

Λύσεις

1.

$$\lim_{x \rightarrow \pm\infty} \left(2 + \left| \frac{\cos x}{x} \right|^{1/3} \right) = 2$$

2.

$$\beta_n = \sum_{k=1}^n \frac{1}{k} - \ln n$$

τότε $\beta_n \rightarrow \gamma \approx 0.5772$, $n \rightarrow \infty$.

3.

$$\inf_{n \in \mathbb{N}} \left\{ \frac{1}{1 + \sqrt{n}} \right\} = 0$$

4.

$$\forall x, y \in \mathbb{R} \quad |x| + |y| \geq |x + y|$$

5.

$$\text{Αν } A \subset \Omega, \text{ τότε}$$

$$A^c = \Omega \setminus A = \{x : x \in \Omega, x \notin A\}$$

6.

$$\bigcup_{n=1}^{\infty} \left(0, \frac{1}{n}\right) = (0, 1) , \quad \bigcap_{n=1}^{\infty} \left(0, \frac{1}{n}\right) = \varnothing$$

7.

$$A \times B = \{(a, b) : a \in A, b \in B\},$$

8.

$$\text{Αν } \mathbf{u}, \mathbf{v} \in \mathbb{R}^2 = \mathbb{R} \times \mathbb{R}, \text{ τότε}$$

$$\mathbf{u} \perp \mathbf{v} \iff \langle \mathbf{u}, \mathbf{v} \rangle = 0 .$$