

## What actions do teachers envision when asked to facilitate mathematical argumentation in the classroom?

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**Abstract** Mathematical argumentation in classrooms has been linked with students' increased understandings of and achievement in mathematics. A key component of mathematical argumentation is the teacher's use of questioning strategies. In particular, effective questioning asks students to explain and justify their work and probes student understanding. However, literature has consistently shown that instead of asking questions that probe student understanding, teachers generally ask leading or recall-oriented questions. Though reform efforts in the US and elsewhere have called for teachers to adopt more effective questioning strategies, the trend in teacher questioning suggested in the literature continues. One possible explanation for this trend is that teachers' interpretation of facilitating mathematical argumentation is not aligned with what reformers in mathematics education envision. Examining teacher-created representations of practice, the present study explores this possible difference in interpretation and presents preliminary evidence suggesting that the teachers' conceptions of facilitating mathematical argumentation may indeed be substantially different from what reformers intend.

**Keywords** Mathematical discussion · Teacher questioning · Mathematical argumentation · Representations of practice

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## Introduction

The encouragement for students to engage in mathematical argumentation can improve their understanding of mathematics (NCTM 2000) and has a positive effect on mathematics achievement (Cross 2009). Various policy documents have advocated teacher facilitation of argumentation, with the most recent call in the US coming from the Common Core State Standards Initiative (CCSSO and NGA 2010). The CCSSI specifically advocates that students should “construct viable arguments and critique the reasoning of others” (p. 6) in mathematics classrooms as part of mathematical practice. Yet, for students to effectively engage in mathematical argumentation, the teacher must serve an active role in facilitating such engagement (Cobb et al. 2001; Douek and Pichat 2003; Forman et al. 1998; Hunter 2007; Yackel and Cobb 1996).

Mercer and Sams (2006) provide some evidence regarding the importance of teachers as facilitators in their studying of conversational ground rules in middle school mathematics classrooms in Great Britain. Mercer and Sams (2006) found that students of teachers trained in providing scaffolds in the form of conversational ground rules showed much higher quality of discussions and argumentation than did control groups. Goos’ (2004) observations of a high school teacher in Australia also suggest that the manner in which a teacher scaffolds a community of inquiry influences the quality of argumentation in a classroom. Similar observations of primary grades teachers’ ability to affect mathematical argumentation were made in New Zealand by Hunter (2007). Therefore, evidence suggests the effectiveness of mathematical argumentation is highly dependent on the manner in which teachers facilitate and scaffold such argumentation. In addition to the classroom norms a teacher establishes, one essential means for scaffolding argumentation is through teachers’ questioning strategies (e.g., Hunter 2007; Yackel and Cobb 1996).

While the literature suggests most teachers ask less engaging, lower cognitive demand questions (Gall 1970; Graesser and Person 1994; Pimm 1987) and there is research on the frequencies teachers ask certain types of questions (e.g., Boaler and Brodie 2004), relatively little research exists on teachers’ interpretation of what facilitation of higher level questioning looks like. Hence, this study explores teachers’ interpretations of what it means to encourage students to “justify their conclusions, communicate them to others, and respond to the arguments of others” (CCSSO and NGA 2010, p. 6). While this quote is from a US policy document, similar expectations and language exist in policy documents worldwide (e.g., Mathematics Achievement Standards, ACARA 2013). Thus, while the present study was conducted with teachers in the US, its implications reach beyond the US in providing an initial means of understanding how teachers interpret such policy language into conceptualizations of teaching practice.

### Teachers’ questioning in mathematics lessons

Research examining teacher questioning has shown the importance of the kinds of questions teachers ask. Hiebert and Wearne (1993) found that the more frequently teachers asked students to describe their solution strategies and explain their responses, the more students engaged in class, and the higher their gains in mathematics achievement were. By probing students to make their thinking public, teachers were able to engage more of their class in mathematical discourse. Students taught by such teachers demonstrated higher mathematics achievement than students who were not asked to describe and explain their mathematical

thinking as frequently. Cross (2009) examined specific scaffolds for facilitating argumentation in grade 9 classrooms and found that when students were consistently prompted (either through written or vocal argumentation) to justify and explain their reasoning, they showed statistically significant and higher gains than students who did not receive such scaffolding.

Rather than looking at individual questions, Franke et al. (2009) investigated teachers' questioning sequences following student-provided explanations. They found that when teachers asked sequences of questions to probe for student understanding, rather than asking isolated follow-up questions, students were more likely to produce fully complete and correct explanations of their mathematics. By pressing students for justification, instead of simply accepting initial explanations of methods, the students demonstrated success in finding the correct answers and providing reasonable explanations.

Martino and Maher (1999) examined teacher actions that facilitated mathematical argumentation and observed strong connections between students' understanding of mathematics and teachers' asking for explanations and justification following certain explanations. Martino and Maher (1999) go on to suggest that, "teacher questioning that is directed to probe for student justification of solutions has the effect of stimulating students to re-examine their original solution in an attempt to offer a more adequate explanation, justification, and/or generalization (p. 75)." Lending support to such findings, Kazemi and Stipek (2001) observed four teachers in the upper-elementary setting and found that while all four teachers appeared to facilitate similar levels of student engagement, certain teachers' facilitation was more meaningful than others. Specifically, when teachers pressed students for explanation and justification, students were observed to develop deeper understandings of the mathematics involved. Further support for such a press of questioning comes from Hunter's (2007) observation of facilitating argumentation in New Zealand classrooms. Hunter observed that by pressing for multiple ways to justify and validate explanations, one teacher was able to effectively engage students in mathematical argumentation.

However, it appears that not every question a teacher asks in a mathematics lesson needs to be of a pressing (for meaning) nature to engage students with the content at hand. Temple and Doerr's (2012) observations of a grade 10 mathematics teacher's classroom indicated variance in the manner of questioning from one lesson to another. In some lessons, the teacher "pushed students to explain their thinking and build on each other's contributions (p. 301)," while in others, the teacher's questioning "(encouraged) the accuracy and precision of expression more than exploration or explanation. Yet both sets of interactional moves appeared to support students' developing facility with the mathematical register (p. 301)." Essentially, Temple and Doerr (2012) observed that the teacher's use of "focusing," or leading questions encouraged student understanding in certain contexts, as did those questions seeking "explanation and justification" or probing questions. Leading questions were effective in aiding student understanding when used to stimulate conversations about previously learned content. Probing questions were observed to be generally productive, but were particularly effective in aiding students make connections between various ways of representing the mathematics. As described by Temple and Doerr (2012), probing questions often included justification requests defined as "questions requiring arguments to support a proposition (p. 303)." Thus, Temple and Doerr's observations suggest that while leading questions can be used effectively, it is only in particular contexts. Probing questions, which facilitate argumentation, are generally productive in most classroom contexts, and even more effective in particular contexts.

The observations by Temple and Doerr (2012) provide a potential explanation of certain findings of Perry et al. (1993). Perry et al. investigated differences in teacher practices, including questioning, in first grade classes across Japan, Taiwan, and the US. Their findings indicate that questions focused on answers and recall are prevalent in all three nations at similar frequencies. However, teachers observed in the US asked significantly fewer questions probing for conceptual knowledge or problem solving strategies. As there have been numerous criticisms of US mathematics achievement at the international level, such findings, as argued by Perry et al. (1993), infer the importance of probing questions. However, Temple and Doerr's (2012) observations show that leading questions are effective in certain classroom scenarios, providing one potential rationale for why US teachers continue to use this strategy so frequently. Supporting the findings of Perry et al. (1993), Hiebert et al. (2003) compared lessons across seven countries, and found US eighth-grade students had low frequencies of student talk, suggesting relatively little opportunity for explaining and justifying. Similar results were reported by Webb et al. (2006) in their observations of middle school mathematics teachers. Webb et al. (1993) found that teachers rarely asked students to explain or justify procedures, but rather took that responsibility upon themselves. Examining the questioning strategies of primary mathematics teachers, Boaler and Brodie (2004) found that teachers typically asked questions seeking recall-oriented information (e.g., answers). However, it should be noted that when compared to teachers using "traditional" curricula, teachers using reform-oriented curricula were observed to have lower frequencies of such question-types and higher frequencies of questions that probed for explanation or focused on conceptual understanding.

The findings from these various studies regarding the frequency of certain teacher questioning approaches are remarkably consistent with general literature about such questioning over the past hundred years (Gall 1970). Reviewing literature current at the time, Gall (1970) noted that since the beginning of the twentieth century, teachers have consistently been observed to engage students with questions seeking recall or answer-specific information. However, teachers have seldom been observed asking questions requiring students to think or describe procedures. Even though Gall's (1970) review of literature covered all academic disciplines and was not particular to mathematics, his findings and those of researchers since suggest that despite decades of reform, little has changed in regards to teachers' general questioning strategies in countries such as the US. Furthermore, since these various studies suggest low frequencies of teachers soliciting justification and argumentation from students (via probing and similar questioning approaches), it is likely that mathematical argumentation is seldom facilitated purposefully by teachers.

### Teacher perceptions of mathematical communication and argumentation

Such descriptive findings regarding the frequency of question types used by teachers are important, as they provide us with a glimpse of how teachers facilitate mathematical argumentation. Yet, an essential question that goes unanswered is why, after decades of research describing effective questioning strategies, are these strategies so infrequently incorporated into classrooms? One possible answer is that teachers' interpretation of

what counts as mathematical argumentation, explanation, and justification are different than those of mathematics education reform advocates. Hill (2005) provides some initial evidence that this is the case.

Examining the reliability of teachers' and researchers' assessment of teaching practices in the classroom, Hill (2005) observed relatively low agreement between these two groups in regard to "mathematical communication and representation." Among other varying issues, Hill suggests that differing definitions and interpretations of key terms may be the reason for such disagreement. While Hill's (2005) observations suggest that teachers may have differing interpretations than researchers, her analysis does not examine in depth the character of such differences. Another source of preliminary evidence comes from Conner's (2007) observations of student teachers' facilitation of argumentation in secondary mathematics classrooms. Conner (2007) noted that the specific conceptions student teachers had of mathematical proof were related to the questions they used to prompt students, how student contributions were treated, and the aspects of mathematical argument generally solicited from students. Therefore, student teachers' conceptions of proof manifested in establishing certain norms for argumentation amongst their students.

Considering the observations of Hill (2005) and Conner (2007), as well as the pervasively infrequent incorporation of probing questions by teachers (e.g., Boaler and Brodie 2004; Gall 1970), we sought to examine teachers' conceptions of effective facilitation of mathematical argumentation through questioning in more depth. To do this, we used *Depict* (Herbst and Chieu 2011), an interactive web-based tool designed to represent classroom scenarios, to engage participants in depicting their actions as a teacher in a hypothetical, high school algebra context. We then examined these depictions for the questioning strategies teachers employed in order to obtain an initial understanding of teachers' conceptions of appropriate facilitation of student argumentation. These efforts allowed us to investigate the following research question:

What questioning strategies do secondary mathematics teachers perceive as facilitating students' mathematical argumentation in a high school algebra context?

## Methods

### Sample and data

We examined data collected from 20 secondary (grades 8–12) mathematics teachers in the Eastern US. Participants were asked to create depictions of classroom teaching. The sample was fairly diverse in terms of gender (5 males and 15 females) and ethnicity (6 Caucasian, 5 African American, and 9 Asian), with a wide range of teaching experience (2 to 36 years,  $M=15.65$  years,  $SD=9.21$ ).

As part of a larger project, teachers completed an online experience designed to teach them how to create their own cartoon-based depictions of classroom scenarios. The software *Depict* is a web-based application embedded in the *LessonSketch* environment ([www.lessonsketch.org](http://www.lessonsketch.org); see Herbst et al. 2013). Users can drag-and-drop images from a library of classroom characters on to a canvas to create comic-like

image sequences (Herbst and Chieu 2011). Towards the end of the experience, participants were asked to create a depiction centered on the Common Core Mathematical Practice Standard # 3 (see Fig. 1). Participants were asked to review an initial slide with a student's explanation of their solution and then to add slides that would depict how they, as the teacher, would encourage students to "justify their conclusions, communicate them to others, and respond to the arguments of others" (CCSSI 2010, p. 6).

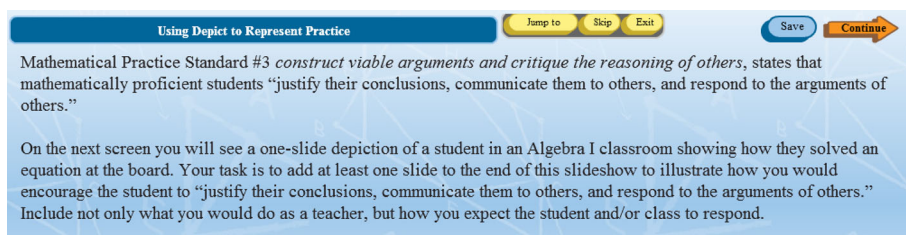
The initial slide provided to teachers, shown in Fig. 2, illustrates an incorrect approach to an explanation of solving a system of equations in a high school algebra classroom. Specifically, the student depicted in Fig. 2 incorrectly equates  $3b+2$  and  $2b+3$ . As part of their task, participants were asked to provide at least one slide to follow the initial student's explanation (see Fig. 1). In creating the additional slides, participants could include additional dialogue, make changes to the board's content, and alternate views of the classroom. Additionally, preceding this particular task, teachers had received training and practice in creating such cartoon-based depictions. Therefore, the slides added to the one initially provided (see Fig. 2) were the focus of our analysis. Depicted actions in these slides were considered as actions perceived by participants as sufficient in facilitating student engagement in mathematical argumentation.

### Additional measures

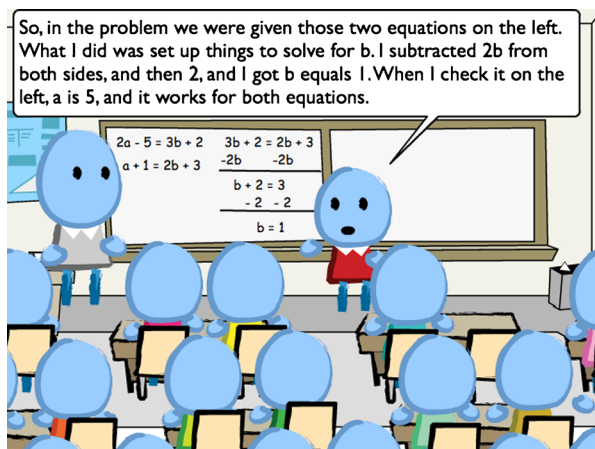
In addition to learning how to create their own cartoon-based depictions of classroom scenarios, participants also completed a general background survey. We used some of the items from that survey as a way to explore potential factors that may relate to participants' depicted questioning actions. These items included the total number years of experience teaching mathematics ( $M=15.65$ ,  $SD=9.21$ ), total number of mathematics content courses completed in university study ( $M=12.70$ ,  $SD=8.54$ ), total number of mathematics pedagogy courses completed in university study ( $M=3.21$ ,  $SD=2.00$ ), and the reported weekly percentage of class time typically devoted to leading "whole class discussions on mathematical topics" ( $M=26.33$ ,  $SD=17.47$ ).

### Analysis

We examined depicted teacher actions within the images created by participants to assess participants' perceptions of questioning facilitative of mathematical argumentation. Specifically, we examined questions asked by cartoon-depicted teachers and considered these depictions as representative of what participants (who constructed



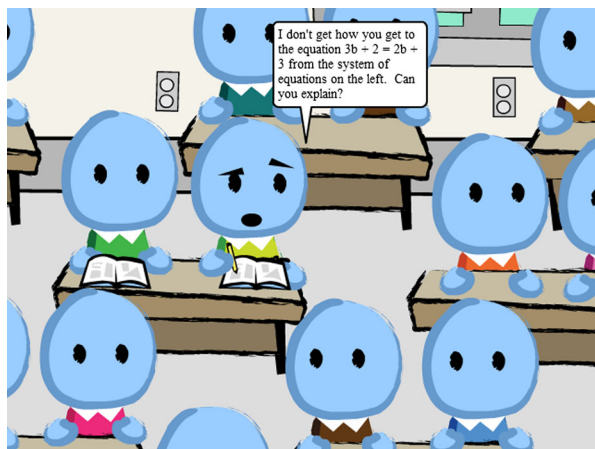
**Fig. 1** Screen shot of prompt and directions given to participants



**Fig. 2** The one-slide stem depiction provided as the starting point for teacher's depictions

the depictions) considered as effectively facilitating Mathematical Practice Standard #3. Much of this coding was based on rubrics constructed by Boaler and Brodie (2004) and Franke et al. (2009). In their rubric, Boaler and Brodie (2004) provide a detailed description of various question types that mathematics teachers may ask students. Thus, this rubric provides a useful means of discerning various individual questions that teachers may pose to students. On the other hand, the classifications used by Franke et al. (2009) are particularly useful in tracing the sequences of questions teachers may use following students' initial explanations. Therefore, the Boaler and Brodie (2004) rubric was used to classify individual questions while the Franke et al. (2009) rubric was used separately to code for sequences of questioning.

We also included three additional codes to examine actions depicted by teachers that were not captured by the aforementioned rubrics. We assigned the code *teacher silence* when the depicted teacher's move represented an expectancy that students act, and not the teacher. Figure 3 provides an example of this code, in which the depiction shows no



**Fig. 3** Example of *Teacher Silence* code from participant 1253, where the depicted teacher did not provide explicit statements

depicted actions from the teacher but rather action from a student. Therefore, the code represents the participants' expectancy that students would act in this manner.

If the depicted teacher did not ask students any questions in a particular slide, but rather made a lecture type statement or indicated the class should "move on," we assigned the code of *teacher statement*. Figure 4 provides an example of this code, where the teacher does not ask any questions of the student following their initial explanation but suggests the class moves on to check the answer. Actions in the form of other questions not assigned the above codes or codes from the two rubrics were assigned the code *other question(s)*.

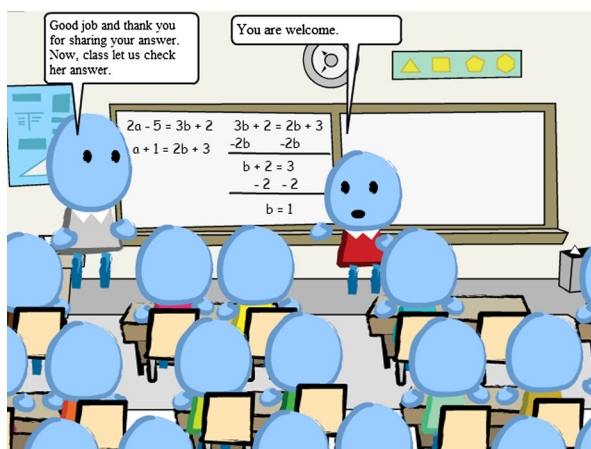
The remaining teacher actions were coded with two rubrics. As previously indicated, the rubric developed by Franke et al. (2009) focuses on whether teachers use sequences of questions to probe a student explanation (see Table 1). Given that teachers were asked to provide facilitative actions following a students' incorrect explanation, we believed this coding rubric would allow for a detailed description of teachers' depicted actions. Following the guidelines of Franke et al., we first looked for sequences of questions that indicated a *probing sequence*. If this code was not present, questions were coded as *general*, *specific*, or *leading*. Data was coded by the first two authors. We had 100 % agreement for *probing sequences*, 97.7 % agreement for *general questions*, 95 % agreement for *leading questions*, and satisfactory agreement for *specific questions* ( $K=0.63$ ).<sup>1</sup> After interrater reliability was assessed, codes were reconciled.

As mentioned earlier, the second rubric we used was developed by Boaler and Brodie (2004) to catalogue the various types of individual questions teachers could ask to facilitate mathematical discussion (see Table 2). Each code in the rubric was considered mutually exclusive from one another (i.e., after being coded as one question type, it was not counted as another question type), but we did allow overlap between this rubric and the one developed by Franke et al. (2009). As with the Franke et al. (2009) rubric, the first two authors coded the data and found generally high reliability. We observed 100 % agreement for *explore mathematical meaning*, *linking and applying*, *extending thinking*, and *establishing context*. We also observed 97.7 % agreement for *inserting terminology* and 95 % agreement for *orienting and focusing*. Satisfactory interrater reliabilities were also observed for *gathering information* ( $K=0.55$ ), *probing* ( $K=0.53$ ), and *generating discussion* ( $K=0.72$ ). Codes were reconciled after reliability was confirmed.

### Some caveats about our measures and analysis

Codes used to examine the questioning actions participants depicted are not necessarily representative of the actions such participants take in their classrooms. Given the nature of the task, assignment of such codes may best represent the researchers' assessment of teacher knowledge as applied to a specific pedagogical context in mathematics (i.e., soliciting argumentation in the depicted Algebra lesson). With regards to participants' self-reported responses to survey measures about classroom practices, we make no

<sup>1</sup> Following recommendations by Cicchetti and Feinstein (1990), positive rater agreement was reported for coding where the Kappa statistic was inappropriate (i.e., a skewness due to little to no negative agreement and overwhelming positive agreement). However, the Kappa statistic was used for coding where such skewness was not observed.



**Fig. 4** Example of *Teacher Statement* code from participant 1212, where the depicted teacher indicated the class should “move on”

claims regarding the accuracy or precision of these reports in comparison with their actual practice, but acknowledge the usefulness of this data in regards to teachers’ perceptions about their practice. Such perceptions relate well, conceptually, to domains of teacher knowledge, and are, thus, suitable to use in exploring relationships with participants’ depicted teaching actions.

## Findings

### Descriptive results

Overall, we coded 44 actions for 20 participants, with an average of 1.9 questioning actions per participant ( $SD=1.48$ , range=1 to 7). The descriptive results were aggregated by participant and are presented in Table 3. These results illustrate high frequencies of participants’ depicted teachers *making statements* (25 %), *generating discussion* (40 %), and *teacher silence* (40 %). These particular action types, however, are relatively passive in regards to the degree of scaffolding provided by teachers to engage students in argumentation. *Teacher statements* essentially represent a way for teachers to take control of the discourse with little to no feedback from students. While such

**Table 1** Teacher questions coding scheme from Franke et al. (2009)

|                                        |                                                                                                                                                                               |
|----------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Probing sequence of specific questions | Consisted of a series of more than two related questions about something specific that a student said and includes multiple teacher questions and multiple student responses. |
| General question                       | Questions not related to anything specific to what a student says.                                                                                                            |
| Specific question                      | Questions that address something specific in a student’s explanation.                                                                                                         |
| Leading question(s)                    | The teacher guides students toward particular answers or explanations and provides opportunities for students to respond.                                                     |

**Table 2** Teacher questions coding scheme from Boaler and Brodie (2004)

|                                                          |                                                                                                                         |
|----------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------|
| Gathering information, leading students through a method | Requires immediate answer rehearses known facts/procedures enables students to state facts/procedures                   |
| Inserting terminology                                    | Once ideas are under discussion, enables correct mathematical language to be used to talk about them                    |
| Exploring mathematical meanings and/or relationships     | Points to underlying mathematical relationships and meaning. Makes links between mathematical ideas and representations |
| Probing, getting students to explain there thinking      | Asks student to articulate, elaborate, or clarify ideas                                                                 |
| Generating discussion                                    | Solicit contributions from other member of the class                                                                    |
| Linking and applying                                     | Points to relationships among mathematical ideas and mathematics and other areas of study/life                          |
| Extending thinking                                       | Extends the situation under discussion to other situations where similar ideas may be used                              |
| Orienting and focusing                                   | Helps students to focus on key elements or aspects of the situation in order to enable problem solving                  |
| Establishing context                                     | Talks about issues outside of math in order to enable links to be made with mathematics                                 |

actions may be justified in certain contexts or sequences, recall that the task specifically asked teachers to depict actions facilitative of argumentation in the algebra class initially shown. Thus, justified uses of *teacher statements* would appear in participants' depicted scenarios that elicited justifications and/or explanations. As can be seen in Fig. 5, *teacher statements* were generally either in isolation or in combination with other actions less facilitative of argumentation. For example, participant 1203 responded to the initial prompt by depicting the teacher acknowledging the student's response in the first slide (*teacher statement*), and then asking the class if they agreed or disagreed (*generating discussion*) to which two students were depicted as responding. Thus, the role of this code in such sequences did little to meaningfully solicit or facilitate mathematical argumentation.

*Teacher silence* and *generating discussion*, as specific actions, were taken as representing passivity towards using the initial students' explanation to facilitate mathematical argument. *Generating discussion* often came in the form of teacher actions asking other students to contribute. Though this could have been a promising move when combined with other forms of questioning, follow-up actions generally included only *teacher silence*, *gathering information*, or *teacher statement* actions (see Fig. 5). Also noticeably lacking are codes from the Franke et al. (2009) rubric. Given that rubric's focus is on examining teachers' follow-up questions about a student explanation, we took this low frequency as indicating a lack of meaningful follow-up by depicted teachers of the initial student's explanation. Specifically, 40 % ( $n=8$ ) of participants included multiple slides and actions, yet none of these represented probing sequences of specific questions. In fact, only two of the eight sequences of actions included the *specific question* code from Franke et al.'s rubric.

In several instances (five of the eight depictions that had been assigned this code), *teacher silence* left it up to students to judge, ignore, or describe what the student stated, and ceded total control to the students, with generally little to no input from the teacher

**Table 3** Findings from our extensions to the coding rubrics

| Source of code           | Question type                                               | Percent of teachers using question type | Percent of total actions using question type* |
|--------------------------|-------------------------------------------------------------|-----------------------------------------|-----------------------------------------------|
| –                        | Teacher statement                                           | 25.0 %<br><i>n</i> =5                   | 13.0 %<br><i>n</i> =5                         |
| –                        | Teacher silence                                             | 40.0 %<br><i>n</i> =8                   | 32.0 %<br><i>n</i> =12                        |
| –                        | Other question(s)                                           | 5.0 %<br><i>n</i> =1                    | 3.0 %<br><i>n</i> =1                          |
| Boaler and Brodie (2004) | 1. Gathering information, leading students through a method | 15.0 %<br><i>n</i> =3                   | 11.0 %<br><i>n</i> =4                         |
| Boaler and Brodie (2004) | 2. Inserting terminology                                    | 5.0 %<br><i>n</i> =1                    | 3.0 %<br><i>n</i> =1                          |
| Boaler and Brodie (2004) | 3. Exploring mathematical meanings and/or relationships     | 0.0 %<br><i>n</i> =0                    | 0.0 %<br><i>n</i> =0                          |
| Boaler and Brodie (2004) | 4. Probing, getting students to explain there thinking      | 15.0 %<br><i>n</i> =3                   | 8.0 %<br><i>n</i> =3                          |
| Boaler and Brodie (2004) | 5. Generating discussion                                    | 40.0 %<br><i>n</i> =8                   | 24.0 %<br><i>n</i> =9                         |
| Boaler and Brodie (2004) | 6. Linking and applying                                     | 0.0 %<br><i>n</i> =0                    | 0.0 %<br><i>n</i> =0                          |
| Boaler and Brodie (2004) | 7. Extending thinking                                       | 0.0 %<br><i>n</i> =0                    | 0.0 %<br><i>n</i> =0                          |
| Boaler and Brodie (2004) | 8. Orienting and focusing                                   | 15.0 %<br><i>n</i> =3                   | 8.0 %<br><i>n</i> =3                          |
| Boaler and Brodie (2004) | 9. Establishing context                                     | 0.0 %<br><i>n</i> =0                    | 0.0 %<br><i>n</i> =0                          |
| Franke et al. (2009)     | Probing sequence of specific questions                      | 0.0 %<br><i>n</i> =0                    | 0.0 %<br><i>n</i> =0                          |
| Franke et al. (2009)     | General question                                            | 0.0 %<br><i>n</i> =0                    | 0.0 %<br><i>n</i> =0                          |
| Franke et al. (2009)     | Specific question                                           | 20.0 %<br><i>n</i> =4                   | 11.0 %<br><i>n</i> =4                         |
| Franke et al. (2009)     | Leading question(s)                                         | 0.0 %<br><i>n</i> =0                    | 0.0 %<br><i>n</i> =0                          |

\*Percentages do not include actions coded as nonsensical (*n*=5)

in the depictions. As shown in Fig. 5, three participants' sole action was teacher silence. Another participant (ID 1150) depicted teacher silence after a *generating discussion* question that provided little scaffolding. Similarly, participant 1119 provided little scaffolding but ended the depiction with a teacher statement providing positive reinforcement. Such sequences demonstrate some involvement by the teacher, but in

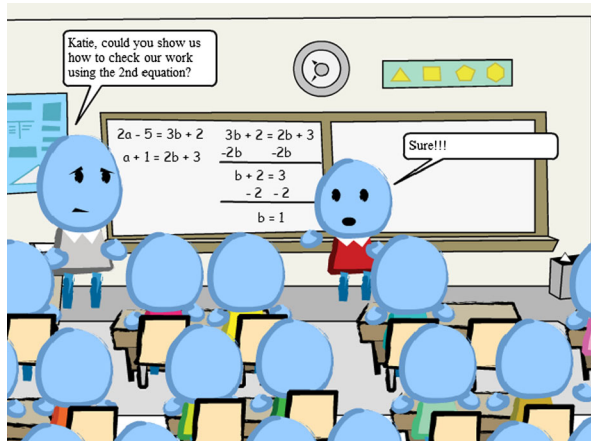
| Participant | Action 1              | Action 2              | Action 3              | Action 4              | Action 5          | Action 6        | Action 7        |
|-------------|-----------------------|-----------------------|-----------------------|-----------------------|-------------------|-----------------|-----------------|
| 1088        | Teacher Silence       |                       |                       |                       |                   |                 |                 |
| 1253        | Teacher Silence       |                       |                       |                       |                   |                 |                 |
| 1264        | Teacher Silence       |                       |                       |                       |                   |                 |                 |
| 1108        | Teacher Silence       | Orienting & Focusing  | Teacher Silence       | Teacher Silence       | Teacher Statement | Teacher Silence | Teacher Silence |
| 1205        | Generating Discussion |                       |                       |                       |                   |                 |                 |
| 1121        | Generating Discussion |                       |                       |                       |                   |                 |                 |
| 1209        | Generating Discussion | Gathering Information |                       |                       |                   |                 |                 |
| 1090        | Generating Discussion | Probing               |                       |                       |                   |                 |                 |
| 1150        | Generating Discussion | Teacher Silence       |                       |                       |                   |                 |                 |
| 1119        | Generating Discussion | Teacher Silence       | Teacher Statement     |                       |                   |                 |                 |
| 1155        | Generating Discussion | Teacher Silence       | Generating Discussion | Gathering Information |                   |                 |                 |
| 1160        | Gathering Information | Teacher Silence       | Gathering Information |                       |                   |                 |                 |
| 1104        | Teacher Statement     |                       |                       |                       |                   |                 |                 |
| 1212        | Teacher Statement     |                       |                       |                       |                   |                 |                 |
| 1203        | Teacher Statement     | Generating Discussion |                       |                       |                   |                 |                 |
| 1126        | Orienting & Focusing  | Other                 |                       |                       |                   |                 |                 |
| 1143        | Orienting & Focusing  |                       |                       |                       |                   |                 |                 |
| 1100        | Probing               |                       |                       |                       |                   |                 |                 |
| 1229        | Probing               |                       |                       |                       |                   |                 |                 |
| 1234        | Inserting Terminology |                       |                       |                       |                   |                 |                 |

Fig. 5 Series of questioning actions depicted by participants

actuality do little to scaffold the mathematics in the discussion. Rather, these moves leave such scaffolding up to the students. While ceding a certain amount of the control in mathematical arguments to the students is generally advocated (e.g., Forman et al. 1998; Yackel and Cobb 1996), there is a risk of ceding too much control and thus decreasing the potential effectiveness of such engagement (e.g., Sfard 2007). As with *generating discussion*, *teacher silence* is an action that, in certain sequences of questioning moves, may be both appropriate and effective. Yet, the prevalence of this code in the present study suggests that teachers may view facilitating mathematical argumentation as equivalent to yielding control of such discussions entirely to the students. However, the literature suggests this is only one of several components necessary for effective facilitation of mathematical argumentation. Teachers must provide certain scaffolds in order for such autonomy-facilitative behavior to be most effective (e.g., Forman et al. 1998; Yackel and Cobb 1996). The expectation that students will adequately facilitate such mathematical arguments without this scaffolding may demonstrate an incomplete understanding of what it means for a teacher to facilitate argumentation.

A few teachers did depict actions that we considered more strongly indicative of facilitating effective argumentation. Particularly, *probing* and *orienting discussion* were each depicted by 30 % of the sample ( $n=6$ ). An example of one participant's depicted action for *orienting and focusing* discussion is shown in Fig. 6. Here, the participant chose an action that focuses on the depicted student's error by asking the student to "check our work" with the second equation provided. By examining the additional slide provided by this participant, in which the student describes procedures, it is evident that the goal of the depicted action was to orient the student towards relating the two equations to each other. Thus, the action simultaneously provides both the student a measure of control in how the procedures are described and the teacher an opportunity to orient the student to a particular aspect of the mathematics at hand.

*Probing* questions operated in a similar manner in that the depicted teacher's actions focused on a particular aspect of the mathematics at hand, but laid the responsibility of verbalizing the mathematics with the depicted student. For example, in one depiction where the participant named the students in accordance with their respective shirt colors, the participant had the teacher ask, "Red, would you explain to Pink why you chose to subtract  $2b$  first?" (participant 1229). Noticeably, the depicted teacher's actions



**Fig. 6** Example of “orienting and focusing” coding from participant 1126, in which the depicted teacher reoriented the discussion to focus on a particular aspect of mathematics

orient the justification to be made as being provided to the student at the board and not specifically to the teacher. In other examples of probing, the teacher pressed for elaboration, but the audience was not specified and this press was therefore implicitly meant for the class as a whole.

### Correlations

In an effort to better understand the characteristics of participants who depicted certain questioning actions, we correlated certain items from a background survey with codes from depicted actions. We elected to calculate Spearman Rho correlations based on the size of the sample ( $n=20$ ) and the categorical nature of the codes for depicted questioning actions. Participants' years of teaching experience was found to have a statistically significant, negative relationship with *teacher silence* ( $\rho=-.58, p<.01$ ), but was not found to have a statistical relationship with the other questioning codes. Participants' number of completed math content courses was also found to have a statistically significant, positive relationship with *teacher silence* ( $\rho=.58, p<.01$ ). Additionally, a statistically significant negative correlation was observed with *probing* ( $\rho=-.45, p<.05$ ). The number of mathematics pedagogy courses a participant had completed had no observed statistical relationship with questioning codes. However, the percentage of reported weekly class time devoted to leading whole class discussions was found to have statistically significant relationships with *teacher silence* ( $\rho=-.55, p<.05$ ), *gathering information* ( $\rho=-.49, p<.10$ ), and *orienting and focusing* ( $\rho=.53, p<.05$ ).

Though these correlational results possess limitations due to sample size and the breadth of some of the items, the results provide interesting context to the descriptive data presented in the previous section. Of particular interest are the results for *teacher silence*, which suggest a negative relationship with years of experience and time devoted to class discussions, but a positive relationship with amount of mathematics content courses a participant has. This suggests a relatively complicated characterization of participants depicting *teacher silence* as an action that facilitates mathematical

argumentation. It suggests that general experience in the classroom and particular experience with facilitating discussions (which includes opportunities for argumentation) may deter teachers from engaging in such an action. However, the finding concerning mathematics content courses should be taken in context.

Since the participants in this study were all secondary mathematics teachers, they had each taken a number of content courses in their university study. Therefore, it may be that possessing an extensive background in the content somehow interacts with teachers' notions of how to facilitate mathematical argumentation in a way that encourages *teacher silence* as a viable action. Further study is needed to investigate this phenomenon.

### Summary of findings

Descriptive statistics of depicted teacher actions suggest that the majority of actions represented passive facilitation of mathematical argumentation. Specifically, actions such as *teacher statements*, *generating discussions*, and *teacher silence* were included in participants' depictions, either in sequences or as stand-alone actions, in two primary ways. First, *teacher statements* were often used for the teacher to take control of the discourse and thus students were not actively engaged in making mathematical meaning. Second, *teacher silence* and *generating discussion* were used in such a way that a specific focus on key mathematical features was not addressed. Thus, these three types of actions represented ways of passively engaging students in mathematical argumentation. Two of these actions (*teacher silence* and *generating discussion*) had negative correlations with participants' reported time spent on whole class discussions in mathematics, and *teacher silence* had a negative correlation with years of teaching experience. Thus, including these actions may have a relationship with a form of novice knowledge of facilitating mathematical argumentation, either due to lack of teaching experience in general or lack of experience with mathematical discourse in particular.

Whereas the aforementioned actions more passively engaged depicted students in mathematical argumentation, certain actions were more facilitative of more active engagement (i.e., *probing* and *orienting and focusing*). These actions solicited explanations and justifications from students explicitly, but also provided a context for expectations of such descriptions to be provided even when such prompts were not conveyed in particular slides. Correlational analysis suggests a negative relationship between *probing* and number of content courses. Recalling both that the participants were all secondary teachers with sufficient numbers of mathematics content courses, and that there was a positive relationship observed with content courses and *teacher silence*, it may be that having a relatively larger amount of mathematics background may cause some teachers to assume that the argument needed is already contained in the answer to the question. Whereas *orienting and focusing* has a positive correlation with participant-reported time spent on class discussions, and other passively engaging actions had negative such correlations (*teacher silence* and *generating discussion*); it may be that more practical experience can correct such implicit assumptions. Given the sample size and exploratory nature of this study, further research is needed to both determine if such a conjecture holds merit, and how this information can be usefully applied to professional development and teacher education.

## Discussion

It has been known for quite some time that US mathematics teachers ask predominately leading answer-driven questions with relatively little focus on asking students to explain or justify their mathematics (Boaler and Brodie 2004; Gall 1970; Hiebert et al. 2003; Perry et al. 1993). While such leading questions can provide contexts for engaging mathematical arguments (Temple and Doerr 2012), students benefit more when teachers ask more probing sorts of questions (Kazemi and Stipek 2001). Hill's (2005) observations suggest the possibility of different interpretations of mathematical argumentation (and other manifestations of communication) between teachers and researchers, yet did not indicate how such different interpretations may manifest themselves in practice. The present study provides such information by examining participating teachers' posited depictions of facilitating mathematical argumentation in a high school algebra context.

The findings presented here provide a stark contrast between researchers and teachers in regards to what mathematical argumentation is supposed to look like. While some participants did depict asking *probing* and *orienting and focusing* questions and these actions did overlap with other questioning actions in certain cases, the majority of participants depicted actions that did not support probing for student understanding. The prevalence of depicted actions such as *teacher silence* and *generating discussion* suggests a tacit assumption by certain participants, by way of their depicted actions, that mathematical argumentation can occur with relatively little scaffolding by the teacher. Indeed, in some classrooms, this might be possible, but this typically occurs only after students have had a great deal of exposure to the process of mathematical argumentation that has been scaffolded by the teacher (e.g., Hufferd-Ackles et al. 2004; Yackel and Cobb 1996). *Teacher silence* and *generating discussions* are worthwhile actions for teachers to take, but neither in and of themselves, nor in certain combinations with other questioning types. For example, the sequences of questions in the current study (see Fig. 5) included both teacher silence and generating discussions actions, but did not press students to explain or justify their mathematics. Both actions demonstrate a ceding of control from the teacher to students; an act that has been consistently linked with productive mathematical discourse and argumentation (Kosko and Wilkins 2012; Forman et al. 1998; Kazemi and Stipek 2001; Yackel and Cobb 1996) but only when a teacher has effectively scaffolded the context of discussions and incorporated such actions to press students for meaning.

Results from the correlational analysis provide some material for us to conjecture why participants may have depicted the actions they did. In the present study, participants who typically depicted actions such as *teacher silence* also reported having fewer years of experience in the classroom and spent less time discussing mathematics in their classrooms. This may suggest that the prevalent use of *teacher silence* is due to a lack of experience or exposure to mathematical discourse settings and general opportunities for mathematical argumentation. Another possibility is that such teachers may fall victim to the conception that "not telling" students facilitates mathematical discourse (Chazan and Ball 1999), whereas Lobato et al. (2005) found that certain "telling" actions function to facilitate students' production of mathematical explanations and justifications. Interestingly, *orienting and focusing* actions were found to have positive relationships with the amount of time participants reported devoting to class

discussions. While these findings may suggest a relationship in regards to experience and exposure, they are just as likely to suggest a relationship with valuing of mathematical argumentation and similar discursive activities. Regardless, further research is necessary to both confirm and unpack in greater detail the trends observed in this study.

Additional research that focuses on teachers' perceptions of how to facilitate mathematical argumentation is needed. Such efforts may benefit from replicating the procedures used in the present study. Stimulated-recall interviews of episodes from teachers' classrooms may also provide a useful context for such study. However, it may be useful to integrate the approaches used in the present study with stimulated-recall to better understand teachers' conceptions of best practices regarding such facilitation. Particularly, when teachers are asked to use a tool such as *Depict* to represent appropriate facilitation of mathematical argumentation (as was done here), they are provided opportunities to showcase instruction they believe to be most appropriate. However, opportunities to engage their students in what they believe to be most appropriate may not always be available in their own schools or classrooms due to certain aspects of the institutional obligation (e.g., administrative restrictions, curricular demands; see Herbst and Chazan 2012). Put simply, *Depict* allows teachers to represent the teaching they believe is best, regardless of whether or not they are able to implement such instruction in their own classrooms and schools.

The present study also provides some suggestions for teacher education. Specifically, more attention needs to be given to the nature of mathematics pedagogical scaffolds for use in facilitating mathematical argumentation. Focusing on questioning types is important, but the findings presented here suggest the need to focus on sequencing of questions as well as individual instances of questioning.

The results of this study, while preliminary and descriptive in nature, are highly relevant. We found that even when asked to explicitly model effective facilitation of student argumentation, a large portion of participating teachers in our sample either depicted actions that did not open up opportunities for argument (*teacher statements*), left such facilitation up to the students with no facilitation by the teacher (*teacher silence*), or asked general questions that sought contributions from other students in the class while ignoring the mathematical contribution made by a student's initial explanation of a procedure. Further analysis suggested that this lack of focus on the initial student's explanation could be due to a lack of general teaching experience or experience with facilitating discussions and mathematical arguments in one's classroom.

A clear implication from our findings is that teachers may not have a clear understanding of what effective questioning strategies look like or how to implement them. Therefore, calls for more explicit instruction for teachers in how to facilitate mathematical argumentation and discussion are well founded (e.g., Boerst et al. 2011; Smith and Stein 2011). Further research is still needed to investigate teachers' knowledge, conceptions, and understandings of questioning strategies and factors that influence them. Only when we fully understand the landscape of teachers' conceptions of mathematical argumentation can we truly be prepared to develop and evaluate our own attempts to improve them.

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Foundation. The character set ThExpians B used in screen shots of depictions created with the Depict tool is (c) The Regents of the University of Michigan.

## References

- Australian Curriculum, Assessment and Reporting Authority [ACARA]. (2013). *Australian Curriculum: Mathematics*. Retrieved at: [www.australiancurriculum.edu.au/Mathematics/](http://www.australiancurriculum.edu.au/Mathematics/)
- Boaler, J., & Brodie, K. (2004). The importance of depth and breadth in the analyses of teaching: A framework for analyzing teacher questions. In D. E. McDougall & J. A. Ross (Eds.), *Proceedings of the 26<sup>th</sup> meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 773–782). Toronto, Canada.
- Boerst, T., Sleep, L., Ball, D., & Bass, H. (2011). Preparing teachers to lead mathematics discussions. *Teachers College Record*, 113(12), 2844–2877.
- Chazan, D., & Ball, D. (1999). Beyond being told not to tell. *For the learning of mathematics*, 19(2), 2–10.
- Cicchetti, D. V., & Feinstein, A. R. (1990). High agreement but low Kappa: resolving the paradoxes. *Journal of Clinical Epidemiology*, 43(6), 551–558.
- Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *The Journal of the Learning Sciences*, 10(1/2), 113–163.
- Conner, A. M. (2007). *Student teachers' conceptions of proof and facilitation of argumentation in secondary mathematics classrooms (doctoral dissertation)*. University Park, PA: The Pennsylvania State University.
- Council of Chief State School Officials (CCSSO), & National Governors Association Center for Best Practices (NGA). (2010). *Common core state standards for mathematics*. Retrieved from Common Core Standards Initiative website: [http://corestandards.org/assets/CCSSI\\_Math%20Standards.pdf](http://corestandards.org/assets/CCSSI_Math%20Standards.pdf)
- Cross, D. I. (2009). Creating optimal mathematics learning environments: combining argumentation and writing to enhance achievement. *International Journal of Science and Mathematics Education*, 9, 905–930.
- Douek, N., & Pichat, M. (2003). From oral to written texts in grade 1 and the approach to mathematical argumentation. In N. A. Pateman, B. J. Dougherty, & J. T. Zilliox (Eds.), *Proceedings of the International Group for the Psychology of Mathematics Education* (Vol. 2., pp. 341–348). University of Hawaii: Honolulu, Hawaii, USA.
- Forman, E. A., Larreamendy-Joerns, J., Stein, M. K., & Brown, C. A. (1998). “You’re going to want to find out which and prove it”: collective argumentation in a mathematics classroom. *Learning and Instruction*, 8(6), 527–548.
- Franke, M. L., Webb, N. M., Chan, A. G., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students’ mathematical thinking in elementary school classrooms. *Journal of Teacher Education*, 60(4), 380–392.
- Gall, M. D. (1970). The use of questions in teaching. *Review of Educational Research*, 40(5), 707–721.
- Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education*, 35(4), 258–291.
- Graesser, A. C., & Person, N. K. (1994). Question asking during tutoring. *American Educational Research Journal*, 31, 104–137.
- Herbst, P., & Chieu, V. M. (2011). Depict: A tool to represent classroom scenarios. Technical report. University of Michigan. Retrieved from <http://hdl.handle.net/2027.42/87949>
- Herbst, P. & Chazan, D. (2012). On the instructional triangle and sources of justification for actions in mathematics teaching. *ZDM The International Journal of Mathematics Education*, 44(5), 601–612.
- Herbst, P., Aaron, W., and Chieu, V. M. (2013). LessonSketch: An environment for teachers to examine mathematical practice and learn about its standards. In D. Polly (Ed.), *Common Core Mathematics Standards and Implementing Digital Technologies* (pp. 281–294). Hershey, PA: IGI Global.
- Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students’ learning in second-grade arithmetic. *American Educational Research Journal*, 30(2), 393–425.
- Hiebert, J., Gallimore, R., Gamier, H., Givvin, K. B., Hollingsworth, H., & Jacobs, J. (2003). *Teaching mathematics in seven countries: results from the TIMSS 1999 video study*. Washington, D.C.: National Center for Education Statistics.
- Hill, H. C. (2005). Content across communities: validating measures of elementary mathematics instruction. *Educational Policy*, 19, 447–475.
- Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education*, 35(2), 81–116.

- Hunter, R. (2007). Can you convince me: learning to use mathematical argumentation. In J. H. Woo, H. C., Lew, K. S., Park, & D. Y. Seo (Eds.), *Proceedings of the 31<sup>st</sup> conference of the International Group for the Psychology of Mathematics Education*, Volume 3 (pp. 81–88), Seoul: PME.
- Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal*, 102(1), 59–80.
- Kosko, K. W., & Wilkins, J. L. M. (2012). Students' quality of mathematical discussion and their self-determination in mathematics. *Investigations in Mathematics Learning*, 4(3), 15–31.
- Lobato, J., Clarke, D., & Ellis, A. B. (2005). Initiating and eliciting in teaching: a reformulation of telling. *Journal for Research in Mathematics Education*, 36(2), 101–136.
- Martino, A. M., & Maher, C. A. (1999). Teacher questioning to promote justification and generalization in mathematics: what research practice has taught us. *Journal of Mathematical Behavior*, 18(1), 53–78.
- Mercer, N., & Sams, C. (2006). Teaching children how to use language to solve math's problems. *Language and Education*, 20(6), 507–528.
- National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- Perry, M., Vander Stoep, S. W., & Yu, S. L. (1993). Asking questions in first-grade mathematics classes: potential influences on mathematical thought. *Journal of Educational Psychology*, 85(1), 31–40.
- Pimm, D. (1987). *Speaking mathematically: communication in mathematics classrooms*. New York: Routledge & Kegan Paul.
- Sfard, A. (2007). When the rules of discourse change, but nobody tells you: making sense of mathematics learning from a commognitive standpoint. *Journal of the Learning Sciences*, 103(4), 357–382.
- Smith, M. S., & Stein, M. K. (2011). *Five practices for orchestrating productive mathematics discussions*. Reston, VA: National Council of Teachers of Mathematics.
- Temple, C., & Doerr, H. M. (2012). Developing fluency in the mathematical register through conversation in a tenth-grade classroom. *Educational Studies in Mathematics*, 81, 287–306.
- Webb, N. M., Nemer, K. M., & Ing, M. (2006). Small-group reflections: parallels between teacher discourse and student behavior in peer-directed groups. *The Journal of the Learning Sciences*, 15(1), 63–119.
- Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, 27(4), 458–477.