Αθρ. διων. συντελ.

Διων Θ

$$\sum_{i=0}^{n}\binom{n}{i}t^{i}=\binom{n}{0}t^{0}+\binom{n}{1}t^{1}+\dots+\binom{n}{n}t^{n}=(1+t)^{n}$$

$t=1 \longrightarrow$

$$\sum_{i=0}^{n}\binom{n}{i}=\underbrace{\binom{n}{0}}_{\cdot}+\underbrace{\binom{n}{1}}_{\cdot}+\dots+\underbrace{\binom{n}{n}}_{\cdot}=2^{n}$$

\# υποσ. συνόλου με n στοιχ.

Cauchy

$$\sum_{i=0}^{n}\binom{r}{i}\binom{s}{n-i}=\binom{r+s}{n}.$$

$$\underbrace{\binom{r}{0}\binom{s}{n}}+\underbrace{\binom{r}{1}\binom{s}{n-1}}+\underbrace{\binom{r}{2}\binom{s}{n-2}}+\dots+\underbrace{\binom{r}{n}\binom{s}{0}}=\binom{r+s}{n}.$$

κοκ

μπλε

$$\binom{n}{0}+\binom{n}{1}+\cdots+\binom{n}{n}=2^n$$

$$\binom{n}{0}^2+\binom{n}{1}^2+\cdots+\binom{n}{n}^2$$

$$=\binom{n}{0}\binom{n}{n-0}+\binom{n}{1}\binom{n}{n-1}+\binom{n}{2}\binom{n}{n-2}+\cdots+\binom{n}{n}\binom{n}{0}$$

$$\binom{n}{k}=\binom{n}{n-k}$$

$$\binom{n}{k}=\frac{n!}{k!(n-k)!}$$

$$=\binom{n+n}{n}=\binom{2n}{n}$$

Τύπος Cauchy

$r=s=n$

τ.μ. Bernoulli(p)

X ⟶ τ.μ.

x ⟶ τιμή

X x

$$p_X(x) = \begin{cases} 1-p, & x=0 \\ p, & x=1 \end{cases}$$

$$E[X] = \sum_x x\, p_X(x) = 1\cdot p + \cancel{0\cdot(1-p)} = p$$

$$Var[X] = \underbrace{E[X^2]}_{?} - \underbrace{\left(E[X]\right)^2}_{p^2}$$

$$E[X^2] = \sum_x x^2 p_X(x) = 1^2\cdot p + 0^2(1-p) = p$$

$$Var[X] = p - p^2 = p(1-p).$$

Διωνυμική κατανομή Bin(n, p)

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, 1, \ldots, n$$

$$E[X] = \underline{np} \qquad Var[X] = \underline{np(1-p)}$$

ποσοστό επιτυχιών

πλήθος επαναλήψεων (δοκιμών)

1ος τρ.) $E[X] = \sum_k k\, p_X(k) = \underbrace{\sum_{k=0}^{n} k \binom{n}{k} p^k (1-p)^{n-k}}$

Ξεκινώ με διων. αναπτ.

$$\sum_{k=0}^{n} \binom{n}{k} t^k = (1+t)^n$$

$$\xRightarrow{d/dt} \sum_{k=0}^{n} k \binom{n}{k} t^{k-1} = n(1+t)^{n-1}$$

$$\xRightarrow{\cdot t} \sum_{k=0}^{n} k \binom{n}{k} t^k = nt(1+t)^{n-1} \qquad t = \frac{p}{1-p}$$

$$\sum_{k=0}^{n} k \binom{n}{k} p^k (1-p)^{n-k} = (1-p)^n \sum_{k=0}^{n} k \binom{n}{k} \left(\frac{p}{1-p}\right)^k = \cancel{(1-p)^n}\, n \frac{p}{\cancel{1-p}} \cdot \cancel{\left(\frac{1}{1-p}\right)^{n-1}} = np.$$

$$Var[X] = \underbrace{E[X^2]}_{?} - \underbrace{(E[X])^2}_{(np)^2}$$

$$E[X^2] = \sum_{k=0}^{\infty} k^2 \binom{n}{k} p^k (1-p)^{n-k}$$

$$\sum_{k=0}^{n} \binom{n}{k} t^k = (1+t)^n \overset{d/dt}{\Longrightarrow} \sum_{k=0}^{n} k \binom{n}{k} t^{k-1} = n(1+t)^{n-1}$$

$$\overset{\cdot t}{\Longrightarrow} \sum_{k=0}^{n} k \binom{n}{k} t^k = nt(1+t)^{n-1}$$

$$\overset{d/dt}{\Longrightarrow} \sum_{k=0}^{n} k^2 \binom{n}{k} t^{k-1} = n(1+t)^{n-1} + nt(n-1)(1+t)^{n-2}$$

$$\Longrightarrow \dots$$

Νοε 11-10:06 πμ

$2^{ος}$ τρ.

$$E[X] = \sum_{k=0}^{n} k \binom{n}{k} p^k (1-p)^{n-k}$$

$$= \sum_{k=1}^{n} k \frac{n!}{k!\,(n-k)!} p^k (1-p)^{n-k}$$

$$= n \sum_{k=1}^{n} \underbrace{\frac{(n-1)!}{(k-1)!\,(n-k)!}}_{\binom{n-1}{k-1}} p^k (1-p)^{n-k}$$

$$= n \sum_{j=0}^{n-1} \binom{n-1}{j} p^{j+1} (1-p)^{n-j-1}$$

$j = k-1$

$$= np(1-p)^{n-1} \sum_{j=0}^{n-1} \binom{n-1}{j} \Big(\overbrace{\frac{p}{1-p}}^{t}\Big)^j = np\,\cancel{(1-p)^{n-1}} \cancel{\left(\frac{1}{1-p}\right)^{n-1}}$$

Γεωμετρική τ.μ.

$$p_X(k) = (1-p)^{k-1} p \;, \quad k = 1, 2, \ldots$$

$$E[X] = \sum_k k\, p_X(k) = \sum_{k=1}^{\infty} k\,(1-p)^{k-1} p$$

$$\sum_{k=0}^{\infty} t^k = \frac{1}{1-t} \xRightarrow{d/dt} \underbrace{\sum_{k=\cancel{0}\,1}^{\infty} k\, t^{k-1} = \frac{1}{(1-t)^2}}$$

$$\sum_{k=1}^{\infty} k\,(1-p)^{k-1} p = p \sum_{k=1}^{\infty} k\,(1-p)^{k-1} = p \cdot \frac{1}{(1-(1-p))^2} = \frac{1}{p}$$

$$E[X] = \frac{1}{p}$$

$$Var[X] = \underbrace{E[X^2]}_{?} - \underbrace{(E[X])^2}_{\left(\frac{1}{p}\right)^2}$$

$$E[X^2] = \sum_{k=1}^{\infty} k^2 (1-p)^{k-1} p$$

$$\sum_{k=0}^{\infty} t^k = \frac{1}{1-t} \xRightarrow{d/dt} \sum_{k=0}^{\infty} k t^{k-1} = \frac{1}{(1-t)^2} \xRightarrow{\cdot t}$$

$$\sum_{k=0}^{\infty} k t^k = \frac{t}{(1-t)^2} \xRightarrow{d/dt} \sum_{k=\cancel{0}1}^{\infty} k^2 t^{k-1} = \frac{1(1-t)^2 + t\,2(1-t)}{(1-t)^4}$$

$$= \frac{1-t+2t}{(1-t)^3} = \frac{1+t}{(1-t)^3} \Rightarrow E[X^2] = \frac{(2-p)p}{p^3} = \frac{2-p}{p^2}$$

$$E[X^2] = \frac{2-p}{p^2}, \quad E[X] = \frac{1}{p}$$

$$\mathrm{Var}[X] = E[X^2] - (E[X])^2$$

$$= \frac{2-p}{p^2} - \frac{1}{p^2} = \frac{1-p}{p^2}.$$

Poisson τ.μ.

$$P_X(k) = e^{-\lambda} \cdot \frac{\lambda^k}{k!}, \quad k = 0, 1, \dots$$

$$E[X] = \sum_{k=0}^{\infty} k \cdot e^{-\lambda} \cdot \frac{\lambda^k}{k!} = \sum_{k=1}^{\infty} \cancel{k} e^{-\lambda} \cdot \frac{\lambda^k}{\cancel{k}(k-1)!}$$

$$\boxed{\sum_{k=0}^{\infty} \frac{t^k}{k!} = e^t.}$$

$$= e^{-\lambda} \sum_{k=1}^{\infty} \frac{\lambda^k}{(k-1)!}$$

$j = k-1$

$$= e^{-\lambda} \lambda \sum_{j=0}^{\infty} \frac{\lambda^j}{j!} = \cancel{e^{-\lambda}} \lambda \cancel{e^{\lambda}}$$

$$= \lambda.$$