Άσκηση Διάλεξη 11

$\bar{\eta} = \frac{1-x}{\sqrt{\varepsilon}}$

$$-\frac{\varepsilon \bar{y}'''(\bar\eta)}{\varepsilon^{3/2}} + \frac{\bar y'(\bar\eta)}{\varepsilon^{1/2}} + (-\varepsilon^{1/2}\bar\eta + 1)\bar y(\bar\eta) = 0 \Rightarrow$$

$$\bar y'''(\bar\eta) - \bar y'(\bar\eta) = \varepsilon^{1/2}\bar y(\bar\eta) - \bar\eta\varepsilon \bar y(\bar\eta)$$

$$\bar y(\bar\eta) = \bar y_0(\bar\eta) + \varepsilon^{1/2}\bar y_{1/2}(\bar\eta) + \varepsilon \bar y_1(\bar\eta) + \dots$$

$$\left(\bar y_0(\bar\eta) + \varepsilon^{1/2}\bar y_{1/2}(\bar\eta) + \varepsilon \bar y_1(\bar\eta)\right)''' - \left(\bar y_0(\bar\eta) + \varepsilon^{1/2}\bar y_{1/2}(\bar\eta) + \varepsilon \bar y_1(\bar\eta)\right)' = \varepsilon^{1/2}\left(\bar y_0(\bar\eta) + \varepsilon^{1/2}\bar y_{1/2}(\bar\eta) + \varepsilon \bar y_1(\bar\eta)\right)$$
$$-\bar\eta\varepsilon\left(\bar y_0(\bar\eta) + \varepsilon^{1/2}\bar y_{1/2}(\bar\eta) + \varepsilon \bar y_1(\bar\eta)\right)$$

ε^0: $\bar y_0''' - \bar y_0' = 0$ $\qquad \bar y_0(\bar\eta) = A_0 e^{-\bar\eta} + B_0 e^{\bar\eta} + C_0$

$\varepsilon^{1/2}$: $\bar y_{1/2}''' - \bar y_{1/2}' = \bar y_0$ $\qquad \bar y_{1/2}(\bar\eta) = A_1 e^{-\bar\eta} + B_1 e^{\bar\eta} + C_1 + \frac{A_0}{2} e^{-\bar\eta}\cdot\bar\eta + \frac{B_0}{2} e^{\bar\eta}\cdot\bar\eta - C_0\bar\eta$

$B_1 = B_0 = 0$ για να μην αυξάνει εκθετικά.

Επίσης για $x=1$ $\bar\eta = 0$ $\qquad \bar y_0(0) = 1 \Rightarrow A_0 + C_0 = 1$

$\qquad \bar y_1(0) = 0 \Rightarrow A_1 + C_1 = 0$

$$\bar y(\bar\eta) = A_0 e^{-\bar\eta} + C_0 + \varepsilon^{1/2}\left[A_1 e^{-\bar\eta} + C_1 + \frac{A_0}{2} e^{-\bar\eta}\bar\eta - C_0\bar\eta\right]$$

για $\bar\eta \to \infty$, $\varepsilon^{1/2}\bar\eta \to 0$ $\qquad \bar y(\bar\eta) = C_0 + \varepsilon^{1/2} C_1$

για $x \to 1$, $\varepsilon \to 0$ $\qquad y_{ef}(x) = e^{1/2} + \varepsilon^{1/2} e^{1/2}$

$\Rightarrow C_0 = e^{1/2}$, $C_1 = e^{1/2}$

$A_0 = 1 - e^{1/2} \qquad A_1 = -e^{1/2}$

$\bar y_0 = (1-\sqrt{e})e^{-\bar\eta} + \sqrt{e}$

$\bar y_{1/2} = -\sqrt{e}\cdot e^{-\bar\eta} + \sqrt{e} + \frac{(1-\sqrt{e})}{2} e^{-\bar\eta}\bar\eta - \sqrt{e}\,\bar\eta = \left[-\sqrt{e} - \frac{1}{2}(\sqrt{e}-1)\bar\eta\right]e^{-\bar\eta} - \sqrt{e}\,\bar\eta + \sqrt{e}$

$$\bar y(\bar\eta) = (1-\sqrt{e})e^{-\bar\eta} + \sqrt{e} + \sqrt{\varepsilon}\left[\left[-\sqrt{e} - \frac{\bar\eta}{2}(\sqrt{e}-1)\right]e^{-\bar\eta} - \sqrt{e}\,\bar\eta + \sqrt{e}\right]$$

$$\Rightarrow y_u = e^{\frac{x^2}{2}}\left[1 + \varepsilon^{1/2}\right] - \sqrt{\varepsilon}\cdot e^{-\frac{x}{\sqrt{\varepsilon}}} + e^{\frac{x-1}{\sqrt{\varepsilon}}}\left[(\sqrt{e}-1)\left(-\frac{3}{2} + \frac{x}{2}\right) - \sqrt{\varepsilon}\sqrt{e}\right]$$

εξωτερική $\qquad$ στρώμα: 0 $\qquad$ στρώμα: 1