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APPROACHES TO THE TEACHING OF CREATIVE AND NON-CREATIVE MATHEMATICAL PROBLEMS

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ABSTRACT. This study investigated the approaches to teaching by three fifth-grade teachers' of creative and non-creative mathematical problems for fractions. The teachers' personal constructs of the two kinds of problems were elicited by interviews through the use of the repertory grid technique. All the teaching was observed and video-recorded. Results revealed that the teachers had slightly distinctive constructs of creative and non-creative problems, and professed a greater preference for creative problems. Based on the teachers' creations of problems in classrooms and related features, the study identified three types of teaching approaches: liberal, reasoning, and skill approaches. The liberal approach appeared to indicate the most appropriate teaching methods for creative problems.

KEY WORDS: classroom practice, creative mathematics teaching, repertory grid technique

INTRODUCTION

Problem solving has been the focus of education (Jonassen, 1997) or of mathematics education, as revealed by the national mathematics curricula in Taiwan (Ministry of Education in Taiwan, 2000), the US (National Council of Teachers of Mathematics, 1995), and England and Wales (Department for Education and Employment, 2000). However, only a few educational studies focus on diversity in problem types in relation to teaching and learning. With the world trend towards the use of constructivist and situated cognition approaches to learning, problem types used in the mathematics classroom have gradually included more real-life, creative, or higher-order-thinking problems. This trend is reflected in the problem types designed by test developers of international education assessment studies, such as the Trends in International Mathematics and Science Study 2003 (International Association for the Evaluation of Educational Achievement, 2005) and the Program for International Student Assessment 2003 (Organization for Economic Co-operation and Development, 2005). Accordingly, the goal of the present study was to focus on the issue of the use of creative and non-creative problems and how teachers interpreted and attempted the teaching of such problems.

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LITERATURE REVIEW

Creative and Non-creative Mathematical Problems

Researchers have identified a variety of types of mathematical problems, such as word and time-consuming problems, and word and calculation problems (Vermeer, Boekaerts & Seegers, 2000). The consideration of these problems is more related to the length of time and the language needed for solving a problem than to the use of creativity in solving it. Another typology for mathematical problems is 'routine' and 'non-routine' problems (McLeod, 1988, 1994). This dichotomy, however, relates more to learners' experiences than to problem types. For example, word problems are likely to be non-routine for students used to solving calculation problems. For example, a study was described in which students were often invited by the teachers to create mathematical games and articles, such as Chinese new-year calendars in the cycle of twelve: for these students, these creative tasks were 'routine'.

Well-structured and ill-structured problems are another taxonomy, as indicated by Nitko (1996) and Jonassen (1997). Well-structured problems are tasks that are clearly laid out, give students all the information they need, and usually have one correct answer that students can obtain by applying a procedure taught in class. The purpose of well-structured problems is to give students opportunities to rehearse the procedures or algorithms taught in class. In contrast, most authentic problems are ill-structured. In order to solve an ill-structured problem, students have to organize, clarify, and obtain information not readily available for understanding the problem. There are likely to be a number of correct answers for an ill-structured problem. Jonassen views well-structured and ill-structured problems as 'a continuum from decontextualized problems with convergent solutions to very contextualized problems with multiple solutions' (p. 67). Jausovec's (1994) study defines well-defined problems as being clearly defined with given states, goal states, and an operator, while ill-defined problems are viewed as having vaguely defined goals, which can only be solved by creative strategies. In the present study, creative problems refer to the extreme end of problems with multiple or divergent (often limitless) solutions, while non-creative problems were those with single or convergent solutions.

Teaching Approaches to Different Types of Problems

Although there are distinct characteristics for extremely creative and non-creative problems, very few studies address the issue of how teachers interpret and teach these different kinds of problems. Stipek et al.'s (1998) study distinguished 'procedurally' oriented problems from 'conceptually' oriented ones. Procedurally oriented problems refer to 'lower-level memorization or rote-learning tasks' or problems of 'routine computations linked to traditional instructional practices', which can be solved correctly by simply applying a set of procedures without understanding mathematical concepts. Conceptually oriented problems, on the other hand, are 'conceptually challenging' or 'high-level' problems, which are 'linked to reform-oriented curriculum. This distinction implies that there is a link between procedurally oriented problems and traditional teaching approaches, and between conceptually oriented ones and reform-oriented teaching in the US.

Boaler's (1998) study had a similar concern. She compared teaching approaches and students' responses between two schools in England. Teaching in the first school was dominated by a traditional, textbook and content-based approach. The individualized booklets introduced students to mathematical procedures, and then to questions for the students to practise. In the other school, using a process-based teaching approach, students only worked on open-ended mathematical projects/problems. They were given some open-ended problems and had to choose one. Then, they were encouraged to develop ideas and formulas, to extend the problems, and to use their knowledge of mathematics. Boaler's study suggests that there is a relationship between the problems used in the classroom and teaching approaches.

An assumption underlying these studies is the inextricable link between teaching approaches and the problem types posed in the classroom. However, these studies failed to address the issue of what teachers actually do with the specific types of problems. In educational practice, most teachers are obliged to teach some problem types that are not based on, or sometimes even in stark contrast to, their own theories or philosophies of teaching. Therefore, they have to activate their creativity to accommodate or break through the environmental constraints.

Creative Teachers in the Educational Settings

Creative teaching, like other creative human activities, can be understood by the four elements of creativity: the person, process, product and place/environment. Research on creative teachers, focusing on the 'person,' tends to interpret the complicated interactions between the four components for the educational settings. Based on a case study of a creative teacher, Lilly & Bramwell-Rejskind (2004) created a diagram, which indicated the interactions between three macro-processes (preparation, connection, and reflective teaching) and four micro-processes (content and temporal constraints; awareness of self and students within the process; feedback from students and colleagues; and values and goals). The analysis was based on a viewpoint of personal creation in the environment. The environment can work as a provider of resources or constraints.

In Fryer's study (1996), the top three items that best identified teachers with positive attitudes to creativity were: (1) a wish to deepen pupils' understanding of the world, (2) a belief that all pupils can be creative, and (3) an attempt to match teaching to each pupil. In relation to the issue of environmental constraints, the teachers' four most preferred criteria for assessing creativity were that a response showed: use of imagination, was original for the pupil, showed initiative, and was pleasing to the pupil. In addition, 'appropriate' ranked 19th among the 24 criteria. This response implied that teachers view creativity as something that should be little judged or controlled by the environment. This finding was consistent with another result in Fryer's study: A 'constrained environment' is seen as the most important factor that hinders the development of creativity. The most significant constraints the teachers indicated were inadequate teaching resources, inadequate preparation time, over-large classes, excessive non-teaching workload, excessive teaching load, and unsuitable accommodation. The most important item, 'inadequate teaching resources,' refers to the shortages of 'mathematics and scientific equipment, up-to-date textbooks and design materials'. In mathematics classrooms, mathematical problems are the most important factors in influencing the students' development of creativity. How teachers interpret and teach the textbook problems and the supplementary problems that they create become a crucial factor in students' learning quality. The present study, therefore, aimed to answer the following questions:

(1) What are teachers' personal constructs of creative and non-creative problems? (Person)

(2) What kinds of teaching methods are used by teachers for creative and non-creative problems, respectively? (Process and Place)
(3) What types of problems are created by teachers during teaching? (Product)

By answering the above research questions, the study wished to find the answer to the question:

(4) What kinds of teaching approaches can be identified by relating teachers' constructs, teaching methods, and their creation of problems for creative and non-creative problems?

METHOD

Participants

The participants were three mathematics teachers, Mr. Mao, Ms. Tang and Ms. An, names which represent pseudonyms in the study. The three teachers were each Grade 5 class teachers in a public primary school in Taipei, Taiwan. Each class had 29 students, and there were approximately the same numbers of boys and girls in each class. There were no significant differences in student attainments in mathematics between the three classes prior to and during the study period. The fractions topic was taught in the three classes for about one week at around the same time using the same textbook.

The Four Problems

The four problems were taken from the participants' textbook and all had been taught by the teachers and attempted by their pupils during the teaching. The two creative problems (Problems 1–2) and two non-creative problems (Problems 3–4) are shown below.

Problem 1: Please use the calculation procedure, $7 \div 5 = 1\ 2/5$ to make a mathematical problem.
Problem 2: Mother made several pizzas and Betty got 3/4 of the pizza. What are the ways by which the pizzas could be divided?
Problem 3: Thirty-six scenery postcards are packed in a box. Divide ten boxes of postcards equally between nine people. How much of a box of scenery postcards will one person get?
Problem 4: Two ribbons (of equal length) are equally divided among six people. How much ribbon will one person get?

Measures

The three teachers were interviewed prior to the commencement of the mathematics teaching in the semester, and all of their teaching of the fractions topic was observed. They were interviewed as soon as possible after completion of their teaching of the topic. Three measures were used for the study of their teaching approaches:

Interview questions about conceptions about teaching. The interview included ten major questions regarding their opinions about teaching methods (e.g., What strategies do you think are the most successful in mathematics teaching?); teaching styles (e.g., What is your favorite style or approach to teaching? What do you think about the relationship between a teacher and his or her students?); learning strategies (e.g., From your observation, what are pupils' strategies for learning mathematics?); learning motives (e.g., In your opinion, what are pupils' motives for learning mathematics?); and learning emotions (e.g., If you find pupils lose confidence in taking the necessary steps to learn more about mathematics, what do you think about them? Have you ever found pupils who lose confidence in mathematics before? How did/will you deal with them?) Certain of the items above were adapted from Gao and Watkins (2001) and adapted to mathematics teaching and this study. The interviews each lasted about 50 min.

Classroom observation. The observer arrived just before the teachers began the mathematics lesson and stayed until the mathematics class was over. Each observation lasted about 40 min. All classroom observations were video-recorded. In order to observe unobtrusively (Hayes, 2000), the researcher sat quietly in an inconspicuous place, being as uninvolved as possible in what was going on in the classroom. However, it should be noted that 'observer effect' or 'reactivity' (Wilkinson, 1995) was inevitable.

Interview questions about teaching tasks. The teachers were interviewed after their teaching of the fractions topic in relation to five questions, concerning their opinions about the fractions topic, their teaching designs, their goals, their teaching strategies and their views on effective teaching. In addition, the repertory grid technique, a procedure first designed by Kelly (Kelly, 1955; Shaw, 1980; Jin, 1998), was utilized to enable an understanding of teachers' underlying *personal constructs* in relation to the problems. The teachers randomly chose three problems from the available problems and separated the three problems into 'two similar problems' and 'one different problem.' They were asked for their constructs of 'similarity' and 'difference' between the problems, and the researcher took notes on their constructs. The procedure was repeated, but gradually became more flexible, for example, using four problems if the teachers were willing, until the teachers stated that they had no more constructs for the four problems. The teachers were then asked to rate each of the four problems in relation to their constructs on a five-point scale, from *5*, as being very close to the constructs of similarity pole, to *1*, as being very close to constructs of the difference pole. They were also asked to rank the four problems according to their preferences. Each teacher was interviewed once and each interview lasted about 70 min.

Data Analysis

The analysis of teachers' personal constructs for the problems centered on two aspects: (1) the teachers' knowledge and (2) their perceptions of the two problem types. Two methods were used to categorize the teachers' constructs developed by means of the repertory grid technique. First, the teachers' constructs were divided into five groups of knowledge: *content knowledge in regard to fractions, other content knowledge* (other mathematical knowledge apart from that about fractions), *pedagogical content knowledge* (the pedagogy related to the teaching of fractions), *general pedagogical knowledge* (not significantly related to the teaching of fractions), and *cognitional knowledge* (knowledge about students' thinking and prerequisite knowledge). These five groups of knowledge were adapted from Lehrer and Franke's (1992) study. Second, cluster analyses, as suggested by Pope and Denicolo (2001), were used to categorize each teacher's constructs for the four problems.

The teacher's interview data were dealt with as case studies. All interviews were transcribed. Constant comparisons (Strauss & Corbin, 1990, 1998; Charmaz, 2000) were made between the results of the interviews and classroom observation and between the teachers. The constant comparison method in the grounded theory refers to the comparison between different people, incidents, time points, data and categories. However, these comparisons need to be analyzed within the context from which the data come (Miles & Huberman, 1994).

Results

Teachers' Personal Constructs of the Four Problems

The following shows the three teachers' respective constructs of the four problems, each followed by an analysis of their construct categories, and the results of the cluster analysis for the four problems, based on their constructs.

Mr. Mao's Repertory Grids

Preference in Rank	1*	2	4	3	
Constructs of Similarity Pole	P1**	P2	P3	P4	Constructs of Difference Pole
P3P4, Equally divided between people; how much each person gets	1***	1	5	5	P2, (The problem) giving you an outcome, your reasoning back
P3P4, Equal division	3(both)	1	5	5	P1, Higher level (after enough practice and learning)
P1P2, After enough learning and practice (The problem gives you an outcome and you reason back to the original situation.)	5	5	2	1	P4, At the beginning stage of building concepts

*Preference in Rank: From 1= like most, to 4=like least, for the four problems
**P1=Problem 1, P2=Problem 2, P3=Problem 3, P4=Problem 4
*** The problem: From 5= very close to the construct of the similarity pole, to 1= very close to the construct of the difference pole

Mr. Mao's Knowledge. Mr. Mao's repertory grids show that he had three constructs, in descending order, e.g., Construct 1 is 'Equally divided between people, how much each person gets' (the entry at the similarity pole) vs. '(The problem) giving you an outcome, your reasoning back' (the entry at the difference pole); Construct 2 is 'Equal division' vs. 'Higher level (after enough practice and learning).' Mr. Mao's three constructs were all a combination of pedagogical content knowledge and cognitional knowledge. Among the six entries of the three constructs, three entries were pedagogical content knowledge and three entries were cognitional knowledge. There was a supplementary entry of pedagogical content knowledge at the similarity pole of the third construct. The categories of Mr. Mao's knowledge are shown below:

(1) *Pedagogical content knowledge: Constructs 1–2 (also cognitional knowledge)*. At the similarity pole, 'Equally divided...' and 'Equal division' were pedagogical content knowledge; teachers can convey the meanings of the fractions problems in their teaching to help

students understand and solve the problems. At the difference pole, '(The problem) giving you an outcome, your reasoning back' and 'Higher level (after enough practice and learning)' were the knowledge of students' ability levels, and how student think, which was cognitional knowledge. Mr. Mao's Constructs 1–2 therefore were a combination of pedagogical content knowledge and cognitional knowledge.

(2) *Cognitional knowledge: Construct 3 (also pedagogical content knowledge)*. In Construct 3, both the similarity and difference poles contained cognitional knowledge, i.e., 'After enough learning and practice' and 'At the beginning stage of building concepts,' which were the knowledge of students' learning processes and cognitive stages. Mr. Mao also added 'The problem gives you outcome and you reason back to the original situation,' which was pedagogical content knowledge, to the similarity pole. Mr. Mao's Construct 3 also was a combination of pedagogical content knowledge and cognitional knowledge, but emphasis was placed on cognitional knowledge here.

Mr. Mao's Clusters for the Problems. Cluster Membership

Case	3 Clusters	2 Clusters
Problem 1	1	1
Problem 2	2	1
Problem 3	3	2
Problem 4	3	2

A cluster analysis was performed based on Mr. Mao's construct ratings in his repertory grids. The results revealed that if the four problems were categorized into three clusters, Problems 3–4 were one group and Problem 1 and Problem 2 formed the other two groups, respectively. If the four problems were categorized into two clusters, Problems 3–4 together, were one group and Problems 1–2 together were the other group. The results showed that Mr. Mao regarded Problems 3–4 as very much similar since both the problems emphasized 'Equal division.' Problems 1–2 could be similar but there were more differences between Problems 1 and 2 than those between Problems 3 and 4. Both Problems 1–2 were 'After enough learning and practice (The problem gives you an outcome and you reason back)' but Problem 2 was the highest level among the four problems.

Ms Tang's Repertory Grids

Preference in Rank	2	1	3	4	
Constructs of Similarity Pole	P1	P2	P3	P4	Constructs of Difference Pole
P1P3, The children who memorize can solve it.	5	1	5	5	P2, Children who memorize, will have difficulty.
P1P2, They are probably unable to formulate a response.	2 (P1 has given all data so children are likely to speak out)	5	1	1	P3, All the necessary data are provided by the problem.
P3P4, All the necessary data are provided by the problem.	5	1	5	5	P2, They can solve the problem after the concepts have been made clear to them.

Ms. Tang's Knowledge. Ms. Tang's first construct focused on cognitional knowledge. The second and third constructs were a combination of pedagogical content knowledge and cognitional knowledge. Among the six entries in the three constructs, four entries were cognitional knowledge and two entries were pedagogical content knowledge.

(1) *Pedagogical content knowledge: Construct 3 (also cognitional knowledge)*. At the similarity pole of Construct 3, 'All the necessary data are provided by the problem' was pedagogical content knowledge; at the difference pole, 'They can solve the problem after the concepts have been made clear to them' was cognitional knowledge.
(2) *Cognitional knowledge: Constructs 1– 2 (also pedagogical content knowledge)*. In Constructs 1–2, 'The children who memorize can solve these,' 'They are probably unable to formulate a response,' and 'Children who memorize, will have difficulty' were cognitional knowledge, which emphasized children's thinking. 'All the necessary data are provided by the problem' at the difference pole of Construct 2 was pedagogical content knowledge.

Ms. Tang's Clusters for the Problems. Cluster Membership

Case	3 Clusters	2 Clusters
Problem 1	1	1
Problem 2	2	2
Problem 3	3	1
Problem 4	3	1

The results of cluster analysis and Ms. Tang's ratings in her repertory grids revealed that she regarded Problems 3–4 as the same. Both Problems 3–4 had provided 'all the necessary data' for problem-solving and could be solved by children who relied on memorization as a study strategy. Problem 1 could be similar to Problems 3–4 if the four problems were categorized into two clusters. Like Problems 3–4, Problem 1 had provided all the necessary data and children who relied on memorization could solve Problem 1 successfully; on the other hand, children might be slightly 'unable to formulate a response' to Problem 1. Ms. Tang regarded Problem 2 as the most unique and difficult among the four problems; she believed that only children with a clear understanding of the concept of fractions could successfully solve Problem 2.

Ms. An's Repertory Grids

Preference in Rank	1	1	1	2	
Constructs of Similarity Pole	P1	P2	P3	P4	Constructs of Difference Pole
P3P4, Fractions are derived from the problem (Fractions are the conclusions)	5	1	5	5	P2, The situation prior to division is understood from the mathematical symbols of fractions.
P1P4, Direct	5	1	5	5	P2, Reasoning the causes from the outcome
P1P2, Solving and expressing through the use of words	5	5	1	1	P4, Solution arrived at through procedure of mathematical calculation
P1P3, Mixed fractions	5	1	5	1	P4, Proper fractions
P1P3, Cheating (or allowing children to think)	5	5	4	4	P4, Simple
P1P2, Greater space for imagination	5	5	3	1	P3P4, Little space for imagination

Ms. An's knowledge. Ms. An provided six constructs: one construct was content knowledge, one construct was pedagogical content knowledge, and the other four constructs were general pedagogical knowledge. There was no combination of different kinds of knowledge in any construct.

(1) *Content knowledge of fractions: Construct 4*. Problems 1 and 3 were 'Mixed fractions' and Problems 2 and 4 were 'Proper fractions,' which were some facts about fractions.
(2) *Pedagogical content knowledge: Construct 1*. Problems 1, 3 and 4 were 'Factions are derived from the problem (Fractions are the conclusions),' while Problem 2 was 'The situation prior to division is understood from the mathematical symbols of fractions.' The knowledge was the meanings of these problems and can be conveyed to students in teaching.
(3) *General pedagogical knowledge: Constructs 2, 3, 5 and 6*. The eight entries of Constructs 2, 3, 5 and 6 were general pedagogical knowledge. For example, 'Direct,' 'Simple' and 'Greater space for imagination' could be applied to general problems and teaching, not focusing on fraction problems.

Ms. An's Clusters for the Problems. Cluster Membership

Case	3 Clusters	2 Clusters
Problem 1	1	1
Problem 2	2	2
Problem 3	3	1
Problem 4	3	1

The results of cluster analysis revealed that Ms. An regarded Problems 3–4 as similar and Problem 2 as the most unique. Ms. An's pattern of cluster membership for the four problems was the same as Ms. Tang's but their construct contents were different. Problems 3–4 were slightly 'Simple' and were 'calculation' problems, with 'fractions' as the 'conclusion.' Problem 1 was similar to Problems 3–4 because Problem 1 was 'Direct' and 'Fractions are derived from the problem (Fractions are the conclusions).' On the other hand, Problem 1 was different from Problems 3–4 as it had 'greater space for imagination' and had to be solved by the use of words. Problem 2 was unique among the four problems as it emphasized 'Reasoning the causes' that were 'The situation prior to division.'

Several themes emerged from the above results:

First, *the differences between the problems:* The results of cluster analyses revealed that when the four problems were divided into three groups, all three teachers perceived the two non-creative problems (Problems 3 and 4) as a group, while Problem 1 and Problem 2 were seen as different. When the four problems were categorized into two groups, Ms Tang and Ms An viewed Problem 2 as different from the other three problems; while Mr. Mao perceived the two creative problems (Problems 1 and 2) as a pair and the two non-creative problems (Problems 3 and 4) as a pair. The three teachers' constructs for the two non-creative problems (Problems 3 and 4) are '*At the beginning stage of building concepts,*' '*Equal division,*' (Mr. Mao), '*Solution arrived at through procedure of mathematical calculation*', and '*Little space for imagination*' (Ms An). Problem 2 was perceived as '*The children who memorize will have difficulty,*' '*They are probably unable to formulate a response,*' '*They can solve the problem after the concepts have been made clear to them,*' (Ms Tang), '*The situation prior to division is understood from the mathematical symbols of fractions,*' and '*Reasoning the causes from the outcome*' (Ms An). For Mr. Mao, Problem 1 was more like Problem 2. For Ms Tang and Ms An, Problem 1 was more like the two non-creative problems (Problems 3–4), although Problem 1 still had some characteristics like Problem 2, '*Solving and expressing through the use of words*,' and '*Greater space for imagination*,' as perceived by Ms An.

Second, *preference for the problems*: the three teachers generally revealed a higher preference for the two creative problems (Problems 1–2) than for the two non-creative problems (Problems 3–4). The rankings in the 'Preference in Rank' of the three teachers' repertory grids indicated that Mr. Mao and Ms Tang preferred Problems 1–2 to Problems 3–4; Ms An viewed Problem 3 as preferable as the two creative problems.

Third, *knowledge groups*: Mr. Mao's and Ms Tang's knowledge groups of constructs appear to be a mixture of pedagogical content knowledge and cognitional knowledge. There is however a slight difference between them: Mr. Mao emphasized pedagogical content knowledge more as, out of the six entries of constructs, three (and a supplementary one) were pedagogical content knowledge. On the other hand, Ms Tang was more focused on cognitional knowledge because, out of her six construct entries, four were cognitional knowledge. Ms An's knowledge groups revealed a difference from Mr. Mao's and Ms Tang's in that she created more constructs, but was more focused on general pedagogical content knowledge. Out of Ms An's 12 construct entries, eight were general pedagogical content knowledge. Ms An also had one construct of (fractions) content knowledge and one construct of pedagogical content knowledge.

Approaches in Teaching the Problems

The three teachers' constructs of the four problems, as described above, were reflected in their teaching practice.

Mr. Mao

A significant characteristic of Mr. Mao is that he perceived Problem 1 to be more like Problem 2 and to be unlike the two non-creative problems. This implies he placed an emphasis on Problem 1, as Problem 2 was perceived by the three teachers as being the most difficult in requiring high-order-thinking among the four problems.

Problem 1 was the last problem in the fractions unit in the participants' textbooks; this might be a reason for the fact that Ms Tang and Ms. An spent less time on Problem 1 than most of the other problems in the textbook. Mr. Mao, by contrast, spent a significant amount of time guiding the children to pose diverse and novel solutions for Problem 1. He gave students time for 'group discussion' and 'whole-class discussion.' In the whole-class discussion, he gave each group enough time to share their creations, and encouraged novel creations through the use of amusement, a positive attitude, and the use of cognitively creative scaffolding (e.g. '*Do you want to combine them*?'). After that, attention was paid to relating the creations of the students to the meanings of each of the numbers in the fractions calculation procedure: '7÷5=1 2/5'. The following is a transcription of Mr. Mao's teaching of Problem 1.

Mr. Mao: Each group is to pose a problem suitable for calculation by this procedure.

.... (Group discussion lasts for about 2 min. Mr. Mao chooses one group to pose its problem.)

Group A: Seven game-boys are distributed among 5 children. How much can each child get?

Mr. Mao: What does the '7' mean?...(Students answer.) What does the '5' mean?... (Students answer.) What does 1 2/5 mean? (Students answer)

Group B poses its problem: Seven beauties are distributed among 5 handsome men. How much can each handsome man get? (Students laugh...)

Group C gives the answer: Seven steaks are distributed among 5 grandpas. How much steak can each grandpa get?

Mr. Mao: When you were discussing it, you were thinking about either 'chocolate' or 'steak.' (Smiles...Asks Group C) What do you want to be distributed? (Group C discusses.) Do you want to combine them?

Group C: Seven steak-shaped chocolates are distributed to 5 grandpas!
(Mr. Mao laughs.)
Mr. Mao: What does the '7' mean?...(Students answer.) ...
Group D gives an answer: Jenny has got 7/5 boxes of pencils. By which possible ways could she get this?
Mr. Mao: This group has given us a problem. Let's solve it.
(Students think...Mr. Mao waits...No one gives any answer.)
Mr. Mao: Can any one change it to a way like those of the other groups?
Students: Seven boxes of pencils are distributed to 5 people.

Students in Mr. Mao's class posed the greatest number of diverse solutions for Problem 1, compared to those in Ms. Tang's and Ms. An's classes. The most difficult example in Mr. Mao's class was '*Jenny has got 7/5 boxes of pencils. By which ways could she get this?*' - a problem type like Problem 2.

After this problem, Mr. Mao posed a problem: "*What are the possible meanings of 'fractions' in your life?*" Like Problem 1, this problem required the children to clarify what the problem means, and there are a diversity of possible answers. Neither Ms. Tang nor Ms. An posed this problem. This problem was not included in the participants' textbook.

For the other problems, Mr. Mao's typical procedure in teaching was: to pose a problem; have the children solve it on their own (but they were allowed to discuss the problem freely with other group members); walk around the tables; find children with different solution methods (and ask them to write them on the blackboard); have the children explain their solutions; summarizing the children's solution methods; and finally giving other solution methods, if any. In the interview, he stated that his emphasis is on 'self-learning' and a 'relaxing atmosphere' in the mathematics classroom. This was reflected in his answer to the interview question, '*What did you expect students to learn from this lesson?*'

I hope that they won't reject mathematics and that it is then possible that they will accept mathematical concepts. ... Mathematics provides opportunities for children to think and use their brains. If the problems are designed well, it will motivate them to think about mathematics. If children can feel that, then they will be interested in learning mathematics. As for this lesson, ... the concept of 'equal division' is the key to understanding ... They are likely to suppose that the former number must be bigger than the latter number, or the 'dividend' must be bigger than the 'divisor'...It is a problem of 'reading comprehension.'

In summary, Mr. Mao's teaching is a mix of a relaxed atmosphere, understanding, and a clear distinction between creative and non-creative problems.

Ms. Tang

Ms. Tang was significant in her special focus on the problem type, represented by Problem 2. As revealed by her repertory grids, she emphasized that the children should get a '*clear understanding*' of mathematics concepts. In answering the question, '*What did you expect students to learn from this teaching?*' she answered: '*Have a clear concept of fractions. I placed a lot of emphases on the concept of unit fractions. ...For there to be a clear concept that the dividend is not necessarily bigger than the divisor.'*

Whenever she found that the children were not clear about the concept of fractions, she changed the non-creative problems to a problem type, similar to Problem 2, '*How can it be divided?*' and asked the students to draw pictures showing 'how things are divided.' Each child drew a picture on his or her own whiteboard. Ms. Tang walked around the tables, choosing children with different solutions, and had them post their whiteboards on the blackboard. Then Ms. Tang directed the whole-class discussion. '*If I can't understand their thoughts from just looking at their solutions, I'll ask. I'll try my best to understand what their problems are. By asking them some questions, I will know whether they really understand the solutions (they produced).*'

Take Problem 2 for example. (Ms. Tang added a condition, ''*Eight people ate the pizzas?')* After Ms. Tang posed the problem, the children spent 9 min. solving the problem on their own by group discussion. Then each group posted their whiteboards on the blackboard.

Group A's whiteboard shows: As some pizzas were divided between-people, and Betty got 3/4=6/8, and, therefore, 6 pizzas were divided between8 people, and each person got 3/4 of a pizza. (They also drew 6 pizzas, each divided between 8 people.)

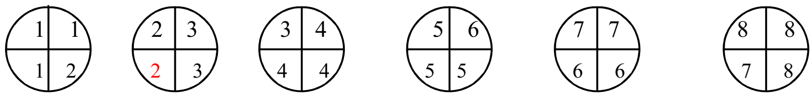
A member of Group A then explains their solutions. After the explanations...

Ms. Tang: Can anyone solve this problem without using the method of 'reduction to the lower terms'? (Waits for a while. No one answers. Ms. Tang finds that each group has used similar solution methods.)

Ms. Tang: Now each group should work to find if there are different ways of thinking about how to solve the problem, other than reducing it to the lower terms....(Group discussion for 4 min.)

Group B's whiteboard shows: Each person can get 3/4 pizza. Therefore, 3/4×8=24/8=6. Answer- 6 pizzas.

Group C's whiteboard shows: 3/4×8=24/8=6



(Ms. Tang asks Group C to explain their solution methods.)

Rose (Group C): We drew 6 pizzas and divided each of them into 4 parts.

Ms. Tang: Does anyone have any questions?...(No one answers.)

Ms. Tang: Then it's my turn to ask questions. (Referring to Group C's drawing.) Can any one explain what the 1, 2, 3, 4, 5, 6, 7, and the 8 mean? (No one answers. Ms. Tang points to a student to answer.)

Alice: '1' means the first person.

Ms. Tang: Not bad. Go on!

Alice: '2' means the second person.

Ms. Tang: Good! Explain more clearly. (The three) '1's mean ...

Alice: The three '1's mean the amount that one person got.

Ms. Tang: Betty is one of the 8 people (who shared the pizzas). She got 3/4 (pizza). This means that Betty is one of them. Does the drawing mean every one got the same (amount of pizza)? ... If it's not the same, how can we say it is equally divided?...It means each pizza is divided into 4 parts, and the three '1's mean what the first person ate. You (referring to Alice) spoke very well. (Ms. Tang gave Alice's group a point.)

(Ms. Tang then asked another student.... The whole of the teaching for Problem 2 took 35 min.)

Ms. Tang was not satisfied with the solution of 'reduction to the lower terms,' as this was 'calculation,' not 'clear understanding.' Challenging the children through the use of serious and consecutive questions was a time-consuming process, but it did make the concepts clear. However, she gave Problem 2 an additional condition, '*8 people ate the pizzas?*' This condition inevitably limited children in the use of more creative solutions, but met her major aim: To build a clear understanding of mathematics concepts in the minds of the children.

Ms. An

Ms. An's teaching generally followed a procedure of: (1) posing a problem, (2) group discussion, (3) each group in turn presenting their solutions and the whole class challenging the solutions excitedly, and finally (4) Ms. An giving comments and points as an award to groups and/or to individual children. Ms. An used a variety of teaching methods as revealed by her repertory grids, with most of the constructs about general pedagogical content knowledge. Ms. An's comments were focused on 'correct solutions' and skills to reduce wrong solutions, as she stated in answering the question, '*What did you expect the students to learn from this teaching?'*

'Six apples are divided between 2 people,' you have to know why it is 6 divided by 2, rather than 2 divided by 6. Here, it (the textbook) doesn't say very much. Therefore, I found that some weak children, when they saw the numbers, they didn't know what they were and just placed them randomly... (Another example is that) 'Twenty biscuits in 4 bags are divided among 5 people'... the children could not solve it well. Many children wrote 20 divided by 5, but the question requested answers about 'bags'. That's why I emphasized in the class, 'Now, it asks about 'bags,' but how about if it asks about 'biscuits'?

This implies Ms. An's emphasis on correct calculation and on surface features of a tricky problem, like Problem 3. This is consistent with the details in her repertory grids. She showed the same preference to Problem 3 as Problems 1–2. She was the only teacher among the three who had a construct about the 'content knowledge of fractions.' While Mr. Mao and Ms. Tang emphasized the need for the children to have clear concepts or understanding, i.e., cognitional knowledge, Ms. An tended to focus on skills to get solutions right and to use general pedagogical knowledge.

Although Ms. An could see that Problems 1–2 required a bigger imaginative space than Problems 3–4, she followed the same teaching procedure for all the four problems. In other words, she did not attempt to build an imaginative atmosphere or use imaginative pedagogy for creative problem-solving. As a result, there were no diverse or imaginative solutions for creative problems, as revealed by Ms. An's statement about students' 'routine' solutions to a problem similar to the Problem 1.

There's a problem in the children's practice book: Use the calculation procedure, 5 ÷ 3, to make a mathematical problem. I found that the students didn't produce diverse answers. Most (students) wrote, 'Five pizzas are divided between three people.' They didn't make problems like 'Ten pizzas are divided between six people.' Not much diversity ... They just wrote out the formula...Little imagination was shown.

A mathematical problem requiring children to use their imagination to solve it also requires teachers to use their imagination to teach and to have a creative mind to challenge children in depth.

At the end of the teaching for the fractions topic, Ms. An gave her students a paper-and-pencil quiz about a non-creative problem, *30 pencils are placed in a box. 4 boxes of pencils are divided between 3 people. How many boxes can one person get?*' (similar to Problem 3 in type). Her emphasis on 'assessment' and 'results' was also revealed in her statement in the interview: '*Very few children got bad results on the test. I feel that that is not bad. Some children who are not good at math also got good results; but, (in fact,) I posed easy problems (for the test)*.' Neither Mr. Mao nor Ms. Tang gave tests in class. Mr. Mao even said in the interview that he didn't think that many tests and related practices were necessary.

Discussion

Three Types of Teaching Approaches to Mathematics Teaching

Using the framework of the four elements of creativity (i.e., 4Ps'), the present study identified three types of teaching approaches to mathematics teaching: liberal, reasoning, and skill approaches (Table I). Teachers with a liberal approach, such as Mr. Mao, emphasized relaxation, self-regulation (Zimmerman, 1989; Puustinen & Pulkkinen, 2001; Fuchs et al., 2003; Stright, Neitzel, Sears & Hoke-Sinex, 2001), imagination, and basic understanding in mathematics classrooms. Mr. Mao preferred to use and work on liberal problems (i.e., extremely ill-structured problems; Jonassen, 1997; Nitko, 1996), stimulating, encouraging, and enjoying his pupils' imaginative and diverse solutions or creations. He worked on non-liberal problems efficiently after making sure pupils generally had a basic understanding of the teaching content. Thus, the class could spare time for a liberal reference to life-related problems. Teachers with reasoning approaches, such as Ms. Tang, tended to focus on a clear understanding of mathematical concepts, i.e., a deep approach to learning (Biggs, 2001). She posed challenging problems or asked scaffolding questions to clarify the meanings of key mathematical concepts. She also tended to change the different types of mathematical problems into a type of 'how' or 'reasoning' problem, in order to make sure students know 'why' (Baroody, 1993). Teachers with a skills approach, such as Ms. An, emphasize getting the answers right by providing solution tips. She paid

TABLE I

Three types of teaching approaches to mathematics

	Liberal approach	*Reasoning approach*	*Skill approach*
PRODUCT			
1. Types of created problems	Liberal problems: Open procedure and open answer	Reasoning problems: Open procedure and closed answer	Skill problems: Closed procedure and closed answer
2. Link between the original and created problem	External link: Outward thinking, often relating to real-life or the beyond	Internal link: Inward thinking, often relating to indepth meanings	Parallel link: Prototypical thinking, imitating the surface structure of the original problems
PLACE			
3. When to create problems	(1) A liberal problem stimulates the creation of another liberal problem (2) For any topic, there is at least one life-related problem	When students lack a clear understanding, any problem can be transformed into a reasoning problem	When giving tips, the teacher creates a well-structured problem as an example At the end of the teaching, the teacher creates well-structured problems as assessment
4. Classroom atmosphere	Relaxing	Cognitive scaffolding	Competing

PERSON			
5. Knowledge- based	A mix of pedagogical content and cognitional knowledge, with more emphasis on pedagogical content knowledge	A mix of pedagogical content and cognitional knowledge, with more emphasis on cognitional knowledge	(1) Content knowledge of subject matters (2) General pedagogical knowledge
6. Aims of teaching	Basic understanding Positive attitude to math	Clear understanding	Good results
7. Beliefs about learning approaches	Self-regulated learning	Deep understanding	Knowledge and practice of solution strategies
PROCESS			
8. Intervention	Encouragement imagination, Exploration of the environment	Promotion of thinking by the use of challenging/ critical questions	Provision of solution skills, Creation of successful experiences

little attention to the concepts underlying the calculation, and only to whether or not the calculation could help get correct answers. Paper-and-pencil tests and practice were useful methods to help children get good results. In the mathematics classroom, She used a variety of general teaching methods (general pedagogical knowledge), such as classroom management, group discussion, peer modeling or direct teaching, which were, however, not related to what was being taught (content knowledge of fractions). In addition, such teachers view mathematical problems from their own perspectives of the problems, not the students.

As outlined in Table I, the feature that best defines the three teaching approaches were the supplementary problems that the teachers created in the classroom. The three approaches in teaching in relation to their created problems also represented their respective characteristics, in regard to the links between the original and the created problems; when to create problems; the classroom atmosphere; knowledge-base, the aims of their teaching; their beliefs about students' approaches to learning, and the focus of focused intervention.

The connection between the problems which the teachers created and teaching constructs/methods for different problems implies that the element of creative control exercised by the teachers tended to overshadow the pre-determined teaching materials. While the teaching of the teachers seems to be constrained by the problems in the textbook, their creative minds struggle and eventually manage to go beyond the boundaries so that they may put their own agendas into practice. Creativity can be regarded as an endeavor, which is situated in society, filtered, judged, encouraged or restricted by the gatekeepers of any field (Lilly & Bramwell-Rejskind, 2004). Personal creativity can lead society, but also can be constrained by society. Creative minds strive to survive and find their own way through a controlled environment.

Implication for Educational Practice

Every person has his or her dominant styles or abilities, and so does any teacher. Teachers tend to transform different types of problems or create supplementary problems to fit their own theories, beliefs, or concerns. If creative reasoning and skill in solving problems are needed in order to develop the repertoire of an ideal learner or fit diverse needs of different students, then it is necessary for teachers to develop an appropriate mind-set and methods for different problem types, as listed in Table I. A creative mind is needed, which can flexibly and appropriately respond to problems and apply corresponding teaching approaches to diverse students and situations.

If creative problems are worth including in mathematics classrooms, so as to cultivate a deep understanding of and a creativity in mathematics, there seems to be a need to develop the use of corresponding teaching methods such as those which are widely emphasized in creative problem-solving teaching programs (e.g., Scott, Leritz & Mumford, 2004). Students need to be motivated so as to think actively and explore problems independently using their imaginations in a relaxing and self-regulatory environment (De Corte, Verschaffel & Op't Eynde, 2000; Pape, Bell & Yetkin, 2003).

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