Ιδιότητες

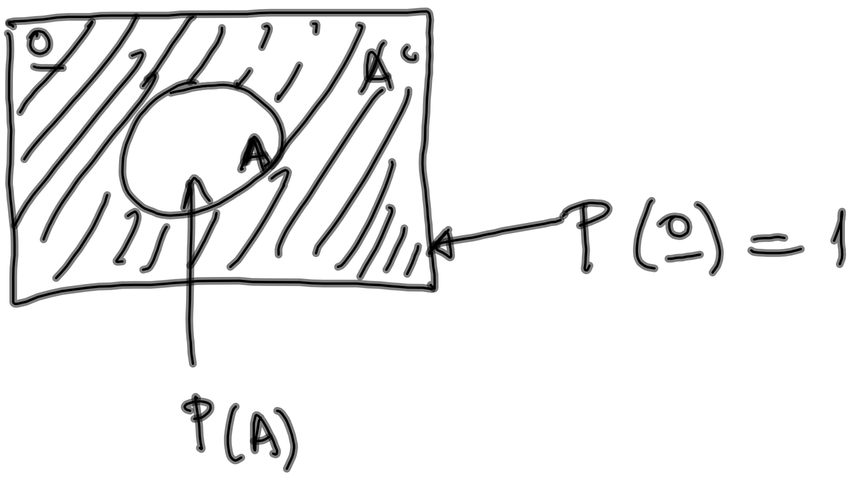
1] $P(\emptyset) = 0$

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)\,,\quad \underline{A_i \text{ ξένα}}$$

$A_i = \emptyset \quad \forall i$

$$0 = \cancel{P(\emptyset)} = \sum_{i=2}^{\infty} P(\emptyset) \Rightarrow P(\emptyset) = 0$$

$$2)\ P(A^c) = 1 - P(A)$$



2] $P(A^c) = 1 - P(A)$

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i), \quad A_i \text{ ξένα}$$

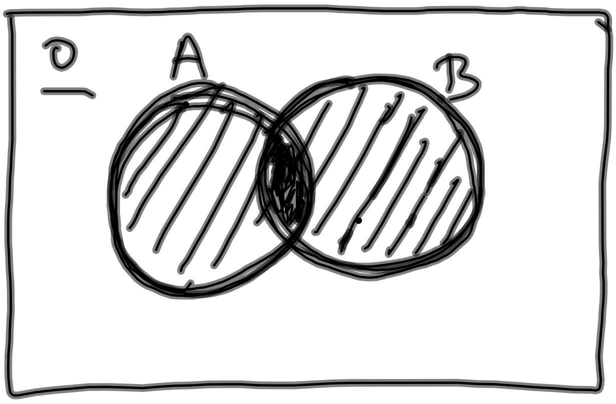
$$A_1 = A, \quad A_2 = A^c, \quad A_3 = A_4 = \ldots = \emptyset$$

$$P(\Omega) = P(A) + P(A^c) + \sum_{i=3}^{\infty} P(\emptyset) \quad (\to 0)$$

$P(\Omega) \overset{1)}{=} 1$

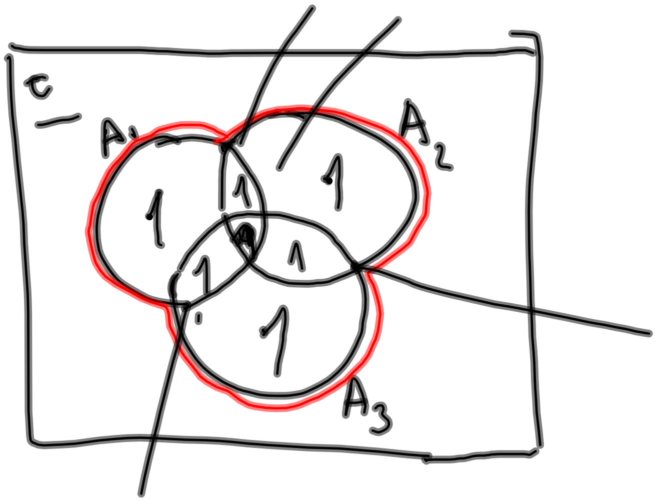
Οκτ 8-3:27 μμ

3] $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



Οκτ 8-3:30 μμ

4) $P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3)$
$- P(A_1 A_2) - P(A_1 A_3)$
$- P(A_2 A_3)$
$+ P(A_1 A_2 A_3)$



4] $P(A_1 \cup A_2 \cup \dots \cup A_n)$

$$= P(A_1) + P(A_2) + \dots + P(A_n)$$

$$- \left(P(A_1 A_2) + P(A_1 A_3) + \dots + P(A_{n-1} A_n) \right)$$

$$+ \left(P(A_1 A_2 A_3) + \dots \right)$$

$$+ (-1)^{n+1} P(A_1 A_2 A_3 \dots A_n)$$

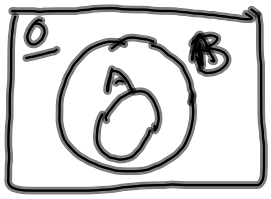
5) $A \subseteq B \Rightarrow P(A) \leq P(B)$

Π.χ.: Βάρος Ανθρώπου

A = Βάρος $\geq$ 70kgr

B = Βάρος $\geq$ 60 kgr

$P(A) \leq P(B)$

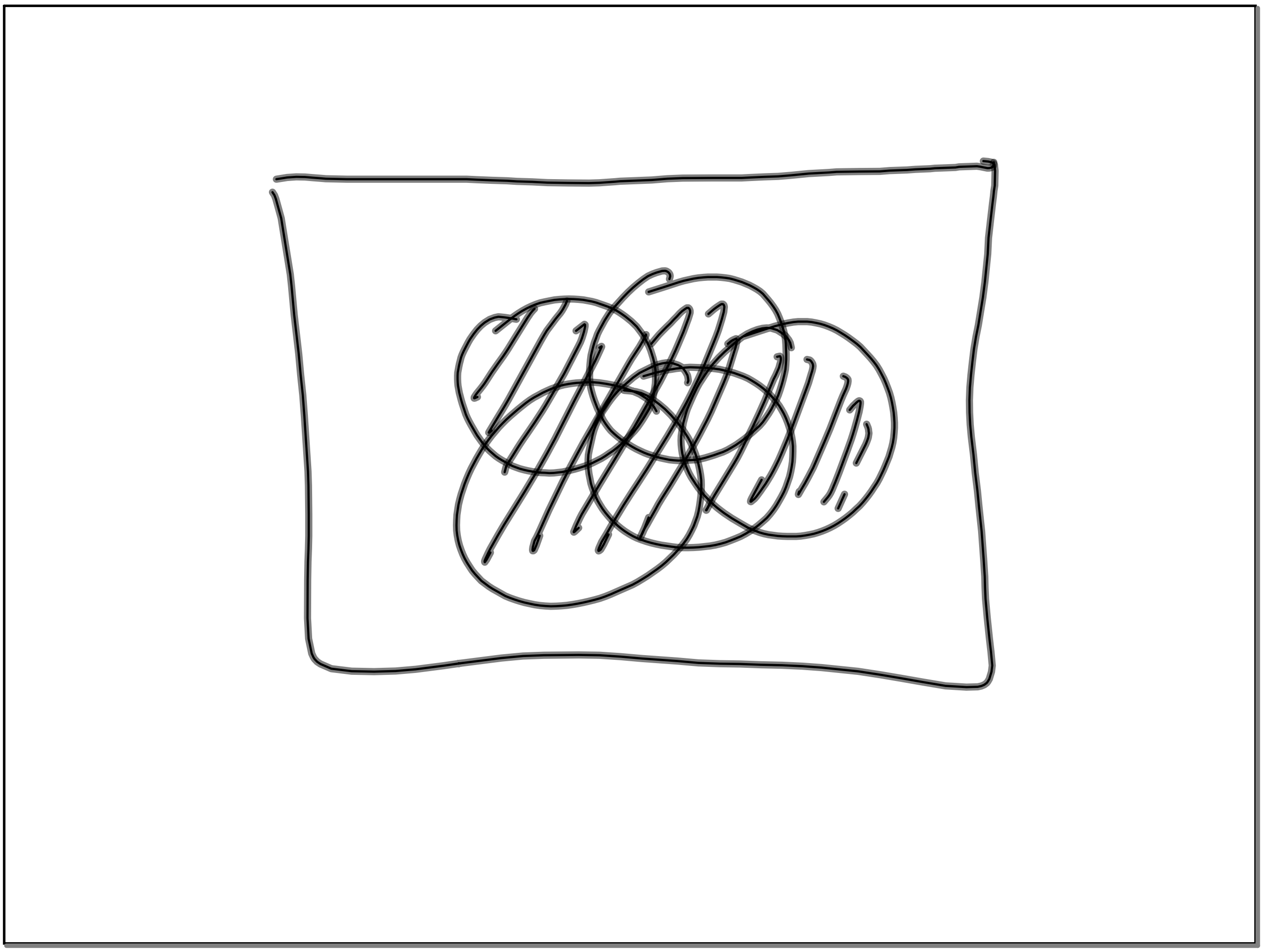


6]

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$$

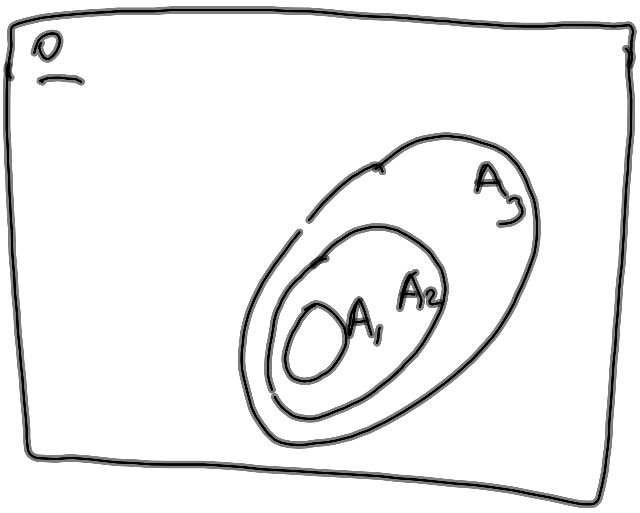
$\leq$ ← Γενικά

$=$ ← Ξένα

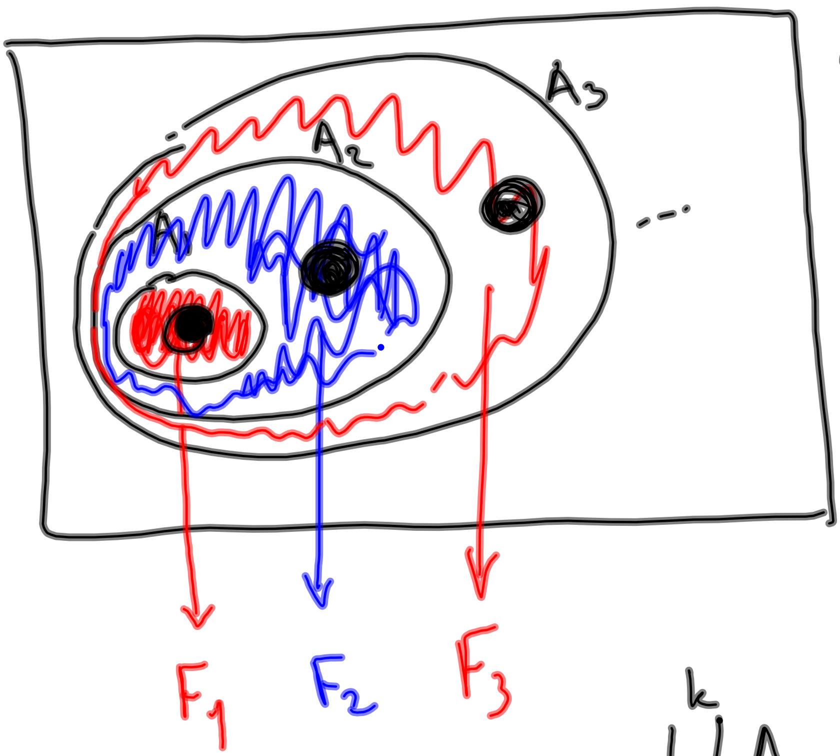


Οκτ 8-3:45 μμ

7] $A_1 \subseteq A_2 \subseteq A_3 \subseteq \dots$



$$P\left(\bigcup_{n=1}^{\infty} A_n\right) = \lim_{n \to \infty} P(A_n)$$



$$P\left(\bigcup_{n=1}^{\infty} A_n\right) = \lim_{n\to\infty} P(A_n)$$

$$F_1 = A_1$$

$$F_i = A_i \setminus A_{i-1}$$

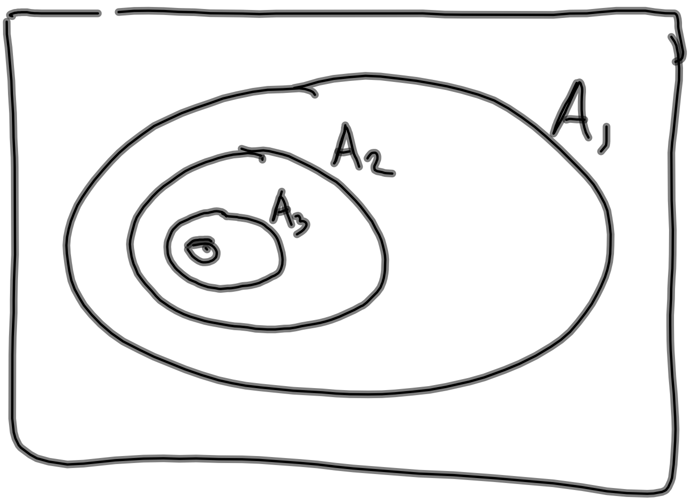
$$\bigcup_{n=1}^{k} A_n = \bigcup_{n=2}^{k} F_n$$

$$\boxed{P\left(\bigcup_{i=1}^{\infty} A_i\right)} \overset{\downarrow}{=} P\left(\bigcup_{i=1}^{\infty} F_i\right)$$

$$\overset{F_i \text{ ξένα}}{=} \sum_{i=1}^{\infty} P(F_i) \overset{\downarrow}{=} \lim_{n\to\infty} \sum_{i=1}^{n} P(F_i)$$

$$= \lim_{n\to\infty} P\left(\bigcup_{i=1}^{n} F_i\right) = \boxed{\lim_{n\to\infty} P(A_n)}$$

8] $A_1 \supseteq A_2 \supseteq A_3 \supseteq \ldots$



$$P\left(\bigcap_{n=1}^{\infty} A_n\right) = \lim_{n\to\infty} P(A_n)$$

$$A_1 \supseteq A_2 \supseteq A_3 \supseteq \dots$$

$$\Downarrow$$

$$A_1^c \subseteq A_2^c \subseteq A_3^c \subseteq \dots$$

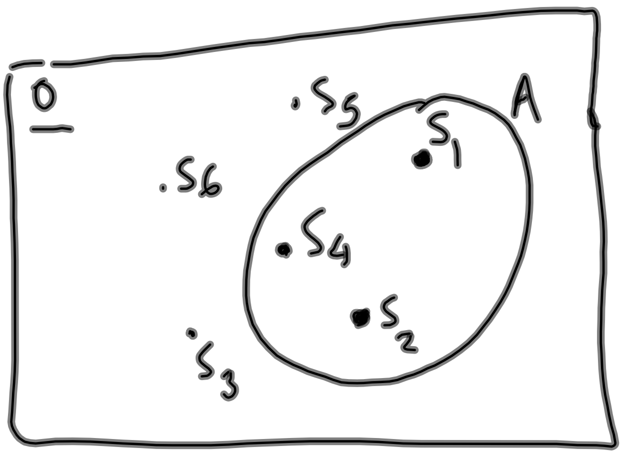
$$P\left(\bigcap_{i=1}^{\infty} A_i\right) = 1 - P\left(\left(\bigcap_{i=1}^{\infty} A_i\right)^c\right)$$

$$= 1 - P\left(\bigcup_{i=1}^{\infty} A_i^c\right) \overset{4)}{=} 1 - \lim_{n\to\infty} P(A_n^c) = \lim_{n\to\infty} P(A_n)$$

Οκτ 8-3:58 μμ

Διακριτός χ.π.

$\underline{\Omega}$: Αριθμήσιμος



$$P(A) = \sum_{s_i \in A} P(\{s_i\})$$

Οκτ 8-4:02 μμ

Φύλο παιδιών 4μελούς οικογ.

P(4 – 0) Χ

P(3 – 1) Μερικοί

P(2 – 2) Πολλοί

- Διαίσθηση; Προσοχή
- Το ερώτημα δεν είναι μαθηματικό
- Μαθηματικά βοηθούν

Πεποίθηση:

Κάθε γέννηση ανεξ.

A με πιθ $1/2$

K με πιθ $1/2$

φύλο παιδιών

$\underline{\Omega} = \{AAAA, AAAK, AAKA, AAKK \ldots KKKK\}$

$|\underline{\Omega}| = 16$

$P(4-0) = P(\{AAAA, KKKK\}) = \frac{2}{16} = \frac{1}{8}$

$P(3-1) = \frac{8}{16} = \frac{1}{2}$

$P(2-2) = 1 - \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$

Οκτ 8-4:33 μμ

Κερνάει / Δεν κερνάει

$\underline{\Omega} = \{ \kappa\kappa\kappa, \kappa\kappa\Gamma, \kappa\Gamma\kappa, \kappa\Gamma\Gamma$
$\Gamma\kappa\kappa, \Gamma\kappa\Gamma, \Gamma\Gamma\kappa, \Gamma\Gamma\Gamma \}$

$P(\text{κερνάει}) = 1 - P(\text{δεν κερνάει})$
$= 1 - P(\{\kappa\kappa\kappa, \Gamma\Gamma\Gamma\})$
$= 1 - \frac{2}{8} = \frac{3}{4}.$

Παιχνίδι Chuck-a-Luck.

Στοιχηματίζει στη ζαριά i

$$\{\text{Κερδίζω}\} = \{\text{Το } i \text{ εμφ. στην } 1^{\eta} \text{ ρίψη}\}$$
$$\cup \{\text{Το } i \text{ εμφ. στη } 2^{\eta} \text{ ρίψη}\}$$
$$\cup \{\text{Το } i \text{ εμφ. στην } 3^{\eta} \text{ ρίψη}\}$$
$$= A_1 \cup A_2 \cup A_3$$

$$P(\text{κερδίζω}) = P(A_1 \cup A_2 \cup A_3)$$

$$= P(A_1) + P(A_2) + P(A_3) - P(A_1A_2) - P(A_1A_3) - P(A_2A_3) + P(A_1A_2A_3)$$

$$P(A_1) = P(A_2) = P(A_3) = \frac{1}{6}$$

$$P(A_1A_2) = P(A_1A_3) = P(A_2A_3) = \frac{1}{36}$$

$$P(A_1A_2A_3) = \frac{1}{6^3}$$

$$P(A_1 \cup A_2 \cup A_3) = 3 \times \frac{1}{6} - 3 \times \frac{1}{36} + \frac{1}{6^3} = \frac{91}{216}$$