

$$\hat{\theta}_j = n\hat{\theta} - (n-1)\hat{\theta}_{(j)}, \quad \bar{\hat{\theta}} = \frac{1}{n} \sum_{i=1}^n \hat{\theta}_{(i)}$$

$$\hat{\theta}_j = \frac{1}{n} \sum_{i=1}^n P_i, \quad P_i = n\hat{\theta} - (n-1)\hat{\theta}_{(i)}$$

$$\hat{\theta}_j = n\hat{\theta} - (n-1)\hat{\theta}_{(j)} =$$

$$= n\hat{\theta} - \frac{(n-1) \sum_{i=1}^n \hat{\theta}_{(i)}}{n} =$$

$$= \frac{n^2\hat{\theta} - (n-1)[\hat{\theta}_{(1)} + \hat{\theta}_{(2)} + \dots + \hat{\theta}_{(n)}]}{n} =$$

$$= \frac{n\hat{\theta} + n\hat{\theta} + n\hat{\theta} + \dots + n\hat{\theta} - (n-1)\hat{\theta}_{(1)} - (n-1)\hat{\theta}_{(2)} - \dots - (n-1)\hat{\theta}_{(n)}}{n} =$$

$$= \frac{[n\hat{\theta} - (n-1)\hat{\theta}_{(1)}] + [n\hat{\theta} - (n-1)\hat{\theta}_{(2)}] + \dots + [n\hat{\theta} - (n-1)\hat{\theta}_{(n)}]}{n} =$$

$$= \frac{P_1 + P_2 + \dots + P_n}{n} = \frac{\sum_{i=1}^n P_i}{n}$$

n=6

4, 3, 7, 0, 5, 9

$x_1$   $x_2$   $x_3$   $x_4$   $x_5$   $x_6$   
3, 4, 5, 6, 7, 9

$$\hat{\theta} = \frac{34}{6} = 5,66$$

$$\hat{\theta}_{(1)} = \frac{4+5+6+7+9}{5} = 6,2$$

$$\hat{\theta}_{(2)} = \frac{3+5+6+7+9}{5} = 6,0$$

$$\hat{\theta}_{(3)} = \frac{3+4+6+7+9}{5} = 5,8$$

$$\hat{\theta}_{(4)} = \frac{3+4+5+7+9}{5} = 5,6$$

$$\hat{\theta}_{(5)} = \frac{3+4+5+6+9}{5} = 5,4$$

$$\hat{\theta}_{(6)} = \frac{3+4+5+6+7}{5} = 5,0$$

$$P_1 = 6 \cdot 5,66 - 5 \cdot 6,2 = 3$$

$$P_2 = 6 \cdot 5,66 - 5 \cdot 6,0 = 4$$

$$P_3 = 6 \cdot 5,66 - 5 \cdot 5,8 = 5$$

$$P_4 = 6 \cdot 5,66 - 5 \cdot 5,6 = 6$$

$$P_5 = 6 \cdot 5,66 - 5 \cdot 5,4 = 7$$

$$P_6 = 6 \cdot 5,66 - 5 \cdot 5,0 = 9$$

$$\hat{\theta}_j = 6 \cdot 5,66 - 5 \cdot \frac{(6,2+6,0+5,8+5,6+5,4+5,0)}{6} =$$
$$= 6 \cdot 5,66 - 5 \cdot \frac{34}{6} =$$

$$= 6 \cdot 5,66 - 5 \cdot 5,66 = 5,66 = \bar{x}$$

Με βάση τις παρατηρήσεις η  $\hat{\theta}_j$  είναι:

$$\hat{\theta}_j = \frac{\sum_{i=1}^6 P_i}{6} = \frac{3+4+5+6+7+9}{6} = \bar{x}$$

$n=6$   
Zur

Διατάξεις  
 $x_1$   $x_2$   $x_3$  |  $x_4$   $x_5$   $x_6$   
3, 4, 5, 6, 7, 9  
 $x_{\frac{n}{2}}$   $x_{\frac{n}{2}+1}$

$$\hat{M} = \frac{5+6}{2} = 5.5$$

$$\frac{1}{2}(x_{\frac{n}{2}} + x_{\frac{n}{2}+1})$$

$$\hat{\theta}_{(1)} = 6$$

$$\hat{\theta}_{(2)} = 6$$

$$\hat{\theta}_{(3)} = 6$$

$$\hat{\theta}_{(4)} = 5$$

$$\hat{\theta}_{(5)} = 5$$

$$\hat{\theta}_{(6)} = 5$$

$$\hat{\theta}_j = \frac{6 \cdot 5.5 - 5 \cdot (3 \cdot 6 + 3 \cdot 5)}{6} =$$

$$= \frac{6 \cdot 5.5 - 5 \cdot 33}{6} =$$

$$= \frac{6 \cdot 5.5 - 5 \cdot 5.5}{6} =$$

$$= 5.5$$

$$n=7$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 : \text{ Διατεταγμένες}$$

$\swarrow \quad \downarrow \quad \downarrow$

$$x_{\frac{n+1}{2}-1} \quad x_{\frac{n+1}{2}} \quad x_{\frac{n+1}{2}+1}$$

$$\frac{n-1}{2}$$

$$\hat{\theta}_{(i)} = \frac{1}{2} \left( x_{\frac{n+1}{2}} + x_{\frac{n+1}{2}+1} \right), \quad i < \frac{n+1}{2}$$

$$\hat{\theta}_{\left(\frac{n+1}{2}\right)} = \frac{1}{2} \left( x_{\frac{n+1}{2}-1} + x_{\frac{n+1}{2}+1} \right), \quad i = \frac{n+1}{2}$$

$$\hat{\theta}_{(i)} = \frac{1}{2} \left( x_{\frac{n+1}{2}-1} + x_{\frac{n+1}{2}} \right), \quad i > \frac{n+1}{2}$$

$$\frac{n-1}{2} + 1 + \frac{n-1}{2} = n-1+1 = n$$