

An asterisk on the problem number indicates that at least a partial answer is given in the back of the book.

11.1-1. Reconsider the California Manufacturing Co. example presented in Sec. 11.1. The mayor of San Diego now has contacted the company's president to try to persuade him to build a factory and perhaps a warehouse in that city. With the tax incentives being offered the company, the president's staff estimates that the net present value of building a factory in San Diego would be \$7 million and the amount of capital required to do this would be \$4 million. The net present value of building a warehouse there would be \$5 million and the capital required would be \$3 million. (This option would be considered only if a factory also is being built there.)

The company president now wants the previous OR study revised to incorporate these new alternatives into the overall problem. The objective still is to find the feasible combination of investments that maximizes the total net present value, given that the amount of capital available for these investments is \$10 million.

- (a) Formulate a BIP model for this problem.
- (b) Display this model on an Excel spreadsheet.
- (c) Use the computer to solve this model.

11.1-2* A young couple, Eve and Steven, want to divide their main household chores (marketing, cooking, dishwashing, and laundering) between them so that each has two tasks but the total time they spend on household duties is kept to a minimum. Their efficiencies on these tasks differ, where the time each would need to perform the task is given by the following table:

	Time Needed per Week			
	Marketing	Cooking	Dishwashing	Laundry
Eve	4.5 hours	7.8 hours	3.6 hours	2.9 hours
Steven	4.9 hours	7.2 hours	4.3 hours	3.1 hours

- (a) Formulate a BIP model for this problem.
- (b) Display this model on an Excel spreadsheet.
- (c) Use the computer to solve this model.

11.1-3. A real estate development firm, Peterson and Johnson, is considering five possible development projects. The following table shows the estimated long-run profit (net present value) that each project would generate, as well as the amount of investment required to undertake the project, in units of millions of dollars.

	Development Project				
	1	2	3	4	5
Estimated profit	1	1.8	1.6	0.8	1.4
Capital required	6	12	10	4	8

The owners of the firm, Dave Peterson and Ron Johnson, have raised \$20 million of investment capital for these projects. Dave and Ron now want to select the combination of projects that will

maximize their total estimated long-run profit (net present value) without investing more than \$20 million.

- (a) Formulate a BIP model for this problem.
- (b) Display this model on an Excel spreadsheet.
- (c) Use the computer to solve this model.

11.1-4. The board of directors of General Wheels Co. is considering seven large capital investments. Each investment can be made only once. These investments differ in the estimated long-run profit (net present value) that they will generate as well as in the amount of capital required, as shown by the following table (in units of millions of dollars):

	Investment Opportunity					
	1	2	3	4	5	6
Estimated profit	17	10	15	19	7	13
Capital required	43	28	34	48	17	32

The total amount of capital available for these investments is \$100 million. Investment opportunities 1 and 2 are mutually exclusive and so are 3 and 4. Furthermore, neither 3 nor 4 can be undertaken unless one of the first two opportunities is undertaken. There are no such restrictions on investment opportunities 5, 6, and 7. The objective is to select the combination of capital investments that will maximize the total estimated long-run profit (net present value).

- (a) Formulate a BIP model for this problem.
- (b) Use the computer to solve this model.

11.1-5. Reconsider Prob. 8.3-4, where a swim team coach needs to assign swimmers to the different legs of a 200-yard medley relay team. Formulate a BIP model for this problem. Identify the groups of mutually exclusive alternatives in this formulation.

11.1-6. Vincent Cardoza is the owner and manager of a machine shop that does custom order work. This Wednesday afternoon, he has received calls from two customers who would like to place rush orders. One is a trailer hitch company which would like to place orders for custom-made heavy-duty tow bars. The other is a mini-car-carrier company which needs some customized stabilizer bars. Both customers would like as many as possible by the end of the week (two working days). Since both products would require the use of the same two machines, Vincent needs to decide and inform the customers this afternoon about how many of each product he will agree to make over the next two days.

Each tow bar requires 3.2 hours on machine 1 and 2 hours on machine 2. Each stabilizer bar requires 2.4 hours on machine 1 and 3 hours on machine 2. Machine 1 will be available for 16 hours over the next two days and machine 2 will be available for 15 hours. The profit for each tow bar produced would be \$130 and the profit for each stabilizer bar produced would be \$150.

Vincent now wants to determine the mix of these products and quantities that will maximize the total profit.

- (a) Formulate an IP model for this problem.
- (b) Use a graphical approach to solve this model.
- (c) Use the computer to solve the model.

11.1-7. Reconsider Prob. 8.2-21 involving a contractor (Susan Meyer) who needs to arrange for hauling gravel from two pits to three building sites.

Susan now needs to hire the trucks (and their drivers) to do the hauling. Each truck can only be used to haul gravel from a single pit to a single site. In addition to the hauling and gravel costs specified in Prob. 8.2-21, there now is a fixed cost of \$50 associated with hiring each truck. A truck can haul 5 tons, but it is not required to go full. For each combination of pit and site, there are now two decisions to be made: the number of trucks to be used and the amount of gravel to be hauled.

- (a) Formulate an MIP model for this problem.
- (b) Use the computer to solve this model.

11.2-1. Select one of the actual applications of BIP by a company or governmental agency mentioned in Sec. 11.2. Read the article describing the application in the referenced issue of *Interfaces*. Write a two-page summary of the application and its benefits.

11.2-2. Select three of the actual applications of BIP by a company or governmental agency mentioned in Sec. 11.2. Read the articles describing the applications in the referenced issues of *Interfaces*. For each one, write a one-page summary of the application and its benefits.

11.3-1.* The Research and Development Division of the Progressive Company has been developing four possible new product lines. Management must now make a decision as to which of these four products actually will be produced and at what levels. Therefore, an operations research study has been requested to find the most profitable product mix.

A substantial cost is associated with beginning the production of any product, as given in the first row of the following table. Management's objective is to find the product mix that maximizes the total profit (total net revenue minus start-up costs).

	Product			
	1	2	3	4
Start-up cost	\$50,000	\$40,000	\$70,000	\$60,000
Marginal revenue	\$ 70	\$ 60	\$ 90	\$ 80

Let the continuous decision variables $x_1, x_2, x_3,$ and x_4 be the production levels of products 1, 2, 3, and 4, respectively. Management has imposed the following policy constraints on these variables:

1. No more than two of the products can be produced.
2. Either product 3 or 4 can be produced only if either product 1 or 2 is produced.
3. Either $5x_1 + 3x_2 + 6x_3 + 4x_4 \leq 6,000$
or $4x_1 + 6x_2 + 3x_3 + 5x_4 \leq 6,000$.

- (a) Introduce auxiliary binary variables to formulate a mixed BIP model for this problem.
- (b) Use the computer to solve this model.

11.3-2. Suppose that a mathematical model fits linear programming except for the restriction that $|x_1 - x_2| = 0, \text{ or } 3, \text{ or } 6$. Show how to reformulate this restriction to fit an MIP model.

11.3-3. Suppose that a mathematical model fits linear programming except for the restrictions that

1. At least one of the following two inequalities holds:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &\leq 4 \\ 3x_1 - x_2 - x_3 + x_4 &\leq 3. \end{aligned}$$

2. At least two of the following four inequalities holds:

$$\begin{aligned} 5x_1 + 3x_2 + 3x_3 - x_4 &\leq 10 \\ 2x_1 + 5x_2 - x_3 + 3x_4 &\leq 10 \\ -x_1 + 3x_2 + 5x_3 + 3x_4 &\leq 10 \\ 3x_1 - x_2 + 3x_3 + 5x_4 &\leq 10. \end{aligned}$$

Show how to reformulate these restrictions to fit an MIP model.

11.3-4. The Toys-R-4-U Company has developed two new toys for possible inclusion in its product line for the upcoming Christmas season. Setting up the production facilities to begin production would cost \$50,000 for toy 1 and \$80,000 for toy 2. Once these costs are covered, the toys would generate a unit profit of \$10 for toy 1 and \$15 for toy 2.

The company has two factories that are capable of producing these toys. However, to avoid doubling the start-up costs, just one factory would be used, where the choice would be based on maximizing profit. For administrative reasons, the same factory would be used for both new toys if both are produced.

Toy 1 can be produced at the rate of 50 per hour in factory 1 and 40 per hour in factory 2. Toy 2 can be produced at the rate of 40 per hour in factory 1 and 25 per hour in factory 2. Factories 1 and 2, respectively, have 500 hours and 700 hours of production time available before Christmas that could be used to produce these toys.

It is not known whether these two toys would be continued after Christmas. Therefore, the problem is to determine how many units (if any) of each new toy should be produced before Christmas to maximize the total profit.

- (a) Formulate an MIP model for this problem.
- (b) Use the computer to solve this model.

11.3-5.* Northeastern Airlines is considering the purchase of new long-, medium-, and short-range jet passenger airplanes. The purchase price would be \$67 million for each long-range plane, \$50 million for each medium-range plane, and \$35 million for each short-range plane. The board of directors has authorized a maximum commitment of \$1.5 billion for these purchases. Regardless of which airplanes are purchased, air travel of all distances is expected to be sufficiently large that these planes would be utilized at essentially maximum capacity. It is estimated that the net annual profit (after capital recovery costs are subtracted) would be \$4.2 million per long-range plane, \$3 million per medium-range plane, and \$2.3 million per short-range plane.

It is predicted that enough trained pilots will be available to the company to crew 30 new airplanes. If only short-range planes