$y_\varepsilon(x) = \frac{1}{1-e^{\frac{2}{\varepsilon}}}\left(e^{-\frac{x}{\varepsilon}} - e^{\frac{x-2}{\varepsilon}}\right)$ $\qquad \lim_{\varepsilon\to 0}\frac{1}{\varepsilon}\int_0^1 y_\varepsilon(x) = 1$

$$\lim_{\varepsilon\to 0}\left|\frac{1}{\varepsilon}\int_0^1 y_\varepsilon(x)(\varphi(x)-\varphi(0))\,dx\right| \le \lim_{\varepsilon\to 0}\underbrace{\left|\frac{1}{\varepsilon}\int_0^\delta y_\varepsilon(x)(\varphi(x)-\varphi(0))\,dx\right|}_{I_1} + \underbrace{\left|\frac{1}{\varepsilon}\int_\delta^1 y_\varepsilon(x)(\varphi(x)-\varphi(0))\,dx\right|}_{I_2}$$

$$I_2 = \left|\lim_{\varepsilon\to 0}\frac{1}{\varepsilon(1-e^{\frac{2}{\varepsilon}})}\int_\delta^1 \left(e^{-\frac{x}{\varepsilon}} - e^{-\frac{x-2}{\varepsilon}}\right)(\varphi(x)-\varphi(0))\,dx\right| \le \lim_{\varepsilon\to 0}\frac{c}{\varepsilon(1-e^{\frac{2}{\varepsilon}})}\left|\int_\delta^1 e^{-\frac{x}{\varepsilon}} + e^{-\frac{x-2}{\varepsilon}}\right|dx = 0$$

$I_2 = 0$ όπου $|\varphi(x)-\varphi(0)| = c$ αφού $\varphi\in C_c(\mathbb{R})$

$I_1 = \lim_{\varepsilon\to 0}\left|\frac{1}{\varepsilon}\int_0^\delta y_\varepsilon(x)(\varphi(x)-\varphi(0))\,dx\right|$ αφού δ κοντά στο 0 λόγω συνέχειας $|\varphi(x)-\varphi(0)| \le \eta$

$$I_1 \le \lim_{\varepsilon\to 0}\frac{\eta}{\varepsilon}\int_0^\delta y_\varepsilon(x)\,dx \le \lim_{\varepsilon\to 0}\eta\,\frac{1}{\varepsilon}\int_0^1 y_\varepsilon(x)\,dx = \eta$$

άρα $\lim_{\varepsilon\to 0}\left|\frac{1}{\varepsilon}\int_0^1 y_\varepsilon(x)(\varphi(x)-\varphi(0))\,dx\right| \le \eta \Rightarrow \lim\frac{1}{\varepsilon}\int_0^1 y_\varepsilon(x)\varphi(x)\,dx \to \varphi(0)$