15-5-2025

Νευρωνικά Δίκτυα και Βαθιά Μάθηση

$$y = f(x) + \varepsilon$$

Training set $\left.\begin{matrix} y_1 & x_1 \\ \vdots & \\ y_n & x_n \end{matrix}\right\} \Rightarrow \hat{y} = \hat{f}(x)$

1) Models : Υποδεικνύουμε $f \in C$

πχ. $C = \{ b_0 + b_1 x \}$

$C = \{$ πολυώνυμα $\}$

$C = \{$ splines $\}$

2) Κριτήρια : MPE (τετρ. σφάλμα)
misclassification
(penalties ridge/lasso...)

Neural network $f(x)$: { activators, layers ... }

Νευρώνες

Σήμα → Γραμμικό → Ενεργοποίηση !! (activation - firing)

→ Τμηματικά γραμμ.

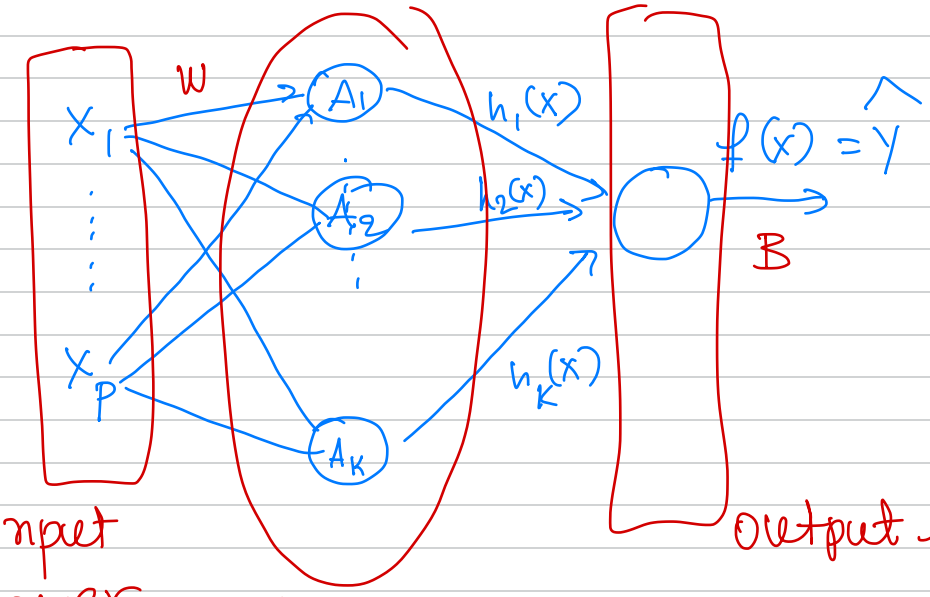
→ Καμπύλη συνάρτηση

⋮

Single Layer Neural Network

$X = (X_1, \dots, X_p)$

feed-forward



$\Updownarrow$ $\hat{y} = f(x)$

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1) Μπορεί να $h_1, \dots, h_K, f$ να είναι οποιεσδήποτε συναρτήσεις;

$$h_1(x) = a_{11}x_1 + \dots + a_{1p}x_p$$

$$h_2(x) = a_{21}x_1 + \dots + a_{2p}x_p$$

$$\vdots$$

$$h_K(x) = a_{K1}x_1 + \dots + a_{Kp}x_p$$

$$f(x) = b_0 + \sum_{l=1}^{K} b_l h_l(x)$$

✓

$$= \sum_{\ell=1}^{K} b_\ell \sum_{i=1}^{P} a_{\ell i} x_i =$$

$$= \sum_{i=1}^{P} \left(\sum_{\ell=1}^{K} b_\ell a_{\ell i} \right) x_i$$

$$= \sum_{i=1}^{P} w_i x_i$$

Υποδίκτυο

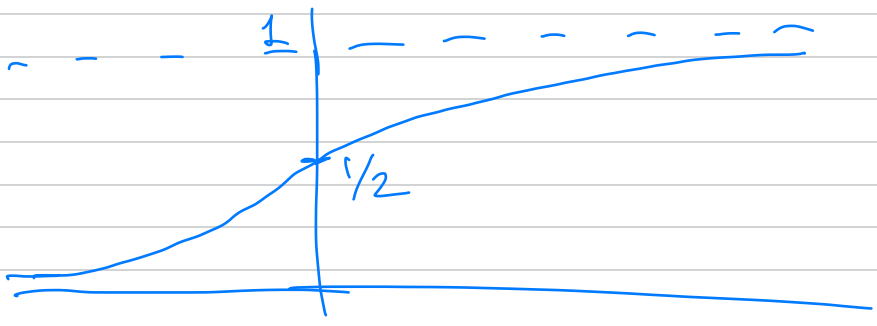
$$f(x) = b_0 + \sum_{k=1}^{K} b_k h_k(x)$$

hidden layer

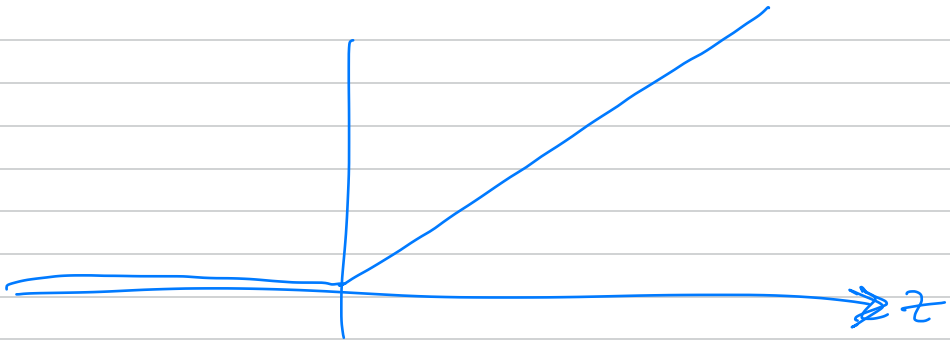
$$A_k = h_k(x) = g\left(w_{k0} + \sum_{j=1}^{P} w_{kj} x_j \right)$$

$g(z)$: activation function

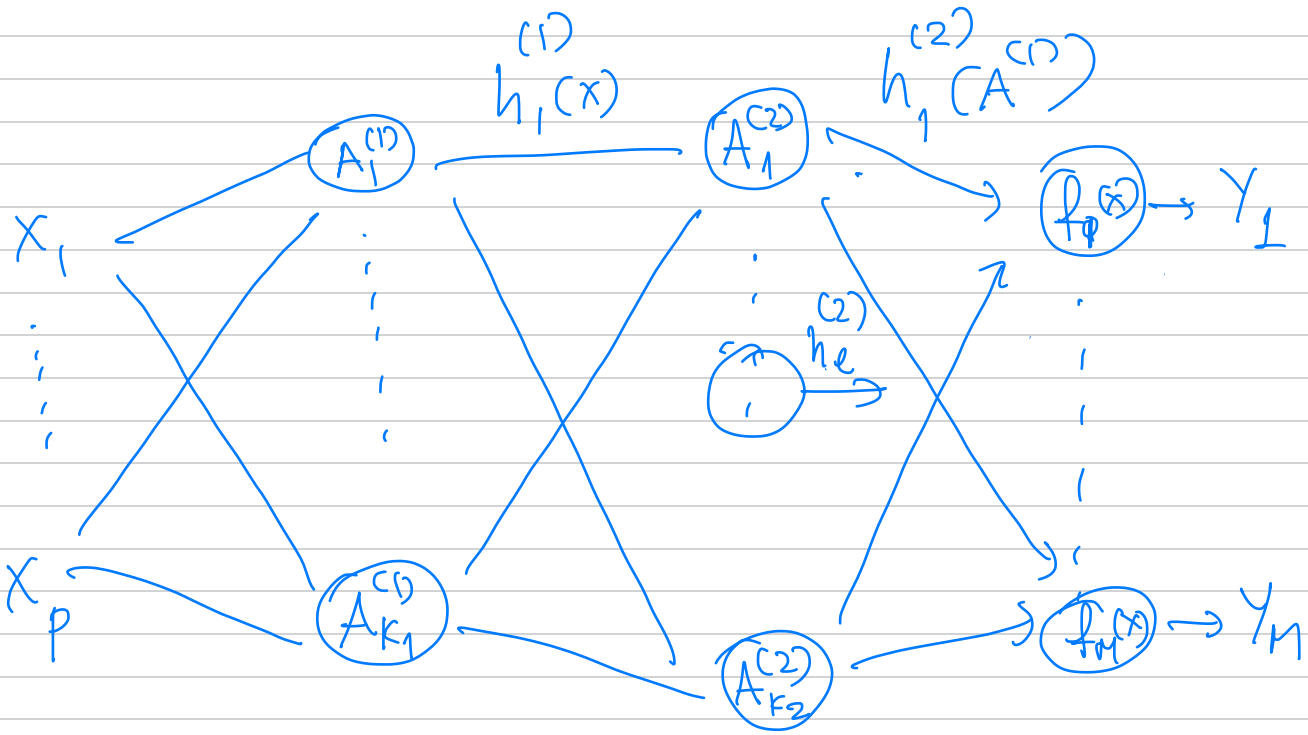
a) Sigmoid : $g(z) = \dfrac{e^z}{1+e^z}$



b) ReLU $g(z) = z^+ = \max(0, z)$



Multi-Layered Networks



L_1

L_1 $k = 1, \dots K_1$: $h_k^{(1)}(x) = g\left(w_{k0}^{(1)} + \sum_{j=1}^{p} w_{kj}^{(1)} x_j\right)$

$K_1 (p+1)$ παράμετροι

L_2 : $l = 1, \dots, K_2$

$$h_l^{(2)}(A^{(1)}) = g\left(w_{l0}^{(2)} + \sum_{k=1}^{K_1} w_{lk}^{(2)} A_k^{(1)}\right)$$

$$K_2 \cdot (K_1 + 1)$$

Output:

$$f_m(x) = b_{m0} + \sum_{l=1}^{K_2} b_{ml} A_l^{(2)} \quad , m = 1, \dots, M$$

$$M \cdot (K_2 + 1)$$

Συνολικός αριθμός 2-layered network

$$= K_1(p+1) + K_2(K_1+1) + M(K_2+1)$$

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1) Αν $Y_1, \dots, Y_M$ ποσοτικές $\quad Y_m = f_m(x)$

2) Y : διακριτή με επίπεδα $\begin{cases} 1 \\ \vdots \\ M \end{cases}$

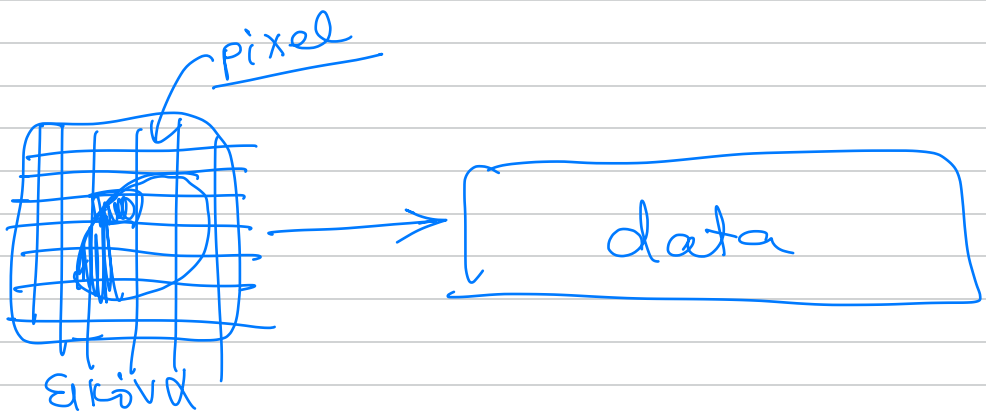
$Y_1 = 1(Y=1), \dots, Y_M = 1(Y=M)$

$$(Y_1, \dots, Y_M) \in \{0,1\}^M, \quad \sum Y_i = 1$$

$$(f_1, \dots, f_M) \to P(Y=m \mid X) = \frac{e^{f_m}}{\sum_{m=1}^{M} e^{f_m}} = \frac{e^{f_m}}{e^{f_1} + \dots + e^{f_M}}$$

Convolutional Neural Networks

(Προσαρμοσμένα για ταξινόμηση εικόνων)



pixel : grayscale (αποχρώσεις)

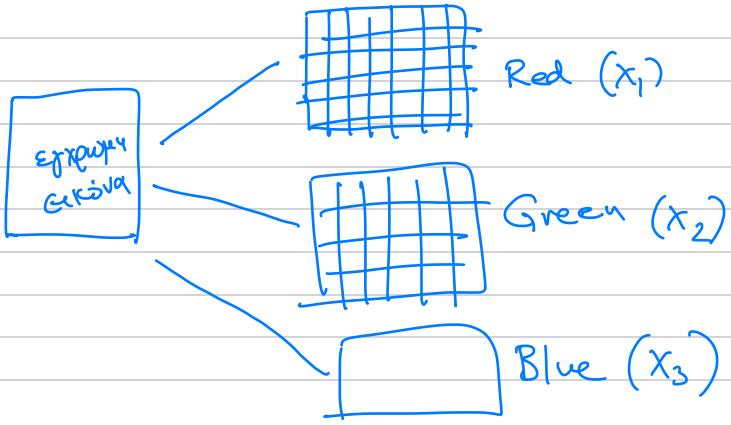
επιλέξω χρώματα → RGB → (x_1, x_2, x_3)

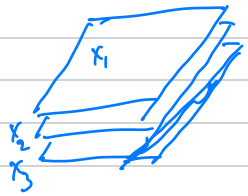
→ CMYK → $(x_1, \dots, x_4)$

pixel → x_1 → channel Red

x_2 " Green

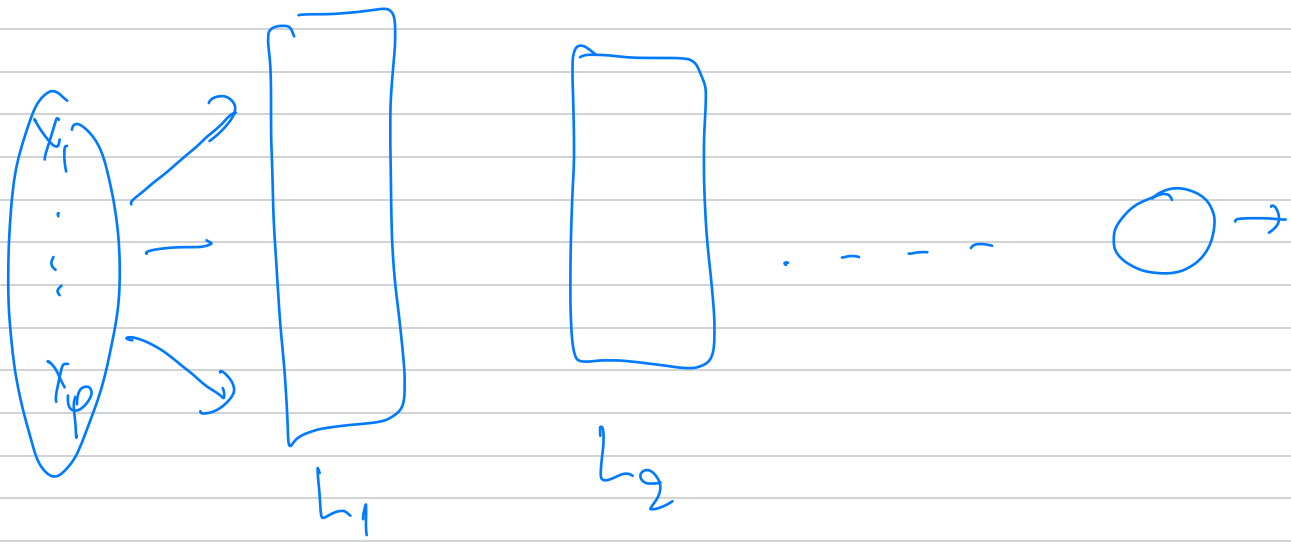
x_3 " Blue





MxN pixels
K channels

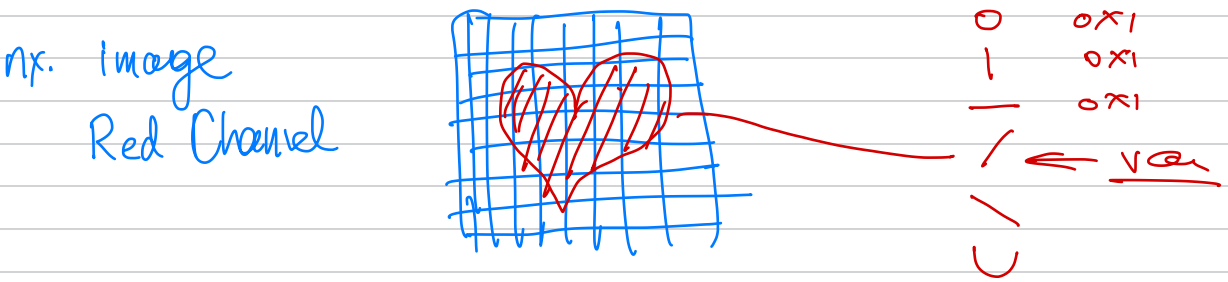
$\Rightarrow$ διάνυσμα διάστ. $\boxed{M \cdot N \cdot K = p}$



L_1 → convolution layer
→ pooling layer

Convolution layer :

Διανυσματική αναπαράσταση



πχ. Image
Red Channel

Έστω μικρές εικόνες 2x2 pixel

Εικόνα $= \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ 3x3 pixel.

$$\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \Rightarrow$$ "—"

convolution

$$\begin{bmatrix} a\alpha+b\beta+d\gamma+e\delta & b\alpha+c\beta+e\gamma+f\delta \\ — & — \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow$$