p &= P_0 \quad &\text{for} \quad Q \leqslant Q^*, \end{aligned} \tag{4}$$

where a is a constant greater than zero and is the decision variable for the seller.[4]

To see this, note that the reaction of the buyer to such a discount schedule depends on the buyer's augmented cost function

$$\begin{aligned} B(Q) &= pD + \frac{D}{Q} A_b + H_b \frac{Q}{2}, \\ &= DP_0 \exp[-a(Q - Q^*)] + \frac{D}{Q} A_b + H_b \frac{Q}{2} \quad \text{for} \quad Q \geqslant Q^*. \end{aligned} \tag{5}$$

The buyer determines the economic order quantity by minimizing the cost function in equation (5), yielding the first order conditions

$$\frac{\partial B}{\partial Q} = -a^* DP_0 \exp[-a^*(Q - Q^*)] - \frac{DA_b}{Q^2} + \frac{H_b}{2} = 0.$$

Since the seller wants the buyer to order in lot sizes of Q^{**}, it selects an a^* that satisfies the above first order equation when $Q = Q^{**}$, i.e.,

$$a^* DP_0 \exp[-a^*(Q^{**} - Q^*)] = \frac{H_b}{2} - \frac{DA_b}{Q^{**2}}. \tag{6}$$

Although (6) will motivate the cost-minimizing buyer to order Q^{**}, it is still necessary to check if the seller's augmented cost function $S(Q^{**})$ is less than $\bar{S}(Q^*)$.

Setting up $S(Q)$ with a discount structure as given in equation (4) yields

$$S(Q) = [P_0 - P_0 \exp[-a^*(Q - Q^*)]]D + \frac{DA_s}{Q} - \frac{H_s Q}{2}. \tag{7}$$

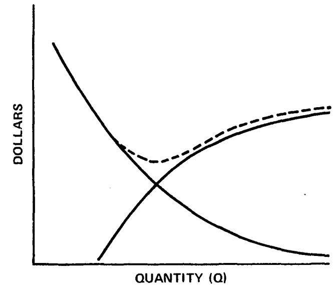

FIGURE 2. Seller's Augmented Cost Function versus Quantity.

[4]Other one and two parameter monotonically decreasing functions (e.g., quadratic and log) were tried, but they did not satisfy all the above conditions. As will be shown subsequently, equation (4) also insures that the seller's augmented cost $S(Q)$ is minimized at Q^{**}. We did not, however, exhaustively explore all possible discount functions.

The first term, which represents the extra costs associated with the quantity discounts, is concave in Q, while the sum of the second and third is a decreasing function in Q. (See Figure 2.) However, as shown in Appendix 1, $S(Q)$ has a minimum at Q^{**} when the value of a^* satisfying the buyer's first order conditions, i.e., equation (6), is substituted into the right-hand side of equation (7) and (a) the buyer orders at least twice a year (i.e. $D > 2Q^{**}$) and (b) the shift in the order quantity is no more than 2.747 times what the buyer would have purchased prior to the quantity discount (i.e. $Q^* > 0.364Q^{**}$).[5] Intuitively, these later two requirements insure that (a) the buyer orders often enough to guarantee the availability of some savings by reducing the number of orders and (b) the shift required to reach the joint optimum is not so great that the seller must offer too large a discount. In summary, whenever these parameter restrictions hold, and the seller sets a^* using (6), it is insured that (i) the cost minimizing buyer will order Q^{**}, (ii) the seller's costs will be reduced, and (iii) the minimum augmented cost $S(Q)$ occurs at Q^{**}.

Since equation (6) provides a mechanism for determining the discount policy $p(Q)$, we next explore how to obtain the specific value(s) for a^*. Let $K_1 = Q^{**} - Q^*$, and

$$K_2 = \frac{1}{P_0 D}\left[\frac{H_b}{2} - \frac{DA_b}{Q^{**2}}\right]. \tag{8}$$

Then equation (6) can be rewritten as:

$$K_2 \exp(K_1 a^*) - a^* = 0. \tag{9}$$

We show in Appendix 2 that the left side of (9) has zero, one or two positive roots and that two roots will exist whenever the seller's ordering costs are between 50 percent and 150 percent of the buyer's ordering costs, i.e., $0.5A_b \leqslant A_s \leqslant 1.5A_b$, and the seller's cost of capital costs is no more than 80 percent of the buyer's holding costs, i.e., $H_s \leqslant 0.8H_b$. Since these parameter values are likely to hold for most real world situations, we assume the two solution case. Moreover, since it is in the best self-interest of the seller to choose the lower of these two feasible discount policies, one would use the smaller of the two solutions of a^* when determining the seller's pricing policy.

Partitioning the Joint Gain Between Buyer and Seller

Although the solution to (9) uniquely defines a pricing policy which leads the cost-minimizing buyer to order Q^{**} and insures that the seller's augmented costs at Q^{**} are less than those at Q^*, no consideration has been given to how the gains are split between the two agents. We do this next by removing the constraint that the discount policy starts at Q^*.

Intuitively, one could argue that if the seller wanted to pass more of the savings to the buyer, it could do so by having the first price break occur before Q^*. Similarly, if the seller wanted to keep more of the gains, the first price break would occur after Q^*.

More formally, let $\bar{Q}$ be the quantity at which the discount policy starts and r the amount of money that the seller deems necessary that it must give to the otherwise cost-minimizing buyer in order to insure that the buyer will alter its ordering policy. Then $r = [P_0 - p(Q^{**})]D - \Delta$, where $\Delta = \bar{B}(Q^{**}) - \bar{B}(Q^*)$, the increase in the buyer's costs associated with increasing its order quantity. Solving for the average price at Q^{**} yields $p(Q^{**}) = P_0 - (\Delta + r)/D$. Then by noting that the discount pricing

[5] The proof also requires two "nonrestrictive" assumptions, these being (a) $A_b \leqslant A_s \leqslant 1.5A_b$ and (b) $H_b \leqslant 1/3P_0$. Both assumptions would seem to hold in most situations.

must yield this average price when evaluated at Q^{**}, one gets

$$P_0 \exp\left[-a^*(Q^{**} - \bar{Q})\right] = P_0 - (\Delta + r)/D. \tag{10}$$

Equations (9) and (10) represent a system of equations in two unknowns, i.e., a^* and $\bar{Q}$. Solving for $\bar{Q}$ and a^* yields:

$$\bar{Q} = Q^{**} - \left[\frac{1 - (r + \Delta)/DP_0}{K_2} \right] \ln \left[\frac{1}{1 - (r + \Delta)/DP_0} \right], \tag{11a}$$

$$a^* = K_2 \left[\frac{1}{1 - (r + \Delta)/DP_0} \right]. \tag{11b}$$

Once r, the amount of savings to be passed on to the buyer, is specified, equations (11a) and (11b) uniquely define a pricing scheme that insures

(a) $B(Q^{**})$ is the lowest augmented cost solution for the buyer,

(b) the allocation of the gain between the agents is as specified by the seller.

Allowing Ordering Costs to Vary with Q

In many situations the buyer's or seller's ordering costs, A_b or A_s respectively, vary with the lot size Q. For example shipping costs often decrease per unit as the ordering quantity increases. One possible way to model these decreasing cost functions which leads to a tractable solution is

$$A_b(Q) = \alpha_1 + \beta_1 Q + \gamma_1 Q^2 \quad \text{and} \tag{12a}$$

$$A_s(Q) = \alpha_2 + \beta_2 Q + \gamma_2 Q^2, \tag{12b}$$

where for $i = 1,2$, $\beta_i > 0$, $\gamma_i < 0$ and $-\beta_i/2\gamma_i$ is greater than the largest amount Q ever ordered by a buyer so as to insure the relevant portions of the functions are increasing in Q. Then from (5) and (12) the buyer's augmented cost function becomes

$$B(Q) = DP_0 \exp\left[-a^*(Q - Q^*)\right] + \frac{D}{Q}(\alpha_1 + \beta_1 Q + \gamma_1 Q_1^2) + \frac{H_b Q}{2}. \tag{13}$$

Minimizing the cost expressed in equation (13) implies that the lot size Q should satisfy the following:

$$\frac{\partial B(Q)}{\partial Q} = -a^* DP_0 \exp\left[-a^*(Q - Q^*)\right] - \frac{D\alpha_1}{Q^2} + D\gamma_1 + \frac{H_b}{2} = 0. \tag{14}$$

By redefining the holding costs to be $H_b + 2D\gamma_1$ and the ordering costs to be α_1, equation (14) becomes identical to equation (6). Hence a solution can be achieved as stated in the previous section even though ordering costs vary with the quantity ordered as specified by equation (12). Again it can be shown in a manner similar to that used before that the cost incurred by the seller under this discount structure will be less than with no discounts, thus satisfying all requirements of the solution. Finally, note that the assumption $H_b > H_s$ has to be modified to $(H_b + 2D\gamma_1) > H_s + 2D\gamma_2$ for a solution to exist.

Multiple Buyer Groups

We now extend the model to handle the case of N groups of buyers of different sizes. Let there be $n_1, n_2, \ldots, n_N$ buyers in each group. We assume the buyer's holding costs, ordering costs and demands are the same within groups although these costs can differ across groups. The basic problem is to find a unified pricing policy where the price $p(Q)$ is monotonically decreasing in Q and yet (a) motivates the individual buyer groups to order more each time and (b) increases the seller's profits.

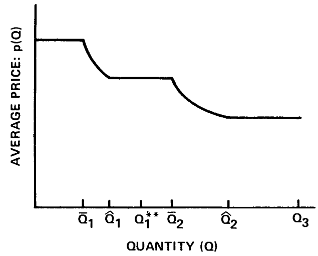

FIGURE 3. Example of a Unified Discount Structure for More Than One Buyer Group.

We were unable to obtain a closed form solution to this problem. However, we are able to provide a simple algorithm which produces an optimal unified pricing policy having the structure shown in Figure 3. More specifically, the discount policy starts at $\bar{Q}_1$ and decreases continuously to some quantity $\hat{Q}_1$ (defined subsequently), the discount policy being specified by parameter a_1^*. The average price is then held fixed between $\hat{Q}_1$ and $\bar{Q}_2$ before it again continuously decreases to some new quantity $\hat{Q}_2$. This pattern continues until $\hat{Q}_N$.

The algorithm starts off with an initial policy constructed by treating each buyer group independently. This policy then is modified to acknowledge that any discount given to buyer group i must be given to larger buyers (i.e., groups $i+1, i+2, \ldots, N$) even though the discount to group i is not needed to motivate these larger buyers to order more units per order. Since the discounts given to smaller buyers may cost the seller more in lost revenue from the larger buyers than the reduction in costs associated with altering the ordering behavior of the smaller buyers group, it is necessary to determine the trade off between these two factors. In this way a unified policy has the effect of only increasing the order size for group i from Q_i^* to $\hat{Q}_i$ instead of Q_i^{**}. The difference between Q_i^{**} and $\hat{Q}_i$ represents the effects of the groups $i+1$ to N on the pricing policy for the ith group.

The actual procedure for determining the optimal pricing policy is done through an iterative sequential search to determine the optimal values of $\bar{Q}_i$, $\hat{Q}_i$ and a_i^*, $i = 1, 2, \ldots, N$. The process continues until the solutions for all $3N$ parameters converge. The first step in the procedure is to calculate a_i^*, $\bar{Q}_i$ and Q_i^{**} from equations (3), (11a), and (11b), treating each buyer group independently. Next the price P_0 is modified downward for all groups except the smallest to account for the fact that a unified pricing structure requires the average price to monotonically decrease in Q. This is done by setting $p_i(\bar{Q}_i) = p_{i-1}(Q_{i-1}^{**})$. This produces a pricing structure similar to Figure 3.

Next the implications of the price discounts given to buyer group i on buyer groups $i+1$ to n are explicitly taken into consideration. This is done by finding the quantity $\hat{Q}_i$ where $\hat{Q}_i \leqslant Q_i^{**}$, which equates the increase in the seller's augmented costs from having the buyers in group i order $\hat{Q}_i$ versus Q_i^{**} with the decrease in the seller's augmented costs associated with increasing the starting price by the amount $[p_i(\hat{Q}_i) - p_i(Q_i^{**})]$ for all groups larger than group i. As shown in Appendix 3, the optimal value of $\hat{Q}_i$ which equates the increase in costs associated with group i with the decrease in costs associated with the larger groups is the solution to the equation

$$\frac{n_i A_s D_i}{\hat{Q}_i^2} = -\frac{n_i H_s}{2} + P_{i-1} a_i^* L_i \exp\left[-a_i^*\left(\hat{Q}_i - \bar{Q}_i\right)\right], \quad \text{where} \tag{15}$$

$$L_i = n_i D_i + \sum_{k=i+1}^{N} n_k D_k \left[\prod_{j=1+1}^{k} c_j\right],$$

$$c_j = p(\hat{Q}_j)/p(\hat{Q}_{j-1}) = P_j/P_{j-1}, \quad \text{and} \quad P_{i-1} = p(\hat{Q}_{i-1}).$$

Since $\hat{Q}_i$ depends on the previously determined values of a_i^* and $\overline{Q}_i$ (as well as the values of a_i^*, $\overline{Q}_i$ and Q_i^{**} for the larger buyer groups), it is necessary to solve equations (11a), (11b), and (15) simultaneously for new values of a_i^*, $\overline{Q}_i$ and $\hat{Q}_i$. This yields the following set of equations:

$$\hat{Q}_i^2 = \frac{2D_i[n_i D_i A_s + L_i A_b]}{L_i H_b - n_i D_i H_s}, \tag{16a}$$

$$a_i^* = \left[\frac{H_b}{2} - \frac{D_i A_b}{\hat{Q}_i^2} \right] \Big/ \left[D_i \left(P_{i-1} - \frac{\Delta_i + r_i}{D_i} \right) \right], \tag{16b}$$

$$\overline{Q}_i = \hat{Q}_i + \frac{1}{a_i^*} \ln\left(1 - \frac{\Delta_i + r_i}{D_i P_{i-1}} \right). \tag{16c}$$

These values are calculated for each buyer group i, $i = 1, 2, \ldots, N$.

Finally, note that the calculations for $\hat{Q}_i$ during the first pass through the N groups assume that all buyer groups larger than i ordered quantity Q_j^{**} where $j > i$. However, after iterating through all N groups, each Q_j^{**} has been modified to $\hat{Q}_j$. Consequently, the aforementioned procedure must be repeated by replacing Q_i^{**} by $\hat{Q}$ and cycling through each of the N buyers groups. This procedure (of replacing the previously determined order quantities with newly determined order quantities) is continued until all $\hat{Q}_i$ converge simultaneously. Empirically this convergence seems to occur very rapidly.[6] The final solutions to (16a), (16b), and (16c) provide the desired policy which will be of the form of

$$p_i(Q) = p_i \exp\left(-a_i^*(Q - \overline{Q}_i)\right), \qquad \overline{Q}_i \leqslant Q \leqslant \hat{Q}_i, \quad \text{and}$$

$$p_i(Q) = p_i(\hat{Q}), \qquad \hat{Q}_i \leqslant Q \leqslant \overline{Q}_{i+1}.$$

An Application

Setting

The figures and setting presented below are based on an actual case, although both have been adjusted somewhat to protect the confidentiality of the data. The problem setting concerns a large U.S. manufacturer distributing its product nationally through a network made up of 1128 privately owned dealers. The manufacturer's wholesale price is \$35 per unit. The industrial norm is for the manufacturer to pay shipping costs. These costs decrease per unit with increases in quantity shipped. Dealers are free to order any quantity over a minimum of 25 units.[7] The actual quantities ordered vary from a minimum of 26 to over 4000.

Analysis of Costs

Past ordering behavior for the 1128 dealers was used to develop seven reasonably homogeneous groups where homogeneity was measured with respect to the number of items purchased per order and annual demand. Although some variance in behavior was found within these seven groups, the rest of the analysis assumes each group to be homogeneous with respect to these two variables.

The average quantity ordered per order for group i was used to estimate Q_i^*, while the average annual demand per dealer within group i was used to estimate D_i. Buyer

[6] In all our runs convergence occurred after 4 or less iterations.

[7] Dealers can order less but they are required to pay the shipping costs for orders less than 26 units.

TABLE 1

Estimates for Q_i^*, H_{bi}, D_i, n_i, A_s and A_b for Each Buyer Group

Group	Q_i^*	H_{bi}	D_i	n_i	A_b	A_s
1	58	\$11.50	362	632	145	81
2	154	11.00	1658	363	283	147
3	381	10.50	4191	85	407	305
4	584	10.00	9228	30	526	445
5	770	9.50	14,966	8	634	573
6	1494	9.25	18,565	4	1176	1063
7	1811	9.00	25,346	6	1305	1275

holding costs, H_{bi}, for each group were estimated using the then current interest rate and other costs associated with holding inventory. These three estimates along with the number of dealers in each group are displayed in the first four columns of Table 1.

Ordering costs for each dealer group were estimated using the first three assumed values. More specifically, equation (2) was used to obtain an estimate of A_{bi} for each set of values of Q_i^*, D_i and H_{bi}. Visual inspection of these estimates, along with prior expectations, indicated that the dealer's ordering costs varied by quantity ordered. Consequently, these costs were modeled within specific ranges to be quadratic in Q. Parameter estimates for the quadratic relationships were obtained as follows. First, as previously noted when ordering costs are a function of both Q and Q^2, and where α_1 and γ_1 are the parameters of equation (12a), $Q_i^{*2} = 2D\alpha_1/(H_b + 2D\gamma_1)$. Rearranging terms yields a linear relationship in α_1 and γ_1, i.e., $H_{bi}/2D = -\gamma_1 + \alpha_1/Q_i^{*2}$. These parameters were then estimated using OLS. Finally the estimates of Q_i^* were adjusted slightly so that the vector of values $(Q_i^*, H_{bi}, D_i, \alpha_1, \gamma_1)$ were compatible, i.e., $Q_i^{*2} = 2D_i\alpha_1/(H_{bi} + 2D_i\gamma_1)$ for all i within a set of groups assumed to have the same α_1 and γ_1.

A separate analysis was performed to determine the seller's ordering and holding costs. This cost function was modeled as quadratic in Q reflecting the decreasing average cost of shipping. The exact function used was $A_s = 40 + 0.7Q - 0.00002Q^2$. The seller's holding costs, reflecting the value of obtaining capital quicker, were estimated to be \$3 per unit per annum.

The estimates of the ordering costs are shown in Table 1 where the values of A_{si} are the *total cost* of getting (and delivering) an order of Q_i^* units and the values of A_{bi} are the *total cost* of placing (and receiving) an order of Q_i^* units. Thus the total cost to the seller of getting and shipping an order increases from \$81 for 58 units to \$1275 for 1811 units while the buyer's costs of placing the order, receiving it, unloading it and putting it in inventory increase from \$145 to \$1305. On a per unit basis, these costs decrease from \$1.40 to \$.66 per unit and \$2.40 to \$.70 per unit respectively. These decreases represent the economies of scale associated with placing larger orders.

Results

Using the procedure outlined previously and by setting $r_i = 0$, estimates of $\bar{Q}_i$, $\hat{Q}_i$, $p_i(\hat{Q})$ and a_i^* were obtained for the seven groups. By setting $r_i = 0$, we made the implicit assumption that each buyer in group i would order $\hat{Q}_i$ instead of Q_i^* as long as the pricing scheme insured that the buyers' augmented costs were no greater than under the circumstances when no discounts are given to that group.

The savings to each buyer and the savings to the seller are displayed in Table 2. As can be seen from this table, buyers in group 1 are no better off ordering $\hat{Q}_1$ (= 62) than Q_1^* (= 58), although the seller saves \$20.60 per buyer. These seller savings in group 1 result primarily from the fact that the average number of orders per buyer per annum is reduced from 6.2 to 5.8. This results in a decrease of approximately 250 orders per year for this group of 632 buyers.

TABLE 2

Group	Savings to Buyer: From Prior Discounts	Savings to Buyer: For Moving to $\hat{Q}_i$	Per Buyer Savings to Seller: From Moving a Buyer to $\hat{Q}_i$	Per Buyer Savings to Seller: From Prior Discounts	Total Seller per Group Savings
1	\$ 0	\$0	\$ 20.60	\$0	\$13,015
2	6.37	0	117.48	(6.37)	40,333
3	84.17	0	81.28	(84.17)	− 240
4	225.50	0	154.14	(225.50)	− 2141
5	438.76	0	178.52	(438.67)	− 2081
6	591.36	0	217.50	(591.35)	− 1495
7	867.71	0	629.93	(867.71)	− 1427
				Total	\$45,963

For group 2, the savings are more dramatic. In this group, the buyers are shifted from ordering 154 units per order to 192 units. The average number of orders placed is reduced from 10.8 to 8.6 and the seller receives a net benefit of \$111.09 per buyer (e.g., \$117.48 − \$6.37) for a total savings of \$40,333. Although the pricing policy developed does not pass on any of these savings to the buyer, the buyers in group 2 save \$6.37 per year as a result of the discount given to the previous buyer group. Similar observations with respect to buyer and seller savings can be made for all subsequent groups. The total savings to the seller for such a plan is \$45,963 per year.

One might question if group 1 buyers would alter their ordering behavior in light of the fact that they are no better off by ordering $\hat{Q}$ (with the discount) than Q^* (without the discount). If it is felt that the buyers would not change their ordering policy without being given a more positive incentive, then the seller would not realize the \$13,015 savings associated with this group. In that case a discount policy can be developed which insures positive savings to group 1 buyers by assigning a positive value to r_1. Such a policy, however, should be analyzed to determine if the total savings to the seller exceeds \$32,948 (i.e., \$45,963 − \$13,015), since any additional discounts given to group 1 must also be passed on to all subsequent buyer groups. If the resulting plan shows savings less than the \$32,948 figure, then it is better for the seller to forfeit the \$13,015 savings and concentrate on the larger buyer groups.

For example, if a priori, it is felt that \$5 per buyer must be passed back to the first buyer group, then the total savings to the seller are reduced by almost \$25,000 to \$20,967. This latter figure is less than the \$32,891 associated with a pricing discount policy which ignores group 1 entirely but still insures that each member in group 2 saves \$6.37.

Finally, we explored the sensitivity of these results to the specific parameter values chosen, by modifying both the buyer's holding and ordering costs, i.e., H_b and A_b. The overall conclusion from these analyses was that the general form of the discount policy remained unchanged. However, the following patterns were observed.

(1) Total savings increased by 6 percent when there was a 10 percent decrease in H_b. The discount policy became shallower with buyers ordering more (at a lower average price at the new Q^{**}). When H_b was increased 10 percent, savings to the seller were reduced by 4 percent and the discount policy became steeper. Interestingly, the buyer now buys at a slightly higher average price.

(2) When A_b was increased by 12 percent, total savings to the seller were reduced by 7 percent. As with the change in H_b, the pricing policy becomes steeper. This time, however, the average price at the new Q^{**} is reduced. Conversely when A_b was decreased by approximately 12 percent, profits to the seller increased by 13 percent. The resulting discount policy was less steep and yielded slightly higher average prices.

Implementation

Implementation of such a pricing schedule is quite straightforward if no quantity discount policy is already in place. However, as in the situation modeled, implementation is somewhat more complex when the manufacturer already has an established discount policy. In such situations the existing quantity discount policy represents the base-line from which any new policy must be evaluated (at least from the eyes of the buyers). This complicated the issue somewhat since each buyer group is not ordering at Q_i^* (the quantity associated with P_0) but some other value. Thus, comparison must be made from this baseline and not the baseline of all buyers ordering Q_i^*. Also it is possible that some buyer groups are currently being extended discounts greater than they would under the suggested plan. In such cases, the seller must be able to communicate the reasons for the change or choose to introduce new pricing list during a period of price fluctuations.

Conclusion

This paper developed a unified pricing policy which motivates the buyer to increase its ordering quantity thereby reducing the joint ordering and holding costs. As a result, the seller is able to reduce its costs while leaving the buyer no worse off and often better off. In this way the seller can achieve better channel coordination without the need to exert any extra "channel power."

In developing such a scheme for any real world application, the analyst will normally have to infer from each buyer's behavior, the buyer's holding and ordering costs. Using such knowledge, the seller can form a number of reasonably homogeneous groups and then use the developed methodology to design a scheme which insures that the relevant buyer groups are economically motivated to increase their ordering quantities.

It should be pointed out that the seller would be much better off if it were allowed to develop different pricing schemes for each buyer or group of buyers and thus not have to give discounts on units which have no effect on the particular buyer's behavior. However, such individual policies may not be acceptable to buyers, since they may demand discounts on units given to smaller buyers. Also having differing pricing policies for different (competing) buyers may open the seller to problems with the Robinson–Patman Act, although these policies can be justified in terms of cost savings. Finally it should be pointed out that the pricing policy described in this paper can be used as a baseline to measure the extent to which buyers pressure the seller to give price concessions in return for the buyer placing a larger order. In other words, although there is an upper limit to the amount a seller can reduce the price (without an accompanying increase in buyer demand) and still be better off, this upper limit is not zero.

Finally, as with most management science applications, the model developed is an abstraction of reality. In our case, the buyer's ultimate demand, D, is treated as a certainty equivalent instead of a random variable and is assumed to be independent of the seller's discount policy. Also, the buyer's ordering behavior is assumed to be captured by a simple EOQ model. These assumptions are abstractions of reality which can be relaxed in future studies. We believe, however, that our model provides an initial beginning.

Appendix 1

In this appendix, we show the conditions needed to insure that the sellers augmented costs are minimized at Q^{**}. To do this we investigate the properties of the second derivative evaluated at Q^{**}.

Differentiating twice equation (7) (after replacing Q^* with $\bar{Q}$) with respect to Q yields

$$\left.\frac{\partial^2 S(Q)}{\partial Q^2}\right|_{Q=Q^{**}} = \frac{2DA_s}{Q^{**3}} - a^{*2}DP_0\exp\left[-a^*(Q^{**}-\bar{Q})\right].$$

Substituting for a^* from equation (6) in the above equation yields

$$\left.\frac{\partial^2 S(Q)}{\partial Q^2}\right|_{Q=Q^{**}} = \left[\frac{2DA_s}{H_b Q^{**3}} - \frac{a^*}{2}(1-k^2)\right]H_b$$

where $k = Q^*/Q^{**}$.

We also know that Δ in equation (11b) is $\bar{B}(Q^{**}) - \bar{B}(Q^*)$ which is equal to $Q^{**}H_b(1-k)^2/2$. Therefore, on substituting for a^* (from (11b)) and Δ in the above equation we get

$$\left.\frac{\partial^2 S(Q)}{\partial (Q^*)}\right|_{Q=Q^{**}} = \frac{2DA_s}{H_b Q^{**3}} - \frac{H_b(1-k)^2/2}{DP_0 - (H_b/2)Q^{**}(1-k^2)(1+x)} \cdot \frac{(1-k^2)}{2},$$

where $x = r/\Delta$.

For $0 \leqslant x \leqslant 1$ and $0.5\,A_b \leqslant A_s \leqslant 1.5A_b$, we can show that

$$(2Q^{**})\left.\frac{\partial^2 S(Q)}{\partial Q^2}\right|_{Q=Q^{**}} \geqslant k^2 - \frac{(1-k)^2(1-k^2)}{2[DP_0/H_b Q^{**} - (1-k^2)]}.$$

Also assuming that $H_b/P_0 \leqslant 0.33$ and $D \geqslant 2Q^*$

$$(2Q^{**})\left.\frac{\partial^2 S(Q)}{\partial Q^2}\right|_{Q=Q^{**}} \geqslant k^2 - \frac{(1-k)^2(1-k^2)}{2[6k-1+k^2]}.$$

Therefore, if k is greater than 0.364, $\partial^2 S(Q)/\partial Q^2|_{Q=Q^{**}}$ will always be greater than zero.

Appendix 2

This appendix shows that one of three possible conditions must hold for the equation $K_2\exp(K_1a^*) - a^* = 0$, i.e. that there exist zero, one or two solutions; the number depending on the values of K_1 and K_2. We do this as follows. First note that $y_1 = K_2\exp(K_1a^*)$ is monotonically increasing in a^*, is convex and has a positive intercept. Moreover $y_2 = a^*$ is an increasing linear function in a^* and has a zero intercept. Consequently the difference between these two functions (i.e., $y_1 - y_2$) is convex in a^* and has a positive intercept. Thus, depending on the constants K_1 and K_2, the function $y_1 - y_2$ crosses the a^* axis zero, one or two times, i.e. equation (9) has zero, one or two solutions.

Case I: *One solution.* One solution exists if and only if at the point where y_1 equaled y_2 the slope of both functions is equal, i.e.,

$$K_2\exp[K_1a^*] = a^*, \quad \text{and,} \quad K_1K_2\exp[K_1a^*] = 1.$$

The only value of a^* to satisfy both these equations is $1/K_1$. Thus a^* as solved from $K_1K_2\exp[K_1a^*] = 1$ must equal $1/K_1$ if only one solution is to exist.

Case II: *No solutions.* Since y_1 is an increasing function in a^*, no solution would exist if the slope of y_1 evaluated at $a^* = 0$ exceeded 1. However even if the slope at $a^* = 0$ is less than 1, it is possible that $y_1 - y_2$ never crosses the a^* axis. In fact if the point where the slope of y_1 is equal to 1 is less than $1/K_1$, the point of one solution, the function $y_1 - y_2$ will not cross the a^* axis and equation (9) will have no solutions. Said more precisely, if the solution to $\partial y_1/\partial a^* = 1$ is less than $1/K_1$ there is no solution to $y_1 - y_2 = 0$.

Case III: *Two solutions.* For two solutions to exist the function $y_1 - y_2$ must cross the a^* axis twice. This will occur whenever on solving for a^* from $K_1K_2\exp[K_1a^*] = 1$, the value of a^* is greater than $1/K_1$.

Since the chances of one solution occurring in any real world situation are remote, we next determine if Case II or Case III would exist in general. (Case I can be thought of as a special case of Case III.)

To satisfy the sufficient conditions for a solution to exist, the solution for a^* on solving from $K_1K_2\exp[K_1a^*] = 1$ should be greater than or equal to $1/K_1$. This requires $a^* = (1/K_1)\ln(1/K_1K_2)$ be greater than or equal to $1/K_1$, which implies that $\ln(1/K_1K_2)$ should be greater than or equal to one, or $1/K_1K_2$ should be greater than or $e = 2.7183$.

Next we investigate the range of parameter values that insure that $K_1K_2 \leqslant 1/e$. Note that

$$K_1K_2 = (Q^{**} - Q^*)\frac{1}{P_0D}\left[\frac{H_b}{2} - \frac{A_bD}{Q^{**2}}\right].$$

Substituting for the values of Q^{**}, H_b and D as assumed in Appendix 1 (i.e., $Q^{**} \leqslant 2.747Q^*$, $H_b/P_0 \leqslant 0.33$

and $D \geqslant 2Q^*$) yields

$$K_1K_2 \leqslant \frac{1.747Q^*}{P_0D}\left[\frac{H_b}{2} - \frac{A_bD}{7.546Q^{*2}}\right] \leqslant \frac{1.747Q^*}{P_0D}\cdot\frac{H_b}{2}\left[1 - \frac{2A_bD}{7.546Q^{*2}H_b}\right],$$

but since $Q^{*2} = 2A_bD/H_b$

$$K_1K_2 \leqslant \frac{1.747Q^*H_b}{P_0D2}\left[\frac{6.546}{7.546}\right] \leqslant 0.2779,$$

hence $1/K_1K_2$ will be greater than 2.7813.

The assumption $Q^{**} \leqslant 2.747Q^*$ used in Appendix 1 to insure minimization at Q^{**} at first seems innocuous, but it imposes certain restrictions on the values of the seller's holding and ordering costs relative to the buyer's. To see this, recall that

$$Q^{**} = \left[\frac{2(A_b + A_s)D}{(H_b - H_s)}\right]^{1/2}$$

Let $A_s = XA_b$ and $H_s - YH_b$ where X and Y are positive constants. The $Q^{**} = Q^*(1 + X)/(1 - Y)$. Since Q^{**} is assumed to be $\leqslant 2.747Q^*$, $(1 + X)/(1 - Y)$ must be $\leqslant 7.546$.

As in Appendix 1 we assume that the value of X is from 0.5 to 1.5, i.e., the seller's ordering costs range from 50 percent to 150 percent of the buyer's ordering costs. This implies that $Y \leqslant 0.8$, i.e., that the seller's cost of capital be less than 80 percent of the buyer's holding costs (which include the cost of capital). When these restrictions are met they ensure the existence of solutions to equation (9).

Appendix 3

This appendix presents a detailed description of how $\hat{Q}_i$ is obtained. First the change in seller's profits due to a reduction of the discount to the smallest buyer group is calculated. This reduction will cause the smallest group to cut back on its order quantity from Q_1^{**}. Then the increase in costs to the seller by having the buyers in group 1 reduce their ordering quantity from Q_1^{**} to Q_1 is

$$S(Q_1) - S(Q_1^{**}) = n_1\left[D_1P_0\exp\left[-a_1^*(Q_1 - \bar{Q}_1)\right] - \frac{A_sD_1}{Q_1} + \frac{H_sQ_1}{2}\right] - S(Q_1^{**}) \qquad \text{(A1)}$$

where

$$S(Q_1^{**}) = n_1\left[D_1P_0\exp\left[-a_1^*(Q_1^{**} - \bar{Q}_1)\right] - \frac{A_sD_1}{Q_1^{**}} + \frac{H_2Q^{**}}{2}\right]$$

This latter term reflects the effects of the price differential and ordering and holding costs on the seller but is not a function of Q_1.

Next the change in profits due to increasing the average price to the larger groups is calculated. This increased price occurs since group 1's average price is now $p_1(Q_1)$ which is greater than $p_1(Q_1^{**})$. Since the price discount for group 2 is calculated from the base price for group 1, the new average price for group 2 would be $p_0\exp[-a_1^*(Q_1 - \bar{Q}_1)]\exp[-a_2^*(Q_2^{**} - \bar{Q}_2)]$. Multiplying the average price for each group by the total demand for the group and adding across groups yields the total revenue. Consequently the total increase in profits is the difference in revenue for the two pricing policies or

$$P_0\exp\left[-a_1^*(Q_1 - \bar{Q}_1)\right]\sum_{i=2}^{N} D_in_i\left[\prod_{j=2}^{i}\exp\left[-a_j^*(Q_j^{**} - \bar{Q}_j)\right]\right] - TR(Q_1^{**}) \qquad \text{(A2)}$$

where $TR(Q_1^{**})$ is the total revenue associated with the pricing policy leading to buyer group 1 operating at Q_1^{**} (which is not a function of Q_1).

Equation (A2) can be simplified by noting the unified pricing policy implies that the price at which buyer group i buys is the base price from which buyer group $i + 1$ discount is calculated. Thus multiplying and dividing $\exp[-a_2^*(Q_2^{**} - \bar{Q}_2)]$ by $p(Q_1)$ and dividing $\exp[-a_j^*(Q_j^{**} - \bar{Q}_j)]$, $j = 3, \ldots, N$, by $p(Q_{j-1}^{**})$ yields $p(Q_2^{**})/P(Q_1)$ and $p(Q_j^{**})/p(Q_{j-1}^{**})$ respectively. Define the ratio of the average price paid by group i to that paid by group $i - 1$ as c_i. Then (A2) can be rewritten as

$$P_0\exp\left[-a_1^*(Q_1 - \bar{Q}_1)\right]\sum_{i=2}^{N} D_in_i\left[\prod_{j=2}^{i} c_j\right] - TR(Q_1^{**}). \qquad \text{(A3)}$$

Adding (A1) to (A3) and differentiating with respect to Q_1 produces the optimal quantity discount policy for group 1 assuming that the larger groups of buyers are all operating at their respective Q_1^{**}'s, i.e., the pricing policy for the larger groups of buyer is not modified. The resulting equation for $\hat{Q}_1$ is

$$\frac{n_1 A_s D_1}{\hat{Q}_1^2} = -\frac{n_1 H_s}{2} + a_1^* L_1 \exp\left[-a_1^*(\hat{Q}_1 - \bar{Q}_1)\right] P_0$$

where $L_1 = n_1 D_1 + \sum_{i=2}^{N} n_i D_i [\prod_{j=2}^{i} c_j]$.

Similarly it can be shown that the optimal values for Q_i, i.e., $\hat{Q}_i$ for group i should satisfy the following equation:

$$\frac{n_i A_s D_i}{\hat{Q}_i^2} = -\frac{n_i H_s}{2} + a_1^* L_i \exp\left[-a_i^*(\hat{Q}_i - \bar{Q}_i)\right] P_{i-1}(\hat{Q}_{i-1})$$

where $L_i = n_i D_i + \sum_{k=i+1}^{N} n_k D_k [\prod_{j=i+1}^{k} c_j]$, c_j is the ratio of prices paid buyer group j and buyer group $j-1$ and $\hat{Q}_N = Q_N^{**}$.[8]

[8]The authors would like to acknowledge the helpful comments of Finn Kydland and two anonymous reviewers who reviewed a prior version of this paper. The ideas for this paper were conceived while both authors were at Carnegie-Mellon University.

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