2ος τρόπος (συνάρτηση μεταφοράς).

$$\hat{G}(z) = C(zI-A)^{-1}B = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} z-1 & -2 \\ 0 & z-1 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 3 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{z-1} & \frac{2}{(z-1)^2} \\ 0 & \frac{1}{z-1} \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{z-1} & \frac{z+1}{(z-1)^2} \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$= \frac{2}{z-1} + \frac{3(z+1)}{(z-1)^2} = \frac{2(z-1)+3(z+1)}{(z-1)^2} = \frac{5z+1}{(z-1)^2} = \frac{5z^{-1}+z^{-2}}{(1-z^{-1})^2}$$

$$u_k = 1 \ (k \geq 0) \Rightarrow \hat{u}(z) = 1 + z^{-1} + z^{-2} + \dots = \frac{1}{1-z^{-1}} = \frac{z}{z-1}$$

$$\therefore \hat{y}(z) = \hat{G}(z)\hat{u}(z) = \frac{5z^{-1}+z^{-2}}{(1-z^{-1})^3} = \frac{2z^{-1}}{(1-z^{-1})^2} + \frac{3z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$$

$$\Rightarrow y_k = \underline{2k+3k^2} \quad (k \geq 0).$$

(β) Πίνακας παρατηρησιμότητας:

$$\Gamma_o = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}, \quad \det(\Gamma_o) = 2 \neq 0$$

$\Rightarrow (A, C)$ πλήρως παρατηρήσιμο.

$$\left.\begin{aligned} y(0) &= Cx(0) \\ y(1) &= CAx(0) \end{aligned}\right\} \Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix} \underline{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow \underline{x}(0) = \frac{1}{2} \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$