The impact of information in service systems with strategic customers

Antonis Economou

National and Kapodistrian University of Athens, Greece

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Part I:

Introduction

The basic queueing models with strategic customers

Queueing problems

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- **Dynamic optimization problems**
  How should we administrate a system? (dynamic control)
  (the administrator makes decisions)
- **Strategic behavior problems**
  How do the agents behave in a system? What can we do to induce a desirable behavior?
  (each agent makes his own decision)

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  (the administrator makes decisions)
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  (Markov Decision Processes)
- **Strategic behavior**
  (each customer makes his own decision)
  Stochastic Processes + Game Theory

Strategic queueing problems

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- To join or balk?

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  - What level of information should be provided?

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  7. R: customer's benefit from completed service.
  8. C: waiting cost per time unit for a customer (it is paid even when he is in service).
  9. Upon arrival, a customer inspects the queue length and decides whether to join or balk.

The basic observable model - Equilibrium I

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$$R - C\frac{n+1}{\mu} \geq 0.$$

The basic observable model - Equilibrium II

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Theorem

*The individual's optimizing strategy for a customer that sees n customers upon arrival is the **threshold** strategy that presribes to join if $n+1 \leq n_e$ with*

$$n_e = \lfloor \frac{\mu R}{C} \rfloor \text{ (Naor's threshold).}$$

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This is the unique equilibrium strategy, but also a dominant strategy.

The basic observable model - Social opt. I

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- Social benefit per time unit, under a threshold strategy n:

$$S_{soc}^{(obs)}(n) = \lambda R \frac{1-\rho^n}{1-\rho^{n+1}} - C\left[\frac{\rho}{1-\rho} - \frac{(n+1)\rho^{n+1}}{1-\rho^{n+1}}\right].$$

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Its unique maximum is attained for

$$n_{soc} = \lfloor x_{soc} \rfloor$$

where x_{soc} is the unique solution to

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Moreover:

$$n_{soc} \leq n_e.$$

Individual optimization leads to longer queues than are socially desired.

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- Profit of the administrator when he uses a fee $p = R - \frac{Cn}{\mu}$ to induce a threshold strategy n:

$$S_{prof}^{(obs)}(n) = \lambda \frac{1-\rho^n}{1-\rho^{n+1}} \left(R - \frac{Cn}{\mu} \right).$$

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The unique profit-optimizing threshold n_{prof} *that maximizes* $S_{prof}^{(obs)}(n)$ *is given by*

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Moreover

$$n_{prof} \leq n_{soc} \leq n_e.$$

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  3. Upon arrival, a customer decides whether to join or balk without observing the queue length.

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- If $R - C\frac{1}{\mu-\lambda q} > 0$, then the best response is to join.
  If $R - C\frac{1}{\mu-\lambda q} < 0$, then the best response is to balk.
  If $R - C\frac{1}{\mu-\lambda q} = 0$, then any strategy is best response.

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$R \leq \frac{C}{\mu}$	0
$\frac{C}{\mu} < R < \frac{C}{\mu-\lambda}$	$\frac{\mu - C/R}{\lambda}$
$R \geq \frac{C}{\mu-\lambda}$	1

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$$S_{soc}^{(un)}(q) = \lambda q R - C\lambda/(\mu - \lambda).$$

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$R \leq \frac{C}{\mu}$	0
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Moreover $q_{soc} \leq q_e$.

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- Then, there exists a unique critical value λ^* such that

$$\lambda_e^{(un)} > \lambda_e^{(obs)}, \text{ for } \lambda < \lambda^*,$$

while

$$\lambda_e^{(un)} < \lambda_e^{(obs)}, \text{ for } \lambda > \lambda^*.$$

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- For low arrival rates, it is better to conceal information from the customers to increase the throughput.

Observable vs. unobservable II

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  The profit maximizer prefers to conceal the queue length for low values of λ and to reveal it for high values.

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  Only a fraction of the customers observe the queue length.

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- Other information structures, extensions, conclusions, bibliography.

Part II:

Strategic customers in models with imperfect information structure

Models with imperfect information structure

Models with imperfect information structure

- $M/M/1$ queue with same dynamics and operational parameters as in Naor's model (λ, μ and ρ).

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- Same reward-cost structure (R and C).

Models with imperfect information structure

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- Upon arrival, a customer decides whether to join or balk based on an 'imperfect' observation of the queue length.

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- 'imperfect' observation means that the customer gets some information about the queue length but not its exact value.

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Theorem

An individual's optimizing strategy for a customer that gets informed that will be placed in the compartment i is the ***threshold*** *strategy that prescribes to join if $i \leq i_e^N$ with $i_e^N = \lfloor x_e^N \rfloor$, with*

$$x_e^N = \begin{cases} \frac{R\mu}{aC} + \frac{1}{1-\rho^a} - \frac{1}{a(1-\rho)} & \text{if } \rho \neq 1, \\ \frac{R\mu}{aC} + \frac{a-1}{2a} & \text{if } \rho = 1. \end{cases}$$

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This is the unique equilibrium strategy within the set of pure strategies.

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Individual optimization leads to longer queues than are socially desired.

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- The equilibrium strategy is unique, if we exclude some very special values of the parameters (R being an integer multiple of $\frac{C}{\mu}$).
- The equilibrium, social optimizing and profit maximizing thresholds can be computed in closed form.

Numerical results I

Numerical results I

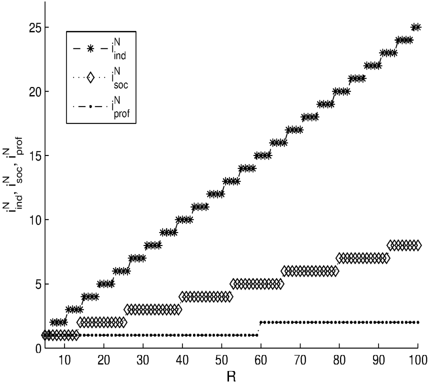

Figure 1: Optimal thresholds with respect to R - N case

Numerical results II

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Numerical results III

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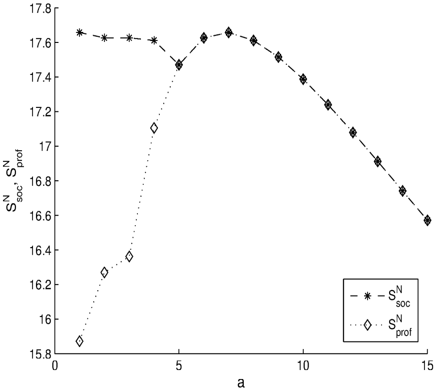

Figure 2: Optimal social benefit and administrator's profit with respect to a - N case

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- Take-away message: The administrator can improve its profit by an adequate selection of the compartment size.

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Part III:

Strategic customers in models with delayed information structure

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- The administrator of the system announces to all customers their positions in the system, every Exp(θ) time units.

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- Joining customers may decide to renege at any later time.

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- A customer stays after the first announcement, if his position n at the system is such that $R - C\frac{n}{\mu} > 0$.
- The best strategy of a customer taking into account the reaction of the others is to stay if his position n at the first announcement is such that

$$n \leq n_e,$$

with

$$n_e = \lfloor \frac{\mu R}{C} \rfloor \text{ (Naor's threshold).}$$

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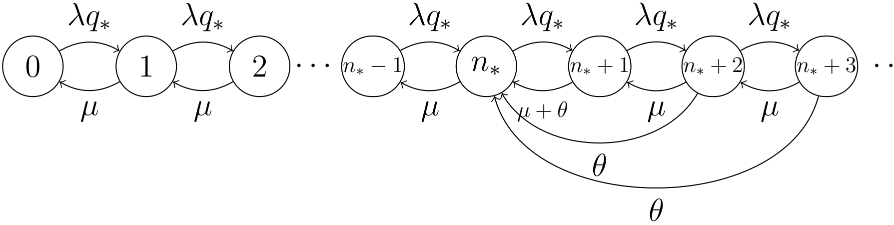

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where

$$\rho_{*1} = \frac{\lambda q_*}{\mu}, \ \rho_{*2} = \frac{\lambda q_* + \mu + \theta - \sqrt{(\lambda q_* + \mu + \theta)^2 - 4\lambda q_* \mu}}{2\mu}$$

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$$\pi_n = \pi_n(n_*, q_*) = \begin{cases} B_* \rho_{*1}^n & \text{if } 0 \le n \le n_* - 1, \\ B_* \rho_{*1}^{n_*} \rho_{*2}^{n-n_*} & \text{if } n \ge n_*, \end{cases}$$

where

$$\rho_{*1} = \frac{\lambda q_*}{\mu}, \ \rho_{*2} = \frac{\lambda q_* + \mu + \theta - \sqrt{(\lambda q_* + \mu + \theta)^2 - 4\lambda q_* \mu}}{2\mu}$$

and

$$B_* = \frac{(1-\rho_{*1})(1-\rho_{*2})}{1 - \rho_{*2} - \rho_{*1}^{n_*+1} + \rho_{*1}^{n_*} \rho_{*2}}.$$

Stationary mean number of customers

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Proposition (continued)

The mean stationary number of customers in system is

$$\mathbb{E}_{(n_*,q_*)}(N) = \frac{(1-\rho_{*2})[(n_*-1)\rho_{*1}^{n_*+1} - n_*\rho_{*1}^{n_*} + \rho_{*1}]}{(1-\rho_{*1})[1-\rho_{*2}-\rho_{*1}^{n_*+1}+\rho_{*1}^{n_*}\rho_{*2}]} + \frac{(1-\rho_{*1})[n_*\rho_{*1}^{n_*} - (n_*-1)\rho_{*1}^{n_*}\rho_{*2}]}{(1-\rho_{*2})[1-\rho_{*2}-\rho_{*1}^{n_*+1}+\rho_{*1}^{n_*}\rho_{*2}]}.$$

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Suppose that the customers follow an (n_*, q_*) *strategy.*
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Suppose that the customers follow an (n_*, q_*) strategy. Consider a tagged customer that finds n customers upon arrival (but he does not know about it). The conditional expected net benefit of the tagged, if he decides to join is

$$\mathcal{U}(n|n_*) = \begin{cases} R - \frac{C(n+1)}{\mu} & \text{if } n < n_*, \\ \left(R - \frac{Cn_*}{\mu} + \frac{C}{\theta}\right)\left(\frac{\mu}{\mu+\theta}\right)^{n-n_*+1} - \frac{C}{\theta}, & \text{if } n \geq n_*. \end{cases}$$

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- $\mathcal{U}(n|n_*)$ does not depend on λ nor on q_*.

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\mathcal{U}(n_*, q_*) &= B_*\left(R - \frac{C}{\mu}\right)\frac{1-\rho_{*1}^{n_*}}{1-\rho_{*1}} \\
&-B_*\frac{C}{\mu}\frac{(n_*-1)\rho_{*1}^{n_*+1} - n_*\rho_{*1}^{n_*} + \rho_{*1}}{(1-\rho_{*1})^2} \\
&+B_*\left(R - \frac{Cn_*}{\mu} + \frac{C}{\theta}\right)\frac{\mu\rho_{*1}^{n_*}}{\mu+\theta-\mu\rho_{*2}} \\
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- $\mathcal{U}(n_*, q_*)$ is decreasing in q_* for any fixed n_*.

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$$\mathcal{U}(n_e, q) = 0$$

in $(0, 1)$, *with respect to* q.

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  The equilibrium throughput is a unimodal funct. of θ. There exists an 'ideal' announcement rate that maximizes the equilibrium throughput.

Other models with delayed information structure

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- Allon, G., Bassamboo, A. and Gurvich, I. (2011) 'We will be right with you': Managing customer expectations with vague promises and cheap talk. *Oper. Res.*

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- Roet-Green, R. (2013) *Information in Queueing Systems with Strategic Customers. PhD Thesis.* Chapter 4: The armchair decision: to depart towards the queue or not.

Part IV:

Strategic customers in models with mixed observation structure

Models with mixed observation structure

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- $M/M/1$ queue with known dynamics and operational parameters (λ, μ and ρ).

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- The best strategy of a customer against any strategy of the others is to join, if his position n given that he joins is such that

$$n \leq n_e,$$

with

$$n_e = \lfloor \frac{\mu R_o}{C_o} \rfloor \text{ (Naor's threshold).}$$

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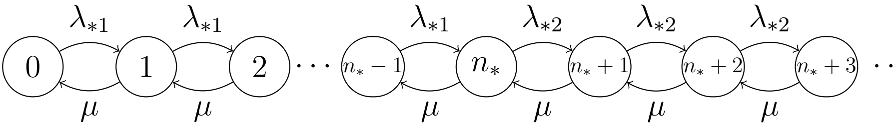

where $\lambda_{*1} = \lambda p_o + \lambda p_u q_*$, $\lambda_{*2} = \lambda p_u q_*$.

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- It is too complicated to reduce it in closed form and to maximize.
- For each $n_* = 0, 1, 2, \ldots, n_e$ we find q_* that maximizes $S(n_*, q_*)$ and then choose the one that gives the overall maximum, namely (n_{soc}, q_{soc}).

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$$PoA = \frac{S(n_{soc}, q_{soc})}{S(n_e, q_e)} :$$

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- Hassin and Roet-Green (2013) Equilibrium in a two dimensional queueing game: When inspecting the queue is costly.

Part V:

Strategic customers in models with partial information structure

Models with partial information structure

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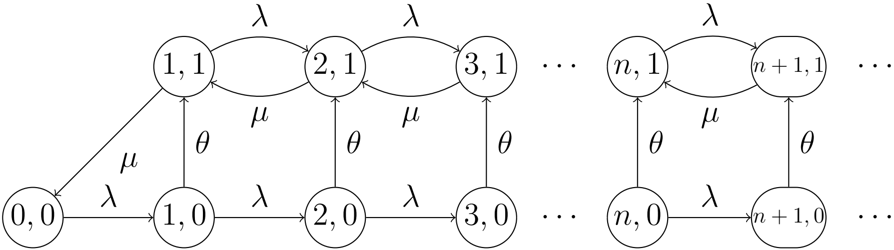

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Numerical results I

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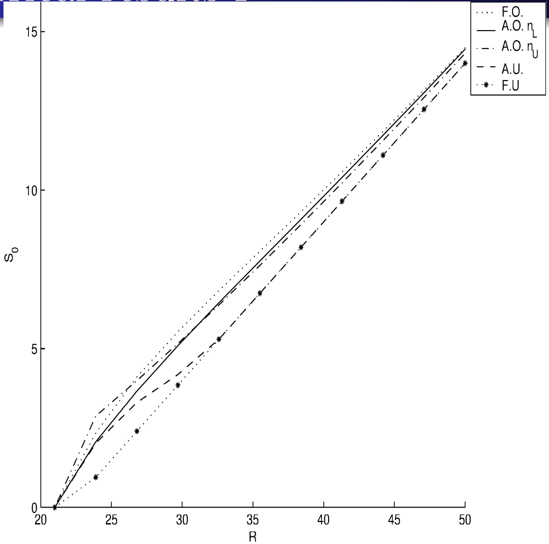

Figure 3: Equilibrium social benefit for various information levels with respect to R.

Numerical results II

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Numerical results II

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- For low values of R the optimal social benefit under ao may surpass the optimal social benefit under fo.

Part VI:
Final remarks

Conclusions

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- This indirect influence of the customers can be less disturbing for them than imposing admission fees etc.
- The throughput of the system can be also controlled by tuning the information provided to the customers.

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Thank you!

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Questions?