Time Series

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MSc in Statistics and Operations Research

Outline - Theory

This course provides theory and practice of time series analysis.

- Presents the basic theory of stationary/non-stationary processes - unit root testing.
- Describes and presents analytically AR(I)MA models and the Box-Jenkins methodology.
- Introduces the class of conditional heteroscedastic models (ARCH/GARCH).
- Presents time series forecasting techniques.
- Illustrative examples applying time series models/techniques to actual economic and financial data.

Outline - Applications

The empirical applications consist of

- unit root testing to economic/financial series (e.g. exchange rates, stock returns)
- modeling and forecasting economic/financial return series
- performance evaluation of fund investments (e.g. mutual and/or hedge fund returns)
- estimation of different risk measures

Examples and applications are performed using the statistical package R.

Stationarity and Unit root Testing

- Examples of time series
- Basic concepts: autocorrelation and stationarity
- Properties of stationary and non-stationary processes
- Unit root testing: augmented Dickey-Fuller test
- Illustration of unit root testing to economic and financial data sets using R
  - Example 1: Unit root testing to exchange rate series (application and useful conclusions)
  - Example 2: unit root testing to financial time series, e.g. stocks and indices (application and useful conclusions)

Stationary Time Series Models

- Moving Average (MA) processes - Autoregressive (AR) processes
- Mixed Autoregressive Moving Average (ARMA) processes
- Properties of ARMA processes - Autocorrelation and partial autocorrelation function
- Stationarity - Stationarity through differencing - Invertibility.

Box-Jenkins Methodology

- Identification step: the role of autocorrelation and partial autocorrelation function.
- Estimation step: maximum likelihood estimation - Exact and conditional likelihood.
- Diagnostic step: residual analysis.
- Prediction step: minimum mean square error forecasting - ARMA forecasting.
- Illustration of applying Box-Jenkins methodology using R - Applications to real economic and financial series.
  - Example 1: modeling and forecasting financial return series (S&P 500 monthly returns and Johnson and Johnson quarterly data).
  - Example 2: modeling and forecasting economic series (GDP of EU countries).

Time Series Models of Heteroscedasticity

- Characteristics of financial time series
- (Generalised) Autoregressive Conditional Heteroscedasticity (ARCH/GARCH) type models
- Maximum likelihood estimation
- Models'diagnostics
- Extensions of the GARCH model
- Variance prediction
- Illustration of estimating GARCH-type models to financial time series using R. Applications to real financial series:
  - Example 1: modeling and forecasting financial return series (S&P 500 monthly returns).
  - Example 2: estimation of different risk measures (e.g Value at Risk).
  - Example 3: performance evaluation of fund investments (e.g. mutual and/or hedge fund investments).

Students' Evaluation

The evaluation of the students' work is based partly on the final exam and partly on projects.

- Final exam will contribute 80 per cent to the final mark.
- Two assignments will contribute 20 per cent to the final mark.
- Note: one needs to pass the final exam (independently of the marks in the homework assignments) in order not to fail the course!

Modeling Approaches - Stationary Time Series models

Autoregressive models [AR(p)]

$$y_t = \delta + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \varepsilon_t$$

Moving Average models [MA(q)]

$$y_t = \mu + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q} + \varepsilon_t$$

Autoregressive Moving Average models [ARMA(p,q)]

$$y_t = \delta + \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p} + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} + \varepsilon_t$$

Assuming: (a) uncorrelated errors, (b) constant variance - homoscedastic errors, (c) normal errors.

Modeling Approaches - Regression Type Models

Explanatory Models - Asset Pricing: build models with the aim to identify important explanatory variables (risk factors) that explain financial returns.

$$y_t = \alpha + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \ldots + \beta_k x_{k,t} + \varepsilon_t$$

Forecasting Models - Return Predictability: built models with the aim to identify important predictive variables that have the ability to forecast financial returns.

$$y_t = \alpha + \beta_1 x_{1,t-1} + \beta_2 x_{2,t-1} + \ldots + \beta_k x_{k,t-1} + \varepsilon_t$$

Assuming:

- uncorrelated errors
- constant variance - homoscedastic errors
- normal errors

Violation of Important Assumptions - Consequences

If the standard assumptions on the error terms are violated:

- Point estimation of model parameters is valid [e.g. least squares, maximum likelihood].
- Statistical inference, which is theoretically based on the above assumptions is not valid [e.g. hypothesis testing, CIs].

Consequences:

- We can not identify accurately which risk factors are really important to explain financial returns and to predict future returns [model selection problem].
- We can not accurately infer the constant α in the regression model (test its statistical significance), which is a measure of the performance or skill of a manager, and the regression coefficients, which quantify the relationship between y_t and the risk factors or predictors.

Autocorrelated errors: Solution via ARMA-type models.
Heteroscedastic errors: Solution via GARCH-type models.

$$y_t = \alpha + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \ldots + \beta_k x_{k,t} + u_t$$

$$u_t = \delta + \phi_1 u_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \sigma_{t-1}^2$$

Account for: (a) autocorrelated errors, (b) heteroscedasticity, i.e. volatility clustering, fat tails, excess kurtosis.

Textbooks

- Hamilton, James D. Time Series Analysis. Princeton, New Jersey: Princeton University Press, 1994.

- Enders, Walter. Applied Econometric Time Series. New York: Wiley, 2010.

- Cowpertwait, Paul S.P., and Metcalfe V. Andrew. Introductory Time Series with R. New York: Springer Texts in Statistics, 2009.

- Cryer, Jonathan D., and Chan Kung-Sik. Time Series Analysis with Applications in R. Springer Texts in Statistics, 2010.

- Gujarati, Damodar N. Basic Econometrics. New York: McGraw-Hill, 2008.

- Harvey, Andrew. Time Series Models. Cambridge: MIT Press, 1993.

- Hendry, David F. Dynamic Econometrics. Oxford: Oxford University Press, 1995.

- Shumway, Robert H. and David S. Stoffer. Time Series Analysis and Its Applications with R Examples. New York: Springer Texts in Statistics, 2011.