ZDM Mathematics Education
DOI 10.1007/s11858-012-0467-1

ORIGINAL ARTICLE

Connecting mathematical creativity to mathematical ability

**Maria Kattou · Katerina Kontoyianni ·
Demetra Pitta-Pantazi · Constantinos Christou**

Accepted: 21 September 2012

Abstract This study aims to investigate whether there is a relationship between mathematical ability and mathematical creativity, and to examine the structure of this relationship. Furthermore, in order to validate the relationship between the two constructs, we will trace groups of students that differ across mathematical ability and investigate the relationships amongst these students' performance on a mathematical ability test and the components of mathematical creativity. Data were collected by administering two tests, a mathematical ability and a mathematical creativity test, to 359 elementary school students. Mathematical ability was considered as a multidimensional construct, including quantitative ability (number sense and pre-algebraic reasoning), causal ability (examination of cause–effect relations), spatial ability (paper folding, perspective and spatial rotation abilities), qualitative ability (processing of similarity and difference relations) and inductive/deductive ability. Mathematical creativity was defined as a domain-specific characteristic, enabling individuals to be characterized by fluency, flexibility and originality in the domain of mathematics. The data analysis revealed that there is a positive correlation between mathematical creativity and mathematical ability. Moreover, confirmatory factor analysis suggested that mathematical creativity is a subcomponent of mathematical ability. Further, latent class analysis showed that three different categories of students can be identified varying in mathematical ability. These groups of students varying in mathematical ability also reflected three categories of students varying in mathematical creativity.

M. Kattou (✉) · K. Kontoyianni · D. Pitta-Pantazi · C. Christou
Department of Education, University of Cyprus,
P.O. Box 20537, 1678 Nicosia, Cyprus
e-mail: kattoum@hotmail.com; kattou.maria@ucy.ac.cy

Keywords Mathematical creativity · Mathematical ability · Alternative models · Fluency · Flexibility · Originality

1 Introduction

Mathematical creativity has recently come to be considered as an essential skill that may and should be enhanced in all students (Mann 2005). Indeed, the creative application of knowledge in specific circumstances (Sternberg 1999), proposing original solutions by employing common mathematical algorithms (Shriki 2010), as well as the ability to find numerous and distinctively different answers in mathematical tasks (Sriraman 2005), are considered to be important elements for success in mathematics.

In this view, mathematical creativity is closely related to deep knowledge in the specific domain (Mann 2005). However, the relationship between mathematical creativity and mathematical ability is still ambiguous. Two relative questions arise: Is there a correlation between mathematical creativity and mathematical ability? Are mathematical abilities prerequisites of mathematical creativity or vice versa? To date, related research has led to conflicting results. Therefore, the aim of this paper is to investigate and clarify the relationship between mathematical creativity and mathematical ability.

The paper is organized as follows. First, the theoretical background is presented, addressing the relationship of ability and creativity in mathematics. Then, the methodology is presented. Afterwards the structures of the alternative models regarding the relationship of mathematical ability and mathematical creativity are presented, followed by the results yielded by the comparison of the alternative models and the comparison among the groups of students.

Published online: 30 September 2012

Finally, the conclusions and limitations of the study are discussed.

2 Theoretical background

"Creativity is closely related to deep, flexible knowledge in content domains" (Silver 1997, p. 75). Indeed, the relationship between creativity and domain-specific ability and/or knowledge has been investigated in numerous studies (e.g. Bahar and Maker 2011; Baran et al. 2011; Mann 2005; Sak and Maker 2006). However, the results of these studies varied depending on the instruments used, the populations studied and the specific domain that was examined (Bahar and Maker 2011). Therefore, it still remains uncertain in which ways domain-specific knowledge and ability affect the creative process or lead to the emergence of the creative product.

Thus, in an effort to explain the variations in research studies an extensive theoretical framework on the domain will follow. The theoretical framework will firstly review studies focused on the relationship between general creativity and knowledge. Then, the literature review will focus on the relationship in a domain-specific context, that of mathematics.

2.1 Creativity and knowledge

Research related to the examination of content knowledge and creative ability makes a distinction between two views: the tension view and the foundation view (Weisberg 1999). The tension view assumes that the relationship between knowledge and creativity is shaped like an inverted U, where maximum creativity occurs at a middle range of knowledge (Weisberg 1999). In other words, knowledge in a field is essential to produce something novel within it, since it provides the foundation from which new ideas can be developed (Weisberg 1999). According to Weisberg (2006), knowledge leads to the production of high-quality work, because the individual concentrates on the "discovery" of ideas rather than on the manipulation of basic skills. However, an excess of knowledge might create a mental barrier to the creative process. If an individual knows well how things work in a field, he/she is unable to propose new ideas beyond stereotypes (Sternberg 2006; Weisberg 1999).

On the other hand, the foundation view implies a positive relation between creative eminence and knowledge (Weisberg 1999). In this view, deep knowledge in a domain is a prerequisite for creative work, since if an individual knows what has already been done in the discipline he/she can move forward. In Weisberg's words, "Rather than breaking out of the old to produce the new, creative thinking builds on knowledge" (Weisberg 1999, p. 226).

2.2 Creativity and knowledge in mathematics

As the relationship between general content knowledge and creativity is still ambiguous, the corresponding relationship in the domain of mathematics is similarly so. On the one hand, a number of researchers (e.g. Hong and Aqui 2004; Sak and Maker 2006) have suggested that content knowledge is a crucial factor for mathematical creativity. Indeed, Sak and Maker (2006) stated that content knowledge is the variable that contributes more than any other variable to students' mathematical creativity. The importance of mathematical creativity to mathematical ability is also proposed by Hong and Aqui (2004) who, in their research study, considered the differences between highly academic and highly creative students in mathematics. The results revealed that creative students in mathematics were more cognitively resourceful than their peers who achieved high grades in school mathematics.

On the other hand, students' knowledge and familiarity with techniques and rules may limit their creative potential (Haylock 1997). Students' perceptions, such as that every question has only one right answer that can be achieved by applying well-known algorithms, may inhibit their imagination and hinder their curiosity and experimentation (Mann 2005). According to Pehkonen (1997), this phenomenon occurs because when knowledge and logic are overemphasized, the left hemisphere of the brain is developed, while at the same time the right hemisphere, which seems to be more related to creativity, is neglected.

2.3 Correlation between mathematical ability and creativity

Attempts to illustrate the existence of correlation between creativity and mathematical ability can be traced back to the 1970s. Among the first studies that investigated the existence of this correlation was the research undertaken by Jensen in 1973 (in Haylock 1987). Specifically, Jensen administered a numerical aptitude test, a mathematical divergent production test (to measure mathematical creativity) and a mathematical achievement test (to measure mathematical ability) in terms both of computation and problem solving, to students of age 11–12. The results of the study revealed high and significant correlations between numerical aptitude and mathematical achievement scores, whereas there was no significant relationship between mathematical creativity and mathematical achievement scores. Haylock's findings (1997) are in accordance with that study. In Haylock's study, students with a similar degree of mathematical achievement had significant differences in their mathematical creativity scores. This result implies that several factors differentiate mathematical creativity from mathematical ability in general.

Similar results were found by Baran et al. (2011), who investigated the relationship between creativity and mathematical ability of 6-year-old students. In this study, data for creative ability were collected using a general creativity instrument (Torrance Tests of Creative Thinking) whereas data for mathematical ability were gathered using a mathematical test, measuring aspects of informal (e.g. fewer/more, counting) and formal mathematics (e.g. numbers, calculations). Results revealed that there was no statistically significant relationship between mathematical ability and creativity or even between mathematical ability and creativity indicators (such as fluency, originality and elaboration).

However, there are some opposing views. For instance, Silver (1997) emphasized that "[mathematical] creativity is closely related to deep, flexible knowledge in content domains" (p. 75), stressing the correlation between the two concepts. These results are in accordance with the findings of two recent studies: one undertaken by Sak and Maker (2006) and the other by Bahar and Maker (2011). In the first research study a mathematical test was used, consisting of open-ended as well as closed mathematical problems, to measure domain-specific mathematical knowledge and divergent production (originality, flexibility, elaboration, fluency). Data analysis revealed that knowledge had a statistically significant contribution in explaining variance in originality, flexibility and elaboration in fourth and fifth graders (Sak and Maker 2006).

As for the research study undertaken by Bahar and Maker (2011), the researchers investigated whether the students' score in a mathematics instrument similar to the one applied in the previous study was correlated with students' score in the Iowa Tests of Basic Skills (ITBS) and the Comprehensive Tests of Basic Skills (CTBS). From the ITBS the mathematics section was used, which included measures of problem solving, data interpretation, math concepts, estimation and computation. The CTBS was used in this study as a measure of mathematical achievement in mathematical concepts, estimation and computation. Bahar and Maker (2011) concluded that there was a strong, significant correlation of originality, fluency, flexibility, elaboration and total mathematical creativity with mathematical achievement in both ITBS and CTBS tests.

2.4 Relationship between mathematical ability and creativity

Apart from the question whether mathematical creativity is related to mathematical ability, another question emerges. The question concerns the nature of this relationship: is mathematical ability a subcomponent of mathematical creativity or is mathematical creativity a subcomponent of mathematical ability?

A number of researchers (e.g. Starko 1994) have suggested that mathematical creativity is important for the development of mathematical ability. According to Starko (1994), students who use the content creatively learn the content well. Creative abilities in mathematics include alternation of representations, comparison of solution strategies, connection of several concepts and ideas, and viewing mathematical content from different perspectives. These aspects are considered as valuable evidence of the development of mathematical reasoning and abilities (Leikin 2007; NCTM 2000). In other words, mathematical creativity "is an essential aspect in the development of mathematical talent" (Mann 2005, p. 29).

This relationship is verified by the work of Bahar and Maker (2011). According to these researchers, a multiple regression analysis verified that fluency, flexibility, elaboration and originality were significant predictors of mathematical ability. In the same line are the findings proposed by Sak and Maker (2006), who observed that the score of mathematical creativity may predict mathematical ability, explaining 20 % of the variance in the mathematics test.

On the other hand, other researchers have suggested that mathematical knowledge is vital for the development of mathematical creativity (e.g. Mann 2005; Nakakoji et al. 1999). The importance of mathematical knowledge and abilities in mathematical creativity was stressed by Mann (2005), who characterized mathematical achievement as the most significant predictor of creative mathematical performance. In his study, Mann (2005) performed a regression analysis to predict the contribution of students' knowledge in their creative mathematical performance. Data analysis showed that the measure of students' achievement in mathematics contributed to the prediction of their performance on a mathematical creativity test. Consequently, he concluded that "students who have not yet attained sufficient mathematical knowledge and skills may be unable to demonstrate creative mathematical thinking" (p. 54).

Other researchers have argued that excellent content knowledge helps individuals to recall, process and integrate information (Chi et al. 1988), as well as to make connections between different concepts and types of information (Sheffield 2009). According to Leung and Silver (1997), an individual may generate an original mathematical idea if he/she is aware of mathematical facts and relations embedded in the situation: he/she is able to decontextualize the problematic situation and transform it into a mathematical one. Indeed, creative work involves a certain amount of pre-existing domain knowledge and its transformation into new knowledge (Nakakoji et al. 1999).

As can be deduced from the above discussion, a variation of research aims, findings and methodology appear in studies focusing on the relationship between mathematical creativity and knowledge/ability. Specifically, among the studies presented above, several of them examined the relationship between mathematical ability and general creativity (e.g. Baran et al. 2011) or focused on the investigation of domain-specific creativity (mathematical creativity) with mathematical ability (e.g. Mann 2005). Variations in the aims of the research studies lead to discrepancies in methodological approaches, tests administered and population. In regard to the tests, researchers administered either general creativity tests or domain-specific creativity tests in order to examine the relationship between creativity and ability. For instance, mathematical creativity tests (e.g. Mann 2005; Jensen 1973, in Haylock 1987) as well as general creativity instruments such as Torrance Tests of Creative Thinking (e.g. Baran et al. 2011) were administered for data gathering. Data for mathematical ability were gathered through mathematical tests, measuring aspects of informal and formal mathematics (e.g. Baran et al. 2011), mathematical achievement and numerical aptitude (Jensen 1973, in Haylock 1987), as well as through well-known tests, such as the ITBS and the CTBS. Moreover, participants' ages varied from 6-year-old students for Baran et al.'s study to 12-year-old students for Jensen's research. Even allowing for these differences, there is no agreement on whether there is a correlation between mathematical creativity and ability (e.g. Sak and Maker 2006; Haylock 1997) and in which way the two concepts are related.

3 Purpose of the study

Despite extensive research on the relationship between mathematical creativity and ability, the research results remain conflicting. Is there a correlation between mathematical creativity and mathematical ability? If the answer is positive, what is the relationship between the two concepts: does creativity affect an individual's mathematical ability or does mathematical knowledge enhance mathematical creativity?

Furthermore, there is a lack of studies that focus on mathematical creativity and mathematical ability; instead researchers often focus on general creativity or problem solving. More specifically, a number of the studies presented earlier did not use a mathematical creativity test; rather they used a general creativity test, in order to investigate the relationship between mathematical ability and general creativity (e.g. Baran et al. 2011). We believe that the use of a divergent test specifically focused on mathematics is necessary in order to investigate mathematical creative ability.

Moreover, previous research studies considered mathematical ability as a uni-dimensional entity, that of computation or problem solving. However, mathematical ability is a multidimensional construct. Krutetskii (1976) considered mathematical ability as a multidimensional construct that consists of spatial conception, arithmetic and operations, proper use of logical methods, formulation of hypotheses concerning cause and effect, and the ability to think analogically. According to Krutetskii's (1976) definition, mathematical ability encompasses all or several of these aspects. Therefore, a mathematical ability test should be comprised of tasks assessing these mathematical abilities rather than concentrating on a single ability, such as computation or problem solving.

The majority of the existing research studies are limited to the investigation of the correlation of mathematical creativity and ability. They do not investigate the deeper relationship between these two concepts. More advanced statistical methods are required to reveal the structure of this relationship.

Based on this discussion, the main objective of this study is to investigate whether there is a relationship between mathematical ability and mathematical creativity, and then to examine the structure of this relationship. Furthermore, in order to validate the relationship between the two constructs, we will examine whether students that differ across mathematical ability have statistically significant differences across the components of mathematical creativity.

4 Methodology

4.1 Participants

The participants in this study were 359 students aged 9–12 years attending elementary schools in Cyprus (Grade 4 students, 143; Grade 5 students, 118; Grade 6 students, 98). All students attended average public schools in Nicosia, in urban and suburban areas. The only requirement for a school to be used in this study was the existence of a computer lab. This requirement was due to the fact that the instruments were presented and solved in electronic form.

4.2 Tests and procedures

To fulfill the objectives of the study two tests were administered to students: one assessing mathematical abilities and one measuring mathematical creativity. The design of the two tests was based on the related theoretical background. In regard to the mathematical abilities test, we considered that mathematical ability is not a uni-dimensional entity, rather it is a multidimensional construct which consists of spatial conception, arithmetic and operations, proper use of logical methods, formulation of hypotheses concerning cause and effect, and the ability to think analogically (Krutetskii 1976). As for the design of the mathematical creativity test, we found guidance from previous research. It appears that previous studies used open-ended problems or multiple solutions tasks and assessed students' creativity based on the fluency, flexibility and originality of their solutions (e.g. Leikin 2007; Levav-Waynberg and Leikin 2009).

The tests used were validated by two content experts and two elementary school teachers, who assessed whether the items were measuring all aspects of mathematical ability and mathematical creativity, respectively. The internal consistency of scores measured by Cronbach's alpha was 0.78 for the mathematical creativity test and 0.70 for the mathematical abilities test. Reliability estimates of 0.80 or higher are typically regarded as moderate to high while alpha of 0.70 is considered as a reasonable benchmark (Murphy and Davidshofer 2001).

The two tests were administered to students in electronic form. Students worked individually for 80 min to complete the tests in the computer lab of their school. What follows is a brief description of the tests used (examples of tasks are presented in Fig. 1).

4.2.1 Mathematical abilities test

The mathematical abilities test consisted of 29 mathematical items measuring the following abilities: (a) quantitative ability, requiring students to focus on quantitative properties, such as number sense and pre-algebraic reasoning; (b) causal ability, asking students to examine cause/effect relations; (c) spatial ability, including field-dependence, paper folding, perspective and spatial rotation problems; (d) qualitative ability, demanding focus on the representation and processing of similarity and difference relations; and (e) inductive/deductive ability, including reasoning problems.

4.2.2 Mathematical creativity test

The mathematical creativity test included five open-ended multiple-solution mathematical tasks, in which students were required to provide: (a) multiple solutions; (b) solutions that were distinct from each other; and (c) solutions that none of their peers could provide.

4.3 Scoring and analysis

The items of the mathematical abilities instrument were marked as correct (1) or wrong (0). The assessment of students' creativity was based on the fluency, flexibility and originality of their solutions (Leikin 2007). For fluency, the number of correct solutions was counted. For flexibility, the number of different types or categories of solutions was measured. Originality was calculated by comparing a student's solutions with the solutions provided by all students and the rarest correct solution received the highest score. The scores in fluency, flexibility and originality were converted to a score ranging from 0 to 1 (with 1 being the highest score). Three different numbers indicated the score in each mathematical creativity item. Below, the way in which a creativity task was assessed is presented.

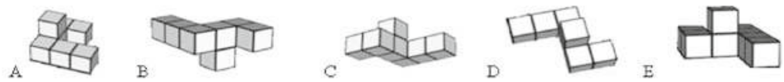
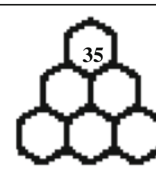
For instance, in the creativity task that is presented in Fig. 1, students were asked to fill in the pyramid in such a manner that the result in the top cell remained as 35. In Fig. 2 the answers provided by one of the students are presented.

In order to assess the creativity of the solutions presented in Fig. 2 we performed the following steps. (a) For the fluency score we calculated the ratio: number of the correct mathematical solutions that the student provided, to the maximum number of correct mathematical solutions provided by a student in the population under investigation. (b) For the flexibility score we calculated the ratio: number of different types of correct solutions (depending on the type of numbers used—e.g. decimals, integers; and the structure of the pyramid—e.g. additive, multiplicative) that the student provided, to the maximum number of different types of solutions provided by a student in the population under investigation. (c) For the originality score, we calculated the frequency of each solution's appearance, in relation to the sample under investigation. A student was given the score 1 for originality if one or more of his/her answers appeared in <1 % of the sample's answers. Correspondingly, a student was given a score of 0.8 if the frequency of one or more of his/her answers appeared in between 1 and 5 %, 0.6 if the frequency of one or more of his/her answers appeared in between 6 and 10 %, 0.4 if the frequency of one or more of his/her answers appeared in between 11 and 20 %, 0.2 if one or more of his/her answers appeared in more than 20 % of the sample's answers. Three different numbers (fluency, flexibility and originality scores) were calculated for each student, indicating the score in each mathematical creativity task. The total fluency, flexibility and originality scores were obtained by adding the respective scores across the five creativity tasks.

4.4 Data analysis

The objective of the analysis was first to articulate and empirically test a theoretical model that addresses the correlation between mathematical creativity and mathematical ability (Model 1; see Fig. 3). In the event that there was correlation between the two concepts then the two

Fig. 1 Examples of tasks from the mathematical abilities and mathematical creativity tests

Spatial Task	Observe carefully the following solid.  If you rotate the solid shown above, which of the following solids could be the result?  A. A B. B C. C D. D E. None of the previous
Qualitative Task	Twenty passengers travel to Larnaca airport by bus. Twelve of them carry a travel bag, eleven a computer bag and six people both types of suitcase. How many passengers carry only a computer bag? A. 5 B. 6 C. 8 D. 11 E. 17
Quantitative Task	George's marbles are three times as many as Spyros' marbles. Andreas has 20 marbles less than George. If Spyros has S marbles, how many marbles does Andreas have? A. $\frac{S}{3} - 20$ B. 3S + 20 C. S + 50 D. S - 50 E. 3S – 20
Inductive/ Deductive Reasoning Task	If the suitcase has stripes, then it belongs to Emmanuel. If the suitcase is yellow, then it does not belong to Emmanuel. The suitcase is yellow. Therefore, A. The suitcase has stripes. B. The suitcase does not have any stripes. C. The suitcase belongs to Emmanuel. D. Cannot be determined.
Causal Task	Inside a vase there are four balls: one yellow, one red, one green and one blue. If I want to pull out of the vase only three balls, by pulling out one at a time, how many possible permutations are there? Write down all the possible permutations. (For example I can pull out a red, a yellow and then a green ball or a green, a yellow and then a red ball. These are only two possible permutations, but there are more.) A. 6 B. 12 C. 18 D. 24 E. None of the previous
Creativity Task	Look at this number pyramid. All the cells must contain one number. Each number in the pyramid can be computed by performing always the same operation with the two numbers that appear underneath it. Fill in the pyramid, by keeping on the top the number 35. Try to find as many solutions as possible. 

alternative theoretical models (Model 2 and Model 3; see Figs. 4, 5) would be compared, in order to clarify the structure of the relationship of mathematical ability and mathematical creativity. Secondly, it was our aim to trace groups of students that differ across the components of mathematical ability.

In regard to the first objective, confirmatory factor analysis (CFA) was conducted in order to investigate the fit of the three models to the data of the present study, using the statistical modeling program MPLUS (Muthén and Muthén 1998). CFA was used to test alternative theoretical models. By using CFA two major advantages emerge: (a) it allows testing the validity of a priori models and related hypotheses, as they arise through a corresponding theoretical framework; and (b) it makes it possible to examine alternative hypothesized models and gives a measure of the "goodness of fit" between competing theoretical models.

The three models assume that latent variables (mathematical creativity, mathematical ability) corresponding to theoretical constructs underlie a set of observed indicators (e.g. fluency, flexibility, originality). The relationships that exist within a set of indicators are explained by the

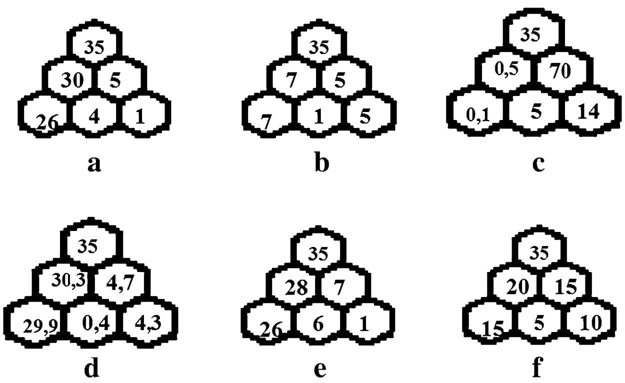


Fig. 2 Indicative answers on a mathematical creativity task

covariances between those indicators and the latent variables (Bollen 1989). Thus, the first step in the CFA procedure starts from a covariance matrix among indicators and decomposes these covariances into the effects of the latent factors upon the observed variables and the random error coefficients (Bollen 1989).

Moreover, the CFA gives some indices of goodness of fit for the model in which the evaluation of models are based. For the purposes of the present study, goodness of fit was based on three fit indices: the comparative fit index (CFI), the ratio of Chi-square to its degree of freedom (x^2/df) and the root-mean-square error of approximation (RMSEA). According to Marcoulides and Schumacker (1996), for the model to be confirmed, the values for CFI should be higher than 0.90, the observed values for x^2/df should be less than 2 and the RMSEA values should be close to or lower than 0.08.

For the accomplishment of the second objective, latent class analysis (LCA) was used to explore whether there were different categories of students in our sample whose achievement could vary according to mathematical ability. It is important to note that LCA, which is part of the mixture growth analysis (Muthén and Muthén 1998), enables specification of models in which one model applies to one subset of the data, and another model applies to another set. Once the latent class model is estimated, subjects can be classified to their most likely class by mean of recruitment probabilities.

In order to investigate differences between groups of students on mathematical abilities due to their different degree of mathematical creativity, analysis of variance (ANOVA) was conducted.

5 Alternative theoretical models

In the framework of the present study, we adopt the definition of creativity as first proposed by Torrance (1995) and transferred into the field of mathematics by Silver (1997) and Leikin (2007). More concretely, Torrance proposed that verbal creative ability is constituted by fluency, flexibility, elaboration and originality. However, the scoring was simplified to include just fluency, flexibility and originality; elaboration was not included, due to the fact that there is a difficulty of getting inter-rater reliability on elaboration (Cramond et al. 2005). In other words, there was no homogeneity among scorers during the assessment of elaboration. Likewise, in the content of mathematics, the

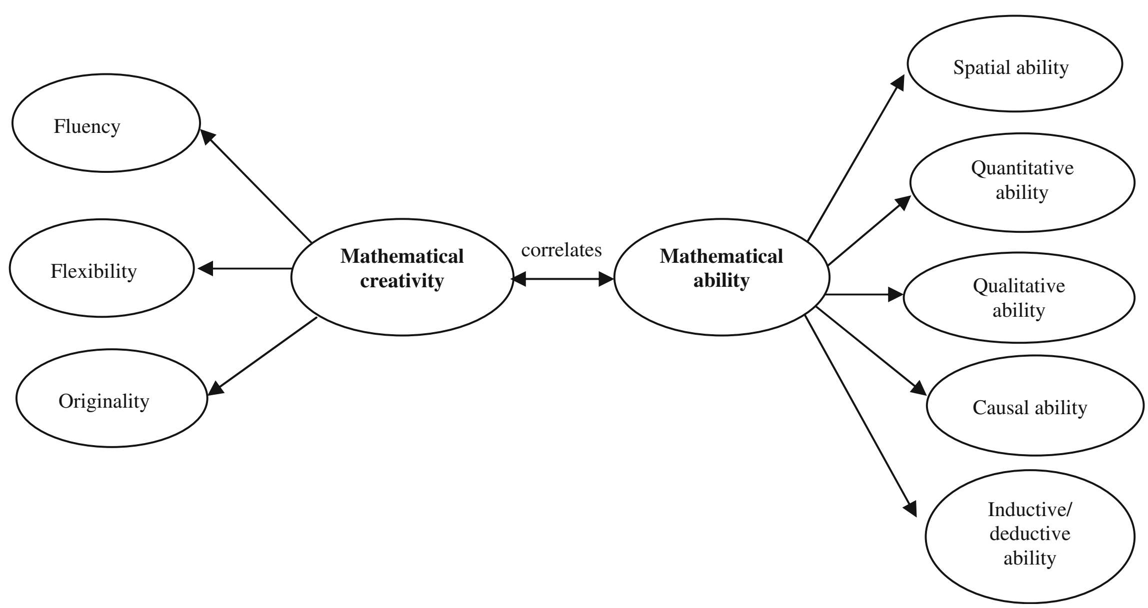


Fig. 3 The structure of the proposed Model 1

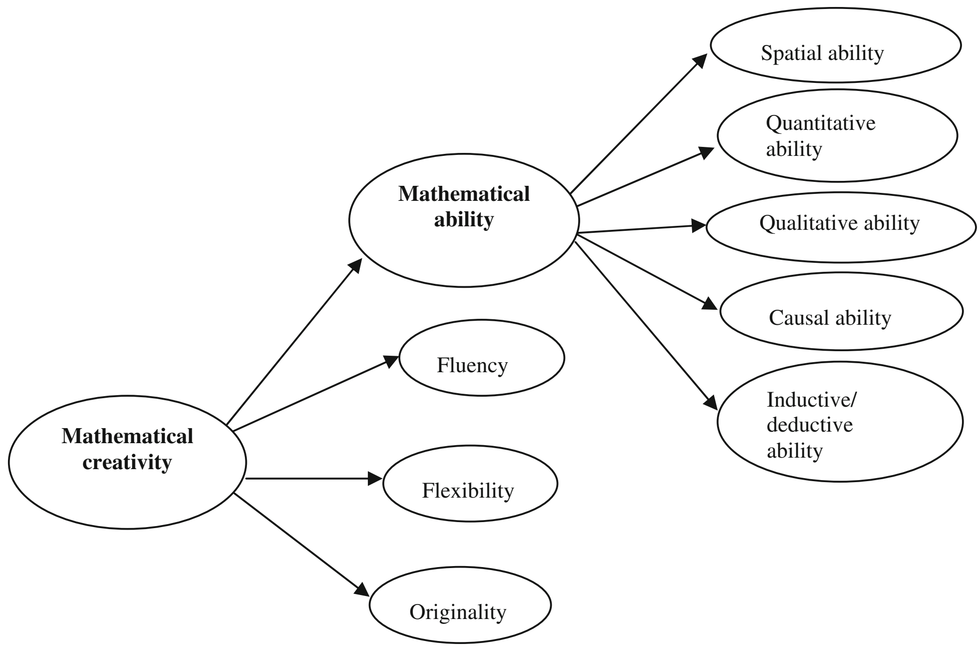


Fig. 4 The structure of the proposed Model 2

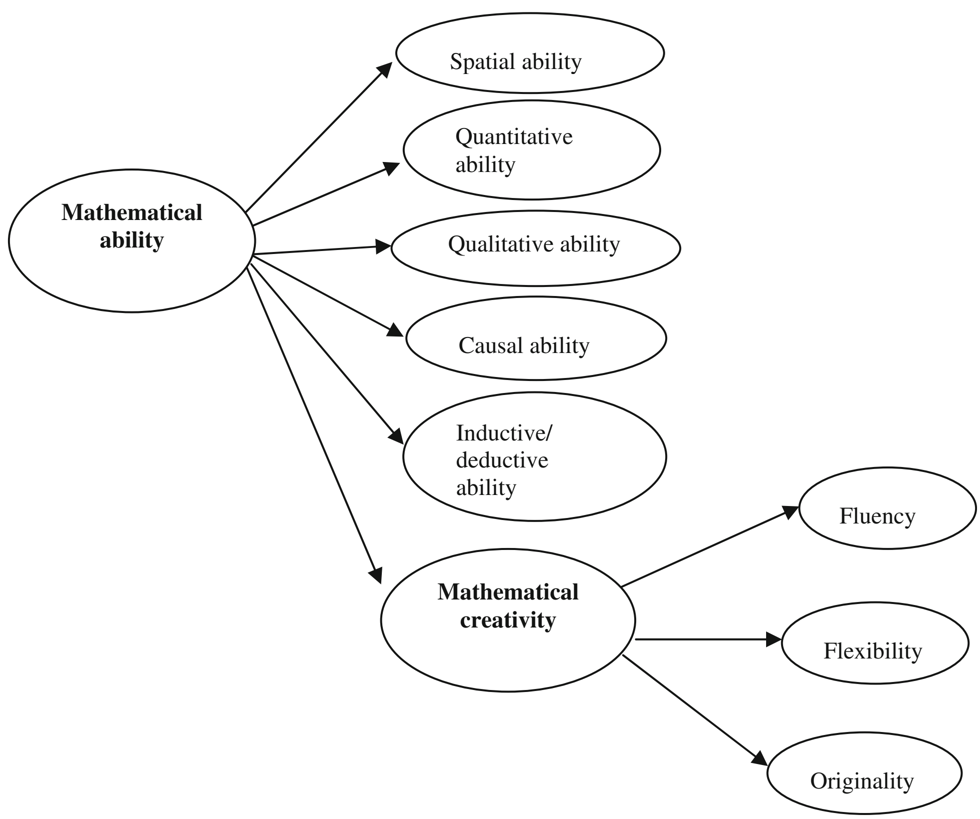


Fig. 5 The structure of the proposed Model 3

researchers (e.g. Leikin 2007; Silver 1997) employed only the concepts of fluency, flexibility and originality, due to the difficulty of determining levels of elaboration in mathematical tasks. According to Leikin (2007), fluency in mathematics refers to the ability to produce many ideas, flexibility refers to the number of approaches that are observed in a solution, and originality refers to the possibility of holding extraordinary, new and unique ideas. As far as mathematical ability is concerned, we defined it as a multi-component construct, which consists of spatial conception (spatial ability), arithmetic and operations (quantitative ability), proper use of logical methods (inductive/deductive reasoning), formulation of hypotheses concerning cause and effect (causal ability), and the ability to think analogically (qualitative ability). These components are based on Krutetskii's (1976) theoretical framework on schoolchildren's mathematical abilities. According to Krutetskii (1976), mathematical ability is seen in terms of a student's ability to: formalize; symbolize; generalize; carry out sequential deductive logic; syncopate or curtail logic or argument; reverse logical thinking or find the converse; be flexible in mathematical methods used; conceptualize spatially; and develop before puberty a "mathematical mind" (pp. 84–88).

5.1 Structure of Model 1

Since there is a controversial discussion regarding the existence of a relationship between mathematical ability and mathematical creativity (Haylock 1997; Jensen 1973 in Haylock 1987), Model 1 addresses the correlation between the two concepts (Fig. 3).

If a correlation is found between mathematical ability and mathematical creativity, we will proceed to examine the relationship between them. More specifically, two additional models will be examined: one that will investigate whether mathematical creativity is a subset of mathematical ability; and one that will examine the reverse, whether mathematical ability is a subset of mathematical creativity.

5.2 Structure of Model 2

Model 2 is based on Balka's idea, which implies that mathematical ability is a component of mathematical creativity. According to Balka's definition (1974, in Mann 2005), the criteria of mathematical creativity include the abilities: (a) to formulate mathematical hypotheses concerning cause and effect in mathematics situations; (b) to determine patterns in mathematical situations; (c) to break from established mind sets; (d) to consider and evaluate unusual mathematical ideas; (e) to sense what is missing in a mathematical problem; (f) to ask mathematical questions; and (g) to split mathematical problems into subproblems.

5.3 Structure of Model 3

In contrast to Model 2, Model 3 examines the reverse relationship: that mathematical creativity is one of the components of mathematical ability. In considering this model we found guidance from a cognitive model created by the Department of Education of Iowa, called "Integrated Thinking Model" (Iowa Department of Education 1989). According to the Integrated Thinking Model, an individual can reach high abilities in a domain if a combination of content knowledge and critical and creative thinking are present (Iowa Department of Education 1989).

6 Results

6.1 Testing the structure of Model 1

Figure 6 presents the structural equation model with the latent variables (mathematical creativity and mathematical ability) and their indicators. This model aims to investigate whether there is a correlation between the two concepts, assuming that mathematical creativity consists of fluency, flexibility and originality and that mathematical ability is comprised of spatial, quantitative, qualitative, causal and inductive/deductive abilities.

The results of the analysis revealed that Model 1 matched the data set of the present study and determined the "goodness of fit" of the factor model (CFI $= 0.990$, $x^2 = 29.269$, $df = 19$, $x^2/df = 1.540$, RMSEA $= 0.039$). The analysis revealed that the statistically significant loadings of fluency ($r = 0.833$, $p < .05$), flexibility ($r = 0.925$, $p < .05$) and originality ($r = 0.793$, $p < .05$) constitute a first-order factor, that of mathematical creativity. Moreover, the five cognitive abilities: spatial ability ($r = 0.306$, $p < .05$), quantitative ability ($r = 0.667$, $p < .05$), qualitative ability ($r = 0.625$, $p < .05$), causal ability ($r = 0.475$, $p < .05$) and inductive/deductive reasoning ability ($r = 0.725$, $p < .05$) can model the performance of students in mathematics. The analysis revealed that there is a positive correlation between mathematical creativity and mathematical ability ($r = 0.610, p < .05$). In other words, when a student's mathematical ability is low, then the student's mathematical creativity is also low; as the mathematical ability increases, mathematical creativity also increases, and vice versa.

Although correlation indicates the degree of a relationship between two variables, more extensive investigation is needed in order to determine the structure of this relationship. Therefore, two additional models will be examined: Model 2 assumes that mathematical ability is a component of mathematical creativity and Model 3 hypothesizes that mathematical creativity is a component of mathematical ability.

6.2 Testing the structure of Model 2

In the second CFA analysis we assumed and examined the validity of a model describing the latent factor, that of mathematical creativity. As highlighted in Fig. 7, the first-order

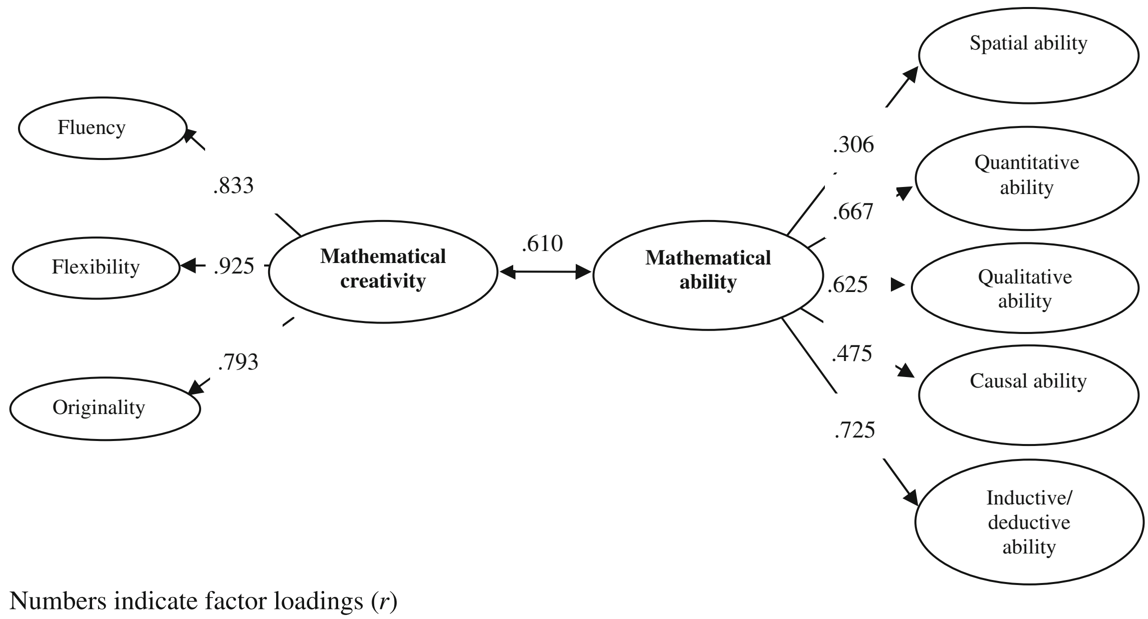


Fig. 6 The confirmation of the structure of Model 1

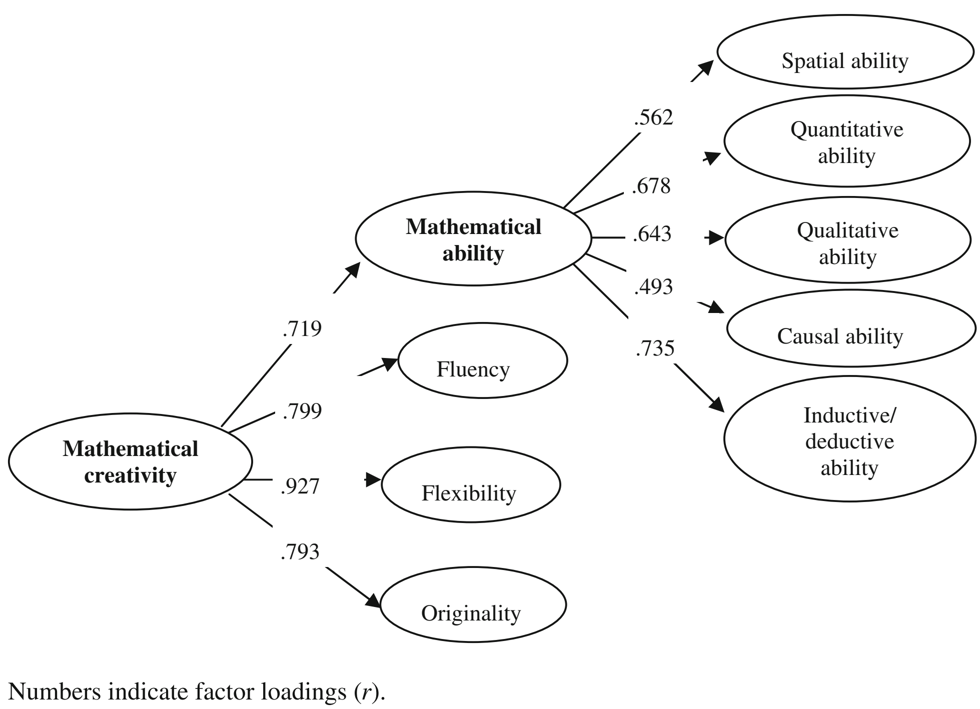
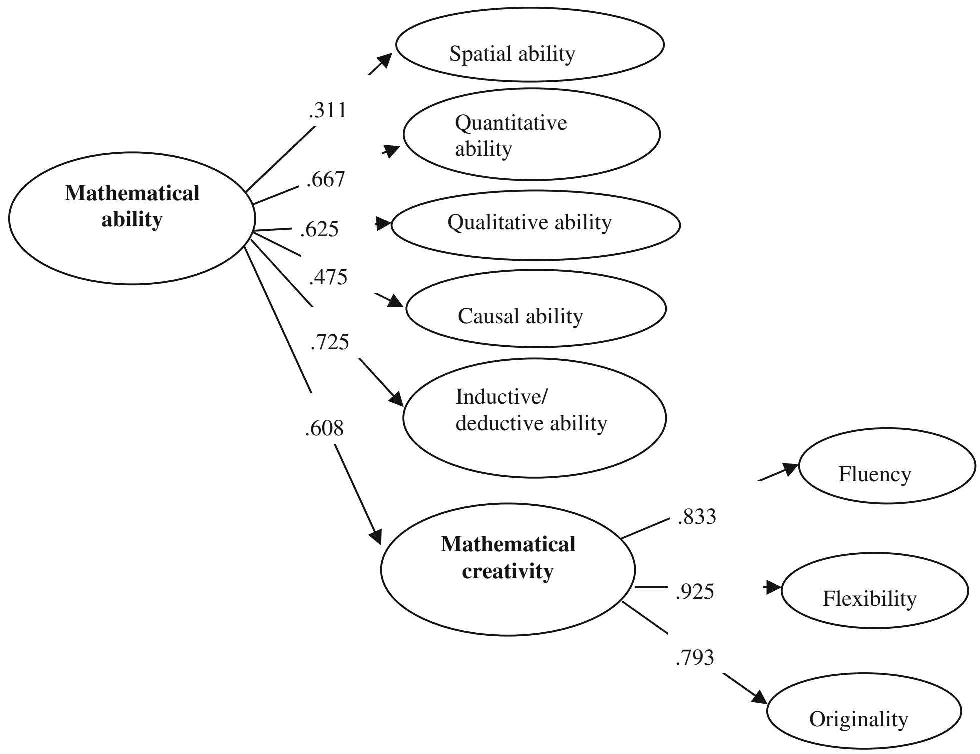


Fig. 7 The confirmation of the structure of Model 2

factors are spatial, quantitative, qualitative, causal and inductive/deductive abilities. These factors were hypothesized to construct a second-order factor, namely mathematical ability. The factor of mathematical ability accompanied with fluency, flexibility and originality comprised the third-order variable, that of mathematical creativity.

CFA showed that each of the variables employed in the present study loaded adequately (i.e. they were statistically significant) on each factor, as shown in Fig. 7. It also showed that the observed and theoretical driven factor structures matched for the data set of the present study and determined the "goodness of fit" of the factor model (CFI = 0.948, $x^2 = 72.399$, $df = 20$, $x^2/df = 3.619$, RMSEA = 0.085). In particular, Model 2 proposes that fluency, flexibility, originality and mathematical ability are subcomponents of mathematical creativity. Thus,



Numbers indicate factor loadings (r).

Fig. 8 The confirmation of the structure of Model 3

according to Model 2 a researcher may measure an individual's mathematical creativity by considering these four constructs. This implies, among other things, that the assessment of mathematical ability allows an estimation of mathematical creativity, and hence provides a strong estimation, due to the high factor loading ($r = 0.719, p < .05$).

6.3 Testing the structure of Model 3

In contrast to Model 2, Model 3 hypothesized that mathematical creativity is a component of mathematical ability. Therefore, in Model 3 mathematical ability is the third-order factor which is comprised of spatial, quantitative, qualitative, causal and inductive/deductive abilities and mathematical creativity. Mathematical creativity is a second-order factor, which is comprised of fluency, flexibility and originality.

Figure 8 presents the structure and the loadings of Model 3. The corresponding loadings determined the "goodness of fit" of the factor model (CFI = 0.991, $x^2 = 29.280$, $df = 20$, $x^2/df = 1.464$, RMSEA = 0.036). According to Model 3, mathematical ability is composed of spatial, quantitative, qualitative, causal and inductive/deductive abilities and mathematical creativity. Model 3 suggests that mathematical creativity is a component that contributes to the development of mathematical ability. In fact, the factor loading of mathematical creativity ($r = 0.608$, $p < .05$) indicates that this component contributes significantly to the factor, since the higher a factor loading of a component is, the more it contributes to the factor.

6.4 Comparing the two alternative theoretical models

Since the aim of the study was to test alternative theoretical models and to discern the one that best describes the structure of the relationship between mathematical ability and mathematical creativity, we compared the two models on a combined basis of their parsimony, fit with existing theory, and the relative goodness of fit to the observed data, as indicated by the corresponding indices. Table 1 presents the fit indices for the two comparative models.

When alternative models are compared for best fit to the data, Chi-square difference test is computed, which indicates the change in the Chi-square statistic relative to the change in degrees of freedom. If the difference is statistically significant ($p < .05$), the model with the lower Chi-square value provides better fit to the data. Given two alternative models with equivalent fit indices, the model that is more parsimonious (i.e. has fewer parameters) is preferred.

According to CFI indices, as they are presented in Table 1, the two models showed acceptable fit on the CFI (>0.90), showing that all models indicators loadings

Table 1 Models' indices

	x^2	*df*	x^2/df	CFI	RMSEA	AIC	BIC
Model 2	72.399	20	3.619	0.948	0.085 0.065–0.107	6,649.751	6,711.884
Model 3	29.280	20	1.464	0.991	0.036 0.000–0.062	6,606.632	6,668.765

determined the "goodness of fit" to the observed data. However, the CFI of Model 3 (CFI = 0.991) was the higher one. The fit of the two models was further compared by computing differences in x^2 and *df* between models. Model 3 ($x^2 = 29.280$, $df = 20$, $p < .01$) fits the data significantly better than Model 2 ($x^2 = 72.399$, $df = 20$, $p < .01$).

Akaike's information criterion (AIC) as well as Bayesian's information criterion (BIC) also shows that Model 3 fits better than Model 2. According to Muthén and Muthén (1998), the best-fitting model is the one with the smallest AIC and BIC indices. Since AIC was 6,606.632 for Model 3, as compared with 6,649.751 for Model 2, while BIC was 6,668.765 for Model 3, as compared with 6,711.884 for Model 2, it is verified that Model 3 fits better than the other model.

It is noteworthy that the loadings of the variables on the latent or observed factors are not considered as indication of better fitting. In other words, although mathematical ability has a stronger factor loading as a component of creativity (see Fig. 7) than mathematical creativity as a component of mathematical ability (see Fig. 8), it is not evidence that Model 2 fits the data better than Model 3. As indicated by all of the fit statistics, Model 3 provided a better fit to the data.

6.5 Categories of students

In order to empower the results that mathematical creativity is included among mathematical abilities, further analysis took place in an effort to investigate the extent to which students in the sample vary according to their mathematical ability. To this end, we examined whether variation on mathematical ability leads to discrepancy on mathematical creativity components. The LCA used a stepwise method—that is, the model was tested under the assumption that there are two, three and four groups of subjects. The best-fitting model with the smallest AIC (6,366.99) and BIC (6,467.96) and the largest entropy (0.733) indices (Muthén and Muthén 1998) was the one with three groups.

Taking into consideration the average group probabilities as shown in Table 2, we may conclude that categories are quite distinct, indicating that each class has its own characteristics. The mean and SD of the three categories of students on the specific mathematical abilities are presented in Table 3.

Table 2 Average latent class probabilities

	Category 1	Category 2	Category 3
Category 1 ($n = 111$)	0.878	0.122	0.000
Category 2 ($n = 189$)	0.102	0.861	0.037
Category 3 ($n = 59$)	0.000	0.083	0.916

Table 3 Mean and SD of the three categories of students in mathematical abilities

	Category 1 $\bar{X}$ (SD)	Category 2 $\bar{X}$ (SD)	Category 3 $\bar{X}$ (SD)	Total $\bar{X}$ (SD)
Spatial ability	1.02 (0.88)	1.32 (1.09)	2.07 (0.96)	1.35 (1.078)
Quantitative ability	0.80 (0.81)	1.54 (1.04)	3.34 (1.03)	1.61 (1.28)
Qualitative ability	0.65 (0.82)	1.25 (0.94)	3.14 (0.96)	1.37 (1.23)
Causal ability	1.02 (0.76)	1.74 (0.83)	2.34 (0.71)	1.61 (0.91)
Inductive/deductive ability	1.11 (0.59)	2.57 (0.67)	3.56 (0.65)	2.28 (1.07)

Table 3 reveals that students in Category 3 outperformed students in Category 1 and Category 2 across all mathematical abilities. Students in Category 2 outperformed their counterparts in Category 1. It is important to note that across the three categories of students, there are statistically significant differences ($p < .05$) among all mathematical abilities. Therefore, it can be deduced that our sample can be grouped in three distinct levels of abilities: Category 1 ($n = 111$) consists of low mathematical ability students, Category 2 ($n = 189$) consists of average mathematical ability students and Category 3 ($n = 59$) consists of high mathematical ability students.

Moreover, data analysis revealed that low mathematical ability students were more able in inductive/deductive ability tasks ($\bar{X} = 1.11$), followed by causal ability tasks

Table 4 Mean and SD of the three categories of students in mathematical creativity

	Category 1 $\bar{X}$ (SD)	Category 2 $\bar{X}$ (SD)	Category 3 $\bar{X}$ (SD)	Total $\bar{X}$ (SD)
Fluency	0.93 (0.44)	1.46 (0.57)	2.04 (0.63)	1.39 (0.66)
Flexibility	1.37 (0.42)	1.83 (0.46)	2.21 (0.38)	1.75 (0.52)
Originality	1.67 (0.66)	2.20 (0.79)	2.76 (0.73)	2.13 (0.82)

($\bar{X} = 1.02$), spatial ability tasks ($\bar{X} = 1.02$), quantitative ability tasks ($\bar{X} = 0.80$) and qualitative ability tasks ($\bar{X} = 0.65$). Average ability students found easier the inductive/deductive ability tasks ($\bar{X} = 2.57$), followed by causal ability tasks ($\bar{X} = 1.74$), quantitative ability tasks ($\bar{X} = 1.54$), spatial ability tasks ($\bar{X} = 1.32$) and qualitative ability tasks ($\bar{X} = 1.25$). In regard to high mathematical ability students, they were more able to solve inductive/deductive ability tasks ($\bar{X} = 3.56$), followed by quantitative ability tasks ($\bar{X} = 3.34$), qualitative ability tasks ($\bar{X} = 3.14$), causal ability tasks ($\bar{X} = 2.34$) and spatial ability tasks ($\bar{X} = 2.07$). These results suggest that the easiest category of tasks for all groups of students was inductive/deductive ability tasks. Low and average ability students found qualitative ability tasks the most difficult ones, whereas high ability students found spatial ability tasks the most difficult type of tasks to handle.

Regarding mathematical creativity, Table 4 presents the differences between the three categories of students across the three components of mathematical creativity, namely fluency, flexibility and originality.

In particular, students with high mathematical ability (Category 3) are also high creative students. Likewise, average mathematical ability students (Category 2) have an average performance across fluency, flexibility and originality; while low ability students (Category 1) have low creative potential in mathematics. The differences on fluency, flexibility and originality are statistically significant across the three groups of students ($p < .05$).

Additionally, across the three categories of students the highest score was the one for originality (Category 1: $\bar{X} = 1.67$, Category 2: $\bar{X} = 2.20$, Category 3: $\bar{X} = 2.76$), followed by the score for flexibility (Category 1: $\bar{X} = 1.37$, Category 2: $\bar{X} = 1.83$, Category 3: $\bar{X} = 2.21$), and finally for fluency (Category 1: $\bar{X} = 0.93$, Category 2: $\bar{X} = 1.46$, Category 3: $\bar{X} = 2.04$).

7 Discussion

Creativity is currently discussed as an essential component of the aim "mathematics for all" (Pehkonen 1997). Given the importance of creativity in school mathematics, several researchers have investigated the relationship between mathematical creativity and mathematical ability (Baran et al. 2011), defining the latter either as school mathematics (e.g. Mann 2005) or content knowledge (Sak and Maker 2006). However, few of them examined the impact of the former on the latter, and vice versa.

Hence, the aim of this study was to articulate and empirically test alternative theoretical models, indicating the relationship between mathematical ability and mathematical creativity. The model that best describes the relationship between mathematical creativity and mathematical ability is the one in which mathematical creativity is a subcomponent of mathematical ability. Hence, among other mathematical processing abilities, such as spatial, quantitative, qualitative, causal, inductive/deductive abilities, mathematical creativity is included. This result is in accord with the results of several researchers (e.g. Leikin 2007; Mann 2005) who stressed that mathematical creativity is a prerequisite for the development of mathematical ability. Furthermore, the results of this study are in agreement with the results by Bahar and Maker (2011) as well as Sak and Maker (2006), who argued that mathematical creativity may predict mathematical ability. Although in the present study we concluded that mathematical creativity is a subcomponent of mathematical ability, the abovementioned studies, as well as ours, suggest that in order to enhance mathematical ability teachers should invest in the development of mathematical creativity.

Moreover, data analysis illustrated that three different categories of students varying according to their mathematical ability can be identified: students with low, average and high mathematical ability. These groups of students reflect three categories of students also varying in mathematical creativity. It can be deduced that students with the highest scores in the mathematics test were also the most creative. Accordingly, students with average scores in the mathematics test had average level of creativity and students with low mathematical scores also had low level of creativity. However, this result is controversial; other studies have found that students with similar degree of mathematical achievement had significant differences in mathematical creativity scores (Haylock 1997).

As the readers interpret the results, they should consider the limitations of the present study. Firstly, the study focused on the relationship between mathematical creativity and mathematical ability without taking into consideration other aspects that may influence this relationship. Other complementary aspects, such as intelligence and personality, should also be investigated in combination with mathematical ability and creativity, in order to form a more accurate picture of this relationship.

Furthermore, although the present study proposed a model for the relationship between mathematical creativity and mathematical ability, longitudinal studies using a representative sample of students are needed to validate the proposed model over time. A further validation of the model could also be achieved by the investigation of the influence of students' age and gender.

Despite the limitations of the study, several implications of the results emerge. Firstly, this research study suggests that the encouragement of mathematical creativity is important for further development of students' mathematical ability and understanding, as proposed by Mann (2005), Starko (1994) and Bahar and Maker (2011). Thus, teachers should not limit their teaching in spatial conception, arithmetic and operations, proper use of logical methods, formulation of hypotheses concerning cause and effect, and the ability to think analogically. Teachers should recognize the importance of creative thinking in order to develop students' mathematical talent and to teach mathematical concepts in a creative way.

Moreover, the assessment of mathematical creativity can provide useful information in regard to students' profile and more specifically to their performance in mathematics. Unfortunately, mathematical tests which are used in schools value mainly speed and accuracy and neglect creative thinking abilities (Mann 2005). For this reason, creative tasks should be included in the assessment methods of mathematics, in order to capture not only students who do well in school mathematics and are computationally fluent but also students who have the potential but have not manifested their abilities yet.

Summing up, by enhancing students' mathematical creativity their mathematical ability will be improved as well. If students are able to confront mathematical situations fluently, flexibly, insightfully and originally, they will be competent to use appropriate mathematical knowledge and processes in other mathematical tasks and problems. Given that students, as future citizens, will face problems that are unknown at present, it is especially crucial for them to be creative in order to efficiently tackle the challenges they will meet.

Acknowledgments This work was funded by the Cyprus Research Promotion Foundation (Grant: ANTHROPISTIKES/PAIDI/0308(BE)/13).

References

Bahar, A. K., & Maker, C. J. (2011). Exploring the relationship between mathematical creativity and mathematical achievement. *Asia-Pacific Journal of Gifted and Talented Education, 3*(1), 33–48.

Baran, G., Erdogan, S., & Çakmak, A. (2011). A study on the relationship between six-year-old children's creativity and mathematical ability. *International Education Studies, 4*(1), 105–111.

Bollen, K. A. (1989). *Structural equations with latent variables*. New York: Wiley.

Chi, M. T., Glaser, R., & Farr, M. J. (1988). *The nature of expertise*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Cramond, B., Matthews-Morgan, J., Bandalos, D., & Zuo, L. (2005). A report on the 40-year follow-up of the Torrance Tests of Creative Thinking: Alive and well in the new millennium. *Gifted Child Quarterly, 49*, 283–291.

Haylock, D. (1987). A framework for assessing mathematical creativity in school children. *Educational Studies in Mathematics, 18*(1), 59–74.

Haylock, D. (1997). Recognizing mathematical creativity in schoolchildren. *ZDM—The International Journal on Mathematics Education, 29*(3), 68–74.

Hong, E., & Aqui, Y. (2004). Cognitive and motivational characteristics of adolescents gifted in mathematics: Comparisons among students with different types of giftedness. *Gifted Child Quarterly, 48*, 191–201.

Iowa Department of Education (1989). *A guide to developing higher order thinking across the curriculum*. Des Moines, IA: Department of Education. Retrieved from ERIC database (ED 306 550).

Krutetskii, V. A. (1976). *The psychology of mathematical abilities in schoolchildren*. Chicago: University of Chicago Press.

Leikin, R. (2007). Habits of mind associated with advanced mathematical thinking and solution spaces of mathematical tasks. In D. Pitta-Pantazi, & G. Philippou (Eds.), *Proceedings of the fifth conference of the European Society for Research in Mathematics Education—CERME-5* (pp. 2330–2339). http://ermeweb.free.fr/Cerme5.pdf. Accessed 17 Sep 2012.

Leung, S. S., & Silver, E. A. (1997). The role of task format, mathematics knowledge and creative thinking on the arithmetic problem posing on prospective elementary school teachers. *Mathematics Education Research Journal, 9*(1), 1–20.

Levav-Waynberg, A., & Leikin, R. (2009). Multiple solutions for a problem: A tool for evaluation of mathematical thinking in geometry. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of sixth conference of European Research in Mathematics Education* (pp. 776–785). Lyon: Institut National de Recherche Pédagogique.

Mann, E. (2005). *Mathematical creativity and school mathematics: Indicators of mathematical creativity in middle school students*. Doctoral dissertation. http://www.gifted.uconn.edu/siegle/Dissertations/Eric%20Mann.pdf. Accessed 17 Sep 2012.

Marcoulides, G. A., & Schumacker, R. E. (1996). *Advanced capacity equation modelling: Issues and techniques*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Murphy, K. R., & Davidshofer, C. O. (2001). *Psychological testing: Principles and application* (5th ed.). Upper Saddle River, NJ: Prentice-Hall.

Muthén, L. K., & Muthén, B. O. (1998). *Mplus user's guide*. Los Angeles, CA: Muthén & Muthén.

Nakakoji, K., Yamamoto, Y., & Ohira, M. (1999). A framework that supports collective creativity in design using visual images. In E. Edmonds & L. Candy (Eds.), *Proceedings of the 3rd conference on creativity and cognition* (pp. 166–173). New York: ACM Press.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Pehkonen, E. (1997). The state-of-the-art in mathematical creativity. *ZDM—The International Journal on Mathematics Education, 29*(3), 63–67.

Sak, U., & Maker, C. J. (2006). Developmental variations in children's creative mathematical thinking as a function of schooling, age, and knowledge. *Creativity Research Journal, 18*(3), 279–291.

Sheffield, L. (2009). Developing mathematical creativity—questions may be the answer. In R. Leikin, A. Berman, & B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students* (pp. 87–100). Rotterdam: Sense Publishers.

Shriki, A. (2010). Working like real mathematicians: Developing prospective teachers' awareness of mathematical creativity through generating new concepts. *Educational Studies in Mathematics, 73*(2), 159–179.

Silver, E. A. (1997). Fostering creativity though instruction rich mathematical problem solving and problem posing. *International Reviews on Mathematical Education, 29*(3), 75–80.

Sriraman, B. (2005). Are giftedness and creativity synonyms in mathematics? An analysis of constructs within the professional and school realms. *The Journal of Secondary Gifted Education, 17*, 20–36.

Starko, J. A. (1994). *Creativity in the classroom*. New York: Longman.

Sternberg, R. J. (1999). *Handbook of creativity*. New York: Cambridge University Press.

Sternberg, R. J. (2006). The nature of creativity. *Creativity Research Journal, 18*(1), 87–98.

Torrance, E. P. (1995). *The 'beyonders' in why fly? A philosophy of creativity*. Norwood, NJ: Ablex.

Weisberg, R. W. (1999). Creativity and knowledge: A challenge to theories. In R. J. Sternberg (Ed.), *Handbook of creativity* (pp. 226–250). Cambridge: Cambridge University Press.

Weisberg, R. W. (2006). Expertise and reason in creative thinking: Evidence from case studies and the laboratory. In J. C. Kauffman & J. Baer (Eds.), *Creativity and reason in cognitive development* (pp. 7–42). New York: Cambridge University Press.