$P_1 = P$ $\quad \theta_1, \theta_2 \quad 1-P = P_2$ $\quad \theta_1, \theta_2, \theta_3, \dots, \theta_k$ $\quad P_1, P_2, P_3, \dots, P_k$

$f(x|\theta_1) \quad f(x|\theta_2)$ $\quad \sum_{j=1}^{k} P_j = 1$, $P_j > 0$

$$f(x) = \sum_{j=1}^{2} P_j f(x|\theta_j)$$

$$L(\theta_1, \theta_2, P|X) = \prod_{i=1}^{n} \left(P \frac{e^{-\theta_1} \theta_1^{x_i}}{x_i!} + (1-P) \frac{e^{-\theta_2} \theta_2^{x_i}}{x_i!} \right)$$

E - M

$z_{ij} = \begin{cases} 1, & \text{αν η } i \text{ παρατήρηση ανήκει στη } j \text{ όμαδα} \\ 0, & \text{διαφορετικά} \end{cases}$

$z_{i1} = \begin{cases} 1, & \text{η } i \text{ παρατήρηση προέρχεται από την κατανομή Poisson με παράμετρο } \theta_1 \\ 0, & \text{διαφορετικά} \end{cases}$

$z_{i2} = \begin{cases} 1, & \sim Pois(\theta_2) \\ 0, & \text{διαφορετικά} \end{cases}$

$$L(\theta_1, \theta_2, P_1, P_2 | X, Z) = \prod_{i=1}^{n} \prod_{j=1}^{2} \left[P_j \frac{e^{-\theta_j} \theta_j^{x_i}}{x_i!} \right]^{z_{ij}}$$

$$l(\theta_1, \theta_2, P_1, P_2 | X, Z) =$$

$$\sum_{i=1}^{n} \left[z_{i1} \log P_1 - z_{i1} \cdot \theta_1 + z_{i1} x_i \log \theta_1 - z_{i1} \log x_i! \right.$$

$$\left. + z_{i2} \log(1-P_1) - z_{i2} \theta_2 + z_{i2} x_i \log \theta_2 - z_{i2} \log x_i! \right]$$

$P_1 = P$, $P_2 = 1-P$

$$\frac{\partial l(\theta_1, \theta_2, P_1, P_2 | X, Z)}{\partial \theta_1} = -\sum_{i=1}^{n} z_{i1} + \frac{1}{\theta_1} \sum_{i=1}^{n} z_{i1} x_i = 0$$

$$\hat{\theta}_1 = \frac{\sum_{i=1}^{n} z_{i1} x_i}{\sum_{i=1}^{n} z_{i1}}$$

$$\hat{\theta}_2 = \frac{\sum_{i=1}^{n} z_{i2} x_i}{\sum_{i=1}^{n} z_{i2}}$$

$$\frac{\partial l(\theta_1, \theta_2, P_1 | X, Z)}{\partial P} = \frac{1}{P_1} \sum_{i=1}^{n} z_{i1} - \frac{1}{1-P_1} \sum_{i=1}^{n} z_{i2} = 0$$

$$P_1 = \frac{\sum_{i=1}^{n} z_{i1}}{\sum_{i=1}^{n} (z_{i1} + z_{i2})} = n$$

$$P_1 = \frac{\sum_{i=1}^{n} z_{i1}}{n}$$

$$W_{i1} = E(z_{i1} | X, \theta_1, \theta_2) = \frac{P_1 f(x_i|\theta_1)}{P_1 f(x_i|\theta_1) + (1-P_1) f(x_i|\theta_2)}$$

$$W_{i2} = E(z_{i2} | X, \theta_1, \theta_2) = \frac{(1-P_1) f(x_i|\theta_2)}{P_1 f(x_i|\theta_1) + (1-P_1) f(x_i|\theta_2)}$$

EM για μείγμα 2
Poisson με παραμέτρους
θ_1, θ_2.

Δώσε αρχικές τιμές για
p_1, θ_1, θ_2

E-step: Υπολόγισε:

$$w_{i1} = E(z_{i1}) = \frac{p_1 \cdot f(x_i|\theta_1)}{p_1 \cdot f(x_i|\theta_1) + (1-p_1) \cdot f(x_i|\theta_2)}$$

$$w_{i2} = E(z_{i2}) = \frac{(1-p_1) f(x_i|\theta_2)}{p_1 f(x_i|\theta_1) + (1-p_1) f(x_i|\theta_2)}$$

M-step: Υπολόγισε:

$$\theta_1^{(new)} = \frac{\sum_{i=1}^{n} w_{i1} \cdot x_i}{\sum_{i=1}^{n} w_{i1}}$$

$$\theta_2^{(new)} = \frac{\sum_{i=1}^{n} w_{i2} \cdot x_i}{\sum_{i=1}^{n} w_{i2}}$$

$$p_1^{(new)} = \frac{\sum_{i=1}^{n} w_{i1}}{n}$$

$$p_2^{(new)} = 1 - p_1^{(new)}$$

Ο αλγόριθμος ΕΜ για
μείξη Κ κατανομών Poisson
με παραμέτρους $\theta_1, \theta_2, \dots, \theta_K$

$$Z_{ij} = \begin{cases} 1, & \text{αν η } i \text{ παρατήρηση} \\ & \text{ανήκε στη } j \text{ ομάδα} \\ 0, & \text{διαφορετικά} \end{cases}$$

$j = 1, \dots, K, \quad i = 1, \dots, n$

$\theta = (\theta_1, \theta_2, \dots, \theta_K)$

$P = (P_1, P_2, \dots, P_K)$

$$l(\theta, P | X, Z) = \sum_{i=1}^{n} \sum_{j=1}^{K} \left[Z_{ij} \log P_j + Z_{ij} (-\theta_j + x_i \log \theta_j - \log(x_i!)) \right]$$

$$\hat{\theta}_j = \frac{\sum_{i=1}^{n} Z_{ij} x_i}{\sum_{i=1}^{n} Z_{ij}}$$

$$\hat{P}_j = \frac{\sum_{i=1}^{n} Z_{ij}}{n}$$

$$W_{ij} = E(Z_{ij}) = \frac{P_j F(x_i | \theta_j)}{\sum_{j=1}^{K} P_j F(x_i | \theta_j)}$$

Δώσε αρχικές τιμές
για $(\theta_1, \theta_2, \dots, \theta_k)$
και $(p_1, p_2, \dots, p_k)$

E-step: Υπολόγισε

$$w_{ij} = E(z_{ij}) = \frac{p_j f(x_i|\theta_j)}{\sum_{j=1}^{k} p_j f(x_i|\theta_j)}$$

M-step: Υπολόγισε

$$\hat{\theta}_j^{(new)} = \frac{\sum_{i=1}^{n} w_{ij} x_i}{\sum_{i=1}^{n} w_{ij}}$$

$$\hat{p}_j^{(new)} = \frac{\sum_{i=1}^{n} w_{ij}}{n}$$