10.13

Σχεδιάστε το επίπεδο φάσης για την

$$x'' = \frac{1}{(3-x)^2} - \frac{4}{x^2} =: f(x)$$

$$\begin{cases} x_1' = x_2 \\ x_2' = \dfrac{1}{(3-x_1)^2} - \dfrac{4}{x_1^2} \end{cases}$$

A. Σημεία Ισορροπίας : $x_2 = 0$, $\frac{1}{(3-x_1)^2} = \frac{4}{x_1^2} \Leftrightarrow x_1 = 2,\ x_1 = 6$

$(2,0)$, $(6,0)$

Γραμμικοποίηση στο $(2,0)$

$$\left.\frac{\partial f}{\partial x}\right|_{(2,0)} = \left.\begin{pmatrix} 0 & 1 \\ \frac{2}{(3-x_1)^3} + \frac{8}{x_1^3} & 0 \end{pmatrix}\right|_{(2,0)} = \begin{pmatrix} 0 & 1 \\ 3 & 0 \end{pmatrix} =: A$$

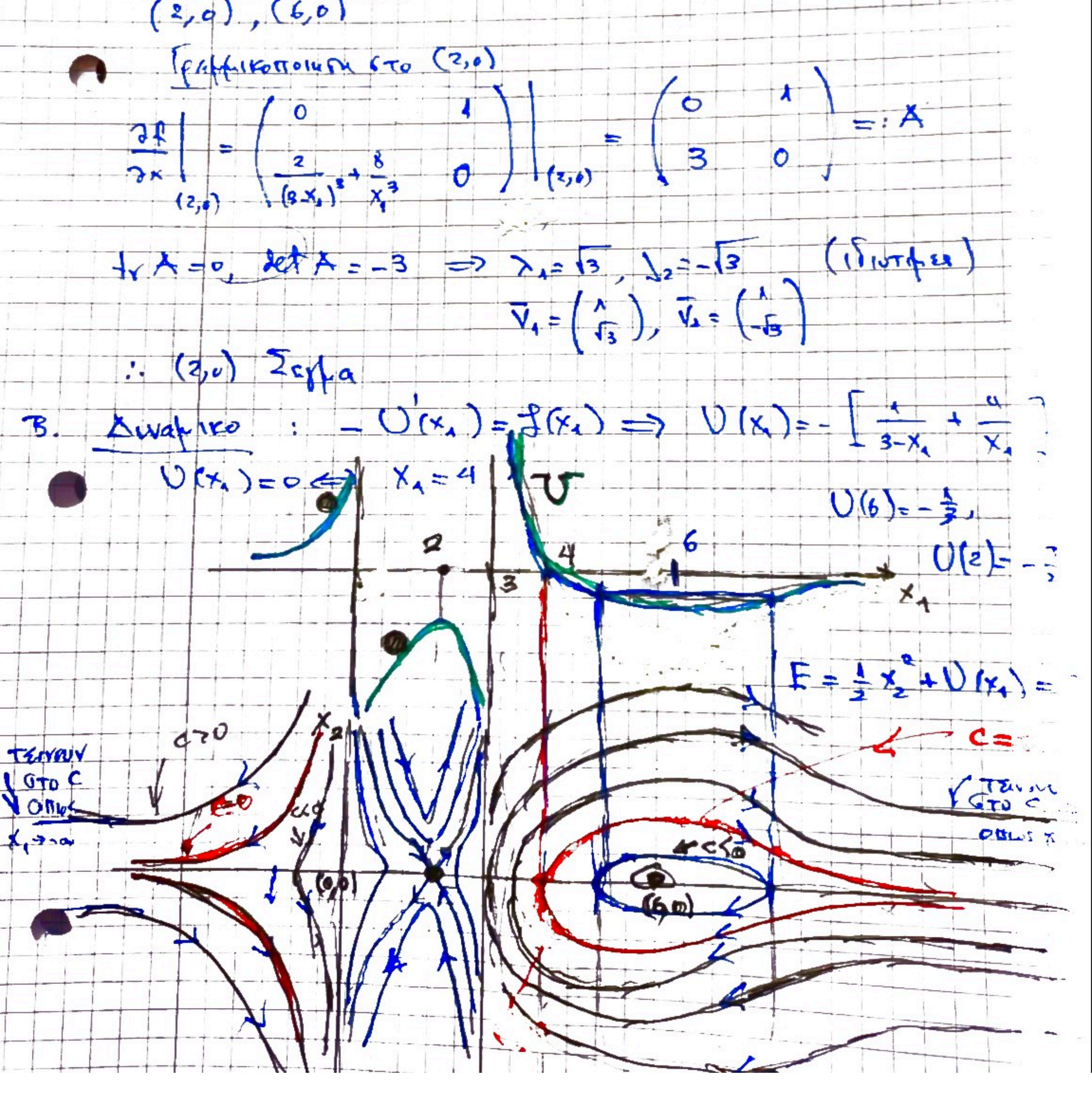
$tr A = 0$, $\det A = -3 \Rightarrow \lambda_1 = \sqrt{3}$, $\lambda_2 = -\sqrt{3}$ (ιδιοτιμές)

$\bar{v}_1 = \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$, $\bar{v}_2 = \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix}$

$\therefore$ $(2,0)$ Σάγμα

B. Δυναμικό : $-U'(x_1) = f(x_1) \Rightarrow U(x_1) = -\left[\frac{1}{3-x_1} + \frac{4}{x_1}\right]$

$U(x_1) = 0 \Leftrightarrow x_1 = 4$



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Προσοχή στα εξής σημεία (10.13)

- Μηδενικά του U, U'
- $c=0$ (διαχωρίζουσες) $c<0$ / $c>0$
- Περιοδικές ($c<0$) ($x_1>3$)
  Μη φραγμένες ($c>0$)
- Για $x_1<0$ $\boxed{c=0}$ διαχωρίζει
- Βελάκια

$\boxed{10.14}$

$$\begin{cases} x' = y + x^3\cos(x^2+y^2) \\ y' = -x + y^3\cos(x^2+y^2) \end{cases}$$

$x = r\cos\theta,\ y = r\sin\theta$

$x' = r'\cos\theta - r\sin\theta\,\theta'$

$y' = r'\sin\theta + r\cos\theta\,\theta'$

$$\begin{cases} r'\cos\theta - r\sin\theta\,\theta' = r\sin\theta + r^3\cos^3\theta\cos r^2 \\ r'\sin\theta + r\cos\theta\,\theta' = -r\cos\theta + r^3\sin^3\theta\cos r^2 \end{cases}$$

Multiply by $\cos\theta$ the 1^{st}, by $\sin\theta$ the 2^{nd}, add:

$r'\cos^2\theta - r\cos\theta\sin\theta\,\theta' = r\sin\theta\cos\theta + r^3\cos^4\theta\cos r^2$

$r'\sin^2\theta + r\sin\theta\cos\theta\,\theta' = -r\sin\theta\cos\theta + r^3\sin^4\theta\cos r^2$

Add

(1) $\boxed{r' = r^3\cos r^2\left(\sin^4\theta + \cos^4\theta\right)}$

Next x by $\sin\theta$ the 1^{st}, $\cos\theta$ the 2^{nd}:

$r'\sin\theta\cos\theta - r\sin^2\theta\,\theta' = r\sin^2\theta + r^3\sin\theta\cos^3\theta\cos r^2$

$r'\sin\theta\cos\theta + r\cos^2\theta\,\theta' = -r\cos^2\theta + r^3\sin^3\theta\cos\theta\cos r^2$

subtract 1^{st} from 2^{nd}:

$r\theta' = -r + r^3\cos r^2\left(\sin^3\theta\cos\theta - \sin\theta\cos^3\theta\right)$

$= -r + r^3\cos r^2\sin\theta\cos\theta\left(\sin^2\theta - \cos^2\theta\right)$

$= -r + r^3\cos r^2\,\frac{1}{2}\sin 2\theta\,(-\cos 2\theta)$

$= -r + r^3\cos r^2\left(\frac{-1}{4}\right)\sin 4\theta$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\sin^2\theta - \cos^2\theta = -\cos 2\theta$

$\sin 2\varphi = 2\sin\varphi\cos\varphi$

$2\sin(2\theta)\cos(2\theta) = \sin 4\theta$

$\therefore$

(2) $\boxed{\theta' = -1 + r^2\cos r^2\left(\frac{1}{4}\right)\sin 4\theta}$

$r^2 = \frac{\pi}{2} + k\pi,\ k = 0,1,2,\dots \Rightarrow r_k = \sqrt{\frac{\pi}{2} + k\pi}$

$r_{k+1} - r_k < r_k \cdot r_k \cdot$ (rough)