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A Linear Systems Primer

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To our Families

To
Melinda and our daughter Lily
and to my parents
Dr. Ioannis and Marina Antsaklis
—Panos J. Antsaklis

To
Leone and our children
Mary, Kathy, John,
Tony, and Pat
—Anthony N. Michel

And to our Students

Preface

Brief Description

The purpose of this book is to provide an introduction to system theory with emphasis on control theory. It is intended to be the textbook of a typical one-semester course introduction to systems primarily for first-year graduate students in engineering, but also in mathematics, physics, and the rest of the sciences. Prerequisites for such a course include undergraduate-level differential equations and linear algebra, Laplace transforms, and modeling ideas of, say, electric circuits and simple mechanical systems. These topics are typically covered in the usual undergraduate curricula in engineering and sciences. The goal of this text is to provide a clear understanding of the fundamental concepts of systems and control theory, to highlight appropriately the principal results, and to present material sufficiently broad so that the reader will emerge with a clear picture of the dynamical behavior of linear systems and their advantages and limitations.

Organization and Coverage

This primer covers essential concepts and results in systems and control theory. Since a typical course that uses this book may serve students with different educational experiences, from different disciplines and from different educational systems, the first chapters are intended to build up the understanding of the dynamical behavior of systems as well as provide the necessary mathematical background. Internal and external system descriptions are described in detail, including state variable, impulse response and transfer function, polynomial matrix, and fractional representations. Stability, controllability, observability, and realizations are explained with the emphasis always being on fundamental results. State feedback, state estimation, and eigenvalue assignment are discussed in detail. All stabilizing feedback controllers are also parameterized using polynomial and fractional system representations. The emphasis in this primer is on time-invariant systems, both continuous and discrete time. Although time-varying systems are studied in the first chapter, for a full coverage the reader is encouraged to consult the companion book titled *Linear Systems*[1] that offers detailed descriptions and additional material, including all the proofs of the results presented in this book. In fact, this primer is based on the more complete treatment of *Linear Systems*, which can also serve as a reference for researchers in the field. This primer focuses more on course use of the material, with emphasis on a presentation that is more transparent, without sacrificing rigor, and emphasizes those results that are considered to be fundamental in systems and control and are accepted as important and essential topics of the subject.

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In a typical course on Linear Systems, the depth of coverage will vary depending on the goals set for the course and the background of the students. We typically cover the material in the first three chapters in about six to seven weeks or about half of the semester; we spend about four to five weeks covering Chapters 4–8 on stability, controllability, and realizations; and we spend the remaining time in the course on state feedback, state estimation, and feedback control presented in Chapters 9–10. This book contains over 175 examples and almost 160 exercises. A Solutions Manual is available to course instructors from the publisher. Answers to selected exercises are given at the end of this book.

By the end of Chapter 3, the students should have gained a good understanding of the role of inputs and initial conditions in the response of systems that are linear and time-invariant and are described by state-variable internal descriptions for both continuous- and discrete-time systems; should have brushed up and acquired background in differential and difference equations, matrix algebra, Laplace and z transforms, vector spaces, and linear transformations; should have gained understanding of linearization and the generality and limitations of the linear models used; should have become familiar with eigenvalues, system modes, and stability of an equilibrium; should have an understanding of external descriptions, impulse responses, and transfer functions; and should have learned how sampled data system descriptions are derived.

Depending on the background of the students, in Chapter 1, one may want to define the initial value problem, discuss examples, briefly discuss existence and uniqueness of solutions of differential equations, identify methods to solve linear differential equations, and derive the state transition matrix. Next, in Chapter 2, one may wish to discuss the system response, introduce the impulse response, and relate it to the state-space descriptions for both continuous- and discrete-time cases. In Chapter 3, one may consider to study in detail the response of the systems to inputs and initial conditions. Note that it is

[1] P.J. Antsaklis and A.N. Michel, *Linear Systems*, Birkhäuser, Boston, MA, 2006.

possible to start the coverage of the material with Chapter 3 going back to Chapters 1 and 2 as the need arises.

A convenient way to decide the particular topics from each chapter that need to be covered is by reviewing the Summary and Highlights sections at the end of each chapter.

The Lyapunov stability of an equilibrium and the input/output stability of linear time-invariant systems, along with stability, controllability and observability, are fundamental system properties and are covered in Chapters 4 and 5. Chapter 6 describes useful forms of the state space representations such as the Kalman canonical form and the controller form. They are used in the subsequent chapters to provide insight into the relations between input and output descriptions in Chapter 7. In that chapter the polynomial matrix representation, an alternative internal description, is also introduced. Based on the results of Chapters 5–7, Chapter 8 discusses realizations of transfer functions. Chapter 9 describes state feedback, pole assignment, optimal control, as well as state observers and optimal state estimation. Chapter 10 characterizes all stabilizing controllers and discusses feedback problems using matrix fractional descriptions of the transfer functions.

Depending on the interest and the time constraints, several topics may be omitted completely without loss of continuity. These topics may include, for example, parts of Section 6.4 on controller and observer forms, Section 7.4 on poles and zeros, Section 7.5 on polynomial matrix descriptions, some of the realization algorithms in Section 8.4, sections in Chapter 9 on state feedback and state observers, and all of Chapter 10.

The appendix collects selected results on linear algebra, fields, vector spaces, eigenvectors, the Jordan canonical form, and normed linear spaces, and it addresses numerical analysis issues that arise when computing solutions of equations.

Simulating the behavior of dynamical systems, performing analysis using computational models, and designing systems using digital computers, although not central themes of this book, are certainly encouraged and often required in the examples and in the Exercise sections in each chapter. One could use one of several software packages specifically designed to perform such tasks that come under the label of control systems and signal processing, and work in different operating system environments; or one could also use more general computing languages such as C, which is certainly a more tedious undertaking. Such software packages are readily available commercially and found in many university campuses. In this book we are not endorsing any particular one, but we are encouraging students to make their own informed choices.

Acknowledgments

We are indebted to our students for their feedback and constructive suggestions during the evolution of this book. We are also grateful to colleagues who provided useful feedback regarding what works best in the classroom in their particular institutions. Special thanks go to Eric Kuehner for his expert preparation of the manuscript. This project would not have been possible without the enthusiastic support of Tom Grasso, Birkhäuser's Computational Sciences and Engineering Editor, who thought that such a companion primer to *Linear Systems* was an excellent idea. We would also like to acknowledge the help of Regina Gorenshteyn, Associate Editor at Birkhäuser.

It was a pleasure writing this book. Our hope is that students enjoy reading it and learn from it. It was written for them.

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