MSc in Statistics and Operational Research
Bayesian Inference Project 1

1. For the data set `RegressionData` consider the simple linear regression model

$$y_i = \alpha + \beta x_i + \epsilon_i, \quad \epsilon_i \sim \text{Normal}(0, \tau^{-1}), \quad i = 1, \dots, 100.$$

Consider the hypothesis test

$$\begin{aligned} H_0 &: \beta = 0 \\ H_1 & \quad \beta \neq 0 \end{aligned}$$

Let $\pi_1, 1 - \pi_1$ be the prior probabilities of hypotheses H_0 and H_1, repectively, and assume the following prior distributions for the model parameters.

$$\begin{aligned} \alpha|\tau &\sim \text{Normal}(\mu_1, (c_1\tau)^{-1}) \\ \beta|\tau &\sim \text{Normal}(\mu_2, (c_2\tau)^{-1}) \\ \tau &\sim \text{Gamma}(p, q). \end{aligned}$$

(i) Consider equal prior probabilities for the two hypotheses and set $\mu_1 = \mu_2 = 0$, $c_1 = c_2 = 0.5$ and $p = q = 0.01$. Compare the two hypotheses and comment on the results.
(ii) Now, consider $\pi_1 = 0.2$, $\mu_1 = \mu_2 = 0$, $c_1 = c_2 = 2$ and $p = q = 1$. Compare the two hypotheses and comment on the results. Perform a prior sensitivity analysis and discuss your findings.

2. Consider Bayesian Inference for the simple linear regression model in Part 1. Let $\theta = (\alpha, \beta)'$ and obtain the conjugate joint prior distribution of θ, τ. Then, using a conjugate prior distribution for the model parameters, calculate the joint posterior distribution of the model parameters, the marginal posterior distribution of τ and the conditional posterior distribution of $\theta \mid \tau$. Use the data set of Part 1 for inference on the model parameters. Plot the marginal posterior distribution of τ and the conditional posterior distributions of $\alpha|\tau$ and $\beta|\tau$. Then, obtain point estimates and 95% credible regions for the model parameters.