

JOHN MONAGHAN

TEACHERS' ACTIVITIES IN TECHNOLOGY-BASED MATHEMATICS LESSONS

ABSTRACT. This paper focuses on high school mathematics teachers and what they do when they use digital technology in their lessons. It is essentially a discursive paper but it uses data from a project on teachers using technology to illustrate points. The main aim of the paper is to present an holistic account of factors influencing teachers' practice. A secondary aim is to present the integration of technology into lessons as a complex issue. Saxe's four parameter model of goal-linked practice is employed to show how different dimensions of teachers' activities interrelate in this complex undertaking. The paper ends with a consideration of approaches related to Saxe's model.

KEY WORDS: high school mathematics, practice, teachers, technology

INTRODUCTION

The focus of this paper is high school mathematics teachers¹ and what they do when they use technology² with their classes. My main motivation is to find an holistic way to examine teachers' practice in these endeavours because of a conviction that the whole undertaking involves a fusion of many factors and analyses suffer if these factors are taken in isolation. A secondary motivation is to react to claims that using technology in teaching is relatively straightforward and/or that teachers using technology 'must' relinquish didactic roles and move to become facilitators. My own experience in working with teachers who endeavour to integrate technology into their classroom practice is that many, but not all, find it a difficult thing to do and that, regardless of ease or difficulty, it is a complex undertaking (Lumb et al., 2000; Monaghan, 2000).

This paper does not aim to locate 'exemplary teacher activity' but aims merely to model the practice of 'ordinary' teachers. I provide an outline of a model of teachers' activities in technology-based mathematics lessons. This model, an application of Saxe's (1991) four parameter model, locates critical influences on teachers' practice which may be used to explore the complexities of integrating technology into teaching.



I use stories and data from a project on teachers using technology to illustrate points, so, after a brief review of literature, I provide a short description of this project. I then outline Saxe's model and, in the next section examine teachers' activities from the point of view of each of Saxe's parameters. The next section focuses on interrelationships between the parameters and compares teachers' pre-lesson planned goals with emergent goals in lessons. Finally I comment on approaches related to Saxe's model.

TEACHERS AND TECHNOLOGY³

With regard to mathematics teachers using technology I examine two themes: (i) a division between those who view technology as something that enables teachers to become less 'didactic' in their interactions with students, and those who perceive change in practice as problematic; (ii) research focused on teachers' practices in mathematics classes is a relatively recent phenomenon and what there is to date has largely focused on forms of knowledge and on beliefs, with little attention to the whole experience of using technology in the classroom.

A division in perceptions of the complexity of integrating digital technology into the classroom can be traced back in the literature to at least 1989 when the *Harvard Educational Review* published a *Symposium: Visions for the Use of Computers in Classroom Instruction*. Of the three original and four response articles I select Schwartz (1989) and Cuban (1989) as examples of poles in this division. The Schwartz article is not naïve and it does acknowledge the critical role of the teacher but it does state that the geometric software considered "provides a setting and an occasion for conjecture and creativity for both student and teacher in the mathematics classroom" (p. 51). Further to this

classroom roles **must** and do shift. It is **no longer possible** for teachers to serve as ex cathedra authorities . . . Teachers and students **must** and do learn to listen carefully to and assess the qualities of one another's arguments. (p. 57, my emphasis)

Cuban, in contrast, does not believe that "the software itself does the job" (p. 219) and finds "the language and thrust of Schwartz's argument for the GEOMETRIC SUPPOSER mildly echoing the feverish search of academics in the 1950s and 1960s for teacher-proof instructional materials." He labels views such as Schwartz's as

'neoprogressive visions' and states that they "are out of synch with organizational realities and go well beyond what ordinary, well-intentioned, good-hearted people can do" (p. 221).

There is a sense in which the 1990s witnessed a progressive if somewhat uneven realisation that teachers teaching with technology is a complex issue. There is evidence for this claim in the professional literature as well as the research literature. Using my own country (England⁴) as an example, The Mathematical Association (a professional association for mathematics teachers) in a 1992 publication (Mathematical Association, 1992), addresses, amongst other things, teaching mathematics with a computer. It states that "Teacher exposition leads **naturally** to discussion between teacher and pupils with the computer display as a focus." (ibid., p. 18, my emphasis) A more recent publication from this organisation adopts a more critical stance, e.g., "Inspection evidence consistently shows very little use of appropriate technology tools in mathematics teaching at all levels." (Mathematical Association, 2002, p. 5).

This change of emphasis during the 1990s is also evident in the academic literature, e.g., compare: "In the implementation of computer-based laboratory explorations, the teacher **must** become a technical assistant, a collaborator, and a facilitator" (Heid, et al., 1990, p. 195, my emphasis) with "The introduction of informatics in mathematics teaching works only when it is perceived as an answer to questions (even though unconscious) already present in teachers' minds." (Bottino and Furinghetti, 1996, p. 132). The term 'facilitator' in the Heid et al. quote is interesting as it is often used in North American literature on technology.⁵ Lerman (1994, p. 148) considers this term as an appropriate metaphor for the role of the teacher within a radical constructivist paradigm and contrasts this metaphor/paradigm with a social constructionist metaphor of teacher as *mediator*. Lerman's positioning of metaphors is, I believe, important in considering different conceptions of teaching. The views expressed in this paper share these *mediator* conceptions of teaching, "the notion of the teacher as someone who plays an active part in the students' learning as mediator, as to texts and classroom social relationships" (ibid., p. 149). I will return to paradigm issues towards the end of this paper and henceforth use the term 'facilitator' when I wish to emphasise constructivist ideology.

I advance this notion of a change in emphasis over time as a general trend, and I can produce counter examples. Going back to the early 1990s Watson (1993, p. 16), for example, notes that

computer activities are primarily dependent on individual teachers' initiatives which do not reflect whole school mathematics teaching practices and Olson (1992, chapter 2) notes that computer activities present problems for teachers when they do not fit with teachers' routines. But 'facilitator writings' still exist, for example Hudson and Borba (1999, p. 20) reporting on an ICME technology working group state "The role of teacher is seen as 'instigator', 'helper' and 'systematiser'", and Zbiek (2002, p. 133) states "Teachers **have** to be technical assistants as well as facilitators and catalysts ...". As a teacher I believe it is good that teachers work in collaborative ways with their students and if technology can assist this, then so much the better. As an academic, my point is merely that claims that this is 'natural' reflect an extremely strange vision of nature and that claims that teachers 'must' do this fall into that old philosophical trap of confusing an 'ought' with an 'is'.

Research focused on teachers' practices in mathematics classes is a relatively recent phenomenon (cf. Noss and Hoyles, 1996, p. 184). With regard to technology, studies which acknowledge the importance of context have largely focused on teachers' beliefs and forms of teacher knowledge. Moreira and Noss (1995), for example, examine primary teachers' attitudes to change in a technology environment following in-service training. They argue that teachers' attitudes to and beliefs about technology use cannot be abstracted from their situations, contexts and cultures, and that time for teachers to develop is an important factor. With regard to content knowledge Lloyd and Wilson (1998), for example, report on how one teacher's deep knowledge of functions allowed him to articulate a variety of relationships in multiple representations of functions which supported meaningful classroom discussions with students. Whilst studies which focus on beliefs or forms of knowledge are important and advance knowledge they do, by design, miss the wholeness of teachers' practices, which I attempt to address in the remainder of this paper. I have, however, one serious criticism of my endeavour: that I focus on teachers at the expense of considering students.⁶

Kendal (2001 – see also Kendal and Stacey, 2001) offers the most holistic account to date, to my knowledge, of teachers' practices in technology-based mathematics lessons. Kendal's analysis of the literature leads her to conclude that "the ways individual teachers take up using technology in their mathematics classrooms is usually consistent with their normal teaching practices" (Kendal, 2001, p. 34). Her empirical study, moreover, focuses on both teachers and

students and examines how teachers' 'privileging' (Wertsch, 1991), teaching styles and attitudes, differentially affects students' learning in CAS lessons but she only implicitly takes practice as the central unit of analysis.

A PROJECT ON TEACHERS USING TECHNOLOGY

Thirteen English mathematics teachers from seven English High Schools, none of whom had made extensive use of technology in their mathematics classes before, made a commitment to move to significant use of technology during one school year. The project explored teachers' lesson planning, classroom interactions, use of written support materials and areas of tension for teachers. The project aimed to be as natural as possible, i.e., teachers doing what they might have done even if they had not been in a project. There was a financial incentive for the Mathematics department to be involved in the project and teachers had to bid to be involved. Applicants who had considerable experience using technology in lessons were not selected. All the teachers selected stated that they intended to try to incorporate technology into their lessons and the project was an opportune incentive to realise their intentions. It is realistic to assume that intentions to incorporate technology into lessons arose in part from government education imperatives that filter down to schools (see Rodd and Monaghan, 2002).

The project team (teachers and university-based researchers) met for a full day before the start of the school term and for three half days during the project year. The initial day included a demonstration of and the opportunity to use a number of different mathematical software systems. This day also included a discussion of how technology might be integrated into the classroom and the types of activities possible. The half days were mainly devoted to accounts from the teachers about what they were doing. Personal relationships in the project team were very good. The teachers focused on the following technology tools: spreadsheets (*Excel*), graphical packages (*Omnigraph*) and calculators, and algebra (*Derive*) and geometry systems (*Geometers' Sketchpad*). Individual teachers chose the tool(s) they thought most appropriate for use with their classes. Teachers chose older classes, students aged 14–17 years, on the assumption that the tools would have the widest use in classes exposed to substantial amounts of algebra.

A legitimate question to ask about the project teachers' activities is, what was my role in their work? Although I set up the project my intention from the outset was to support the teachers in what they wanted to do. I was available for technical assistance and to suggest ideas for activities but I do believe that such help was minimal, i.e., as far as is practically possible in a research project, these teachers did what they wanted to do. The project had an ethical principle that students' scholastic interests should not be jeopardised, i.e., teachers were clear that they did not need to use technology for the sake of being in the project. Only two of the teachers, however, made little use of technology over the course of the year and eight of the teachers made extensive use of technology over the course of the year. Seven of these teachers 'blocked' technology lessons, i.e., they had a period of several weeks where they used technology in most of the lessons in this period, followed by a longer period in which they did not use technology other than scientific calculators in lessons. One teacher had a scheduled computer-based lesson every week of the project year and her students used the computers in virtually all of these lessons.

Data was collected by teachers and university-based researchers, including: observation of and accounts by teachers of their approaches to and use of technology; weekly journal entries by teachers with relevant lesson plans and course materials; teacher and student interviews; video-taped classroom observations of each teacher (one lesson at the start of the year which did not involve technology and three lessons spread out over the year which did involve the use of technology).

The project group of teachers is a useful group of people for the purpose of building my model of teachers' activities in that it is a group of people with similarities and differences. They shared some common characteristics: specialist mathematics teachers in state schools with limited prior experience of using technology in mathematics lessons but who were willing to attempt to make extensive use of technology in their lessons for the school year the project ran. On the other hand the 13 teachers had a range of years of experience, had different motives for using the technology and no more than two taught in the same school.

SAXE'S FOUR PARAMETER MODEL

Saxe's (1991) model⁷ centres on 'emergent goals' in terms of four parameters (see Figure 1). I have found that the term 'goals' in this

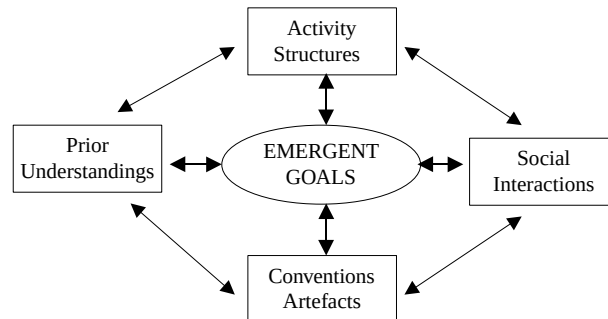


Figure 1. Saxe's four parameter model.⁸

model can confuse people because Saxe's use of the term is somewhat different to the everyday use of the term. In everyday use 'goals' are often equated to conscious desired outcomes. In a teaching situation these may be equated to aims or objectives. I find it is better to think of Saxe's emergent goals as little, and often unconscious, goals ('must do this' situations) that come into being and fade away.

Not only do individuals shape and reshape their goals as practices take form in everyday life, but they also construct goals that vary in character as a function of the knowledge that they bring into practices ... Goals, then, are emergent phenomena, shifting and taking new forms as individuals use their knowledge and skills alone and in interaction with others to organize their immediate contexts. (ibid., 16–17)

The *activity structures* parameter "consists of the general tasks that must be accomplished in the practice- and task-linked motives" (ibid., p. 17). Explicating exactly what a task in a mathematics lesson is a non-trivial undertaking. For the moment I simply note that teachers, in their specific school settings, come to know, through their identification with the practice in their school, what counts as a 'valid-in-their-school mathematical task'⁹ and that generating tasks for technology-based lessons is often problematic. A second aspect of activity structure is the 'activity cycle' of a lesson, which is not a Saxian construct and requires a few words by way of clarification. Engeström et al. (1995, p. 320) claim that "activity systems have cyclic rhythms and long historical half-lives." Magajna (2001, p. 73), commenting on mathematics lessons, notes that "The observed task structure consists of several nested cycles. The most prominent level is the cycle of exercises and exercise-like pieces of theoretical explanations." At this

point I simply note that lessons using technology may take on a structure/cycle different to that of non-technology lessons.

The *social interactions* parameter is concerned with relationships between participants, i.e., teachers and students, in lessons and how these relationships may influence participants' goals. As noted in the 'teachers and technology' section, some research suggests that teachers enter new social relationships with students (as facilitators) in technology-based lessons. If this is the case, then these new social relationships will impact on teachers' emergent goals and should be observable in dialogic interactions between teachers and students.

The *conventions and artefacts* parameter addresses "the cultural forms that have emerged over the course of social history" (Saxe, 1991, p. 18). In the case of teachers using technology in their lessons an obvious artefact to consider is the software and hardware they use. There is, I believe, a sense in which the obvious can blind us to wider issues and I will seek to address teachers using mediational means when I return to this parameter in the next section.

The *prior understandings* parameter, in this context, may suggest a focus on teachers' mathematical content knowledge but this is only a small part of the focus. Prior understandings concern teachers' pedagogical knowledge and institutional knowledge, "the prior understandings that individuals bring to bear on cultural practices both constrain and enable the goals they construct in practices" (ibid., p. 18). The term 'individuals' is important because the different levels of experience mean that participants in practice "bring to bear different arithmetical understandings on practice-linked problems and consequently their goals differ" (ibid., p. 18).

VIEWING TEACHERS' ACTIVITIES THROUGH THE LENS OF SAXE'S FOUR PARAMETER MODEL

In this section, I consider teachers' activities from the point of view of each of Saxe's (1991) four parameters.

Activity Structures

I did not need to analyse video tapes to appreciate that technology and non-technology lessons, by and large, had different activity structures (tasks and cycles) – this was obvious. Nevertheless, the

video tapes provide evidence for my observations. All of the initial 13 observed non-technology lessons had a very similar 'macro' cycle: teacher exposition and examples followed by students doing questions, which were similar to the teachers' examples, from a textbook. Lessons went through several of these cycles with answers to exercises given at the end of each cycle. Teachers did, however, interrupt the cycle, and provide further examples and/or questions, if they perceived that a number of students had misunderstood the work. At the 'micro' level there was, however, variation in teacher-student discourse both during teacher exposition and while students were doing questions.

Askew et al. (2000), in an analysis that draws heavily on Saxe's model, add a new dimension to the four parameter model, 'numeracy orientations', which allows them to focus on what I have called the 'micro' level. An important part of their analysis is concerned with who directs the work students are doing. I do not pursue this line of enquiry because my data is not suitable for this type of analysis (the teachers wore cordless microphones in the video-taped lessons and I only picked up the voices of students as they were talking to the teacher). This is a pity as this is an important line of enquiry.

The tasks which students engaged with in the observed non-technology lessons were textbook exercises. From the point of view of the teacher, textbook tasks are 'safe' tasks, i.e., they are part of their established practice and teachers' activities around these tasks are justified through conforming to practice. The tasks and cycles of the technology-based lessons varied considerably across the 13 teachers and, in most cases, varied over time for each teacher. I will describe several approaches that more than one teacher adopted. Several of the issues raised are further explored in the section on 'social interactions'.

Three of the teachers clearly mimicked the cycle of non-technology lessons. Two of these teachers only used graphic calculators. Exposition in these graphic calculator lessons included instruction on keystrokes and tasks were 'closed' in that there was a specific set of keystrokes which would produce the desired answer. Both of these teachers continued with this format in all three observed technology lessons. My interpretation of this, supported by interviews, was that they felt secure in following established practice, and were able to do this. The third teacher used a long narrow computer room which allowed her and the pupils to see each other. The room had a board and in the first observed technology lesson she instructed them in a

manner (cycle and closed tasks) that was startlingly close to the observed non-technology lesson. She, however, changed this format in subsequent observed technology lessons, moving to 'open' tasks. The cycle of these subsequent lessons was common to many of the observed technology lessons of other teachers: short introduction to the task, worksheets which directed student activity, teacher circulating around the class ensuring the students were working and attending to mathematical or technical issues. In reflecting on this change of format she focused on the students, "they were following instructions and eventually they reached a point where they could then work by themselves on the task". All of the other teachers adopted this 'technology cycle' from the outset, though there was variation in the tasks set and in their interactions with the students.

The tasks set in technology-based lessons were, by and large, quite different to 'normal' tasks, as evidenced by the fact that 11 of the 13 teachers¹⁰ initially felt they could not employ textbooks as a resource in technology-based lessons. Some clearly intended this to be so, for example the teacher who stated that he wished to reduce exposition time, to relieve students from tedious calculations and to have a stronger focus on concepts. All of the teachers except the two who only used graphic calculators (described above) set open tasks in at least some technology-based lessons and setting open tasks was the norm for some of these teachers. Open tasks were often extended investigations (and often without a single correct answer) and these contrast strongly with textbook tasks which are usually a series of similar short questions with a single correct answer. Some teachers found that tasks in technology-based lessons led their students to focus on the technology and at least three of the teachers felt an 'is this maths?' tension when their students attended to technological details at, in their opinion, the expense of the mathematics. As Laborde (2001) notes, the design of tasks for the integration of technology with mathematics is a non-trivial undertaking. My point here is that students' interpretation of tasks set can affect their teachers' emergent goals. I come back to this issue in the section on 'conventions and artefacts'.

Prior Understandings

Mathematics teachers' 'prior understandings' incorporate a range of beliefs, knowledge and social practices: how they view learning and teaching; what mathematics is to them and their mathematical and

technological proficiency; how they plan their lessons. In reflecting on project teachers' practice with regard to this parameter I am aware of a certain fluidity and creativity in non-technology-based lessons and a degree of rigidity and domination by the technology in technology-based lessons. I elucidate my ideas below by noting commonalities and diversity.

Although there was variation in project teachers' stated beliefs about what mathematics students should learn and how students learn this, a common theme in all interviews was that students learn facts, skills and processes through practice and apply them in solving problems. They stated that they viewed this as largely independent of whether or not the lesson used technology. The teachers had a clear understanding of how a normal lesson should unfold and although some wrote lesson plans most did not – they simply made brief notes on mathematical content to be covered. Indeed the plans of those who wrote plans were really just amplifications of other teachers' notes – exercises to be done from textbooks noted and answers worked out. In talking about their lessons the teachers were often vague: "Without ICT the lessons are kind of just there, if you know what I mean. I've got my own room and all my resources are in there so it tends to be what I feel like on the day." – their knowledge of their teaching was tacit knowledge rather than articulate knowledge (Olson, 1992, chapter 3). Information and communications technology (ICT) lessons, however, were planned in detail and this took a lot of time, especially early ICT-based lessons. There is a straightforward explanation for this in terms of teachers' understanding of their practice: their tacit understanding of a 'normal' lesson was moulded in thousands of hours of practice but incorporating technology involved a rethink of this practice. As one teacher said:

Back to being like a student teacher to be honest because I felt happier writing things down. I think it's a kind of a confidence thing. When you're just teaching a normal lesson without ICT you don't, you're prepared for any eventuality without realising it but when you're doing ICT and it's the first time you've done a topic like that you're not.

Teachers' tacit goals in non-technology lessons were partially formed in relation to their use of a textbook which provided the basis of most lessons. But teachers were flexible when learning problems appeared and re-instructed the class or directed them to questions they had written on the board. Further to this there was no obvious

'privileging' of scientific calculator technology in that there were instances where teachers suggested calculator use and instances where they forbade calculator use. In technology-based lessons textbooks, as mentioned, were largely not used and student learning was largely directed by worksheets and dominated by a single technology (*Excel*, *Derive*, etc.). These lessons were, in one sense, less flexible than non-technology lessons. This was especially evident in the lessons of teachers who had to book a computer room in advance and again there is a straightforward explanation: if you have booked a computer room, especially one without a board, then you are not very likely to say 'let's not use the computers for this'. The dominance of the technology leads to emergent goals directed at 'surviving in the classroom'. As Cuban (1989, p. 221) says, "teachers teach the way they do simply to survive the impossibilities inherent in the workplace".

Finally it should be noted that prior understandings includes teachers' subject knowledge. It was anticipated at the outset, due to suggestions by Heid (1995) and Zbiek (1995), that use of technology might generate mathematical output that the teachers did not understand and that this could impact on teachers' practice, e.g., "I'm not going to do that topic with the class because I can't explain the software's solution". However, despite being consciously aware of this possibility there were no observed instances of this happening. Use of technology certainly generated unexpected output at times, e.g., a student's graphic calculator produced a graph which looked like the graph of $y = -\sin x$ when he keyed in $\sin x$. The teacher was initially confused but he did not appear concerned. His emergent goal at this point was to locate why this output was produced, which he eventually did: the student had keyed in the y -axis end points as '5 to -5' instead of '-5 to 5'.¹¹ There were also many instances where teachers directed students to specific operations because they, the teachers, were familiar with these operations but there were no observed instances where, like Heid (1995), technology work revealed weaknesses in teachers' mathematical knowledge.

Conventions and Artefacts

The tools people use in doing mathematics transform ways of working and of reasoning (Vygotsky, 1978, p. 54; Wertsch, 1998, pp. 23–72). One need only consider pencil and paper versus calculator arithmetic to appreciate this. This journal has played an important

role in making the subject of tool transformations explicit, e.g., papers by Artigue (2002) and Healy and Hoyles (2001). Considering teaching with digital tools does not simply mean considering the software and hardware used. Laborde (2001) notes the importance, and the problems of, task design. Teachers, moreover, use (or do not use) written resources (such as textbooks and worksheets) and social and mathematical conventions in their classroom practice. Project work highlighted differences in teachers' practice with regard to the use of resources and conventions between technology and non-technology lessons, and within technology lessons. I comment on software, hardware and written resources below. I briefly comment on mathematical conventions with regard to how software transforms mathematics. There is much more that can be said about conventions but this would require considerable space and I feel I say enough in what follows to make my point.

Software is too large a category to make general claims about. Project teachers' experience was that specific software had a differential effect on the organisation of their teaching. A teacher who primarily used *Derive* with his class found using a computer graphing package (*Omnigraph*) much more akin to his 'normal' lessons. This had several dimensions. To use *Derive* he had to devote several lessons to attend to simple mastery of the commands and syntax but this was a five minute task with *Omnigraph*. Perhaps more important than this is that each program differentially affected the way he interpreted the mathematics his students were doing. He viewed *Omnigraph* as a tool to do straightforward tasks that fitted in with his 'normal' lesson ideas. He viewed *Derive*, however, as a 'monster' that could do virtually everything and this forced him to completely re-interpret his lesson plans. This was a disquieting experience for him. A similar experience was evident with a teacher who primarily used the spreadsheet *Excel* but who made some use of *Omnigraph*. Her lesson planning focused on mathematical ideas not particularly concerned with technology, but 'techno-mathematical' ideas became the focus of the lessons. One lesson, for example, focused on ratio. She set an investigation where three children were left a sum of money in a will to be distributed over a number of years in the ratio of their ages. The students', and subsequently her, foci (emergent goals) in this lesson were on getting the spreadsheet cells right, not only the correct equation but a suitable cell format e.g., from a student's work "the end three columns on the right were formatted for currency". She commented after the lesson that she was unhappy with this focus on 'cell arithmetic'.

These two examples are instances of tool use transforming mathematics – and the problem in both cases was that the teacher was unprepared for this transformation. The transformation to cell arithmetic with *Excel* produces an epistemological shift from hand ratio calculations to spreadsheet formulas and formats (conventions). The curriculum she was required to teach to (DfEE/QCA, 1999) included “divide a quantity in a given ratio” (ibid., p. 59), “represent repeated proportional change using a multiplier raised to a power” (ibid., p. 60) and “set up and use equations to solve word and other problems involving direct proportion” (ibid., p. 62). But to this teacher in this class the students were not dividing and their equations were not derived from relationships expressed using the proportion sign ($\propto$). A note in this curriculum document states “Pupils could use a spreadsheet to construct and use formulae.” (ibid., p. 62) This spreadsheet suggestion fails to recognise the tension teachers may experience between the expected and transformed mathematics. Indeed the expected and transformed mathematics could be (and were in the case of this teacher) seen as ‘valued’ and ‘questionable’ mathematics.

The experiences of the teacher using *Derive* are presented in detail in Lumb et al. (2000). With regard to his ‘monster’ comment above I restrict myself to the following points: the range of the tool’s application, what processes the tool displays, and unexpected conventions. In contrast to *Derive* the range of application of a graphing package is clear and limited: it plots graphs and performs geometric transformations. *Derive* does this and much more. This teacher’s problem was ‘what do I do and not do with this tool?’. I now comment on the processes displayed. He wanted to use this tool to help students learn elementary differential calculus. The aim in teaching differentiation at this level involves giving the students experiences that enable them to appreciate limit ideas and geometric interpretations but also to be able to carry out by-hand differentiation using standard rules such as the product rule. For example, the by-hand process students are expected to display, for $f(x) = x \sin x$, is $f'(x) = \frac{d}{dx}(x) \sin x + x \frac{d}{dx}(\sin x) = \sin x + x \cos x$. There are many ways to differentiate this function using *Derive* but none of these methods display the essential (to the teacher) intermediate step of $\frac{d}{dx}(x) \sin x + x \frac{d}{dx}(\sin x)$. With regard to mathematical conventions this teacher found obtaining inverse functions time consuming and irritating. The usual expression $f^{-1}(x)$ does not work in *Derive*. It is necessary instead to instruct the tool to ‘solve’ $y = f(x)$ for x (the

tool's method for rearranging a formula to isolate a specific variable). This caused the mathematical focus of the lesson to be put aside in order to focus on how to perform the technical operation. As with the teacher using *Excel* this teacher experienced a tension between the expected and transformed mathematics.

Hardware also had a differential effect on the organisation of teaching. I comment on two aspects: the location of computers and graphic calculators versus computers. Project teachers with and without computers in their classes planned use of, and spontaneously used, computers in different ways. Teachers who did not have computers in their class mainly planned 'all or nothing' computer-based lessons, i.e., all the lesson on the computer or no use of the computer at all. This seriously affected their lesson planning (both of the individual lessons and teachers' long-term lesson schedules, i.e., how the computer lessons fitted into their overall scheme of work). Further to this teachers using computer rooms without a board for writing on generally devoted the last part of the lesson prior to the computer-based lesson to explaining what was going to be done with the computers in the next lesson. At one level this may be a trivial change in teachers' practice but, on another level, it seriously affected their practice. With regard to graphic calculators I restrict my comments to noting that the two teachers who made exclusive use of graphic calculators were the two teachers who made the least changes in their practice and the activity structure of their lessons – they taught from the front directing students' keystrokes in much the same way that they otherwise directed their written mathematics from the board.¹²

With regard to written resources all of the project teachers made considerable use of a textbook in their 'normal' lessons. All but two of the teachers in the early months of the project felt that textbook work was inappropriate in technology based lessons. As the year went on three other teachers found that they were able to use the textbook for technology based work and another found that she could write worksheets based on textbook work. Three tried using software-specific published materials but did not make significant use of these, stating that the materials did not match with what they wanted to do.

A common feature of teacher-produced worksheets in the early months of the project was that initial worksheets were technology orientated in two ways: focusing on the technology itself (commands such as 'open', 'highlight', 'drag') and then focusing on how mathematics is done on the technology, e.g., how to input mathematical

expressions or scale graphs. There is a sense in which this is quite a natural development: if you want to do mathematics on a computer package, you first have to learn how to use the package.

The teachers' experience of (not) using textbooks and worksheets points to significant changes in practice. Teachers in many countries plan their lessons around their textbook. Recent research in the USA states that "textbooks have been identified as a primary resource – and often the *only* source – of the content planning performed by expert and novice high school mathematics teachers" (Nathan and Koedinger, 2000, p. 181). But in this project there is evidence that most project teachers are putting their textbooks aside and writing worksheets that are, at least initially, focused on ensuring their students press the correct buttons.

Social Interactions

There was some commonality but also diversity in the social interactions of teachers and students in these English high school mathematics classes. These schools are hierarchical 'communities of practice' where students wear a uniform and are placed in specific mathematics classes on the basis of their performance in various tests. Lessons have beginnings, focused activities and endings (Kyriacou, 1991, chapter 4) and students, by and large, do the work that the teacher tells them to do. Wertsch's (1998) characterisation of teacher–student discourse in formal instructional settings as being generally organised around "sequences composed of *initiation* by the teacher, followed by a *reply* by the student, followed by an *evaluation* by the teacher" (ibid., p. 69) applied to the beginnings of all the observed non-technology classes.

Within this commonality, however, teachers exhibited diverse routines (Olson, 1992) and diverse verbal and non-verbal ways of interacting with students. I comment on observed non-technology lesson beginnings, in particular the couple of minutes during which students are getting their books out, to illustrate this diversity. The students are sitting in desks which face the teacher's desk, which is at the front of the class. During this time some of the teachers stood by their desks and watched the students as they got their books out of their bags but did not talk to the students. When they perceived that the class was ready (books out and looking at the teacher) they made their opening remarks. Other teachers during this period walked around the room while the students were getting their books out

making 'small talk' to students as they walked around. When they perceived that the class was ready they went to the front and started the lesson.

The big question here is does the use of technology effect social interactions in the classroom and, if so, in what ways? Before addressing this question I reiterate the point made immediately above: that despite some common features of non-technology mathematics lessons there is diversity in routines and interactions, i.e., there is no standard form of teacher-student interactions in technology-based lessons. Nevertheless, comparisons of teachers' overt interactions can be noted. Farrell (1996) suggests that teachers take on new roles as consultants and fellow investigators. She used the Systematic Classroom Analysis Notation (SCAN) (Beeby et al., 1979) in her study and I used this to analyse the video tapes of observed technology and non-technology lessons. Details of my analysis are available in Monaghan (2001). In this paper I simply report on areas where there were noteworthy differences in teachers' behaviour in technology-based lessons, compared to non-technology lessons.

Teachers spent considerably less time in 'teacher-whole class exposition' in technology lessons. As mentioned in the 'activity structure' sub-section above, all 13 video-taped non-technology lessons were of the form 'teacher exposition followed by students working on exercises'. There were many reasons for the reduction in whole class exposition in technology lessons. Some teachers consciously saw using technology as a means of changing their practice so that they spent less time on exposition. Eight of the teachers using computers had to book a computer room with the expectation that the students will use the computers most of the time. Further to this, six of these eight teachers prepared their classes for the work in the computer room in the lesson prior to the computer room lesson.

Teachers spent much more time speaking to two or more students (as opposed to speaking to an individual) in technology lessons. This largely reflects the fact that the availability of computers forced students to work with two or more to a machine. It is interesting to note, however, that even when students worked in pairs in non-technology lessons the teacher talk was largely directed to one of the pair but in technology-based lessons the teacher talk was largely directed to all the students around a computer. This appears to be an important general difference concerning technology lessons and it interrelates with the fact that technology is often not just a tool for

doing the mathematics but a medium for *expressing* the mathematics (Kent, 1999). This statement may require clarification. A tool may be used in a task where the processes and product of the task are recorded on a sheet of paper or an artefact, e.g., a compass to draw a circle or to assist in making a polyhedron. In many mathematical software systems, however, the tool may not only perform mathematical actions but the product of these actions may be recorded on the tool and this record may be the basis for student reflection on their actions. Further to this, when it acts as a medium in this way it is a shared medium for all using the same computer. This is one reason why computers can be conducive to collaborative student work. This interrelates with the tasks that are set and can give the teacher an opportunity to set extensions to tasks. An instance of this in project teacher work occurred when a group of three students completed a task that produced variously positioned quadratic graphs on their computer screen. The teacher said, "Good, now reflect those graphs in the x -axis" and then walked away. This was much more challenging task on a graph plotter than on paper and the teacher gave no guidance but the students succeeded in doing it.

Claims that use of technology influences social interactions in mathematics lessons are often based on student-teacher discourse (see, for example, Farrell, 1996). SCAN analysis revealed five areas where big differences in teachers' utterances in technology and non-technology lessons were noted. In technology lessons teachers spent less time coaching/eliciting ideas from students, less time explaining mathematical ideas and more time explaining technological ideas (the last point is rather obvious and I will say no more on this). They also made more assertions and instructions/initiating remarks in technology lessons. The greater number of assertions and instructions was largely due to a propensity in technology-based lessons for six of the teachers to move quickly around the class attending to technical problems, e.g., "copy cell B3 to D3". I interpret this as teachers' emergent goals being to ensure that technical difficulties did not prevent students attending to the task.

That teachers spent less time in technology lessons eliciting ideas from students and explaining mathematical ideas is interesting. With regard to spending less time explaining mathematical ideas my impression from observations was that this arose by default because every technology lesson did attend to technological explanations. With regard to less time coaching/eliciting ideas from students the explanation is not so straightforward. This category was interpreted

as the teacher pointing out mathematical features without revealing the answer. This certainly occurred in technology lessons, e.g., a teacher using *Derive* pointing to a minimum point on a function and asking “What is the gradient of the tangent here?”, but it occurred much less often than it did in observed non-technology lessons. There are many possible explanations for this, including: focusing on technology at the expense of mathematics; the teachers were relative novices to using technology in their lessons and had not (yet) internalised routines that included giving mathematical hints in these lessons.

To return to the big question posed above, the use of technology did affect social interactions in observed classrooms and did so in a variety of ways: teacher exposition; addressing pairs or groups of students rather than individuals; and greater/lesser use of several linguistic categories in student–teacher discourse. Several aspects strike me as worthy of comment. First, as I reflect on individual teachers (rather than global patterns of interaction) I am convinced that teachers who may be described as ‘didactic’ in non-technology lessons, remain didactic in technology lessons. Second, although there are some commonalities in non-technology lessons there was no common pattern to social interactions across all the observed technology lessons. Indeed, technology lessons were notable by their diversity. Third, many of the noted differences in social interactions in technology lessons appear for very obvious reasons, e.g., less time spent on whole class exposition due to having booked a computer room. Finally, my interpretation of the changes that occurred in technology lessons does not suggest that these teachers were on a developmental path towards adopting new roles such as ‘fellow investigator’.

INTERRELATIONS AND GOALS

If the discrete presentation of the four parameters above suggests that they affect teachers’ emergent goals in isolation, then I have not made myself clear: the parameters are interrelated aspects of teachers’ actions in activities which require a single unit of analysis. In the first subsection below I attempt to make these interrelations explicit by systematically linking each parameter to the others. The examples are taken from situations described in the previous section. In second subsection below I focus on emergent goals in practice (and what

unexpected goals these may be *vis a vis* teachers' planned goals). I include this subsection in case my focus on parameters has obscured the centrality of practice-linked goals in Saxe's model.

Interrelations Between the Parameters

Activity structures (cycles and tasks) are intertwined with social interactions. This is manifest both in attempts to preserve established practice and in situations where established practice changes. The three teachers who mimicked the cycle of non-technology lessons attempted to preserve established practice and social interactions. It could be argued that they, like the teacher in Zbiek's (1995) study, wanted to remain in control. But it could also be argued, with a focus on activity-systems instead of a focus on teachers' volitions, that this cycle was preserved because it embodied social interactions of practice which had become routinised. At the other extreme there were the six teachers who moved quickly around student pairs in computer rooms 'techno trouble shooting'. The activity cycle and the social interactions in these lessons were very different to these teachers' non-technology lessons: the teachers literally cycled the classes as software technicians. The tasks teachers present to their students are also intertwined with social interactions and again there were instances of attempting to retain the status quo of established practice and being forced to move beyond established practice. The tasks teachers are used to giving to their students are, by and large, textbooks tasks which the teachers 'control'. Although 11 of the 13 teachers initially abandoned their textbooks in technology lessons, the worksheet tasks they wrote still had this element of control or guidance (cf. Laborde, 2001, p. 299), i.e., the power structure of the classroom remained.

This last statement reveals links between the activity structure and the conventions and artefacts parameters. Textbooks and worksheets are artefacts (and it is significant that textbooks are not produced by teachers whilst worksheets for technology lessons are invariably produced by teachers). Textbooks, in non-technology lessons, and worksheets written for student use in technology-based lessons structure teacher (and student) activity in different ways¹³ (cf. the discussion above on cycles in technology and non-technology lessons). Another link between activity structure and conventions and artefacts is the way mathematics is written. Without technology teachers are, within the bounds of accepted mathematical practice, free to write mathematics as they want. With technology teachers

must write mathematics as the software requires it to be written. In the case of the teacher using *Derive*, noted in the previous section, this led to lessons (and tasks) devoted to mastering the syntax of the system; in the case of *Excel*, teachers designed tasks based on 'cell arithmetic'.

This last statement reveals links between the activity structure and prior understandings. Consider, for example, the teacher mentioned in the last section who was uncomfortable with the focus on cell arithmetic, and was questioning 'is this maths?'. This question was legitimate for her because her prior understanding of what (school) mathematics is developed in a particular English academic historical context. In considering this issue I find it hard to separate considerations of activity structures, artefacts and prior understanding. Tool use (in the widest sense of the word tool – mediational means) is intricately tied up with mathematics teachers 'being' or identity as mathematics teachers. Kiernan (1992), addressing high school teachers' algebra teaching practices, states that little is known about these practices other than "they tend to follow the chapters in the textbook, emphasizing with explanations some of the given examples and then assigning homework exercises" (ibid., p. 394). Textbooks, moreover and in general, embody socially legitimised mathematics. Technology tools, despite the fact that teachers are encouraged by governments to make use of computers, are not invested with the same social priority. So lessons that focus on how to use cell arithmetic in *Excel* or how to express an inverse function in *Derive* create legitimate tensions for teachers. But mathematics teaching is about teaching as well as mathematics and prior understandings of what teaching is are questioned when technology is used. All of the teachers in this project spent an abnormally high amount of time selecting or designing suitable tasks for technology-based lessons.

Although I have argued that the claims of some radical constructivists that using technology turns teachers into facilitators and co-collaborators, is a myth, teachers' use of technology certainly does impact on social interactions. But this impact does not mechanistically result from the presence of a plastic and metal artefact; it is tied up with practice and change of practice and this is one reason why the interrelationship between the parameters of Saxe's model is important. What I have called 'techno trouble shooting' is linked with the positioning of the artefacts and teachers' technical/mathematical/pedagogical 'mastery and appropriation' (prior understandings, see Wertsch (1998, p. 27)) of the artefacts as mathematical tools.

Diversity in changed practice is, to this way of thinking, an expectation of the use of technology by the teacher: so both the teachers who used graphic calculators and appeared to parallel their prior practices and the teacher who said “Reflect those graphs in the x -axis”, can be accounted for in this model. Popperians and those who believe that a model that accounts for anything is empty will be critical of such a claim. So be it, diversity in practice is a reality that needs to be accounted for.¹⁴

Teachers’ Goals and Emergent Goals

Lest the above analysis, with its focus on the parameters and the interrelations between them, relegated the importance of practice-linked emergent goals I devote this section to the topic of goals. It would be somewhat repetitive to go over all the points above, so in the following I focus on tools and tasks and trace the implications with reference to one teacher.

Digital technology is only one of the tools that a teacher may use and, when used, it does not act in isolation but impacts on, and is impacted on by, the use of other tools. Textbooks and worksheets are tools which have emerged in this project as having significant interrelations with technology use (by and large the hegemony of textbooks in non-technology lessons gave way to an hegemony of worksheets in technology lessons). Awareness of changes in practice from these different tools is an important factor to keep in mind given mathematics teachers’ dispositions towards the use of textbooks. This ‘tool shift’, textbooks to technology, interrelates with the activity structure (tasks and cycles) of lessons and may conflict with teachers’ prior understandings of ‘a good mathematics lesson’. Consider, for example, the teacher referred to above in the subsection on ‘conventions and artefacts’ who was uncomfortable with the focus on cell arithmetic. What is interesting here is that she voluntarily planned this task and wrote a worksheet which resulted in a focus on cell arithmetic and this discomfort only emerged in practice because her emergent goals in the lesson were shaped by the need to get the spreadsheet cells right.

I now take this further by focusing on teachers’ views on the appropriateness of tasks and how specific tasks interrelate with other parameters in practice. I have commented that tasks set in technology-based lessons were different to ‘normal’ tasks: often by intention, often more open, often more conceptual. These tasks arose from

many sources: books, courses, journal articles and colleagues. The teachers planned and tried out these tasks on their own prior to lessons. I would like to focus on goals in teacher planning and emergent goals in practice. The following is an extract from a report by the teacher referred to immediately above.

We¹⁵ started by giving the students structured tasks to do such as solving simultaneous equations graphically. We then moved on to less structured work – usually an investigation . . . These investigations were much more successful with set 1 than with set 2 . . . As a result of these difficulties we adapted the assignments to reduce the number of printouts and to make the tasks more prescriptive.

The variation in the structuredness of the tasks could be simply put down to the teacher learning from her mistakes. I accept that there is an element of ‘trial and error’ in any change in practice but locating the error and tracing its ramifications takes the analysis further. The ‘error’ in the early structured tasks was that they were too easy for the students. The teacher did not see this when she went through the work herself before the lesson but commented on this after the lesson. This is an understandable error, she did not know what level to pitch the lesson because it was new practice. The effect of this error was that a large proportion of the students finished their work midway through the lesson and sent their work to the printer. The computer room had a single printer which was slow and the students’ work had multiple pages. Her emergent goals in the second half of the lesson centred on managing the printer queue and managing the behaviour of the students waiting for their work to be printed.

RELATED APPROACHES TO SAXE’S MODEL

My proclivity towards Saxe’s model as a way to view teacher activity is, admittedly, bound up with my own development and I recognise that there are other ways to view teacher activity. Central to my interests in developing a Saxian account of teacher activity is a desire to find an holistic model. Werstch (1991) argues against analysing individual mental functioning in isolation and argues for analyses which are not limited by the boundaries of psychology, sociology or whatever. His statement “we have many isolated, often arcane pieces to a larger puzzle, but we have no coherent, integrative picture of the whole” (ibid., p. 1) could be applied to most of the literature on the use of technology in the teaching and learning of mathematics. This

Saxian approach shares with Wertsch an attempt to preserve “as many dimensions of the general phenomenon under consideration as possible, thereby allowing one to move from one dimension to another without losing sight of how they fit together into a more complex whole.” (ibid., p. 121).

Saxe’s approach is essentially an activity theoretic approach, although Saxe does not say this himself. Both Saxe and activity theory take human practice as central to any account of human development. Vygotsky (see especially 1978, chapter 6) develops a psychology which explains mental development through an analysis of the interaction of humans and the objects of their lived-in environment as mediated by cultural means, especially tools and signs. Vygotskian activity theory argues that all activity is culturally mediated activity; cultural artefacts are central to Saxe’s analysis.

The development of activity theory as expounded by Leont’ev (1978) sheds useful light on Saxe’s goals and parameters and my account of mathematics teachers’ activities in technology-based lessons. I comment on Leont’ev’s contribution with regard to actions, motives and goals (NB Leont’ev’s goals are conscious, not emergent, goals). Leont’ev sees the basic components of activity as actions (processes) that realise activity, are generated by motives and are directed towards goals. Motives are central to Leont’ev’s theory, they are the objects of activity (ibid., pp. 62–63) and that which distinguishes one activity from another. Motives are to activity as goals are to actions. Leont’ev illustrates these ideas with an example of a primitive hunter-gatherer tribe, an activity motivated by the need for food where a goal may be making trapping equipment. I will explore this in the next paragraph with regard to the project teachers and Saxe’s parameters,¹⁶ a situation where motives are much less clear than in hunter-gatherer situations.

Saxe’s parameters influence teachers’ goals and actions but the parameters, with the possible exception of ‘prior understandings’, are not independent of the activity – indeed, they arise from the motive of the activity. Teachers’ conscious goals are not a problematic issue and actions directed towards these goals may involve setting up the technology, providing the task on a worksheet, etc. Motives, however, are problematic and I think it is often not clear what the object of technology-based classroom mathematical activities are. Important questions here are ‘Does the introduction of technology ‘fit’ with the motive of the classroom activity?’ and ‘Does the introduction of technology change the motive?’. The answer to the second

question is an almost certain 'yes' – why use technology if the motive of classroom activity is unchanged? The answer to the first question, in this project and in my opinion, varied across the teachers but whatever the answer the parameters are consistent with each other and with the motive of the activity. In the case of the teacher who asked 'is this maths?' (see the subsection above on 'Interrelations between the parameters') what was happening in the technology-based lesson was not consistent with the teacher's understanding of the motive of the activity.

Saxe's approach also has commonalities with French work on instrumentation, about which the editor of this journal (Volume 7, No.2, 2002) has called for further debate. Artigue (2002) focuses on the dialectics between conceptual and technical work in the teaching and learning of mathematics using computer algebra system (CAS) environments, but the focus on CAS is not essential to her method of analysis. Her focus is on 'instrumented techniques', not the teacher, but this is unimportant as regards commonalities in methods of approach. Central to her analysis are the anthropological approach to didactics of Chevallard and the ergonomic approach to instrumentalisation of Vérillon and Rabardel (1995).

Chevallard's anthropological approach, according to Artigue, is a socio-cultural approach which views mathematical objects as entities which arise from institutional 'praxeologies' (practices). Praxeologies are described in terms of the "task in which the object is embedded; the techniques used to solve this type of task; ... the discourse which is used to both explain and justify these techniques; and the 'theory' which provides a structural basis for the technological discourse" (Artigue, 2002, p. 248). Institutional routinisation of specific techniques is accompanied by a weakening of associated theoretical discourse and by internalisation of associated knowledge which becomes 'natural', becomes a simple act. Technology upsets the traditional equilibrium between conceptual and technical work and the objects of mathematical activity. Drawing on Balacheff, Artigue argues that new needs emerge which are not easily recognised if the emergent goals are simply viewed in relation to techniques and praxeologies associated with that previously privileged by traditional pencil and paper mathematics.

The important focus on the institution is novel here but otherwise there are many commonalities: tasks and activity structures, techniques and conventions, discourse and social interactions, emergent needs

and goals. My (Saxe's) foci add to Artigue's analysis by addressing teachers *per se* and by introducing emergent goals alongside her (Balacheff's) notion of emergent needs.¹⁷ The other component of Artigue's analysis is the approach to instrumentation of Vérillon and Rabardel (1995). This is a post-Vygotskian consideration of the internalisation of tool use which is quite consistent with Saxe's analysis.

I have commented on the similarities of various approaches to position and elucidate aspects of Saxe's approach. I do not think that the fact that there are similarities is surprising. Indeed, it would be surprising if different post-Vygotskian approaches did not have similarities.

CONCLUSIONS, IMPLICATIONS AND FURTHER RESEARCH

The integration of technology into mathematics teachers' classroom practice is a complex undertaking regardless of whether teachers find this an 'easy' transformation of practice or not. Policy makers and pre- and in-service teacher trainers would be well advised to note this complexity. Saxe's model provides one way of viewing teachers' technology practice 'as a whole'. It allows us to consider the interrelations between tools, conventions, social interactions, prior understandings and activity structures in activities where unexpected emergent goals come into being and pass away. The model accounts for diversity in practice. It does not provide expected outcomes. In particular it does not provide pre- and in-service teacher trainers with models for teacher development. If training, however, includes reflection on practice and/or case studies, then an appreciation of the dialectics of emergent goals may help teachers to appreciate the dynamics of practice. A case in point is the teacher considered in the sub-section on planned and emergent goals. She resolved to reduce the number of student-generated printouts but her printer queue problem was not the 'cause' of her problem.

There are three important issues that I am dissatisfied with, but am currently unable to attend to, in my analysis and which require further research or consideration. The first is that this 'holistic' account is not holistic in that students, who most definitely do feature in teachers' practice, are hardly considered here. The second is that the central issue of motives is underdeveloped in this account. The third

is the issue of the relationship between the parameters and teachers attitudes to, or ways of using, technology.

ACKNOWLEDGMENTS

The work reported on here was supported by the Economic and Social Research Council, Award Reference Number R000222618. Thanks to Michèle Artigue and three reviewers who made a number of points which helped me improve this paper. Thanks also to Steve Lerman, Zlatan Magajna and Richard Noss for comments on an early draft. Last, but definitely not least, thanks to the project teachers and Peter Johnson for a fun and an interesting year together.

NOTES

- ¹ Strictly speaking 'teachers' here refers to the 13 teachers described in the third section.
- ² 'Technology' in this paper refers to digital technology. I avoid using the word 'computer' because I include (graphic) calculators in my considerations.
- ³ The purpose of this section is to position the ideas in this paper rather than to provide a comprehensive review of the literature, which would have a disadvantage of being very long. Being a discursive paper further literature is considered in subsequent sections.
- ⁴ I do not think that England is unusual in this respect.
- ⁵ The NCTM *Standards* being a case in point. See, for example, the most recent *Standards* documents, <http://standards.nctm.org/document/chapter2/techn.htm>, though I believe an examination of earlier *Standards* would reveal an even more frequent use of this term.
- ⁶ One reviewer commented that this was an unnecessary self criticism. I feel, however, that this is an important point. Teachers activities are interrelated with their students' attitudes and behaviour. See, for example, Lumb et al. (2000), a paper arising from this project where a teacher using *Derive* changed his activities because his students commented on their concerns that they could not use a computer in the examination. Although I can relate isolated instances like this, I do not have sufficient data to explore this interrelation.
- ⁷ Saxe's model has three components: analysis of practice-linked goals; form-function shifts in cognitive development; and the interplay of learning across contexts. My focus in this paper is on the first component.
- ⁸ Strictly speaking there should be additional arrows linking 'prior understanding' to 'social interactions' and 'activity structures' to 'conventions/artefacts'.
- ⁹ I do not know if there is such a thing as a 'valid mathematical task'. I use the term 'valid-in-their-school mathematical task' to suggest that mathematics teachers in a particular school do, in general, implicitly agree that certain tasks are warranted. This may be an English characteristic in that high schools have hierarchical mathematics departments which produce 'schemes of work' which teachers base classroom tasks

around. Schemes of work are usually linked to national curricula and high stakes examinations which strongly influence departments' and teachers' task choices.

¹⁰ Over the project year three other teachers found that they were able to use the textbook for ICT-based work and another found that she could write worksheets based on textbook work.

¹¹ This example is a nice illustration that emergent goals have an affective dimension. This teacher was clearly delighted that he located and solved this problem and we laughed about it many months later. I do not think this teacher would have worried if he had not solved the problem but I can imagine that some teachers would worry about this (which is Heid's point).

¹² Z. Magajna (private correspondence) notes that Saxe's parameters are perhaps more dependent/related to declared or implicit ways of using technology than to the technology itself. Important questions here are: what are teachers attitudes to technology use in mathematics classes? And how do different ways of regarding technology use in classrooms influence the parameters? These questions relate to issues concerning teachers' 'identity' (Wenger, 1998). I do not develop this here other than to note that the tools teachers use in the classroom to do mathematics impinge on issues of identity. Lins (2000, 2002) work is also relevant here. She comments (Lins, 2000, p. 52) that "technology is simultaneously independent of human action and yet irrelevant without it". In (Lins, 2002, p. 58) she develops the concept of 'software of the teacher', i.e., that your *Cabri* and my *Cabri* might be two distinct *Cabris*.

¹³ I am not suggesting here that there are just two distinct ways but claiming that there are a family of ways for both and that there is considerable diversity, as well as some commonality, in these families of ways.

¹⁴ I could extend the argument here to considerations of 'agency'. I will not do this in the main body of this paper but sketch an argument in this footnote. Pickering (1995) considers agency with regard to technological and scientific practice and what he terms the 'mangle of practice'. The agencies he considers are material agency (tools), social agency (people) and disciplinary agency (in this case Mathematics). These interrelate in non-mechanistic ways, which he terms the 'dance of agency'. My description of the interrelationship between social interactions, prior understandings and conventions and artefacts is a dance of agency.

¹⁵ 'We' refers to two teachers in this school. My comments below this extract refer to one of these teachers.

¹⁶ I am indebted to Z. Magajna for helping me clarify my ideas on this matter.

¹⁷ There are similarities and differences between Balacheff's emergent needs and Saxe's emergent goals (other than obvious linguistic ones). Both concern unplanned foci of attention in practice due to tool-related transformation of practice. The theoretical positions of emergent needs and emergent goals, however, differ. Saxe's emergent goals are at the centre of his theory and his parameters are constructs whose purpose is to aid an analysis of these practice-linked goals. Instrumented techniques, and their epistemic and pragmatic values, however, are at the centre of Artigue's work, as I understand it, and emergent needs contribute to her unravelling of the problematic status of instrumented techniques (though she acknowledges that these needs are not independent of the techniques).

REFERENCES

- Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning* 7(3), 245–274.
- Askew, D., Rhodes and Brown (2000). Numeracy practices in primary schools: Towards a theoretical framework. In T. Rowlands and C. Morgan (Eds.), *Research in Mathematics Education*, Vol. 2, (pp. 63–76). London: British Society for Research into Learning Mathematics.
- Beeby, T., Burkhardt, H. and Fraser, R. (1979). *Systematic Classroom Analysis Notation for Mathematics Lessons*. Nottingham: Shell Centre for Mathematical Education.
- Bottino, R. and Furinghetti, F. (1996). The Emerging of teachers' conceptions of new subjects inserted into mathematics programs: The case of informatics. *Educational Studies in Mathematics* 30, 109–134.
- Cuban, L. (1989). Neoprogressive visions and organizational realities. *Harvard Educational Review* 59, 217–222.
- DfEE/QCA (1999). *Mathematics; The National Curriculum for England*. London: Department for Education and Employment/Qualifications and Curriculum Authority.
- Engeström, Y., Engeström, R. and Kärkkäinen, M. (1995). Polycontextuality and boundary crossing in expert cognition: Learning and problem solving in complex work activities, *Learning and Instruction* 5, 319–336.
- Farrell, A. (1996). Roles and behaviours in technology-integrated precalculus classrooms. *Journal of Mathematical Behaviour* 15, 35–53.
- Healy, L. and Hoyles, C. (2001). Software tools for geometric problem solving: Potentials and pitfalls. *International Journal of Computers for Mathematical Learning* 6(3), 235–256.
- Heid, M.K. (1995). The interplay of mathematical understandings, facility with a computer algebra program, and the learning of mathematics. *Proceedings of the 17th Annual Meeting of the N. American Chapter of PME* (pp. 221–225). Columbus, OH.
- Heid, M.K., Sheets, C. and Matras, M.A. (1990). Computer-enhanced algebra: New roles and challenges for teachers and students. In T. Cooney (Ed.), *Teaching and Learning Mathematics in the 1990s, NCTM 1990 Yearbook* (pp. 194–204). Reston, Va.: NCTM.
- Hudson, B. and Borba, M. (1999). The role of technology in the mathematics classroom. *Micromath* 15(1), 19–23.
- Kendal, M. (2001). Teaching and Learning Introductory Differential Calculus with a Computer Algebra System. Ph.D. Thesis. University of Melbourne.
- Kendal, M. and Stacey, K. (2001). The impact of teacher privileging on learning differentiation with technology. *International Journal of Computers for Mathematical Learning* 6(2), 143–165.
- Kent, P. (1999). Some notes on expressiveness. Available at <http://www.lonklab.ac.uk/came/events/weizmann/>
- Kiernan, C. (1992). The learning and teaching of school algebra. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 390–419). New York: Macmillan.
- Kyriacou, C. (1991). *Essential Teaching Skills*. Hemel Hempstead: Simon & Schuster Education.
- Laborde (2001). Integration of technology in the design of geometry tasks with cabri-geometry. *International Journal of Computers for Mathematical Learning* 6(3), 283–317.

- Leont'ev, A.N. (1978). *Activity, Consciousness, and Personality*. Englewood Cliffs: Prentice-Hall.
- Lerman, S. (1994). Metaphors for mind and metaphors for teaching and learning mathematics. *Proceedings of the 18th International Conference for the Psychology of Mathematics Education*, Vol. III (pp. 144–151). University of Lisbon.
- Lins, B. (2000). The Importance of premises: From an essentialist to an anti-essentialist view of ICT in mathematics education. *Proceedings of the British Society for Research into Learning Mathematics* 20(3), 49–53.
- Lins, B. (2002). Towards new trends on the role of users of technology: A look at some research fields. *Proceedings of the British Society for Research into Learning Mathematics* 22(3), 55–60.
- Lloyd, G. and Wilson, M. (1998). The impact of a teacher's conception of functions on his implementation of a reform curriculum. *Journal for Research in Mathematics Education* 29(3), 248–274.
- Lumb, S., Monaghan, J. and Mulligan, S. (2000). Issues arising when teachers make extensive use of computer algebra. *International Journal of Computer Algebra in Mathematics Education* 7(4), 223–240.
- Magajna, Z. (2001). *Geometric Thinking in Out of School Contexts*, Doctoral Dissertation, School of Education. Leeds: University of Leeds.
- Mathematical Association (1992). *Computer in the Mathematics Curriculum*. In W. Mann and D. Tall (Eds.), Leicester: The Mathematical Association.
- Mathematical Association (2002). *ICT and Mathematics: A Guide to Learning and Teaching Mathematics* (pp. 11–19). The Mathematical Association. Web document available on http://www.m-a.org.uk/education/teachers_teaching_with_technology/
- Monaghan, J. (2000). End of Award Report: 'Moving from occasional to regular use of ICT in secondary mathematics classes'. http://www.regard.ac.uk/regard/home/index_html and search on Monaghan.
- Monaghan, J. (2001). Teachers' classroom interactions in ICT-based mathematics lessons. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th Conference of the International Groups for the Psychology of Mathematics Education*, Vol. 3 (pp. 383–390).
- Moreira, C. and Noss, R. (1995). Understanding teachers' attitudes to change in a logomathematics environment. *Educational Studies in Mathematics* 28, 155–176.
- Nathan, M. and Koedinger, K. (2000). Teachers' and Researchers' Beliefs about the Development of Algebraic Reasoning. *Journal for Research in Mathematics Education* 31(2), 168–190.
- Noss, R. and Hoyles, C. (1996). *Windows on Mathematical Meanings: Learning Cultures and Computers*. Dordrecht: Kluwer Academic.
- Pickering, A. (1995). *The Mangle of Practice: Time, Agency and Science*. Chicago, IL: University of Chicago Press.
- Rodd, M. and Monaghan, J. (2002). Graphic calculator use in leeds schools: Fragments of practice. *Journal of Information Technology for Teacher Education* 11(1), 93–108.
- Saxe, G.B. (1991). *Culture and Cognitive Development: Studies in Mathematical Understanding*. Hillsdale NJ: Laurence Erlbaum Associates.
- Vérillon, P. and Rabardel, P. (1995). Cognition and artifacts: A contribution to the study of thought in relation to instrumented activity. *European Journal of Psychology of Education* 10(1), 77–101.
- Vygotsky, L.S. (1978). *Mind in Society*. Cambridge, MA: Harvard University Press.

- Watson, D.M. (Ed.) (1993). *The Impact Summary*. London: DFE & King's College London.
- Wenger, E. (1998). *Communities of Practice: Learning, Meaning, and Identity*. New York/Cambridge: Cambridge University Press.
- Wertsch, J.V. (1991). *Voices of the Mind*. Cambridge, MA: Harvard University Press.
- Wertsch, J.V. (1998). *Mind as Action*. New York/Oxford: Oxford University Press.
- Zbiek, R.M. (1995). Her Math, their Math: an In-service Teacher's Growing Understanding of Mathematics and Technology and Her Secondary Students' Algebra Experience. *Proceedings of the 17th Annual Meeting of the N. American Chapter of PME* (pp. 214–220). Columbus, OH.
- Zbiek, R.M. (2002). Influences on mathematics teachers' transitional journeys in teaching with CAS. *International Journal of Computer Algebra in Mathematics Education* 9(2), 129–137.

Centre for Studies in Science and Mathematics Education
University of Leeds
Leeds LS2 9JT, UK
E-mail: j.d.monaghan@education.leeds.ac.uk