The problem is to determine at what times (if any) the tractor should be replaced to minimize the total cost for the tractors over 3 years. (Continue at the top of the next column.)

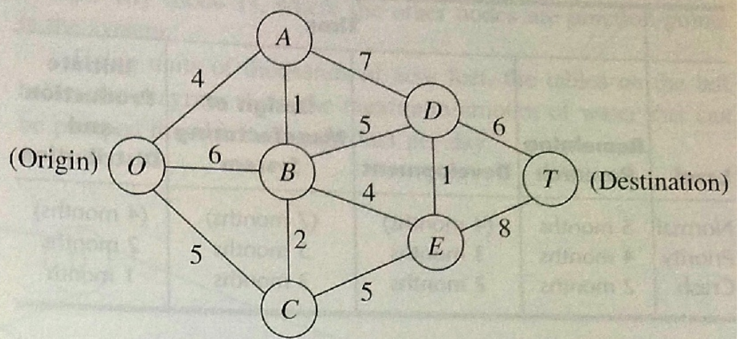
(a) Formulate this problem as a shortest-path problem.

(b) Use the algorithm described in Sec. 9.3 to solve this shortest-path problem.

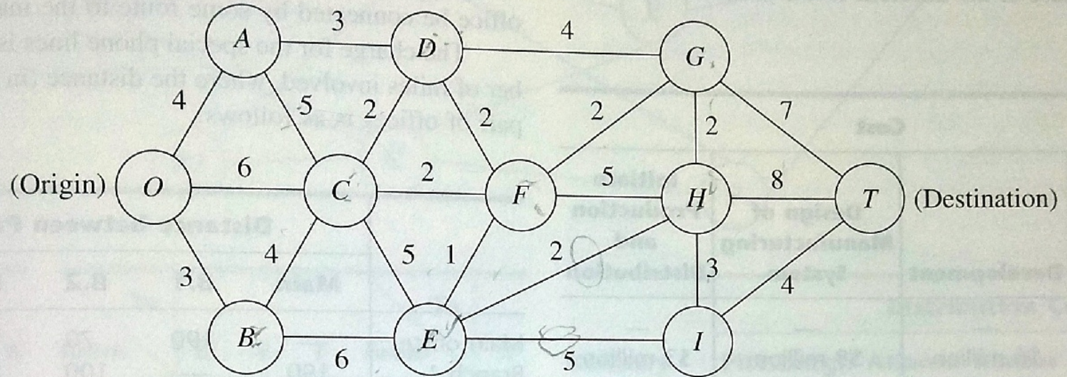
C (c) Formulate and solve a spreadsheet model for this problem.

9.3-3.* Use the algorithm described in Sec. 9.3 to find the *shortest path* through each of the following networks, where the numbers represent actual distances between the corresponding nodes.

(a)



(b)



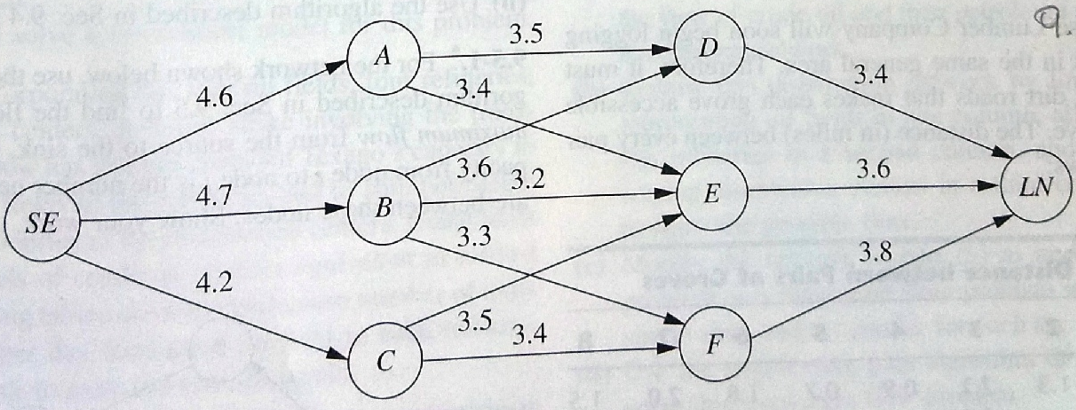
9.3-4. Formulate the shortest-path problem as a linear programming problem.

9.3-5. One of Speedy Airlines' flights is about to take off from Seattle for a nonstop flight to London. There is some flexibility in choosing the precise route to be taken, depending upon weather conditions. The following network depicts the possible routes under consideration, where SE and LN are Seattle and London, respectively, and the other nodes represent various intermediate locations. The winds along each arc greatly affect the flying time (and so the fuel consumption). Based on current meteorologic reports, the flying times (in hours) for this particular flight a shown next to the arcs. Because the fuel consumed is so expe sive, the management of Speedy Airlines has established a poli of choosing the route that minimizes the total flight time.

(a) What plays the role of "distances" in interpreting this probl to be a shortest-path problem?

(b) Use the algorithm described in Sec. 9.3 to solve this shorte path problem.

C (c) Formulate and solve a spreadsheet model for this proble



9.3-6. The Quick Company has learned that a competitor is planning to come out with a new kind of product with a great sales potential. Quick has been working on a similar product that had been scheduled to come to market in 20 months. However, research is nearly complete and Quick's management now wishes to rush the product out to meet the competition.

There are four nonoverlapping phases left to be accomplis including the remaining research that currently is being condu at a normal pace. However, each phase can instead be conducte at a priority or crash level to expedite completion, and these are only levels that will be considered for the last three phases. The ti required at these levels are given in the following table. (The ti in parentheses at the normal level have been ruled out as too lo

Level	Time: Remaining Research	Development	Design of Manufacturing System	Initiate Production and Distribution
Normal	5 months	(4 months)	(7 months)	(4 months)
Priority	4 months	3 months	5 months	2 months
Crash	2 months	2 months	3 months	1 month

Management has allocated $30 million for these four phases. The cost of each phase at the different levels under consideration is as follows:

Level	Cost: Remaining Research	Development	Design of Manufacturing System	Initiate Production and Distribution
Normal	$3 million	—	—	—
Priority	$6 million	$6 million	$9 million	$3 million
Crash	$9 million	$9 million	$12 million	$6 million

Management wishes to determine at which level to conduct each of the four phases to minimize the total time until the product can be marketed subject to the budget restriction of $30 million.

(a) Formulate this problem as a shortest-path problem.

(b) Use the algorithm described in Sec. 9.3 to solve this shortest-path problem.

9.4-1.* Reconsider the networks shown in Prob. 9.3-3. Use the algorithm described in Sec. 9.4 to find the *minimum spanning tree* for each of these networks.

.4-2. The Wirehouse Lumber Company will soon begin logging ight groves of trees in the same general area. Therefore, it must develop a system of dirt roads that makes each grove accessible from every other grove. The distance (in miles) between every pair of groves is as follows:

Grove	Distance between Pairs of Groves: 1	2	3	4	5	6	7	8
1	—	1.3	2.1	0.9	0.7	1.8	2.0	1.5
2	1.3	—	0.9	1.8	1.2	2.6	2.3	1.1
3	2.1	0.9	—	2.6	1.7	2.5	1.9	1.0
4	0.9	1.8	2.6	—	0.7	1.6	1.5	0.9
5	0.7	1.2	1.7	0.7	—	0.9	1.1	0.8
6	1.8	2.6	2.5	1.6	0.9	—	0.6	1.0
7	2.0	2.3	1.9	1.5	1.1	0.6	—	0.5
8	1.5	1.1	1.0	0.9	0.8	1.0	0.5	—

Management now wishes to determine between which pairs of groves the roads should be constructed to connect all groves with a minimum total length of road.

(a) Describe how this problem fits the network description of the minimum spanning tree problem.

(b) Use the algorithm described in Sec. 9.4 to solve the problem.

9.4-3. The Premiere Bank soon will be hooking up computer terminals at each of its branch offices to the computer at its main office using special phone lines with telecommunications devices. The phone line from a branch office need not be connected directly to the main office. It can be connected indirectly by being connected to another branch office that is connected (directly or indirectly) to the main office. The only requirement is that every branch office be connected by some route to the main office.

The charge for the special phone lines is $100 times the number of miles involved, where the distance (in miles) between every pair of offices is as follows:

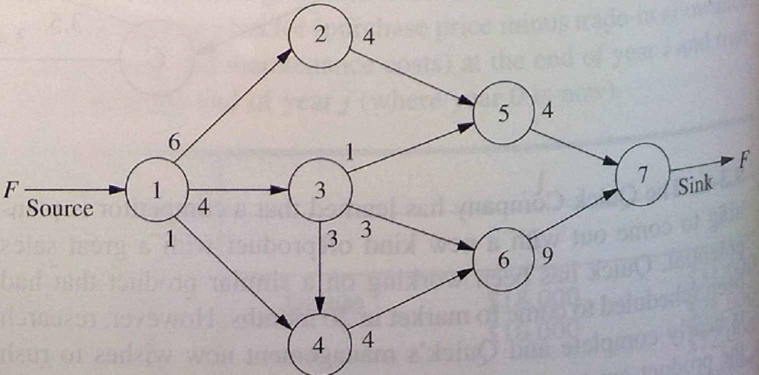
	Distance between Pairs of Offices: Main	B.1	B.2	B.3	B.4	B.5
Main office	—	190	70	115	270	160
Branch 1	190	—	100	110	215	50
Branch 2	70	100	—	140	120	220
Branch 3	115	110	140	—	175	80
Branch 4	270	215	120	175	—	310
Branch 5	160	50	220	80	310	—

Management wishes to determine which pairs of offices should be directly connected by special phone lines in order to connect every branch office (directly or indirectly) to the main office at a minimum total cost.

(a) Describe how this problem fits the network description of the minimum spanning tree problem.

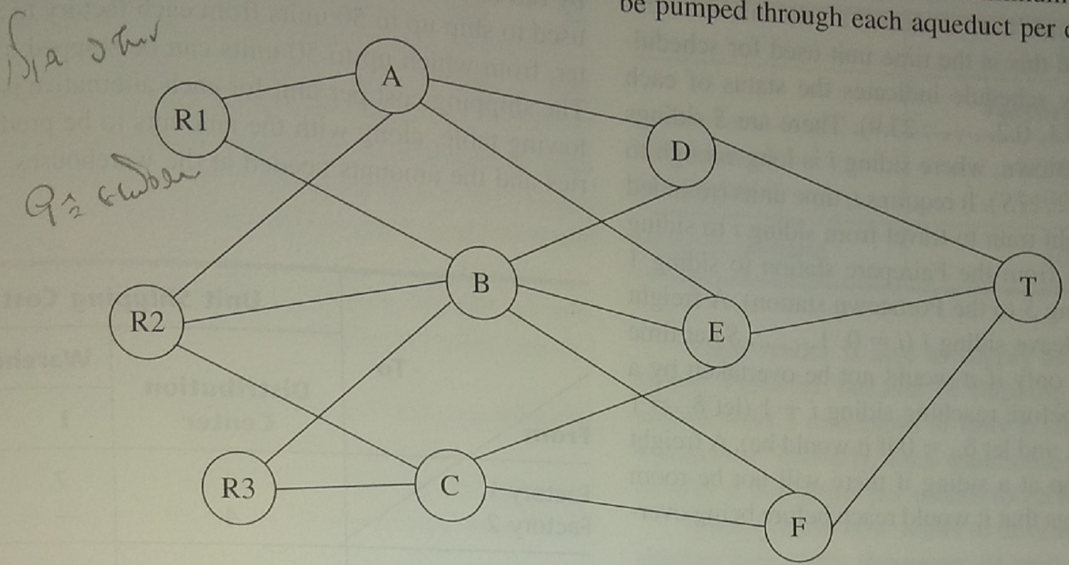
(b) Use the algorithm described in Sec. 9.4 to solve the problem.

9.5-1.* For the network shown below, use the augmenting path algorithm described in Sec. 9.5 to find the flow pattern giving the *maximum flow* from the source to the sink, given that the arc capacity from node i to node j is the number nearest node i along the arc between these nodes. Show your work.



9.5-2. Formulate the maximum flow problem as a linear programming problem.

9.5-3. The diagram below depicts a system of aqueducts that originate at three rivers (nodes R1, R2, and R3) and terminate at a major city (node T), where the other nodes are junction points in the system.



From \ To	A	B	C
R1	75	65	—
R2	40	50	60
R3	—	80	70

From \ To	D	E	F
A	60	45	—
B	70	55	45
C	—	70	90

From \ To	T
D	120
E	190
F	130

Using units of thousands of acre feet, the tables on the left below the diagram show the maximum amount of water that can be pumped through each aqueduct per day.

The city water manager wants to determine a flow plan that will maximize the flow of water to the city.

(a) Formulate this problem as a maximum flow problem by identifying a source, a sink, and the transshipment nodes, and then drawing the complete network that shows the capacity of each arc.

(b) Use the augmenting path algorithm described in Sec. 9.5 to solve this problem.

C (c) Formulate and solve a spreadsheet model for this problem.

9.5-4. The Texago Corporation has four oil fields, four refineries, and four distribution centers. A major strike involving the transportation industries now has sharply curtailed Texago's capacity to ship oil from the oil fields to the refineries and to ship petroleum products from the refineries to the distribution centers. Using units of thousands of barrels of crude oil (and its equivalent in refined products), the following tables show the maximum number of units that can be shipped per day from each oil field to each refinery, and from each refinery to each distribution center.

	Refinery			
Oil Field	**New Orleans**	**Charleston**	**Seattle**	**St. Louis**
Texas	11	7	2	8
California	5	4	8	7
Alaska	7	3	12	6
Middle East	8	9	4	15

	Distribution Center			
Refinery	**Pittsburgh**	**Atlanta**	**Kansas City**	**San Francisco**
New Orleans	5	9	6	4
Charleston	8	7	9	5
Seattle	4	6	7	8
St. Louis	12	11	9	7

The Texago management now wants to determine a plan for how many units to ship from each oil field to each refinery and from each refinery to each distribution center that will maximize the total number of units reaching the distribution centers.

(a) Draw a rough map that shows the location of Texago's oil fields, refineries, and distribution centers. Add arrows to show the flow of crude oil and then petroleum products through this distribution network.

(b) Redraw this distribution network by lining up all the nodes representing oil fields in one column, all the nodes representing refineries in a second column, and all the nodes representing distribution centers in a third column. Then add arcs to show the possible flow.

(c) Modify the network in part (*b*) as needed to formulate this problem as a maximum flow problem with a single source, a single sink, and a capacity for each arc.

(d) Use the augmenting path algorithm described in Sec. 9.5 to solve this maximum flow problem.

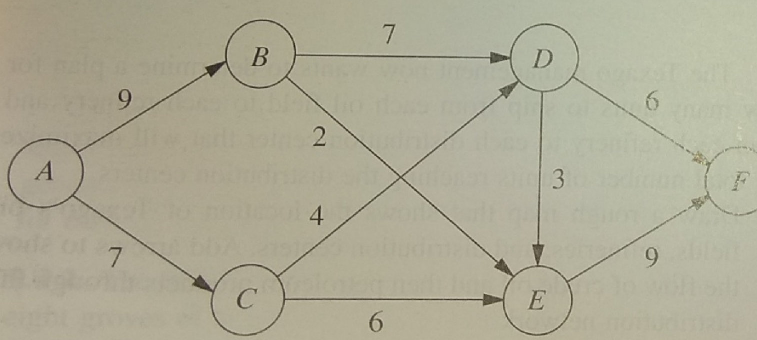
C (e) Formulate and solve a spreadsheet model for this problem.

9.5-5. One track of the Eura Railroad system runs from the major industrial city of Faireparc to the major port city of Portstown. This track is heavily used by both express passenger and freight trains. The passenger trains are carefully scheduled and have priority over the slow freight trains (this is a European railroad), so that the freight trains must pull over onto a siding whenever a passenger train is scheduled to pass them soon. It is now necessary to increase the freight service, so the problem is to schedule the freight trains so as to maximize the number that can be sent each day without interfering with the fixed schedule for passenger trains.

Consecutive freight trains must maintain a schedule differential of at least 0.1 hour, and this is the time unit used for scheduling them (so that the daily schedule indicates the status of each freight train at times 0.0, 0.1, 0.2, . . . , 23.9). There are S sidings between Faireparc and Portstown, where siding i is long enough to hold n_i freight trains ($i = 1, \ldots, S$). It requires t_i time units (rounded up to an integer) for a freight train to travel from siding i to siding $i + 1$ (where t_0 is the time from the Faireparc station to siding 1 and t_s is the time from siding S to the Portstown station). A freight train is allowed to pass or leave siding i ($i = 0, 1, \ldots, S$) at time j ($j = 0.0, 0.1, \ldots, 23.9$) only if it would not be overtaken by a scheduled passenger train before reaching siding $i + 1$ (let $\delta_{ij} = 1$ if it would not be overtaken, and let $\delta_{ij} = 0$ if it would be). A freight train also is required to stop at a siding if there will not be room for it at all subsequent sidings that it would reach before being overtaken by a passenger train.

Formulate this problem as a maximum flow problem by identifying each node (including the supply node and the demand node) as well as each arc and its arc capacity for the network representation of the problem. (*Hint:* Use a different set of nodes for each of the 240 times.)

9.5-6. Consider the maximum flow problem shown below, where the source is node A, the sink is node F, and the arc capacities are the numbers shown next to these directed arcs.



(a) Use the augmenting path algorithm described in Sec. 9.5 to solve this problem.
(b) Formulate and solve a spreadsheet model for this problem.

Reconsider the maximum flow problem shown in Prob. 9.5-6. [Formu]late this problem as a minimum cost flow problem, including the arc $A \to F$. Use $\bar{F} = 20$.

9.6-2. A company will be producing the same new product at two different factories, and then the product must be shipped to two warehouses. Factory 1 can send an unlimited amount by rail to warehouse 1 only, whereas factory 2 can send an unlimited amount by rail to warehouse 2 only. However, independent truckers can be used to ship up to 50 units from each factory to a distribution center, from which up to 50 units can be shipped to each warehouse. The shipping cost per unit for each alternative is shown in the following table, along with the amounts to be produced at the factories and the amounts needed at the warehouses.

From \ To	Unit Shipping Cost			
	Distribution Center	Warehouse 1	Warehouse 2	Output
Factory 1	3	7	—	80
Factory 2	4	—	9	70
Distribution center		2	4	
Allocation		60	90	

(a) Formulate the network representation of this problem as a minimum cost flow problem.
(b) Formulate the linear programming model for this problem.

9.6-3. Reconsider Prob. 9.3-1. Now formulate this problem as a minimum cost flow problem by showing the appropriate network representation.

9.6-4. The Makonsel Company is a fully integrated company that both produces goods and sells them at its retail outlets. After production, the goods are stored in the company's two warehouses until needed by the retail outlets. Trucks are used to transport the goods from the two plants to the warehouses, and then from the warehouses to the three retail outlets.

Using units of full truckloads, the following table shows each plant's monthly output, its shipping cost per truckload sent to each warehouse, and the maximum amount that it can ship per month to each warehouse.

From \ To	Unit Shipping Cost		Shipping Capacity		
	Warehouse 1	Warehouse 2	Warehouse 1	Warehouse 2	Output
Plant 1	\$425	\$560	125	150	200
Plant 2	\$510	\$600	175	200	300

For each retail outlet (RO), the next table shows its monthly demand, its shipping cost per truckload from each warehouse, and the maximum amount that can be shipped per month from each warehouse.

From \ To	Unit Shipping Cost			Shipping Capacity		
	RO1	RO2	RO3	RO1	RO2	RO3
Warehouse 1	$470	$505	$490	100	150	100
Warehouse 2	$390	$410	$440	125	150	75
Demand	150	200	150	150	200	150

Management now wants to determine a distribution plan (number of truckloads shipped per month from each plant to each warehouse and from each warehouse to each retail outlet) that will minimize the total shipping cost.

(a) Draw a network that depicts the company's distribution network. Identify the supply nodes, transshipment nodes, and demand nodes in this network.

(b) Formulate this problem as a minimum cost flow problem by inserting all the necessary data into this network.

C (c) Formulate and solve a spreadsheet model for this problem.

C (d) Use the computer to solve this problem without using Excel.

9.6-5. The Audiofile Company produces boomboxes. However, management has decided to subcontract out the production of the speakers needed for the boomboxes. Three vendors are available to supply the speakers. Their price for each shipment of 1,000 speakers is shown below.

Vendor	Price
1	$22,500
2	$22,700
3	$22,300

In addition, each vendor would charge a shipping cost. Each shipment would go to one of the company's two warehouses. Each vendor has its own formula for calculating this shipping cost based on the mileage to the warehouse. These formulas and the mileage data are shown below.

Vendor	Charge per Shipment
1	$300 + 40¢/mile
2	$200 + 50¢/mile
3	$500 + 20¢/mile

Vendor	Warehouse 1	Warehouse 2
1	1,600 miles	400 miles
2	500 miles	600 miles
3	2,000 miles	1,000 miles

Whenever one of the company's two factories needs a shipment of speakers to assemble into the boomboxes, the company hires a trucker to bring the shipment in from one of the warehouses. The cost per shipment is given in the next column, along with the number of shipments needed per month at each factory.

	Unit Shipping Cost	
	Factory 1	Factory 2
Warehouse 1	$200	$700
Warehouse 2	$400	$500
Monthly demand	10	6

Each vendor is able to supply as many as 10 shipments per month. However, because of shipping limitations, each vendor is able to send a maximum of only 6 shipments per month to each warehouse. Similarly, each warehouse is able to send a maximum of only 6 shipments per month to each factory.

Management now wants to develop a plan for each month regarding how many shipments (if any) to order from each vendor, how many of those shipments should go to each warehouse, and then how many shipments each warehouse should send to each factory. The objective is to minimize the sum of the purchase costs (including the shipping charge) and the shipping costs from the warehouses to the factories.

(a) Draw a network that depicts the company's supply network. Identify the supply nodes, transshipment nodes, and demand nodes in this network.

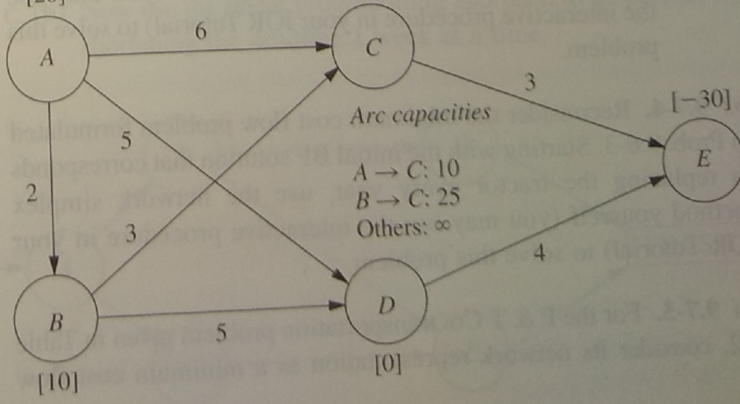
(b) Formulate this problem as a minimum cost flow problem by inserting all the necessary data into this network. Also include a dummy demand node that receives (at zero cost) all the unused supply capacity at the vendors.

C **(c)** Formulate and solve a spreadsheet model for this problem.

C **(d)** Use the computer to solve this problem without using Excel.

D **9.7-1.** Consider the minimum cost flow problem shown below, where the b_i values (net flows generated) are given by the nodes, the c_{ij} values (costs per unit flow) are given by the arcs, and the u_{ij} values (arc capacities) are given between nodes C and D. Do the following work manually.

(a) Obtain an initial BF solution by solving the feasible spanning tree with basic arcs $A \to B$, $C \to E$, $D \to E$, and $C \to A$



(a reverse arc), where one of the nonbasic arcs ($C \to B$) also is a reverse arc. Show the resulting network (including b_i, c_{ij}, and u_{ij}) in the same format as the above one (except use dashed lines to draw the nonbasic arcs), and add the flows in parentheses next to the basic arcs.

(b) Use the optimality test to verify that this initial BF solution is optimal and that there are multiple optimal solutions. Apply one iteration of the network simplex method to find the other optimal BF solution, and then use these results to identify the other optimal solutions that are not BF solutions.

(c) Now consider the following BF solution.

Basic Arc	Flow	Nonbasic Arc
$A \to D$	20	$A \to B$
$B \to C$	10	$A \to C$
$C \to E$	10	$B \to D$
$D \to E$	20	

Starting from this BF solution, apply *one* iteration of the network simplex method. Identify the entering basic arc, the leaving basic arc, and the next BF solution, but do not proceed further.

.7-2. Reconsider the minimum cost flow problem formulated in rob. 9.6-1.

) Obtain an initial BF solution by solving the feasible spanning tree with basic arcs $A \to B$, $A \to C$, $A \to F$, $B \to D$, and $E \to F$, where two of the nonbasic arcs ($E \to C$ and $F \to D$) are *reverse* arcs.

(b) Use the network simplex method yourself (you may use the interactive procedure in your IOR Tutorial) to solve this problem.

-3. Reconsider the minimum cost flow problem formulated in b. 9.6-2.

Obtain an initial BF solution by solving the feasible spanning tree that corresponds to using just the two rail lines plus factory 1 shipping to warehouse 2 via the distribution center.

(b) Use the network simplex method yourself (you may use the interactive procedure in your IOR Tutorial) to solve this problem.

.7-4. Reconsider the minimum cost flow problem formulated b. 9.6-3. Starting with the initial BF solution that corresponds placing the tractor every year, use the network simplex d yourself (you may use the interactive procedure in your utorial) to solve this problem.

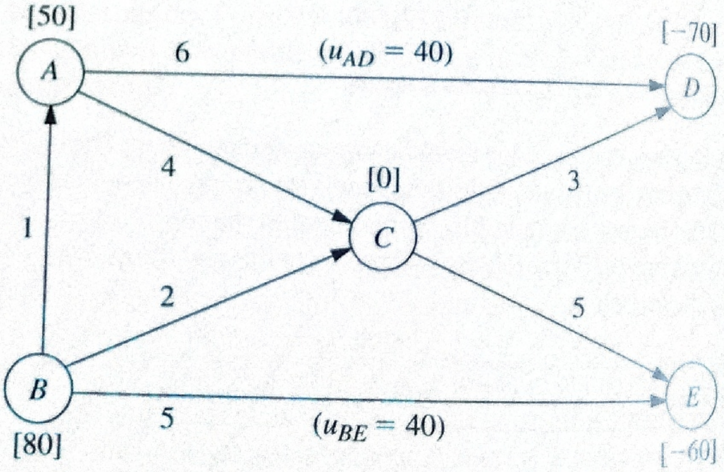
7-5. For the P & T Co. transportation problem given in Table nsider its network representation as a minimum cost flow problem presented in Fig. 8.2. Use the northwest corner rule to obtain an initial BF solution from Table 8.2. Then use the network simplex method yourself (you may use the interactive procedure in your IOR Tutorial) to solve this problem (and verify the optimal solution given in Sec. 8.1).

9.7-6. Consider the Metro Water District transportation problem presented in Table 8.12.

(a) Formulate the network representation of this problem as a minimum cost flow problem. (*Hint:* Arcs where flow is prohibited should be deleted.)

D,I **(b)** Starting with the initial BF solution given in Table 8.19, use the network simplex method yourself (you may use the interactive procedure in your IOR Tutorial) to solve this problem. Compare the sequence of BF solutions obtained with the sequence obtained by the transportation simplex method in Table 8.23.

D,I **9.7-7.** Consider the minimum cost flow problem shown below, where the b_i values are given by the nodes, the c_{ij} values are given by the arcs, and the *finite* u_{ij} values are given in parentheses by the arcs. Obtain an initial BF solution by solving the feasible spanning tree with basic arcs $A \to C$, $B \to A$, $C \to D$, and $C \to E$, where one of the nonbasic arcs ($D \to A$) is a *reverse* arc. Then use the network simplex method yourself (you may use the interactive procedure in your IOR Tutorial) to solve this problem.



9.8-1. The Tinker Construction Company is ready to begin a project that must be completed in 12 months. This project has four activities (*A*, *B*, *C*, *D*) with the project network shown next. The project manager, Sean Murphy, has concluded that he cannot meet the deadline by performing all these activities in the normal way. Therefore, Sean has decided to use the CPM method of time-cost trade-offs to determine the most economical way of crashing the project to meet the deadline. He has gathered the following data for the four activities.