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SURVEY PAPER

Metacognition and mathematics education: an overview

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Abstract

This special issue includes contributions discussing the assessment and training of metacognition that appear promising for the purpose of positively influencing the learning process of students' learning of mathematics. More specifically, contributors explore, illustrate and scrutinize available research evidence for its relevance and effectiveness in the specific curricular field of mathematics education. After an introduction and discussion of the individual input, we explore the scientific progress in the area of the theoretical framework and conceptualizations of metacognition, the relationships between metacognition and mathematics performance, the various effects upon ability levels, the measures to assess metacognition, and the interventions that aim to improve mathematics performance. This special issue ends with a reflection on practical suggestions for mathematics education.

Keywords Metacognition · Survey · Mathematics education · Assessment · Intervention

1 Introduction

The classic papers of Flavell (1976, 1979, 1987) have introduced the concept called 'metacognition'. Flavell presented the concept as a kind of meta-knowledge, at first referred to as 'meta-memory' (Flavell 1971). This definition was later extended and Flavell (1976) defined 'metacognition' as: "one's knowledge concerning one's own cognitive processes and products or anything related to them … [and] the active monitoring and consequent regulation and orchestration of these processes" (p. 232). A similar taxonomy was presented by Brown (1978a, b, 1987). She distinguished two types of metacognition: "knowledge" about one's own cognitive activities (person, task and strategies) and "regulation" of one's own cognitive activities.

Up till now, there is no agreement whether the key aspects of metacognition as higher order cognition, notably metacognitive knowledge and skills, are related only to mathematics (field-specific concept) or if they should be conceptualized as broad constructs regulating all cognitive activities (general concept). On the one hand, previous studies have revealed that the quality of metacognitive skills is a general person-related property throughout a learning environments, rather than a mathematics-specific phenomenon (Veenman et al. 1997). On the other hand, expert problem solvers were able to assess and update their mental representations in familial domains, yet were not any more capable than novices to apply these metacognitive skills in unfamiliar domains. For a comparison of domain-specific and domain-general views on metacognition we refer to Veenman et al. (2004).

From a developmental point of view, metacognition starts to develop itself during preschool or during early-school years, reaching its full bloom at the age of 12 (Kuzle 2018; Veenman et al. 2006; Whitebread et al. 2005), continuing to play a role for adolescents (Hidayat et al. 2018) with low-effort skills (e.g., problem identification), preceding the development of high-effort skills (e.g., plan making and self-regulation; Shute 1996).

Metacognition seems to be one of the most important predictors of mathematical performance (e.g., Depaepe et al. 2010; Kuzle, 2018; Ohtani and Hisasaka 2018). Metacognition is related to 'propensity' factors enabling children (e.g., intelligence) and/or making them willing (e.g., motivation) to learn, as well as to 'opportunities' that expose children to learning content (Byrnes and Miller 2007, 2016; Wang and Byrnes 2013).

The exposure to 'a metacognitive enriched environment' might not always be optimal, since primary and secondary school teachers often lack the requisite knowledge about

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metacognition, and on top of that seem reluctant to promote metacognition (Dignath and Büttner 2018). Depaepe et al. (2010) have demonstrated that teachers rarely, if ever, deal with the 'how' and 'why' of using a particular metacognitive skill, although exactly doing so might be beneficial for instructional design. Dignath and Büttner (2018) confirmed that teachers teach mainly cognitive and very few metacognitive strategies, pointing to the fact that teachers might need extra training and explicit instructions concerning metacognition.

Since Flavell (1976) introduced the term 'metacognition', there has been a debate on what metacognition actually is and on how it may be understood, assessed and trained. For this special issue, we have selected contributions that appear promising for the field of mathematics education. Obviously, this is not the first attempt to come up with such input. In this respect, we refer to previous reviews on metacognition, for instance the studies of Schneider and Artelt (2010), Gascoine and colleagues (2017), Donker and colleagues (2014), de Boer and colleagues (2018), and Ohtani and Hisasaka (2018).

In their review of studies on the learning of mathematics students, Schneider and Artelt (2010) expressed their support for the substantial impact of metacognition on mathematics performance, reporting that it accounted for about 15–20% of common variance. This finding was consistent with that of Ohtani and Hisasaka (2018), who found a positive correlation ($r=0.28$) of metacognition with academic performance, when they checked for intelligence as a disturbing 'propensity' variable.

The meta-analytic review of Ohtani and Hisasaka (2018) indicated that the relationship between metacognition and mathematics is moderated by the choice of measurement tools. Especially on-line tools have strong correlations ($r=0.53$) with academic performance. Similar results were found for all on-line tools (think aloud: $r=0.54$; log files $r=0.51$). Accuracy measures ($r=0.43$) and interviews ($r=0.45$) correlated moderately with academic performance. Broad-based measures ($r=0.21$) exhibited lower correlation coefficients relative to those observed for experimental tasks ($r=0.44$; Ohtani and Hisasaka 2018). Although off-line methods correlated only weakly with academic performance ($r=0.23$), with questionnaires showing a particularly low correlation ($r=0.19$; Ohtani and Hisasaka (2018), and Gascoine et al. (2017) demonstrated in their systematic review that 61% of the studies on children aged 4–16 years used self-report measures.

Schneider and Artelt (2010) stated that all learners did benefit from metacognitive instruction procedures. Veenman (2013) added that metacognitively competent students should not receive additional metacognitive training, since such training might interfere with their spontaneous use of adequate metacognitive skills. The meta-analysis of intervention studies (from 2000 till 2012) of Donker and colleagues (2014) confirmed the substantial effect sizes for metacognitive knowledge instruction in the field of writing (Hedges'g = 1.25), science (0.73), mathematics (0.66) and comprehensive reading (0.36), with no differential effects for students with different ability levels. The positive effects of metacognitive strategy instruction on student performance was confirmed in the thematic review of de Boer and colleagues (2018). However, their moderator analyses showed that especially low SES students benefitted the most, taking into account all long-term measures, from metacognitive strategy instruction. In addition, Dennis and colleagues (2016) demonstrated in their meta-analysis that controlling task difficulty might be an additional instructional component for teaching and empowering students with learning difficulties for mathematics.

To conclude, metacognition continues to be a predictor of mathematics performance. Although the topic has evolved, according to Stillman and Mevarech (2010), from a "hot topic" to a more "mature field", a grounded assessment continues to be advisable. In addition, inconsistent findings are reported concerning the question of whether all or some students benefit more from metacognitive strategy instruction. This special issue aims to extend this discussion in some important ways. First, we consider research findings from the point of view of mathematics education. Second, the various contributions reflect not only upon the strengths, but also on the pitfalls of metacognition to improve mathematics education, with related suggestions for further research.

2 The various contributions

Studies that appear promising for influencing mathematics learning by students, are described, and discussed. On the one hand, examining these studies, we discuss some implications for improving the assessment of student learning, and for improving such learning, in the domain of mathematics education. On the other hand, we looked within the research community of metacognition and self-regulation for (teams of) scholars who explicitly and consistently attempted to assess or enhance mathematics learning by students. Using this information, we selected a list of contributors for this present special issue. Each author (team) was asked to write a chapter about metacognition and mathematical education.

We included inputs (a) that scrutinise the tools to assess metacognition (Lingel et al. 2019; Veenman and van Cleef 2019), (b) that explore and discuss the evidence on instructional behavior and interventions available (Ader 2019; Hacker et al. 2019; Lucangeli et al 2019; Shilo and Kramarski 2019; Temur et al. 2019) or (c) that refer to the understanding or modelling of metacognition and prediction for mathematics learning (Baten and Desoete 2019; Desoete et al. 2019; Zhoa et al. 2019; Vorhölter 2019). Papers on typical performers as well as on atypical performers (individuals with mathematical learning disabilities or problems) were included.

2.1 Tools to assess metacognition

Lingel and colleagues (2019) contributed through a study of German children in Grade 7 ($n=109$) comparing metacognitive monitoring measures. All participants were assessed on mathematics performance and marks were obtained for the last biannual report. In addition, predictions and postdictions were used to assess students' confidence in problem-solving using two kinds of scales. The authors investigated whether the choice of response format and judgment scale made a difference and if calibration measures represented the same concept or if different metacognitive monitoring procedures highlighted different aspects of the metacognition-performance-relationships.

Veenman and colleague (2019) studied in the Netherlands whether all measurement methods were suitable for assessing metacognition. They compared the outcome of five instruments to measure metacognition related to mathematical problem-solving of 30 secondary-school students aged 14–15 years. All participants completed two prospective questionnaires, solved five difficult math problems while thinking aloud and being observed, and completed one retrospective questionnaire. In addition, an indicator of mathematics performance was obtained from the students' school. The results on these five measures were compared to investigate what instruments are to be preferred.

2.2 Evidence concerning instructional behaviour and interventions

Ader (2019) examined whether a professional development program was capable of enhancing metacognitive awareness during mathematical activities by primary school teachers in Turkey. More specifically, Ader investigated whether encouragement by the teachers of their students' self-regulated learning and metacognition changed through participating in a professional development program aiming at the improvement of their quality of implementation of mathematical tasks. Twelve class observations took taken place over a 9-month period. Metacognition was assessed, based on work by Zimmerman (2008) and Schraw (1998), using on-line observations and using frequency ratings of teacher planning, monitoring and evaluating behaviour. In addition, the researchers rated whether the teachers gave students opportunities for practicing metacognitive behaviour.

Hacker and colleague (2019) contributed through two intervention studies in the western part of the United States, with children who were at risk for mathematical learning disabilities from Grades 4 to 6. The first study was a study with children randomly assigned to a treatment or control condition ($n=59$). The second study used a single case multiple-baseline design across five special education teachers who were providing mathematics instruction ($n=32$). The intervention was a supplement curriculum that used a multiple-component strategy intervention to improve the proficiency and reasoning with fractions. There was a language-based approach, building on children's understanding of mathematical language through collaboration, representation, writing-to-learn, self-explanation and peer review in order to enhance the understanding of and working with fractions. Hacker and colleague explored whether children benefitted from such metacognitive strategy instruction or not, and whether this could help struggling learners with and without disabilities to make significant gains in solving mathematical problems involving fractions.

Lucangeli and colleagues (2019) contributed through an intervention study with Italian children with dyscalculia ($n=25$) or difficulties in mathematics ($n=9$) from Grade 2 of primary school till the first year of secondary education. All participants completed a calculation and problem-solving test and their ability to reflect on their mental activities was assessed with a metacognitive questionnaire. Lucangeli and colleagues addressed the issue of whether students benefitted from the metacognitive intervention and whether an individual training program using pencil-and-paper computer-assisted tasks and metacognitive instructions and strategies under supervision of specialized psychologists during 16 weeks, could foster the development of self-regulatory and control skills and the mathematical achievement of children with an atypical mathematical development.

Shilo and Kramarski (2019) reported the results of an intervention study in Israel. They examined whether a self-questioning intervention program with metacognitive focal question prompts (what, how/when and why) was able to foster discourse on mathematics and on planning, monitoring and reflection by 32 mathematics teachers and by their 824 fifth grade students. A mathematical-metacognitive discourse program based on Zimmerman's cyclical model (2008) and IMPROVE self-questions (Mevarech and Kramarski 1997) was used, attributing importance to metacognitive awareness regarding conceptualization, verbalization, use of strategies of control processes across the solution phases. The discourse model was based on mathematical domain-specific knowledge as well as on metacognitive planning, monitoring and reflection of the teacher-student prompted discourse. Two video discourse lessons were recorded in each of the participating 32 classes. The level of mathematical-metacognitive discourse was analyzed through examination of discourse sentences (teachers and students) and case studies were used as illustrations.

Temur and colleagues (2019) highlighted what preschool teachers in Turkey spontaneously do to enhance metacognitive strategies during mathematical activities. They observed the metacognitive knowledge, planning, monitoring, controlling and evaluation of 15 preschool teachers. The instructional behaviour of these teachers was observed and recorded, and subsequently coded.

2.3 Models and prediction for mathematics

Zhao and colleagues (2019) were interested in the domain-specificity or domain-general hypothesis of metacognition. They included some empirical data designed for understanding the latent structure of metacognition. Their first study was conducted with 232 moderately performing senior High School students (on average 17 years old) in China. In the second study, 214 senior High School students (also 17 years old on average) participated. Participants in study 1 completed 6 items of a metacognitive questionnaire in the field of reading. In addition, they completed a physics metacognition inventory (26 items) that was adapted to mathematics, measuring cognition knowledge, information management, evaluation, debugging and planning. Participants in study 2 completed the Learning Strategies of Reading and Mathematics tasks from PISA including 12 items about reading behavior and 12 about mathematics and memory.

Vorhölter (2019) contributed through a study in Grade 9 in Germany ($n = 347$) of the conceptualization of group competencies for modelling specific metacognitive strategies for small groups (groups consisting of three or four students). The study was part of a larger project to evaluate a teaching unit designed for fostering modelling competencies of students, including their metacognitive modelling competencies. The perceptions of students and teachers of metacognitive strategies were evaluated. The class was divided into two groups in order to analyse the impact of the teacher's approach. Half of the teachers participated in three teacher training sessions. Students worked in 240 small groups (groups consisting of three or four students) during geometric and arithmetic class, without having experience in working on modelling problems before the study started. In every class, the students were introduced to a modelling problem and then worked on it in groups. After completing this problem, students were asked to complete a questionnaire on metacognitive modelling strategies (for planning and monitoring referred to as 'proceeding', for overcoming difficulties referred to as 'regulation' and for evaluating the modelling process) without speaking to each other. Furthermore, 57 groups were videotaped and 57 students belonging to 17 different groups were interviewed about their perception of the metacognitive strategies they had used. In addition, interviews with 15 teachers took place. After completing the questionnaire, students presented their work in class and teachers expanded on useful or used strategies or mathematical methods.

Desoete and colleagues (2019) investigated whether the same predictors were related to mathematical accuracy and fluency in Belgian elementary school children from Grades 1 to 6 ($n = 2842$). Metacognition was assessed with computerized self-reports. The accuracy of postdiction was calculated based on the comparison of the postdiction and the real performance. In addition, the question of whether motivation and prior mathematics knowledge might be used as predictors for mathematical accuracy and fluency was addressed. Finally, differences between performance groups (poor, moderate and good mathematical task solvers) were analyzed.

Baten and colleague (2019) performed two studies in Belgium. Study 1 was carried out in the general population, involving 63 children between 9 and 12 years old. Study 2 was conducted with 145 elementary school children (72 participants with and 73 participants without mathematical learning disabilities). The role of metacognitive postdiction accuracy and autonomous and controlled motivation in mathematics was explored. The postdiction accuracy index was computed by calculating the absolute value of the difference between the score on procedural calculation test and the self-estimation of the result by the participants.

3 Some preliminary observations

Metacognition remains an important predictor for mathematics performance. In this section we summarize some implications for the conceptualization, the study of individual differences, the assessment tools, and for the training of metacognition.

3.1 Conceptualizations of metacognition

Although the role of metacognition for mathematics education is based on theoretical and empirical work that has been realized during the last five decades, different definitions, conceptualizations and models continue to be popular and the overlap between metacognition and self-regulation (executive functions) has not faded away. There is still no agreement or shared definition. The definitions used in this special issue are briefly summarized below.

Lucangeli and colleagues (2019) defined 'metacognitive knowledge', referring to Cornoldi (2015) as the individual attitudes, knowledge and emotions about mind functioning, helping to understand how different cognitive processes work and which cognitive processes are involved in learning mathematics. 'Metacognitive control' was described as allowing children to esteem task difficulty, plan sub-actions and strategies and auto-check for possible errors in their thinking and performance.

Lingel and colleagues (Lingel et al. 2019) referred to metacognition as the knowledge about the nature of cognitive tasks and about strategies for coping with such tasks, including executive skills related to monitoring and self-regulation of one's own cognitive activities. Self-regulation included planning, directing and evaluating one's behavior.

Ader (2019) addressed the relationship between self-regulation and metacognition, referring to Adagideli and colleagues (2015). These researchers highlighted the affective elements in motivation and social regulation in addition to the cognitive component parts (Ader 2019; Robson 2010), underlining that self-regulated learning makes use of metacognition by engaging in and monitoring reflective, analytical forms of thinking.

Shilo and Kramarski (2019) defined metacognition as a person's self-knowledge of the cognitive processes necessary for understanding the task and the solution strategy (Flavell 1979), and as second-order cognition and thoughts about thoughts, and knowledge about knowledge or reflections about actions (Zohar 2012). They argued that while mathematical discourse refers to domain-specific knowledge that creates a cognitive level, mathematical-metacognitive discourse reflects both the domain-specific and domain-general aspects that create a meta-level process.

In line with Brown (1987) and Flavell (1978), Zhao and colleagues (2019) distinguished, two major components of metacognition, notably metacognitive knowledge and metacognitive skills including metacognitive monitoring and self-regulation. They were especially interested in the domain-generality versus domain-specificity of metacognition.

Desoete and colleagues (2019) differentiated metacognitive knowledge from metacognitive skills and stated that metacognitive accuracy in postdiction might be used as a relevant indicator of metacognition and (later) mathematics performance.

Baten and colleague (2019) stated that the importance and unique explained variance of metacognition might be overestimated, tackling this issue by exploring the combined effect of metacognition and motivation as inclination predictors in order to obtain a more holistic insight into mathematics development.

Temur and colleagues (2019) tapped declarative, procedural and conditional metacognitive knowledge. Declarative knowledge referred to knowledge related to one's general processing abilities. The knowledge concerning how to solve problems successfully was called procedural knowledge. Conditional knowledge meant knowledge concerning when to employ specific strategies. Temur and colleagues (2019) assessed different categories of metacognitive knowledge for mathematics activities.

Veenman and van Cleef (2019) described metacognitive knowledge as declarative knowledge that does not automatically lead to the appropriate strategic behavior (Veenman 2017). In addition, they referred to metacognitive skills as the executive function of metacognition or the procedural knowledge required for the actual regulation of and control over one's learning activities (Veenman 2006).

Vorhölter (2019) focused on group metacognition and on 'metacognitive group strategies' referring to all the strategies that are introduced and used to solve a problem in groups.

In addition, two papers (Vorhölter, 2019; Zhoa et al. 2019) focused on models trying to explain the structure of metacognition. Firstly, Vorhölter (2019) found that their German questionnaire was not task-dependent. Secondly, Zhao and colleagues (2019) revealed in their dataset of Chinese teenagers that a bi-factor model was the best model to explain the structure of metacognition assessed with questionnaires. 'Domain-general' metacognition explained 46.9% of the variance. Mathematics related metacognition explained 17.9% of the variance of mathematics performance. There seemed to be a transition from domain-specific metacognition to more general metacognition, with reading-related metacognition transforming more quickly than mathematics-related metacognition into a more general form of metacognition. In addition, problem-solving, and mathematics learning strategies mediated in the relationship between general metacognition and learning performance.

Taken together, there is plenty of evidence for declarative metacognitive knowledge and procedural or self-regulation skills as separate predictors of mathematical achievement, although both components have been assumed to be interactive. A relevant issue remains whether metacognition is general or field-specific and whether this changes over time.

3.2 Relationships between metacognition and mathematics performance

In line with previous reviews (de Boer and colleagues 2018; Donker and colleagues 2014; Gascoine and colleagues 2017; Schneider and Artelt 2010; Ohtani and Hisasaka 2018), all papers of this special issue investigated the importance of metacognition for mathematics.

Desoete and colleagues (2019) revealed that the metacognitive skills of Belgian children were related to accuracy in mathematics during the whole elementary school period. With R^2 varying from 0.03 (in Grade 2 about 4% explained variance) to 0.16 (in Grade 6 about 16% explained variance), metacognitive skills were significant predictors of mathematical accuracy in all grades. In addition, children evaluated their own performances as worse when they were slower in Grade 3 and 4. Baten and colleague (2019) confirmed this parallel relationship between the metacognitive postdiction skills of Belgian elementary school children from Grades 4 to 6 and their mathematical accuracy and speed, which leads us to the practical advice that teachers ought to pay attention to the componential nature of mathematics and to the accuracy of self-judgments of children when they analyze metacognition in mathematics education. In addition, Hacker and colleague (2019) highlighted an improvement in the accuracy of fraction calculations and of the confidence in those calculations through a metacognitive intervention, within which children become self-regulated learners. However, in the study of Vorhölter (2019) there was no impact of teacher training that aimed at promoting students to use metacognitive modelling strategies in small groups. Finally, Veenman and colleague (2019) reaffirmed the findings of their previous study, that metacognitive skills accounted for about 40% of the variance in mathematics performance (Veenman 2006). Their current study showed a significant correlation between mathematics problem solving and the systematic observation measure ($r=0.52$) as well as with the think-aloud protocol ($r=0.71$) and the retrospective questionnaire ($r=0.34$) for secondary-school students (aged 14–15 years old) in the Netherlands.

To conclude, in line with Verschaffel et al. (2017) there still is strong probative support that metacognition plays a significant role in mathematics performance. However, we should not forget to distinguish between the assessment tools (see also Sect. 3.4), in line with the componential nature of mathematics (Dowker 2015), study results concerning fact retrieval speed and procedural calculation accuracy (Baten and Desoete 2019; Desoete et al. 2019), fractions, modelling (Vorhölter 2019) and problem solving (Veenman and van Cleef 2019), all of which might differ.

3.3 Differential effects for ability levels

Although metacognition appears useful to explain individual differences in mathematics performance, it remains unclear whether children that are low mathematics performers will also perform differently on metacognition, compared with peers who are moderate or good mathematics performers. This special issue enlarges this discussion by looking into recent studies that use metacognition as an instructional design for various ability groups.

In line with Donker and colleagues (2014), Lingel and colleagues (2019) demonstrated that the degree of overconfidence reported about German children in Grade 7 does not differ as a function of achievement level. Although low achievers showed more bias in their predictions, there was no relation between achievement and postdictions. Low-achieving students were able to postdict their performance quite accurately, showing the same level of metacognitive monitoring as high-achieving students.

However, in contrast with Donker and colleagues (2014), Desoete and colleagues (2019) revealed that poor Belgian mathematics performers from Grades 1 to 6 were less accurate compared to age-matched peers, than to well-performing peers. Baten and colleague (2019) added that children with mathematical learning disabilities in Grades 4 to 6 in Belgium differed as far as postdiction accuracy and autonomous motivation from age matched peers without learning disabilities were concerned.

Everything taken together, children with different ability levels might have heterogeneous problems (Pieters et al. 2015). Additional studies seem indicated.

3.4 Assessment of metacognition

Schneider and Artelt (2010) already mentioned the large number of different measures that were being used to assess metacognition. Some decade later, this still is the case. When we compare the instruments used in this special issue, it becomes clear that Lucangeli and colleagues (2019), Zhao and colleagues (2019) and Vorhölter (2019) used off-line measures to assess declarative metacognitive knowledge and self-regulatory activities. On-line measures were used by Ader (2019), Shilo and Kramarski (2019) and Temur and colleagues (2019) to tap procedural metacognitive knowledge during mathematics performance. Veenman and van Cleef (2019) compared on-line and off-line measures. Lingel and colleagues (2019) added to our knowledge by comparing calibration measures. Desoete et al. (2019) and Baten and Desoete (2019) explored procedural metacognitive knowledge employing their postdiction accuracy paradigm. The assessment of metacognition and the difficulties this entails were touched upon by all contributors to this special issue.

We first describe the off-line measures used in this special issue. Lucangeli and colleagues (2019) developed a metacognitive questionnaire for children with dyscalculia or difficulties with mathematics from Grade 2 of primary school till the first year of secondary education. Zhao and colleagues (2019) examined metacognition with a physics metacognition inventory that was adapted to mathematics, measuring knowledge of cognition, information management, evaluation, debugging and planning. Vorhölter (2019) validated a questionnaire about metacognitive modelling strategies (for planning and monitoring, referred to as 'proceeding', for overcoming difficulties referred to as 'regulation' and for evaluating the modelling process) to assess metacognition in small groups in Grade 9 in Germany.

Lingel and colleagues (2019) used calibration measures to compute the correlation between mathematics performance and the metacognitive off-line judgment in German children in Grade 7. The decision to use open- or closed-response format explained up to 17.8% of variance of absolute accuracy measures. Visual analogue scales led to more accurate calibration. Judgments scales explained up to 6.7% of the variance of prediction accuracy. There was a higher accuracy for postdictions than for predictions, but there was overconfidence for both pre- and postdictions. It seems to make sense that researchers report multiple accuracy measures for statistical and interpretative reasons. Baten and colleague (2019) studied postdiction accuracy by calculating the absolute value of the difference between the scores on the procedural calculation test and the self-estimation of the result by the participants from Grades 4 to 6. In the same perspective, Desoete and colleagues (2019) used computerized self-reports by elementary school children in Belgium. Postdictions and real performances were compared.

On-line measures were used by Ader (2019), Shilo and Kramarski (2019) and Temur and colleagues (2019) to tap metacognition during mathematics performance. Ader (2019) analyzed observations and ratings of the frequency of teacher planning, monitoring and evaluating behaviour. Shilo and Kramarski (2019) focused on the level of mathematical-metacognitive discourse sentences (teachers and students). Temur and colleagues (2019) coded metacognitive knowledge, planning, monitoring, controlling and evaluation of 15 preschool teachers.

Veenman and van Cleef contributed through a comparative study using different measures for 14- to 15-year-old secondary school children in the Netherlands. They revealed a significant correlation ($r = 0.83$) between the scores on think-aloud protocols and the results of observations, pointing out the highly convergent validity of the on-line measures. They also stated that, although not significant, thinking aloud tended to be a better predictor than observational methods in terms of variance accounted for concerning mathematics performance. The prospective questionnaires and the on-line measures did not significantly correlate. The retrospective questionnaire did correlate significantly with the think-aloud protocol results ($r = 0.40$) but not with the observation data ($r = .16$).

We can conclude that there still is no 'gold standard' for the assessment of metacognition, making study outcome difficult to compare. Lingel and colleagues even assumed that different tools might measure different constructs. In addition, also the choice of response form affected study outcome. Visual analogue scales led to more accurate calibrations and there was higher accuracy concerning postdictions than for predictions. Veenman and colleague added that, according to their data, on-line instruments (especially think-aloud protocols) are to be favored over off-line instruments for the assessment of metacognitive mathematics skills.

3.5 Interventions on metacognition

Several intervention programs have been developed aiming at improving the metacognition of children. In addition, initiatives have been taken to reform and innovate mathematical classroom practices by instructional approaches and in-service teacher training aiming at dealing with metacognitive skills (Depaepe et al. 2010). In this special issue the outcomes of some recent interventions have been described and assessed.

The studies of Temur and colleagues (2019) and of Vorhölter (2019) emphasized the need for teacher in-service training. Teachers need to be empowered to make children able and willing to learn from the 'opportunities' to which they are exposed in learning content. These findings are in line with Dignath and Büttner (2018), who confirmed that teachers mainly teach cognitive strategies spontaneously, and very few metacognitive strategies, indicating the fact that they need explicit instruction concerning metacognition. This advice has been endorsed, based on their study, by Temur and colleagues (2019), who found that preschool teachers in Turkey lacked the language skills needed to explain and individualize feedback to young children. In addition, these teachers failed to make children think, solving problems routinely instead of having young children learn from these problems. In addition, preschool teachers did not support children's comments on each other's thinking, and they did not summarize what was learned at the end of the activity. Hence, many chances to focus on metacognition were lost.

In this issue three successful training programs and interventions are described. Firstly, Lucangeli and colleagues (2019) successfully showed that an individual training program for children, from Grade 2 of primary school till the first year of secondary education, that used pencil-and-paper computer-assisted tasks and metacognitive instructions and strategies under the supervision of specialized psychologists during 16 weeks, enhanced the accuracy of mental and written calculation, knowledge and number production, and the speed of performing the calculations. Secondly, Hacker and colleague (2019) demonstrated the effect of two intervention studies in the western part of the United States on children with, and at risk for, mathematical learning disabilities in Grades 4 to 6. Study 1 revealed gains in computational accuracy, mathematical reasoning, and in the number of rhetorical elements in writing compared to the control condition. Study 2 showed an improvement of the accuracy of fraction calculations and of the confidence in those calculations, the quality of the mathematical reasoning and their genre knowledge. These are promising results to help children to become self-regulated learners. Finally, Shilo and Kramarski (2019) contributed through a successful intervention study in Israel. The experimental group was exposed to mathematical-metacognitive discourse with a focus on metacognitive processes (i.e., planning, monitoring and reflection). The control group received mathematical discourse with a focus on domain knowledge construction (i.e. declarative, procedural, and explanatory knowledge). The study illustrated more conceptual verbalization related to planning and reflection in the experimental group, whereas the control group scored more highly regarding declarative knowledge. This finding was validated through the case study in the experimental group, indicating a high verbal (explanatory/reflective) dialogue between the teacher and the students. Shilo and Kramarski (2019) argued that despite the norms to prepare students for engaging in the discussion, learners tended to use mathematics discourse in a merely descriptive way focusing on 'what' and 'how'. They directed their discourse towards local processes in learning actions, without global explanation/reflection to understand the goal, strategies and decisions of 'why' the solution process was used. The mathematical-metacognitive support in the experimental group was effective in raising both kinds of discourse.

However, two studies have been included in this special issue (Ader 2019; Vorhölter 2019) focussing without full success on instructional approaches to enhance children's metacognitive learning outcomes. Ader (2019) studied whether a professional development program in Turkey might help preschool teachers to focus more on metacognition while dealing with young children. The program had no substantial effect on the encouragement by teachers of the self-regulated learning and metacognition of their students, however, the program had some positive effect on the improvement of the quality of implementation by the teachers concerning mathematical tasks. Ader partly attributed the absence of more general positive effects to the fact that none of the teachers were specialists in mathematics, referring to Spruce and Bol (2015), who emphasized the need for teaching teachers to self-regulate first, and then provide them with the tools to share knowledge with their students. The study of Vorhölter (2019) also did not mention any impact through three teacher training sessions aimed at encouraging the use by students of metacognitive modelling strategies in small groups.

We conclude that there have been several positive outcomes from metacognitive intervention studies. However, not every attempt or program was equally successful, stressing the need for empirical research that systematically and thoroughly deals with the conditions needed to address the development of metacognitive abilities at school. Shilo and Kramarski (2019) argued that metacognitive knowledge is mostly tacit, remaining unconscious until teachers are challenged to use that knowledge explicitly by naming strategies, modelling, and thinking aloud. In addition it might be that teachers need intensive training aiming at a fundamental change in their belief systems, explicitly addressing the how and why of aspects of metacognitive skills, as was already suggested by Depaepe and colleagues (2010).

4 Concluding remarks for mathematics education

All contributors were asked to include some suggestions for mathematics education.

Lucangeli and colleagues (2019) reached the conclusion that psychoeducational interventions enriching metacognitive and mathematical achievements through error analysis may be an effective way to promote both the development of self-regulatory and control skills and mathematical achievement in children from Grade 2 of elementary school till Grade 1 of secondary school.

Hacker and colleagues (2019) showed how 4th till 6th Grade students with mathematical learning disabilities might acquire computational accuracy and mathematical reasoning due to a metacognitive writing-to-learn intervention. In addition, also the accuracy of fraction calculations by students and their confidence in those calculations might be improved, stressing/highlighting the value of instructional methods helping these students to become self-regulated learners.

Lingel and colleagues (2019) drew our attention to the difficulty of metacognitive tests in open-ended response conditions and to the inaccuracy of a four-point rating scale to map differences concerning judgments. Finally, the study showed that seventh graders in mathematics education are less overconfident when they use postdictions compared to when they have to predict their mathematics competence.

Ader (2019) highlighted that an instructional approach aiming to improve the quality of mathematical tasks in classes by critical reflection on their own practice and learning by the students but without explicit focusing on self-regulation or metacognition, was not enough to have the teachers spontaneously encouraging the students' metacognition.

Shilo and Kramarski (2019) illustrated that student mathematics failures do not always result from a lack of metacognitive knowledge, but might also be due to students' unawareness of how to activate their knowledge and justify or verbalize their actions and conclusions. They disclosed encouraging outcomes through an explicit mathematical-metacognitive discourse training program in Grade 5.

Zhao and colleagues (2019) helped us to understand the structure of metacognition and its relationship to problem-solving strategies for reading and mathematics. They revealed that it is possible to lack metacognitive monitoring skills for mathematics, although this might not be the case for reading tasks. In addition, they demonstrated that learning strategies and problem-solving were mediators between general metacognition and learning performance in senior high school students.

Temur and colleagues (2019) observed metacognition-based activities of preschool teachers in Turkey and found that teachers did not have adequate knowledge or awareness regarding metacognition. They lacked the language to give feedback, make young children think, and comment on other children's' learning. In addition, teachers did not summarize what was learned at the end of the activity. Hence, there was a need for explicit in-service preschool teacher training dealing with the 'what', 'how' and 'why' of metacognitive skills.

Desoete and colleagues (2019) indicated overlapping yet different predictors for mathematical accuracy and fluency entailing the practical advice for teachers to pay attention to both aspects of mathematics. In addition, they observed that poor mathematics performers were less intrinsically motivated, less metacognitively accurate, and that they overestimated their performances more often than well-performing peers in all grades, stressing the importance of paying attention to these aspects in mathematics education.

The studies of Baten and colleague (2019) led to the advice that teachers ought to pay attention to the accuracy of self-judgments of children and differentiate between controlled and autonomous motivation when analyzing motivation in mathematics education.

The importance of choosing valid measures to assess metacognitive skills and strategy use in mathematics has been dealt with by Veenman and colleague (2019). They concluded that on-line methods (and especially think-aloud protocols) are needed to assess metacognitive skills in mathematics.

Vorhölter (2019) revealed the value of a questionnaire to be used in groups but also the trap of a very short (three-teacher training sessions) intervention, which was not beneficial to enhance metacognitive strategies in Grade 9.

All things considered, children who lag behind in terms of metacognitive development may be at risk for study delay in mathematics. Testing their metacognitive skills might be advisable. Since metacognition is teachable (Baten et al. 2017; Lucangeli et al. 2019; Hacker et al. 2019; Shilo et al. 2019), instructional designs can be used exposing children to metacognitive-mathematical discourse, offering them information aimed at explicitly addressing the 'how' and 'why' of metacognitive skills. Nelson and Powell (2017) revealed that for children with mathematical learning difficulties the approach should be error-based (as in the intervention of Lucangeli and colleagues 2019) and sequenced from easy to difficult. Finally, the studies have revealed that mere observation or taking time to reflect was not sufficient (Desoete and Veenman 2006). Metacognitive skills are to be explicitly taught since they do not develop spontaneously from implicit exposure (Ader 2019; Desoete and Veenman 2006). Summarizing, metacognition needs to be taught explicitly in order to develop and to enhance mathematical skills.

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