

USING DYNAMIC SOFTWARE TO FOSTER PROSPECTIVE TEACHERS' PROBLEM SOLVING INQUIRY

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Mathematical tasks are essential components that help prospective and practicing teachers to develop mathematics and didactic knowledge. What type of reasoning do problem solvers exhibit when they use a dynamic software to solve textbook tasks? We document the extent to which the use of the tool can offer prospective and in-service teachers the opportunity to construct and explore a task's dynamic model where visual, empirical, and geometric reasoning complement and enhance formal approaches. As a result of our research, it was evident that the use of the tool not only offers novel ways to think of the tasks, but also the nature of routine problems can be transformed into a series of nonroutine activities.

INTRODUCTION

There is an agreement within the mathematics education community that the use of technology plays an important role in the students' learning of mathematics. However, the extent to which teachers develop the experiences that allow them to recognize and incorporate the systematic use of the tools in their actual practice continues to be an important theme in teachers education programs. One way for prospective and inservice teachers to know the potential associated with the use of a particular tool is to get them involved in problem solving experiences that enhance the use of tools in solving and discussing routine problems found in regular textbooks (Santos-Trigo & Camacho-Machín, 2009). This report is part of a wider research project that aims to analyse and document the use of digital technology in teachers' education and students' learning. One essential phase during the development of the project was to characterize what ways of reasoning and problem solving strategies emerge when routine tasks such as those that appear in textbooks are approached through the use of digital tools. In this study, we focus on analyzing the problem solving sessions developed within a community formed by mathematicians, mathematics educators and prospective and inservice high school teachers which aimed to characterize and discuss ways of thinking and reasoning that the members of the community showed while using a dynamic software (Geogebra) to represent, explore, and solve a set of textbook problems. Thus, the research question that oriented our inquiry was:

- *What are the features of mathematical thinking and reasoning that distinguish and characterize a problem solving approach that relies on the use of a dynamic software to deal with routine or textbook problems?*

CONCEPTUAL FRAMEWORK.

To frame the study, we rely on two complementary approaches to explain ways for people to take decisions during the development of their activities. Specifically, we recognize that teachers take several important decisions during the preparation and development of their mathematical lessons. These involve the selection of problems, the introduction and discussions of concepts, ways to organize or structure learning activities, and ways to answer, evaluate, and orient students' comments and participation. How do teachers support and carry out choices and decisions related to the framing and development of a mathematical lesson? Schoenfeld (2011) proposes a framework to characterize and interpret ways in which people in different domains engage in, and develop practices associated with such domains or fields. "People's decisions making in well practiced, knowledge-intensive domains can be fully characterized as a function of their orientations, resources, and goals" (p. 182). Kahneman (2011) identifies two systems to account for or explain the decisions and choices that people make: "System 1 operates automatically and quickly, with little or no effort and no sense of voluntary control. System 2 allocates attention to the effortful mental activities that demand it, including complex computations. The operations of System 2 are often associated with the subjective experience of agency, choice, and concentration (p. 20). Hence, it becomes important to document the extent to which the opportunities and experiences that prospective teachers develop and encounter in their education influence the development of their practices.

How do teachers construct their orientations or beliefs, dispositions, values and resources to pursue their goals? What is the role of teachers' initial preparation and experience to achieve instructional goals that are consistent with mathematical practices? To delve into the teachers' preparation implies to recognize that there are multiple paths or programs and traditions to prepare prospective teachers around the world. In some cases, the faculty of education and the mathematics departments jointly share the responsibility to prepare teachers; other programs are part of a school or institutions (*e.g.*, normal schools) dedicated exclusively to the education of teachers. Both teaching models recognize the need and importance for teachers to develop the mathematical and didactic knowledge that can help them structure and implement proper conditions for students to learn the subject. However, the extent to which prospective and inservice teachers develop the mathematical sophistication needed to structure a sound mathematical lesson, to interpret students' ideas or comments and guide their learning has recently been questioned and generating an ongoing debate.

Even (2011) states that the assumption that "advanced mathematics studies would enhance teachers' knowledge of mathematics, which in turn will contributed to the quality of classroom instruction" needs to be reexamined in terms of what means for

teachers to have adequate subject-matter knowledge to become an expert teacher and how a teacher can develop and use that knowledge in her teaching.

Many practicing teachers, for different reasons, have not learned some of the content they are now required to teach, or they have not learned it in ways that enable them to teach what is now required. ... Teachers need support if the goal of mathematical proficiency for all is to be reached. The demands this makes on teacher educators and the enterprise of teacher education are substantial, and often under-appreciated (Adler, et al., 2005, p. 361).

To shed light on the role of advanced mathematical knowledge in teachers' classroom decisions, Zazkis and Mamolo (2011) provide examples where teachers' awareness of that knowledge becomes useful to orient the development of a lesson. They use the construct *horizon knowledge* to refer to "teachers' advanced mathematical knowledge which allow them a "higher" stance and broader view of the horizon with respect to specific features of the subject itself (inner horizon) and with respect to the major disciplinary ideas and structures...occupying the word in which the object exist (outer horizon)" (p. 10). For instance, they describe a class event where Mrs. White asked her Grade 3 students to count the number of triangles formed by drawing segments from each vertex of a regular pentagon to the other vertices. When students provided their answers (32, 27), Mrs. White noticed that both were wrong and her recognition was based on using a mathematical result she had studied in her university course of algebra. Thus, Zazkis and Mamolo suggest that it is important for teachers to take advance mathematics courses in order to wide their mathematical horizon and to use that knowledge during the development of their teaching practices.

The ways in which prospective teachers study mathematics courses play a crucial role in developing resources and contents to be used in their teaching. That is, it is not sufficient for prospective teachers to take advance courses; but they also need to reflect on ways to connect mathematical results with other problems or situations. In this context, the systematic use of diverse digital tools can provide prospective teachers with the opportunity to enhance and extend problem-solving approaches that involve the use of paper and pencil. For instance, with the use of a dynamic software they can construct models of mathematical problems where objects or elements can be moved within the model to observe patterns of parameters that emerge as a result of moving those objects. In this context, it becomes important for prospective and practicing high school teachers to work and discuss ways in which the use of the tool helps them transform some routine problems into a set of activities that fosters mathematical reflection and connections between concepts. These problem-solving experiences are useful for teachers to explore, extend, and discuss mathematical concepts that later will shape their ways to frame and implement problem solving activities in their classrooms.

THE CONTEXT, PARTICIPANTS AND METHODS

Our research group formed by two mathematics educators, two mathematicians, and six high school prospective (three) and inservice teachers (three), has focused its agenda on analysing the types of reform that teachers' educational programs need to consider in order to incorporate the systematic use of digital technology to develop both mathematics and didactical knowledge. Our initial literature review allowed us to recognize that there is scant information on the characterization of ways of reasoning that subjects or learners can construct as a result of using a particular tool and how the use of several tools can help them enhance their problem solving approaches. That is, teachers need information regarding what types of representations and explorations of tasks or problems can be constructed with the use of a particular tools and the kind of arguments, including visual and empirical, that can be used to develop mathematical understanding and to support tasks solutions.

Our point of departure was to work on mathematics problems that appear in textbooks with the use of a dynamic software and discuss them, within the group, features of mathematical reasoning that characterize and are consistent with this approach. Thus, we focus on contrasting the software approach that involves visualizing and supporting mathematical results through the use of empirical and geometric arguments with the analytic approach that relies on the use of algebra to identify mathematical relations. In this process, the software properties are helpful in visualizing the behaviour of particular parameters or relations without making explicit the algebraic model. We contend that while the use of the dynamic tool demands that problem solver think of the problem in terms of properties and their geometric meaning to construct a dynamic model; the analytic approach asks the problem solver to represent and explore the problem through an algebraic model.

The problem solving sessions and the unit of analysis. The group sessions were first developed through four three-hours meetings where we initially worked on a set of textbook problems and discussed, within the group, the role that the use of the software played in representing, exploring, and solving each problem. During each meeting one or two of the members of the group presented a task and possible ways to represent it through the dynamic software to the group. At this stage, the rest of the group got involved in open discussions that included asking for clarification issues, concept explanation or proposing other ways to approach the task. Later, we continued the group discussion via online conferences. In this report, **the unit of analysis** is the group's work exhibited while working on, and discussing the textbook problems. We do not intend to describe in detail the contribution of each group member to the solution; instead, we focus on what the group as a whole agreed and identified as important ideas associated with the problem solving process. To illustrate the common mathematical features that emerged during the solution process, we relied on a set of textbook tasks that appears at the end of unit that

involves the study of perimeters and areas of triangles. It is important to mention that the initial task was to analyse the list of problems in terms of concepts involved and possible strategies needed to approach each problem.

We found that the format and wording of the textbook problems, in general, asked to find a particular answer to each problem and there was little opportunity for learners to think of multiple approaches or go beyond the asked solution. Here, we regrouped the problems and identified those that could be extended and connected with key mathematical ideas.

An Initial Prompt.

Two textbook problems in which students are asked to find areas and perimeters of particular triangles were slightly changed and posed as:

- a. Draw two triangles that share one side AB (base) and whose third vertex lies on a line that is parallel to segment AB. By observing the figure, do those triangles have the same area/perimeter (explain)? With the use of the software, observe what happens to the area and perimeter values of the triangles when the third vertex is moved along line L.

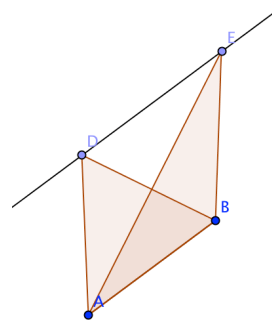


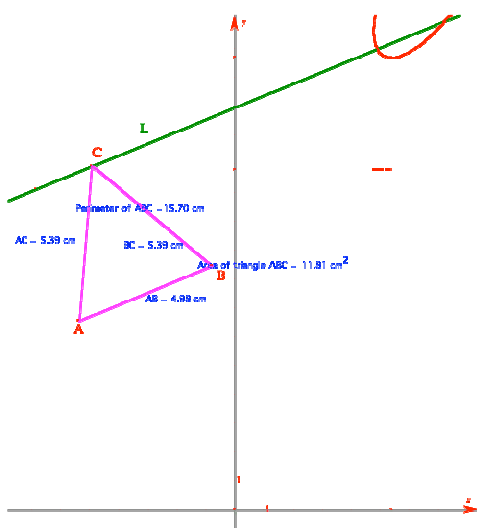
Figure 2: What happens to the triangle's area when point C is moved along line L?

- b. Can you construct three triangles that share a common side (base) and also the same perimeter?

The group discussion of the tasks was framed around two problem-solving principles:

- All tasks are conceptualized as opportunities for learners to connect or extend initial statements.
- Solving the tasks involves looking for different ways to represent, explore and solve them and contrasting mathematical qualities associated with the solution process.

For example, during the process of dealing with the first task, the representation shown in Figure 3 was generated and it became a source of introducing several concepts into the discussion.



What happens the triangles' area and perimeters when point C is moved along line L? How can you graph the area and perimeter behaviours? Is it possible to identify a position where the triangle ABC reaches it minimum perimeter? How can we prove it?

Figure 3: Graphic representation of the perimeter and area of generated triangles.

During the discussion, it was recognized that the use of the dynamic software, became important to move objects within the representation and to quantify particular parameters (side, area, perimeter, angles, etc.) embedded in the representation and observe their behaviour. The use of the tool also allows the problem solver to graph particular functions or relations without defining the algebraic model explicitly. For example, the graph of the variation of perimeter as a function of the length of side AC and the corresponding perimeter as point C is moved along line L. Here, conjecture emerged:

From all triangles that are formed by moving point C along line L, the triangle with minimum perimeter is located when point C is the intersection of line L and the perpendicular bisector of AB. That is when triangle ABC is isosceles.

Other concepts that were addressed while discussing this task include:

- The concept of height of a triangle and the calculation of the area.
- The concept of variation (graph of perimeter) and constant function.
- The concept of perpendicular bisector and its relation to the perimeter variation.
- The concept of infinity (how many triangles can be generated while moving point C along L?)
- The use of the Cartesian system and ways to support conjectures.

Comment: Thinking of this type of task in terms of paper and pencil approach often leads learners to use a formula (triangle area, in this case) to conclude that the area of two triangles that share the same base and height is the same. However, with the use of the tool, the dynamic model of the task offers the problem solver the opportunity

to observe invariances or parameters behaviours that often become a set of conjectures that needs to be supported mathematically. The visual representation and empirical data associate with the parameters' behaviours were important to formulate conjectures that later needed to be proved or rejected. In addition, dragging points or objects within the configuration provided useful information to focus the group's attention on key elements or properties that help us construct arguments to support those conjectures.

The first task also provided the context to address problem b). The idea was to construct a family of triangles with a common base and the same fixed perimeter. With the use of the software, Figures 4 and 5 represent two ways to explore the problem:

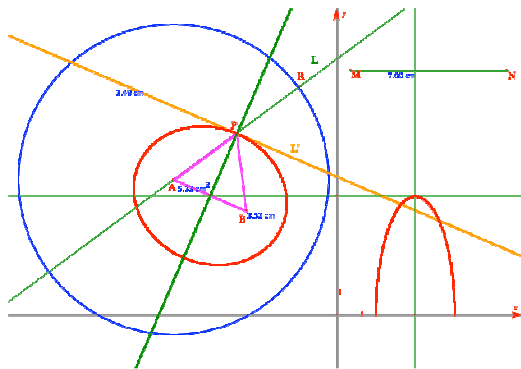


Figure 4: Drawing the condition through the perpendicular bisector

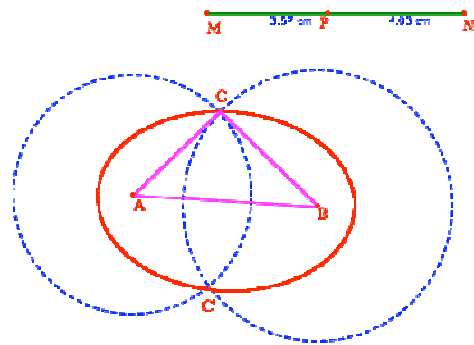


Figure 5: Another way to explore the problem

What is the locus of point C when point P is moved along segment MN?

In Figure 4, segment AB represents the common triangle side and segment MN the sum of the other sides of the triangle. That is, the perimeter of triangle ABC is the sum of segment AB and segment MN. Point A is the centre of a circle with radius segment MN and AR is a line passing by R (which is any point on that circle). L' is the perpendicular bisector of segment BR which intersects line AR at point P. The locus of point P when point R is moved along the circle determines the set of points that are the candidates to locate the vertex C to form a family of triangles with a fixed perimeter. Indeed, the locus is an ellipse since $PB = PR$ (definition of perpendicular bisector) and AR is the radius of a circle. Figure 5 represents another way to explore the same problem: Segment AB is the common side and MN is the sum of the other two sides of the triangle. P is a point on segment MN. Two circles are drawn: One with centre point A and radius MP and other with centre at point B and radius PN. These circles intersect each other at points C and C'. The locus of point C when point P is moved along segment MN is an ellipse and each point on this locus determines and becomes the third vertex of triangle ABC with fixed perimeter. Thus, Figures 4 and 5 were two dynamic models used to represent the problem and the source to discuss:

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Figure 6: when point B is moved outside the circle, the locus is a hyperbola.

Comment: A main teachers' instructional goal is to encourage their students to extend and connect the contents they study with other ideas and situations. There is evidence that the task dynamic model exploration allows the problem solver to relate contents that in a traditional curriculum appear in different units. For example, the second task that asked for the construction of three triangles with the same perimeter became a platform to generate the conic sections that appear in an entire course of analytic geometry. In addition, the area behaviour of the family of triangle generated in figure 5 was analysed graphically to determine the triangle that reaches the maximum area. That is, the use of the tools offers problem solvers the opportunity to relate and connect concepts and themes that emerge as a result of moving objects within the representation or configuration of the task.

DISCUSSION AND REFLECTIONS

In retrospective, we recognize that the problem solving process involved while working on the tasks show several features of the type of reasoning and mathematical thinking that emerge as a result of approaching the tasks with the use of a dynamic software. It was observed that there are several strategies and ways to think of and explore the problems that are associated with the use of the tool. Dragging objects within the model is a key strategy to identify and explore mathematical relations. The controlled movement of objects demands that problem solvers develop a set of heuristic that are crucial to explore and analyze the behaviors of objects relations. For example, drawing circles or extending segments to move

lines on the plane or points along lines are strategies associated with the movement of the model. Assuming the problem as solved is a key heuristic that is enhanced with the use of technology. Here, by analyzing elements associated with the solution, problem solvers can obtain useful information to explore the problem dynamically. In addition, finding loci of particular objects not only becomes important to identify patterns of behavior of some parameters in the model; but also the same loci are tools to find mathematical relations. There is evidence that the use of computational technology (a dynamic software) can offer learners the possibility of transforming some routine problems, found in regular textbooks, into a set of tasks where they can exhibit and contrast different ways of reasoning about the problem that involve visual, empirical, and formal approaches. In this context, the use of dynamics tools plays an important role for conceptualizing the tasks as an opportunity for learners to engage into an inquiring or inquisitive process that goes beyond reporting a particular solution. For example, the dynamic models of the two tasks become a platform to explore not only different forms to represent emerging relations; but also ways to extend and connect the initial statement of the tasks. In this process, it is possible to generate graphic behaviour of particular relations (perimeter variation and locus of particular objects) without defining the algebraic model. In addition, concepts like perpendicular bisector and loci of points became relevant to explain and justify those relations. In general terms, the use of the tool offers problem solvers the opportunity to examine graphically relations that later can be explored and contrasted algebraically. In this context, the use of the tool complements or extends mathematical reflection that learners engage in algebraic approaches. Discussing the tasks and the use of a dynamic software throughout the problem solving sessions provided us, as a research group, important information to think of possible routes for teachers to frame and implement problem solving activities in the classroom. Our current project goal is to include more high school teachers in our group and to work on their own textbook problems in order to discuss ways in which the use of technology can help them orient their teaching practices. By reflecting on main issues addressed in the initial research question, we learned that:

- a) Many of the routine problems or exercises that appear in textbooks could be transformed, with the use of the tool, in a series of activities to engage learners in problem solving experiences. Thus, teachers do not need to think of a set of sophisticated problems to use the tool in problem solving approaches.
- b) Problem solvers or learners can develop certain experience in the use of the tool by experimenting and discussing strategies that involve moving or dragging objects within the representation, finding loci of particular objects, quantifying attributes such as areas, lengths, perimeters, and by contrasting other learners' approaches to the problems.

- c) The discussion of concepts and ways used to represent and explore the tasks became crucial to relate, integrate and extend the initial contents associated with the tasks. In this process, there is also an opportunity to review and extend some of the concepts or themes that appear while dealing with the tasks.
- d) Working on the routine tasks with the use of the tools helped us design worksheets that can be used to orient teachers in the process of transforming the tasks into a platform to formulate conjectures and ways to explore and support them.

CONCLUDING REMARKS

In responding the research question posed initially, we argue about the importance for researchers and inservice teachers to work and discuss within a community what the use of the tool enhances in terms of ways of representing, exploring, solving and extending routine mathematical tasks. As a result, typical tasks found in textbooks offers a point of departure to construct dynamic models or tasks in which problem solvers can identify and explore not only different mathematical routes (contrasted with algebraic approaches) but also possible didactic ways to emphasize the coordinated use of empirical, visual, and graphic approaches.

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